# BUILD X: ALGORITHMS

## MARTIN CHAPMAN

Yellow

Orange

Red

Magenta

Violet

Blue

Cyan

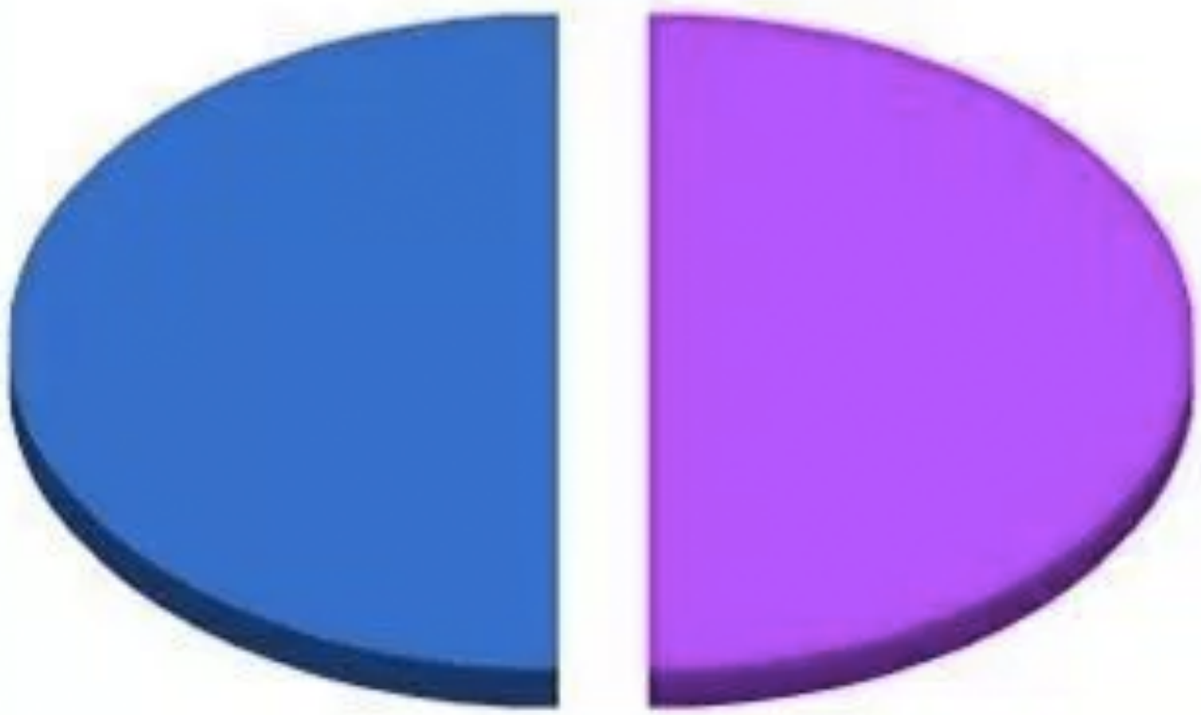
Green

# GRAPHS (sort of)


VIA 9GAG.COM
A MATT DAMON MATT DAMON
MATT
DAMON
MATT DAMON
MATT DAMON
THE MATT DAMON
IN CINEMAS SEPTEMBER 30 IN 3D


DON'T LISTEN TO THEM
I THINK YOU'RE FINTASTIC

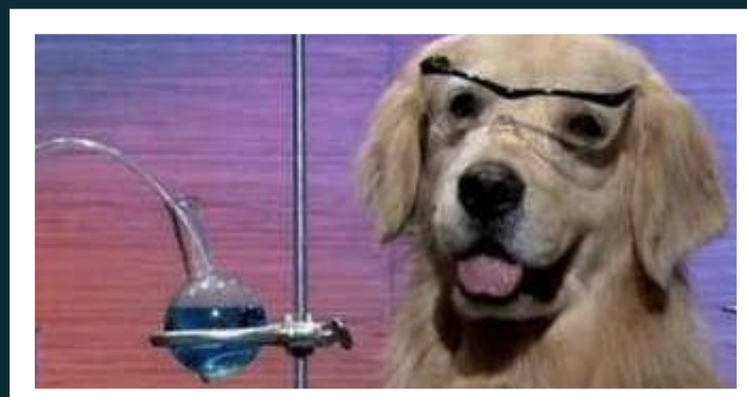
# What I think about in math class



THE F[redacted] IS THAT

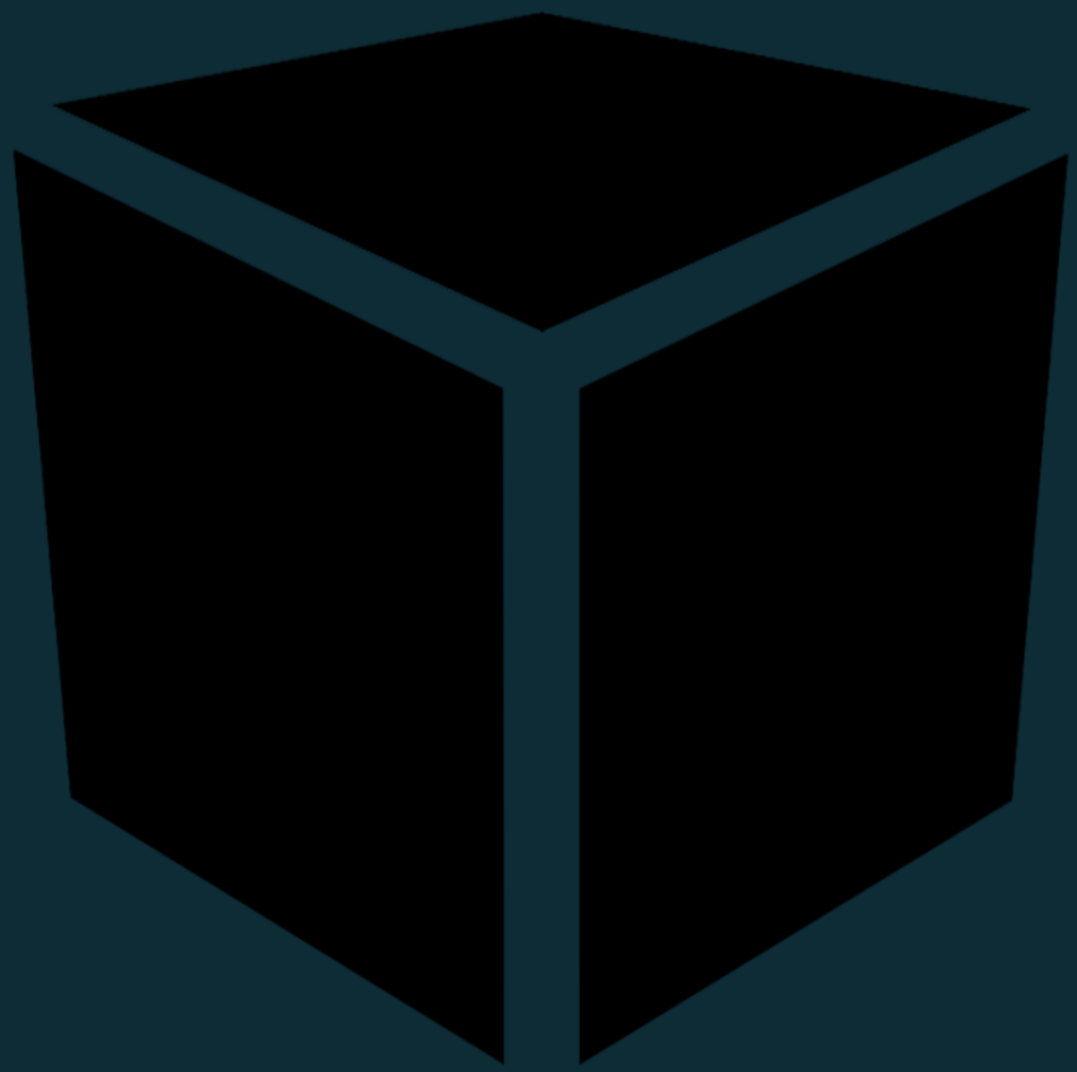
THE F[redacted] IS THIS


I HAVE NO
IDEA WHAT
I'M DOING



Magenta 211 54 130 #d33682

?

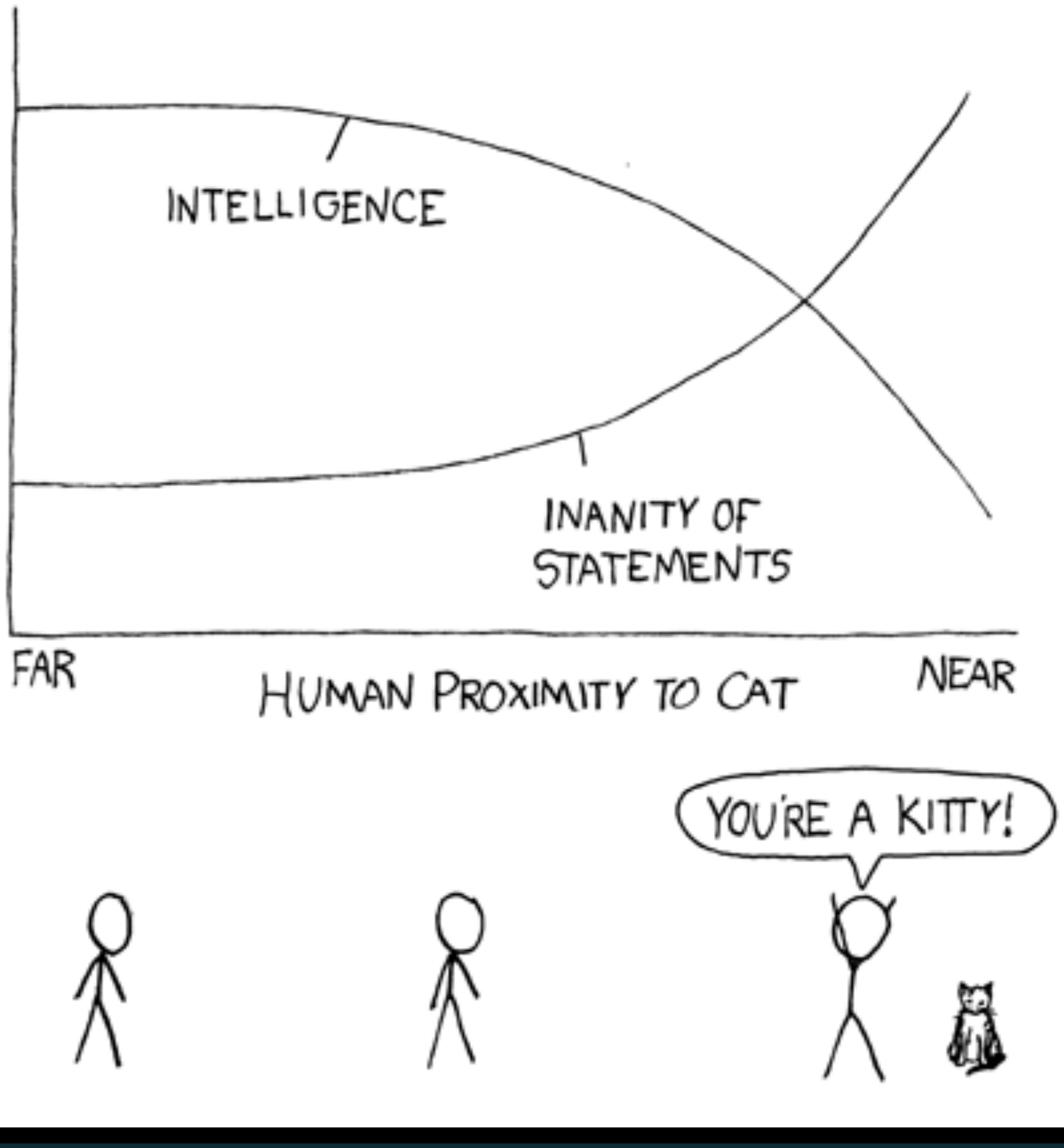


# BUILD X:
# ALGORITHMS


# GRAPHS (sort of)

Yellow 181 137 0 #b58900

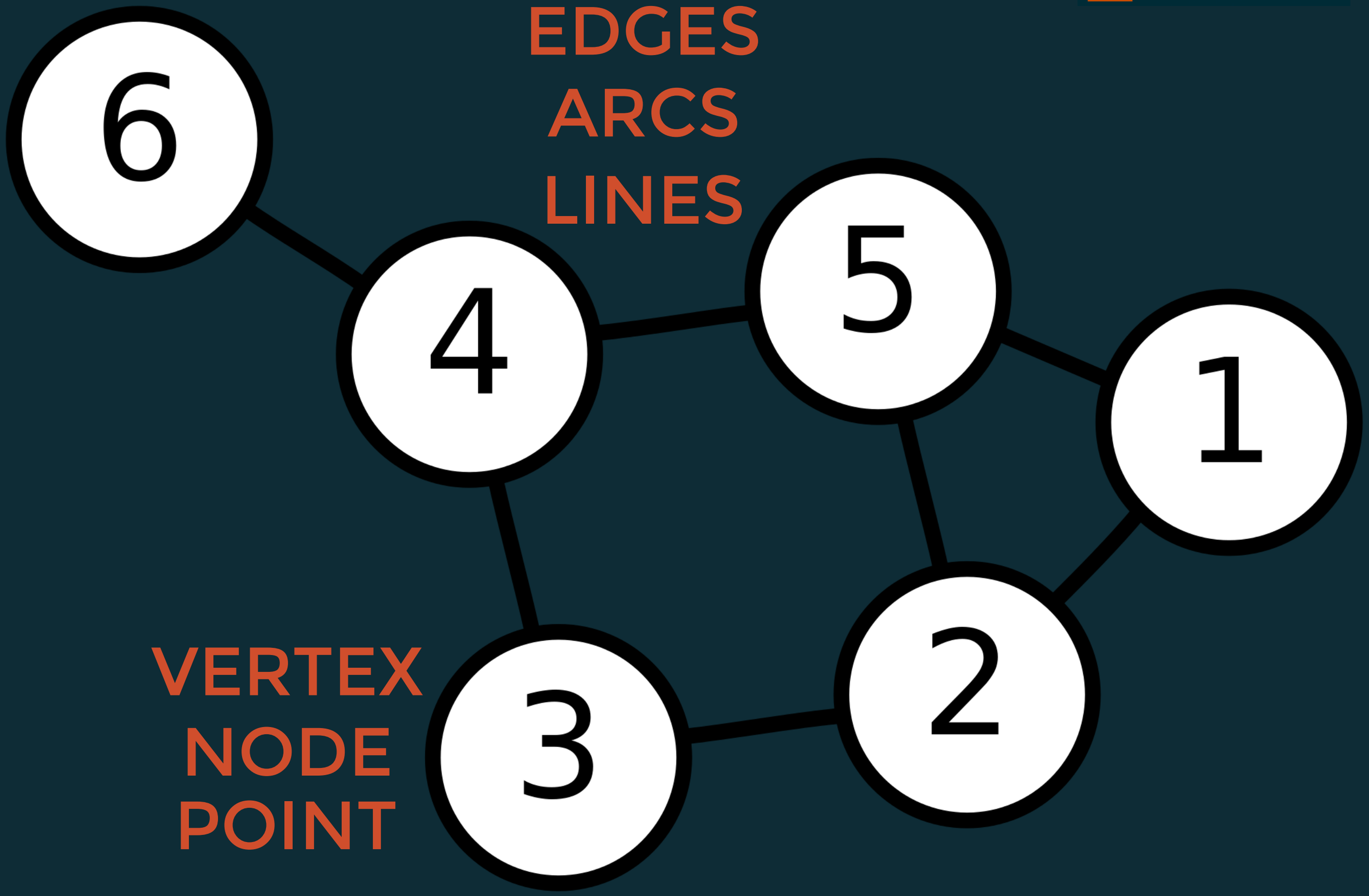
# GRAPH


INTELLIGENCE
INANITY OF STATEMENTS
FAR
HUMAN PROXIMITY TO CAT
NEAR
YOU'RE A KITTY!

Orange 203 75 22 #cb4b16

EDGES
ARCS
LINES

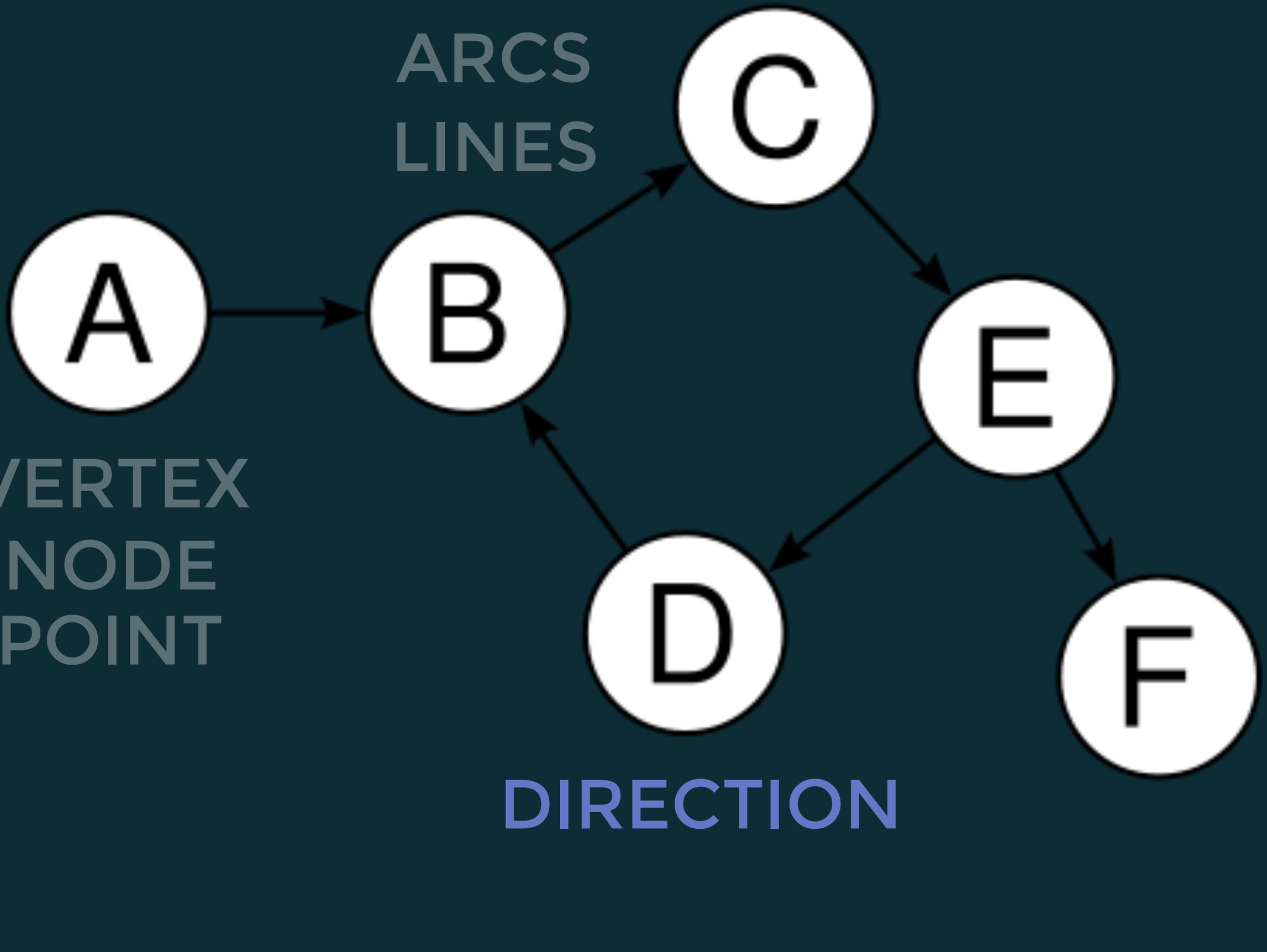
VERTEX
NODE
POINT



Violet 108 113 196 #6c71c4

EDGES
ARCS
LINES

VERTEX
NODE
POINT




DIRECTION

Blue 38 139 210 #268bd2

EDGES
ARCS
LINES

NO
CYCLES

A

B

C

E

VERTEX
NODE
POINT

G

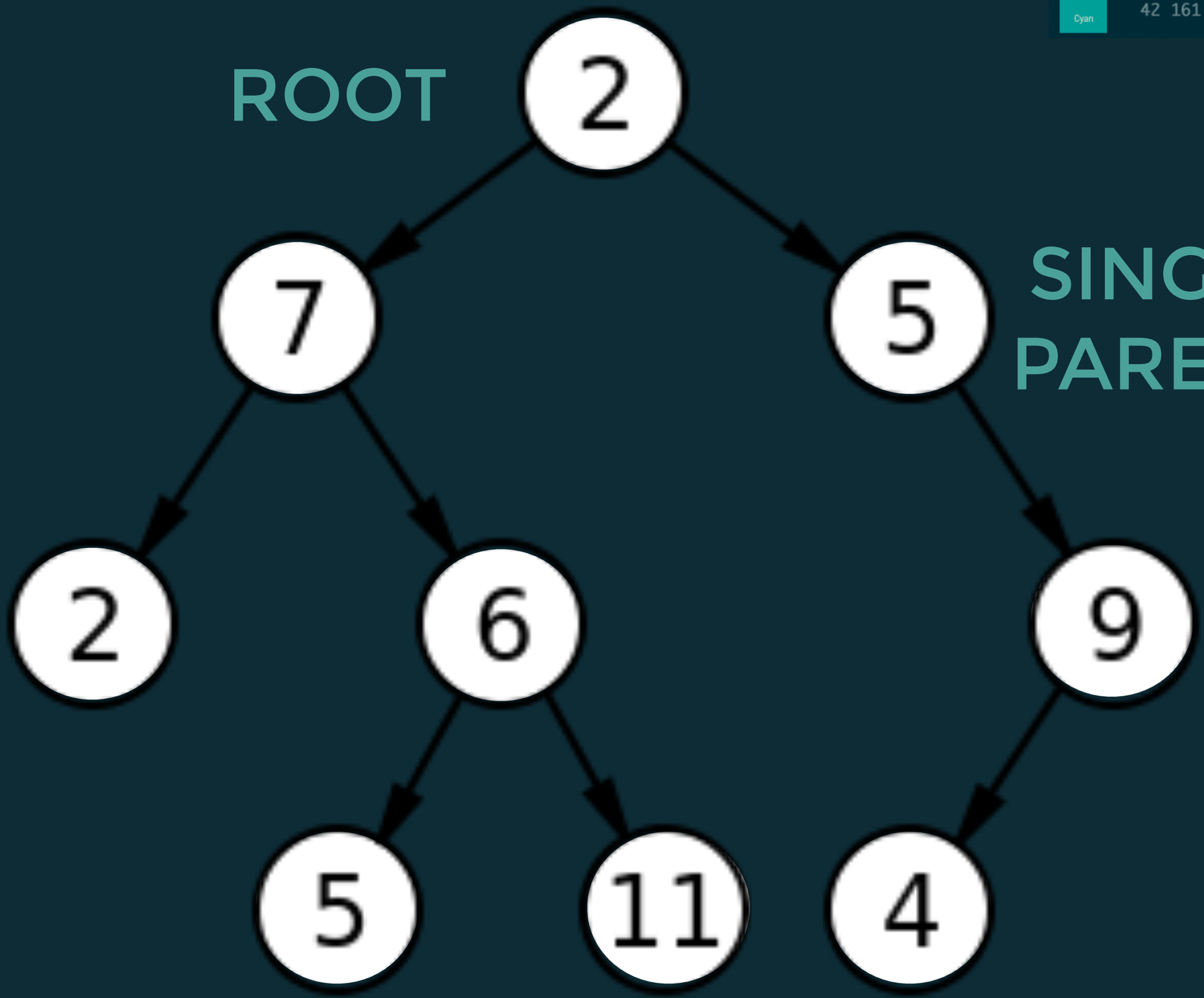
D

F

DIRECTION

Cyan 42 161 152 #2aa198

ROOT




SINGLE PARENT

Red 220 50 47 #dc322f

# TREE

(other definitions exist)

Green 133 153 0 #859900

ROOT

2

7

5

SINGLE PARENT

AT MOST TWO CHILDREN

2

6

9

5

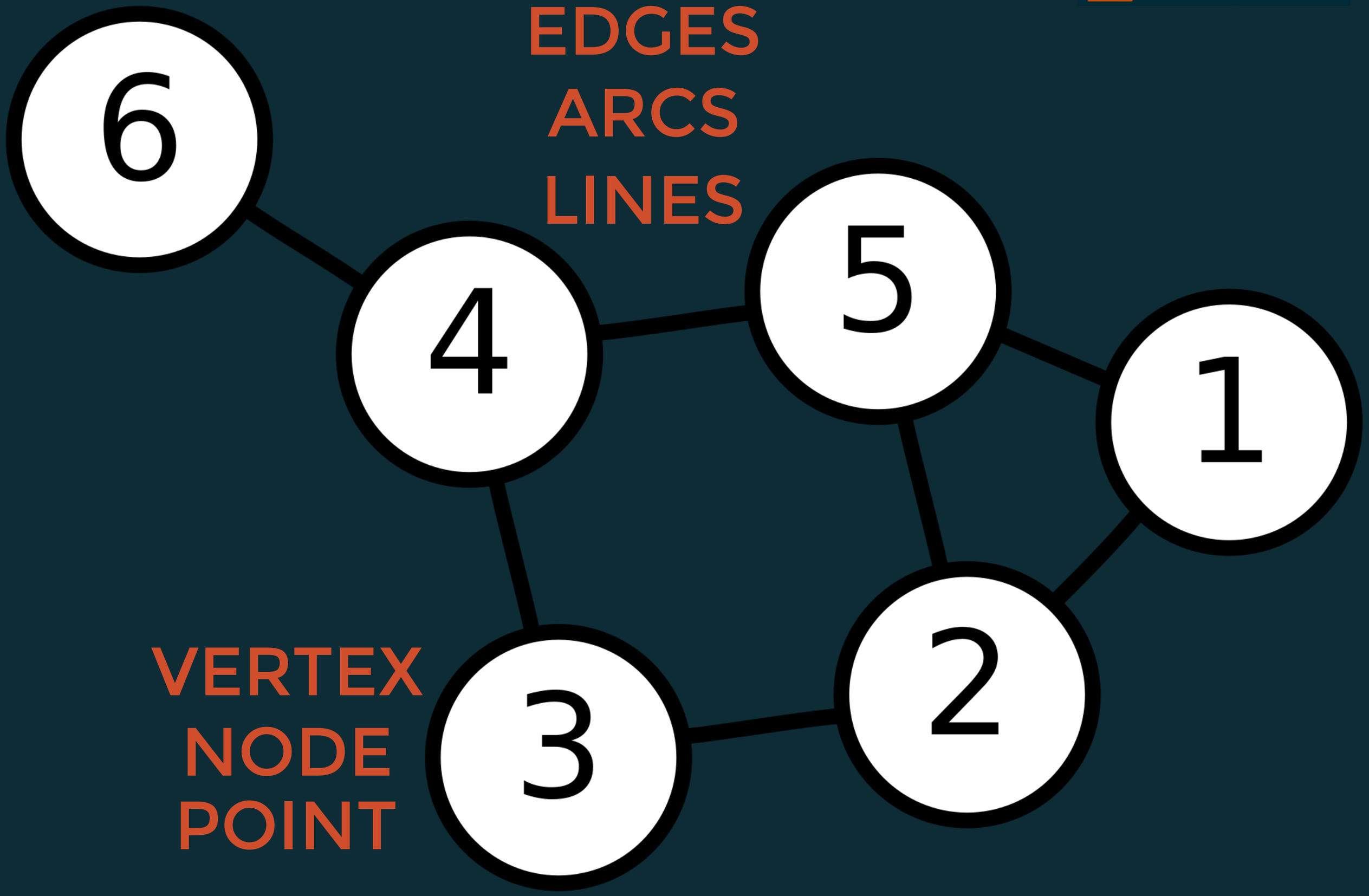
11

4

Orange 203 75 22 #cb4b16

EDGES
ARCS
LINES

VERTEX
NODE
POINT



Green 133 153 0 #859900

# HOW DO WE CONSTRUCT A GRAPH?

Cyan 42 161 152 #2aa198

# TOPOLOGY AND GENERATORS

http://graphstream-project.org/doc/Generators/

Magenta 211 54 130 #d33682

# RANDOM

Béla Bollobás. Random Graphs. Springer, 1998.

Magenta 211 54 130 #d33682

# SCALE-FREE

Albert-László BY Barabási and Eric Bonabeau.
Scale-free Networks.
Scientific American, 5(5):50–59, 2003.

Yellow 181 137 0 #b58900

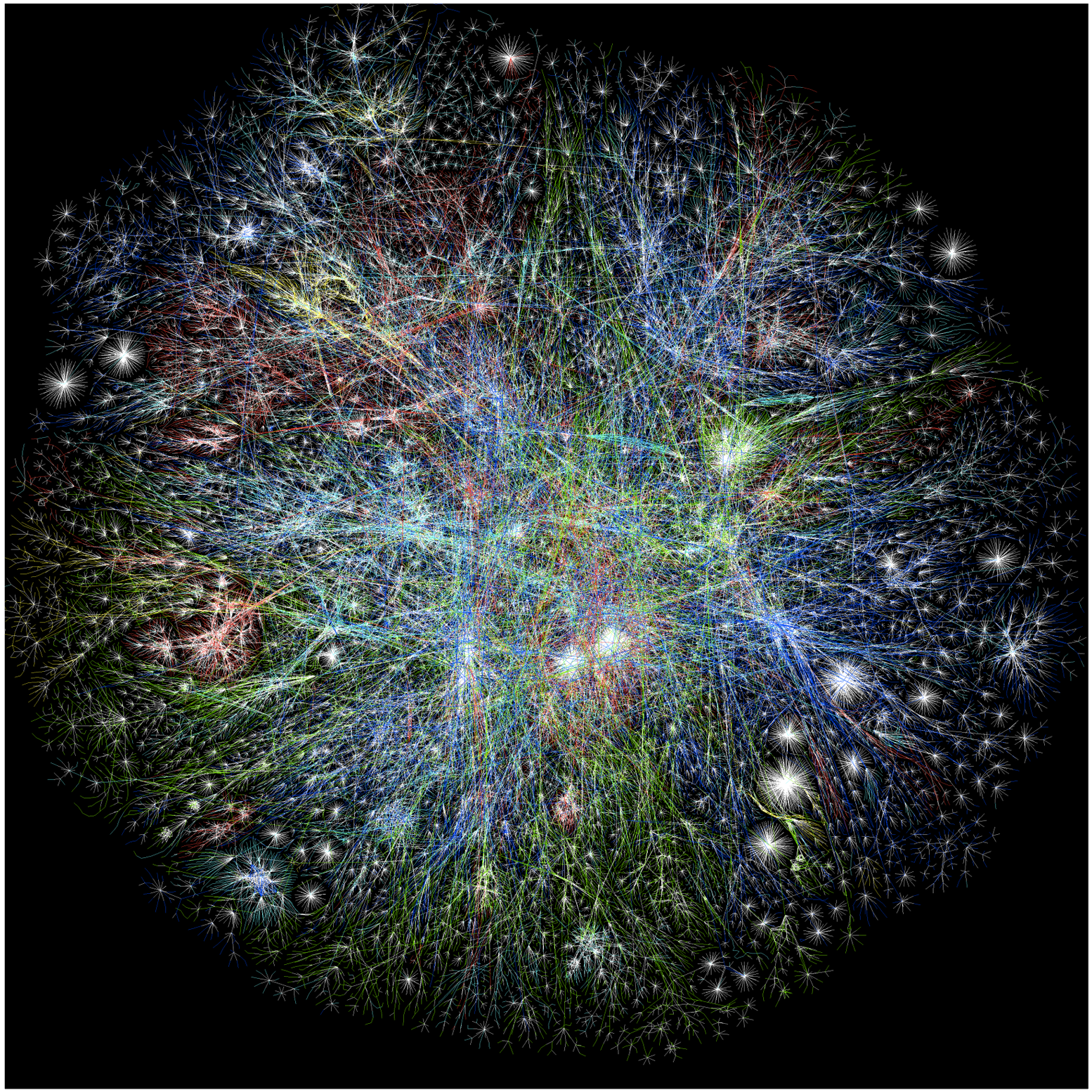
https://github.com/martinchapman/BuildX(.git)

Magenta 211 54 130 #d33682

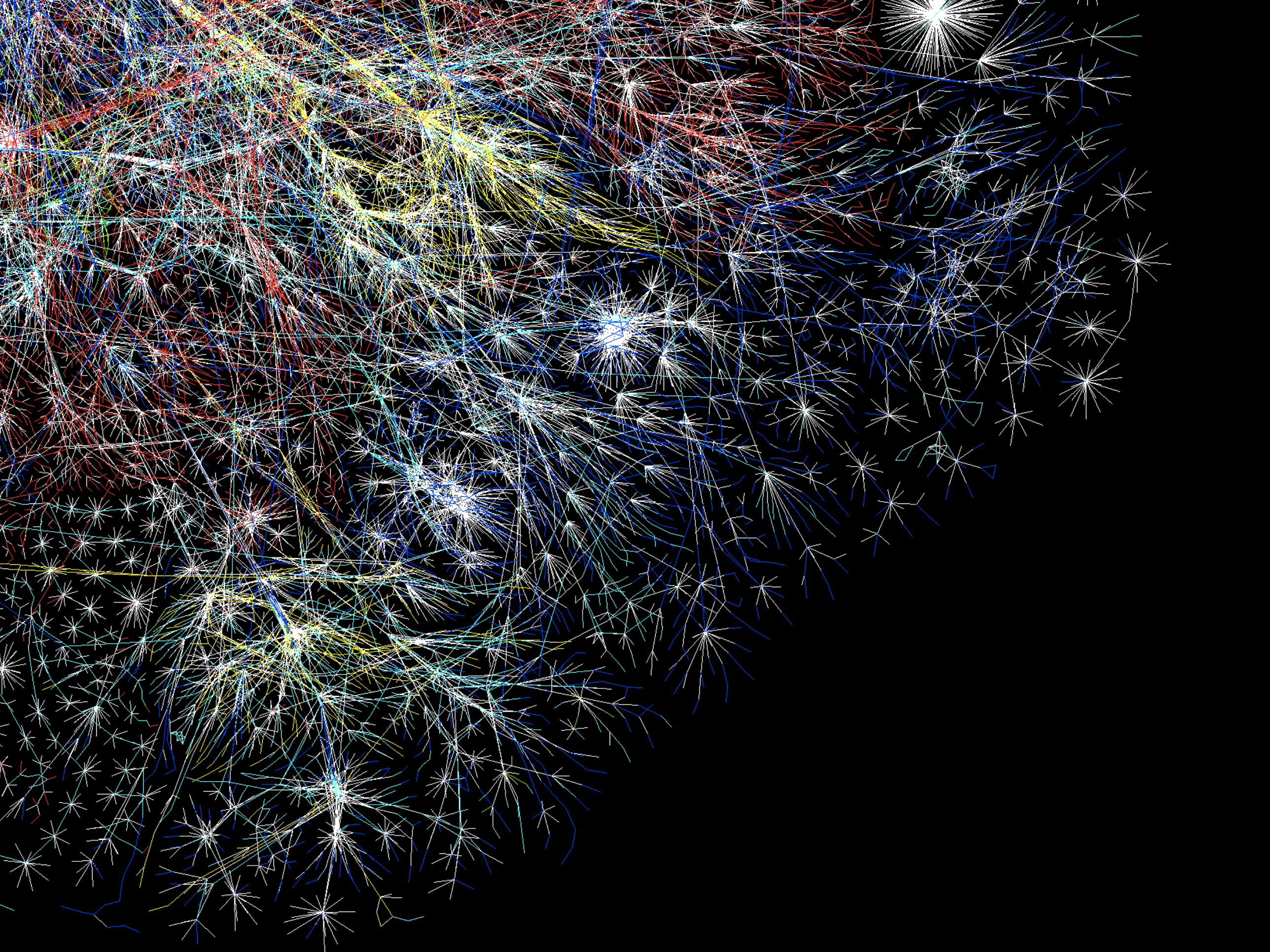
# SCALE-FREE

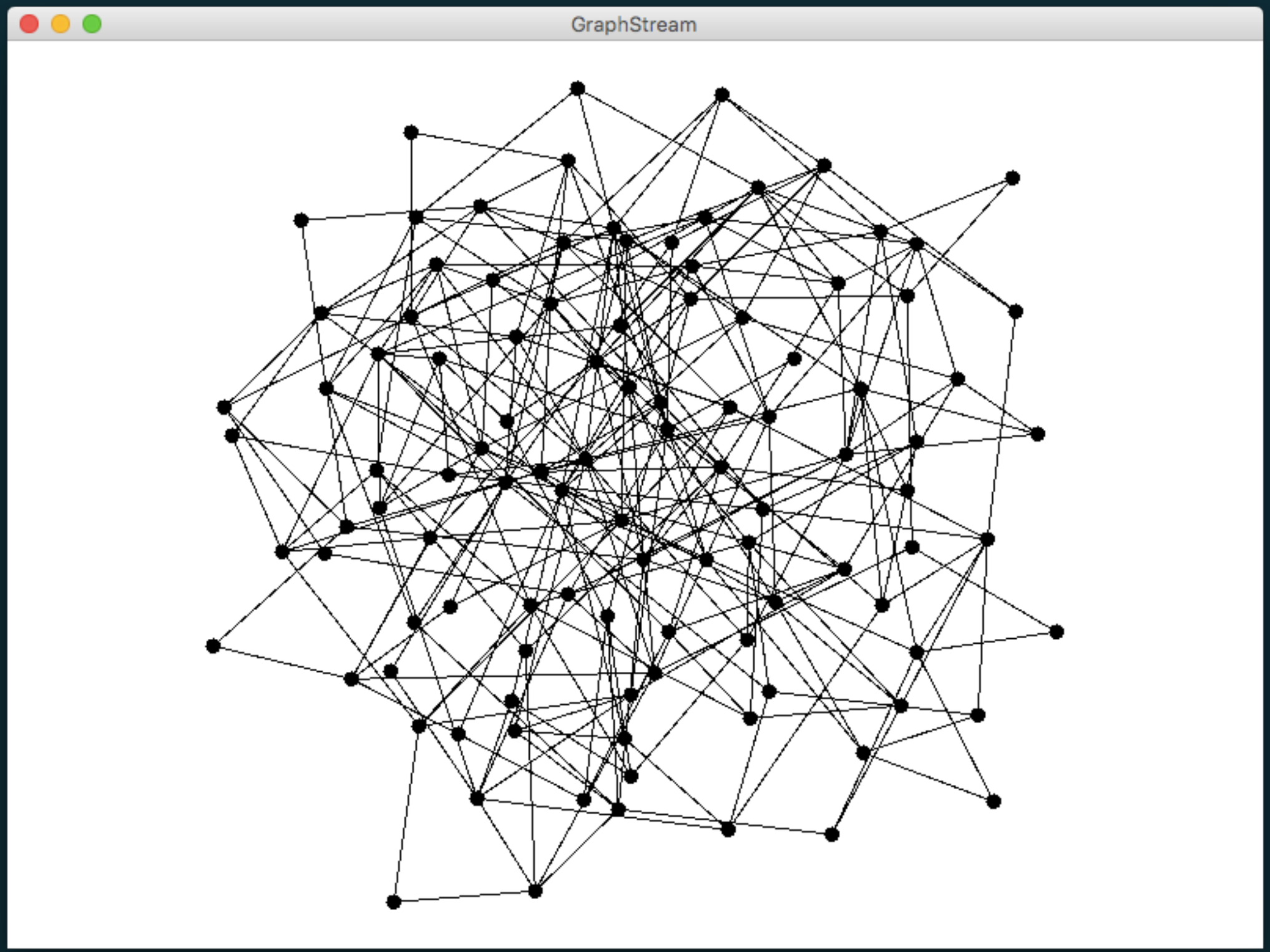
Michalis Faloutsos, Petros Faloutsos, and Christos Faloutsos.
On Power-law Relationships of the Internet Topology.
In Proceedings of
The 1999 Conference on Applications, Technologies, Architectures, and Protocols for Computer Communications
(SIGCOMM99), pages 251–262, 1999.

Yellow 181 137 0 #b58900



http://opte.org/maps/

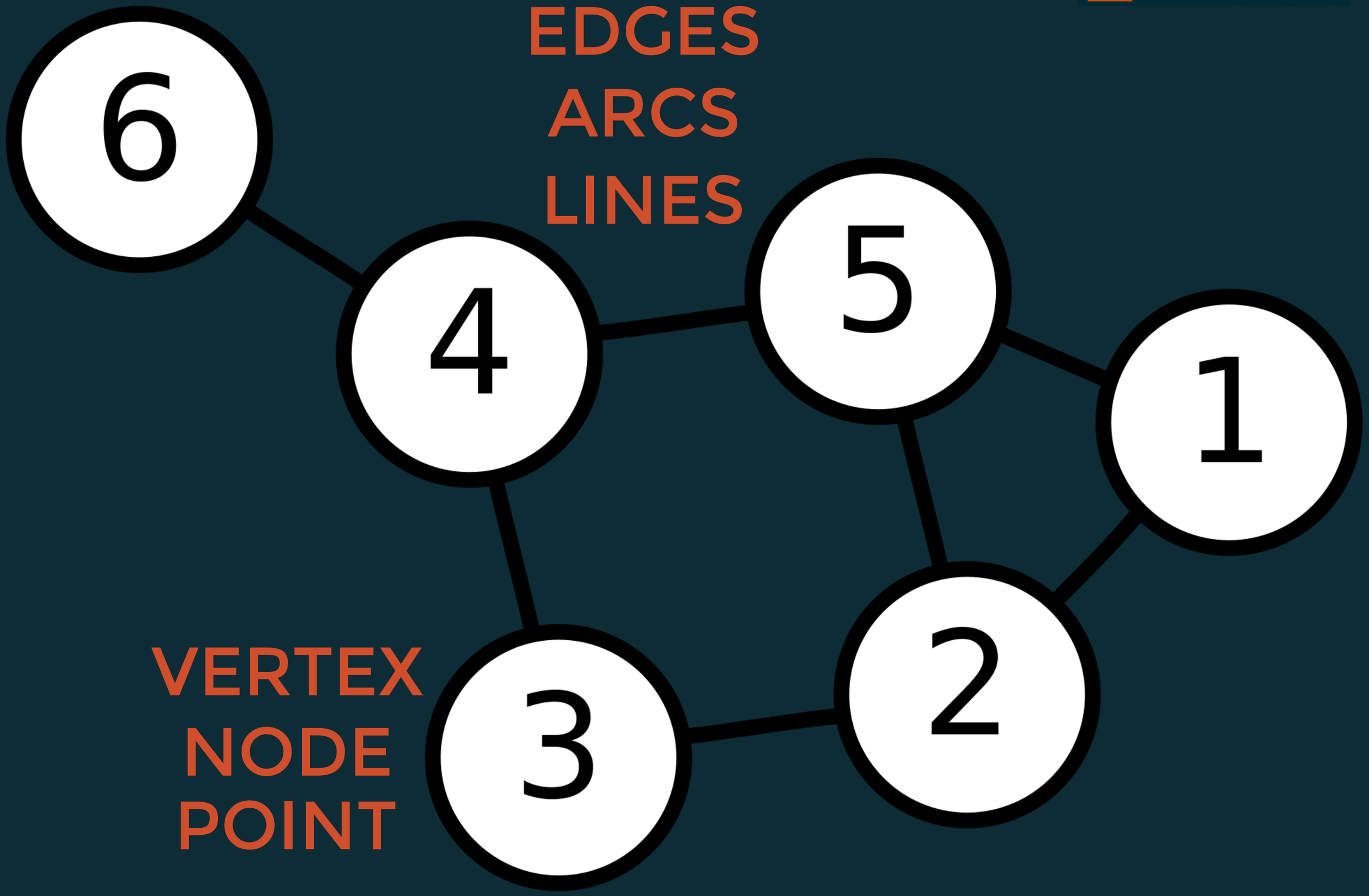



GraphStream

Orange 203 75 22 #cb4b16

EDGES
ARCS
LINES

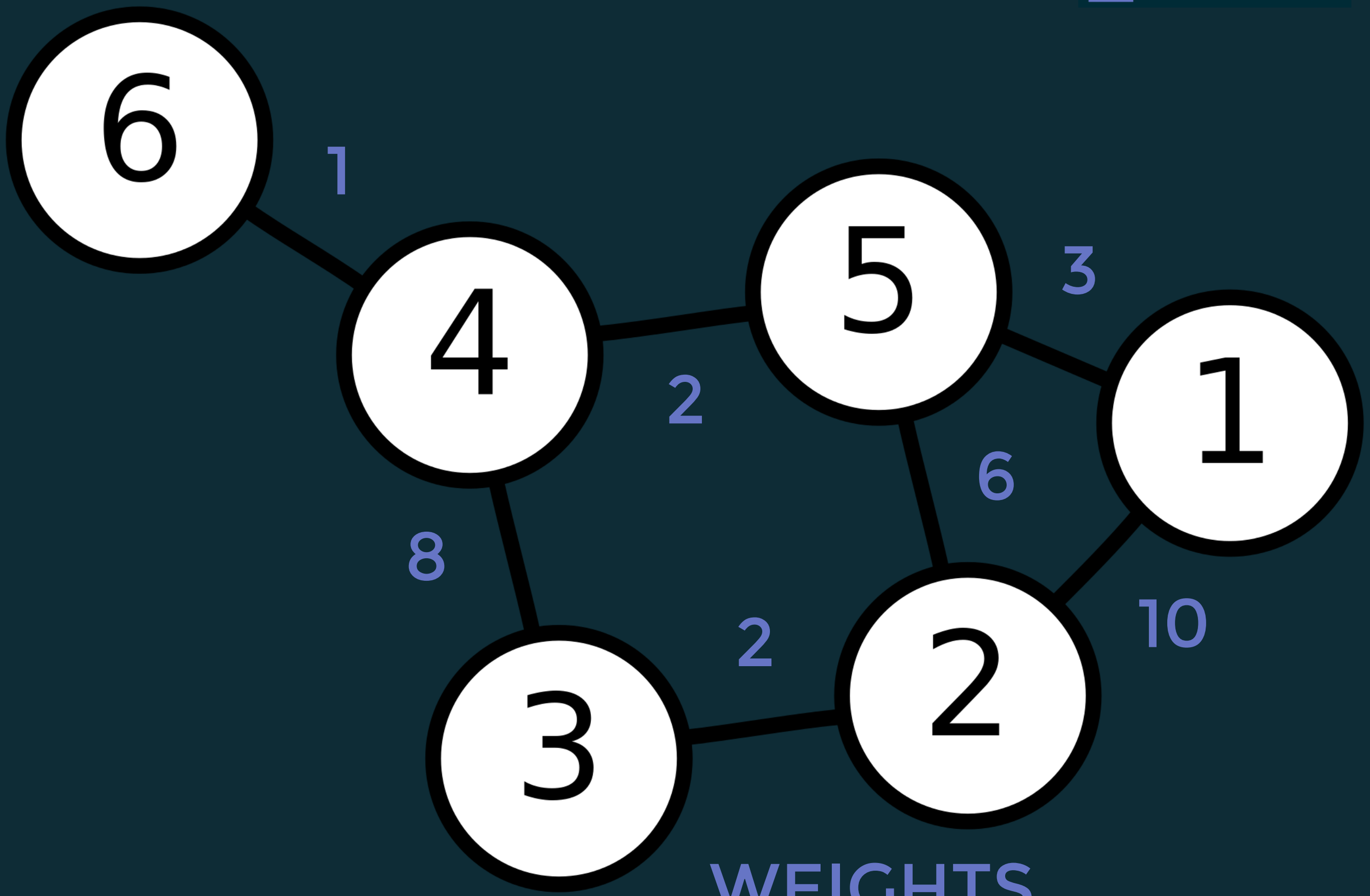
VERTEX
NODE
POINT



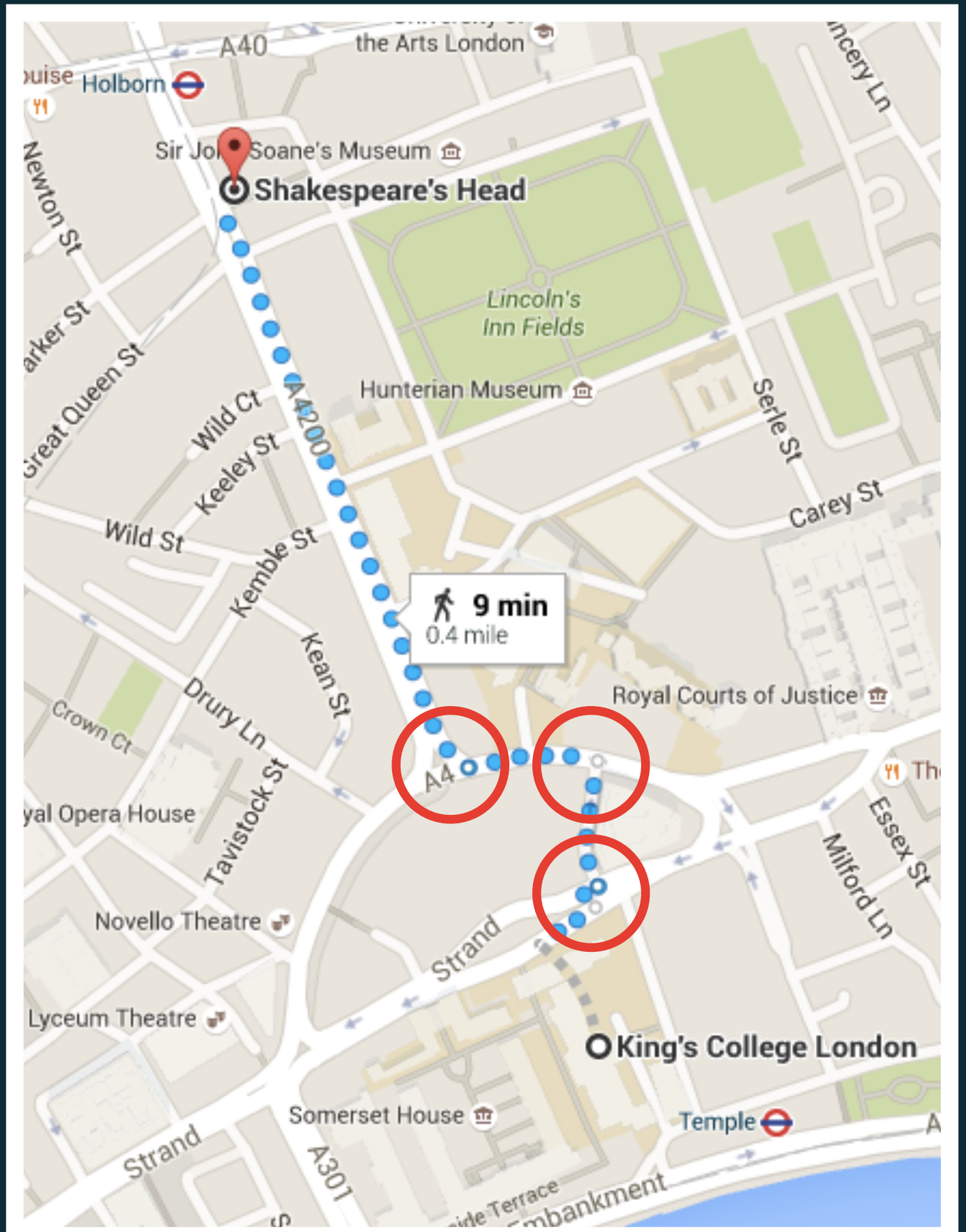
Green 133 153 0 #859900

# WHAT CAN WE DO WITH THIS?

Violet 108 113 196 #6c71c4

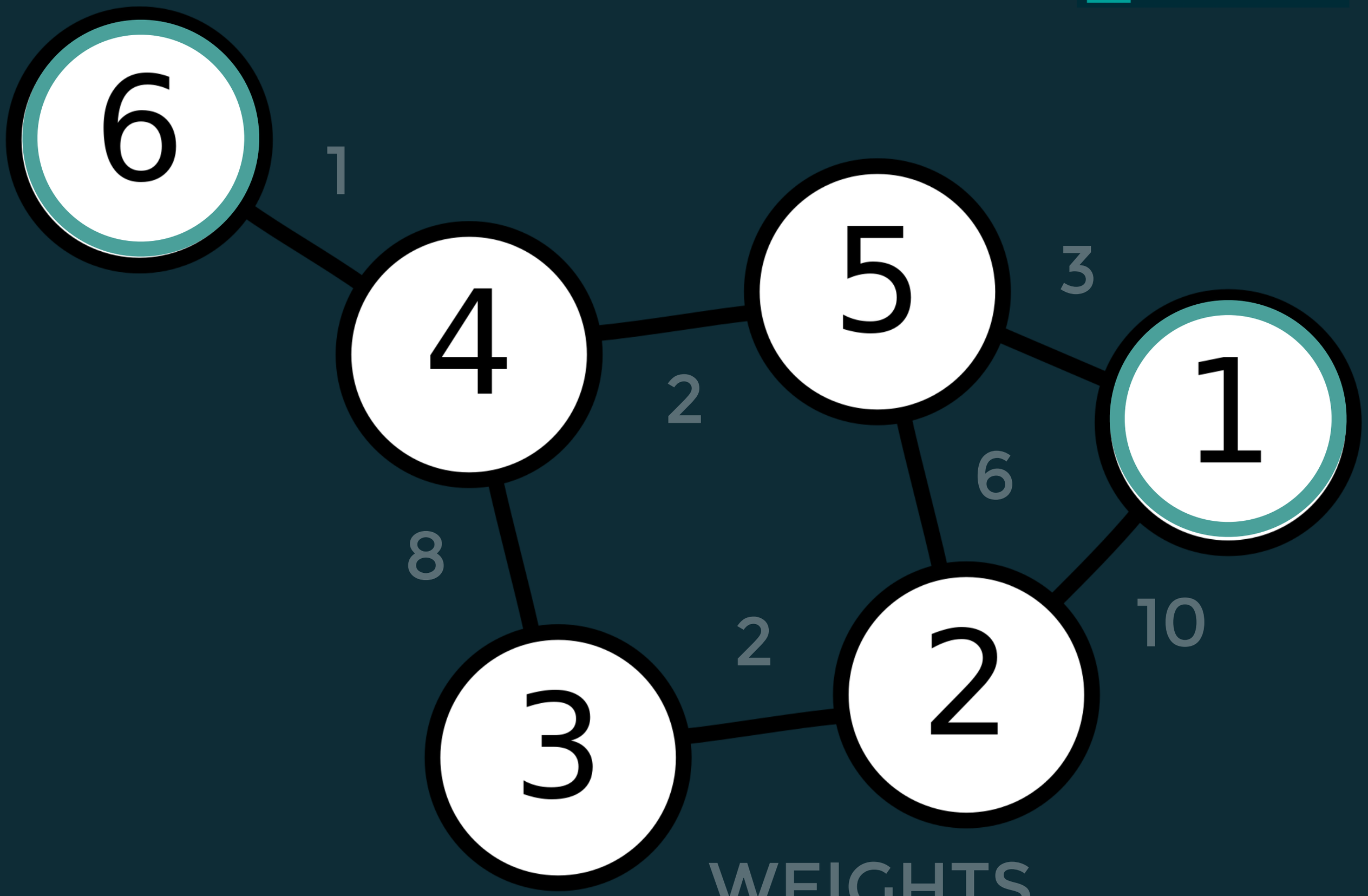


Red 220 50 47 #dc322f




Decision points

Cyan 42 161 152 #2aa198

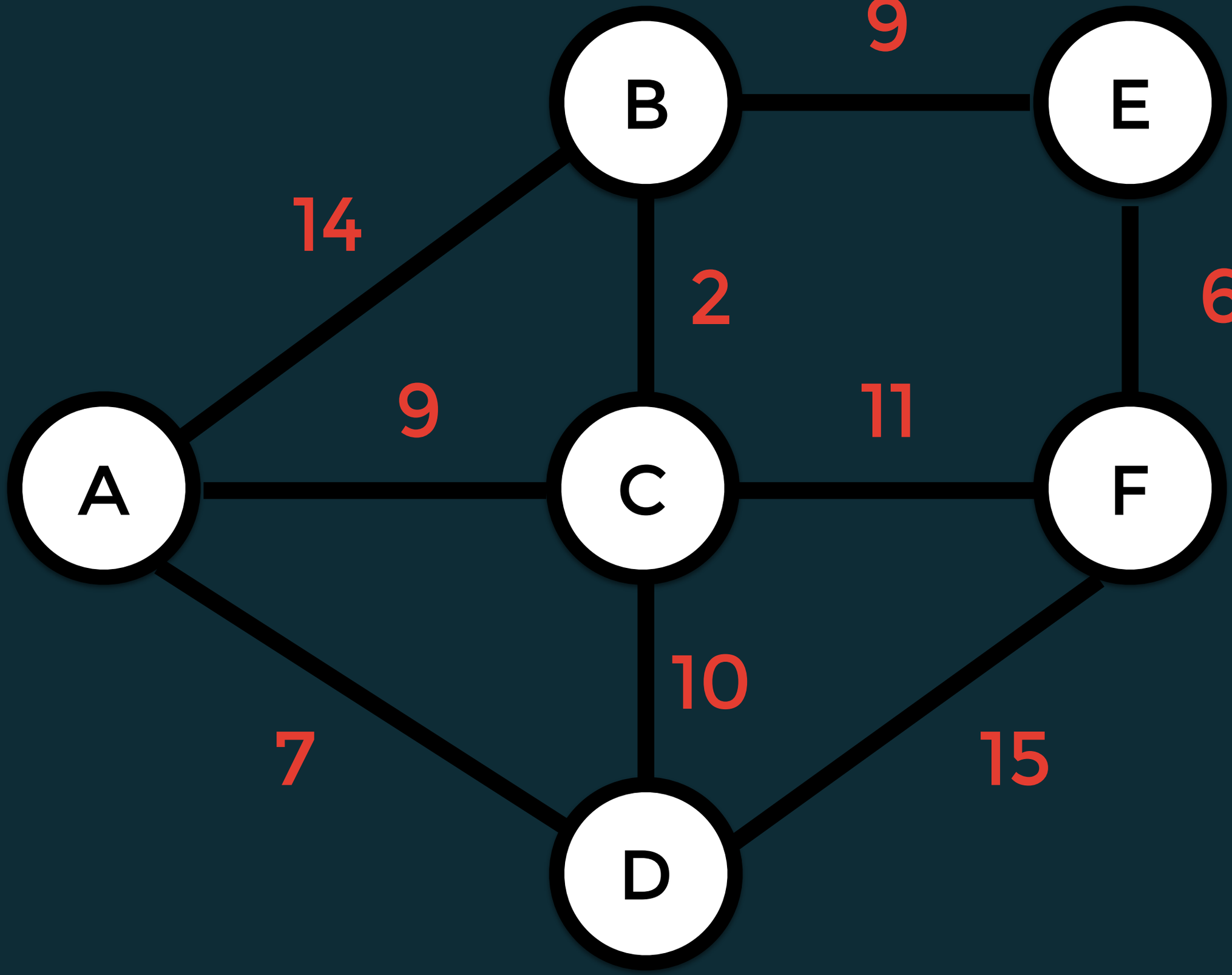


Cyan 42 161 152 #2aa198

# SHORTEST PATH

http://graphstream-project.org/doc/Algorithms/Shortest-path/

Red 220 50 47 #dc322f



Violet 108 113 196 #6c71c4

∞

9

B

E

∞

14

2

6

0

∞

9

11

A

C

F

∞

10

7

15

D

∞

Orange 203 75 22 #cb4b16

14

9

B

E

∞

14

2

6

0

9

9

11

A

C

F

∞

10

7

15

D

7

Cyan 42 161 152 #2aa198

14

9

B

E

∞

14

2

6

0

9

9

11

A

C

F

∞

10

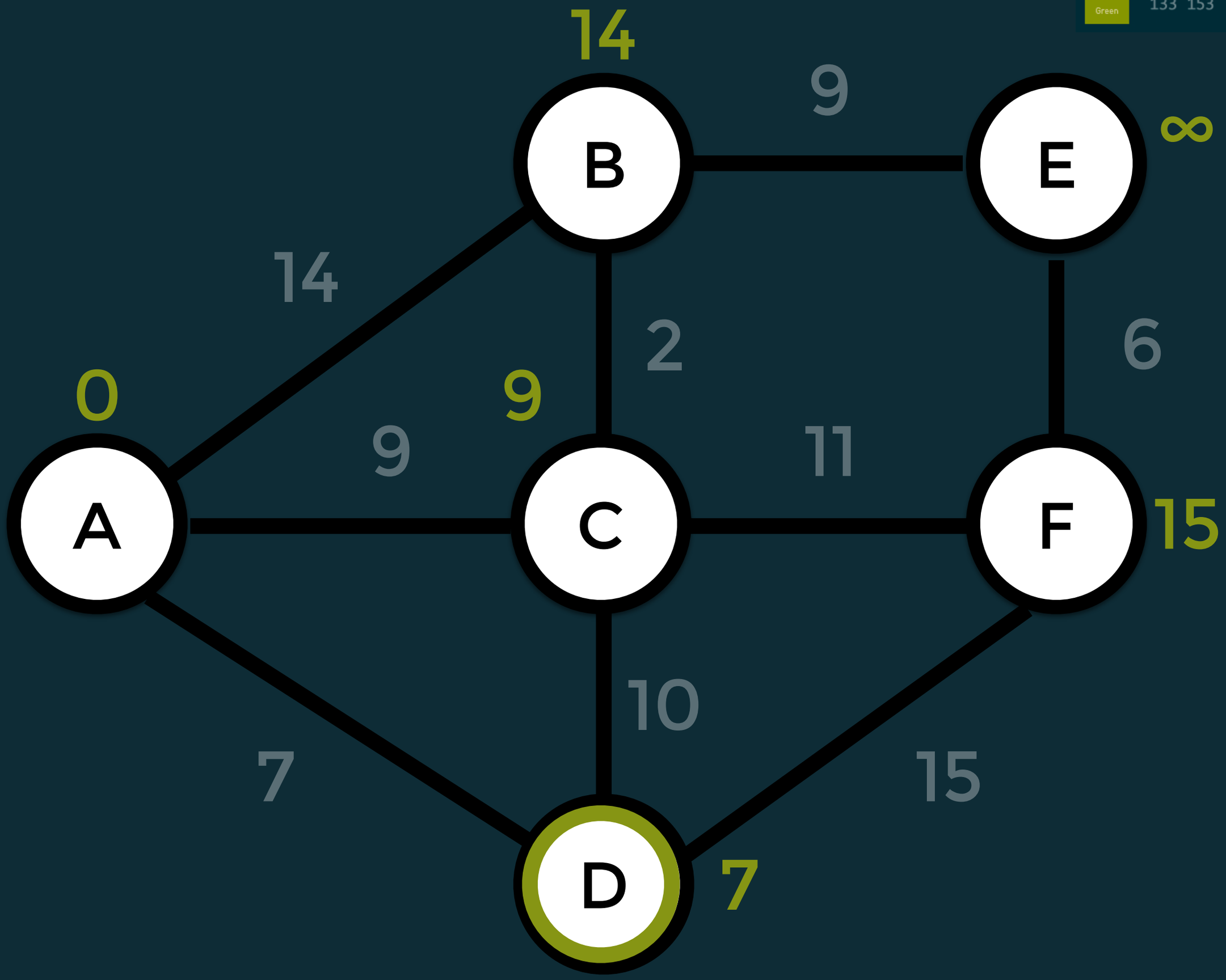
7

15

D

7

Green 133 153 0 #859900



Violet 108 113 196 #6c71c4

# COMPLEXITY

Running time as a function of the input









Green 133 153 0 #859900

$$(V \times \log V) + E$$

Many different permutations on this

https://www.cs.cornell.edu/courses/cs312/2002sp/lectures/lec20/lec20.htm

Blue 38 139 210 #268bd2

$$(V \times \log V) + E$$

```
(* Dijkstra's Algorithm *)
let val q: queue = new_queue()
  val visited: vertexMap = create_vertexMap()
  fun expand(v: vertex) =
    let val neighbors: vertex list = Graph.outgoing(v)
        val dist: int = valOf(get(visited, v))
        fun handle_edge(v': vertex, weight: int) =
          case get(visited, v') of
            SOME(d') =>
              if dist+weight < d'
              then ( add(visited, v', dist+weight);
                     incr_priority(q, v', dist+weight) )
              else ()
          | NONE => ( add(visited, v', dist+weight);
                      push(q, v', dist+weight) )
    in
      app handle_edge neighbors
    end
in
  add(visited, v0, 0);
  expand(v0);
  while (not (empty_queue(q)) do expand(pop(q))
end
```

Cyan 42 161 152 #2aa198

```
// Always deal with the next closest node first (via the estimate)
Node node = getClosestFromEstimate( unsettledNodes );
```

```
if ( getDistanceEstimate( node ) < getDistanceEstimate( minimum ) ) {

    minimum = node;

}
```

```
            else ()
        | NONE => ( add(visited, v', dist+weight);
                    push(q, v', dist+weight) )
    in
      app handle_edge neighbors
    end
in
  add(visited, v0, 0);
  expand(v0);
  while (not (empty_queue(q)) do expand(pop(q))
end
```

So, in reality, our intuitive Java implementation is likely to differ in complexity.

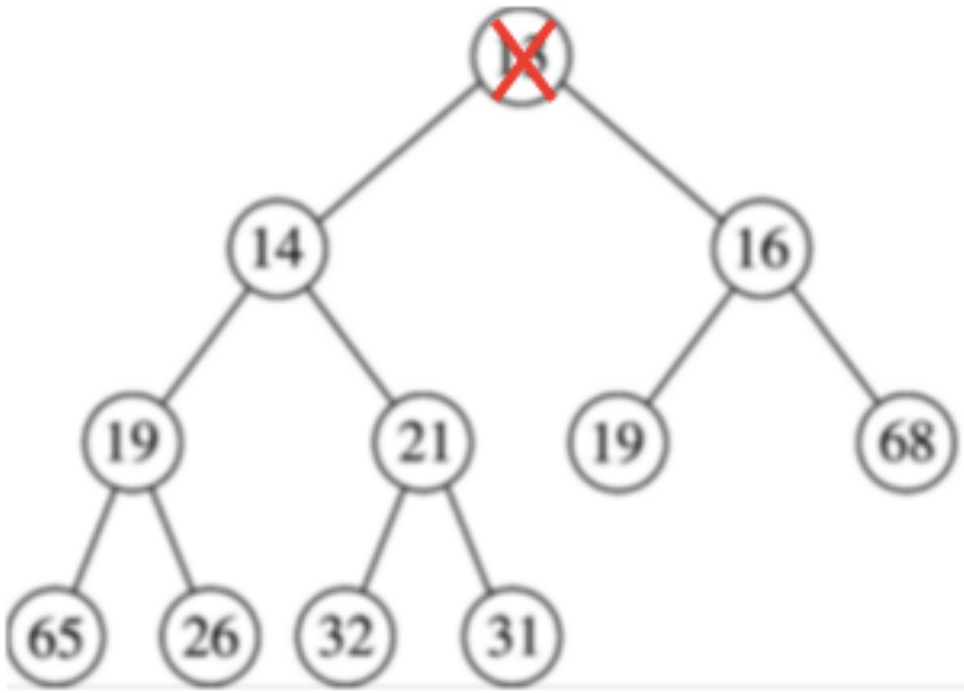
Red 220 50 47 #dc322f

$$(V \times \log V) + E$$

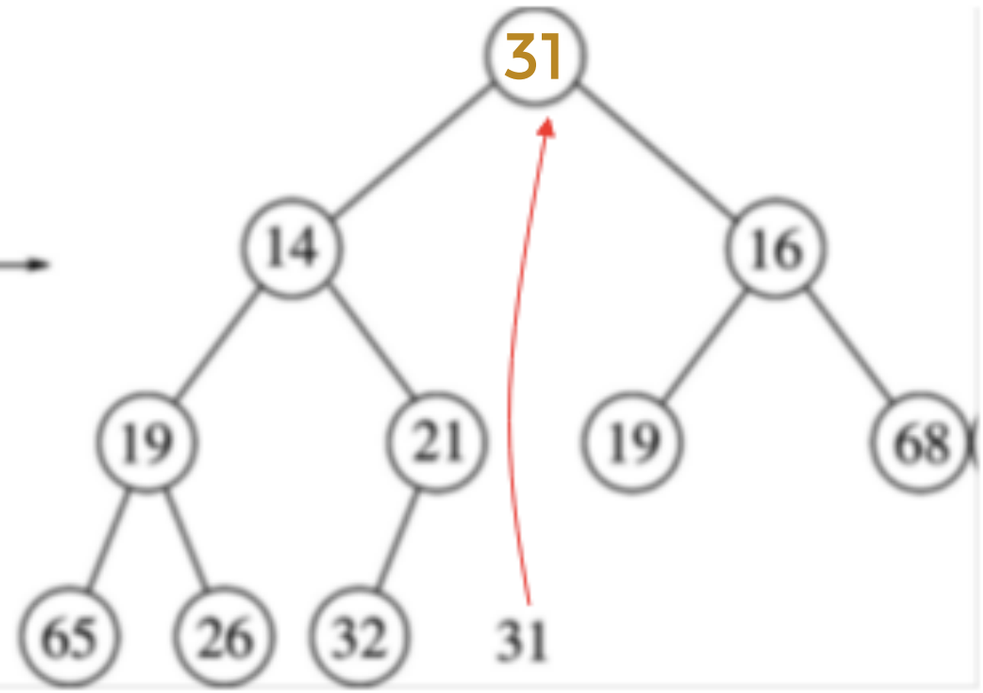
Reshuffling after a pop

Yellow 181 137 0 #b58900

## Remove value at the root




## Move 31 (last element) to the root



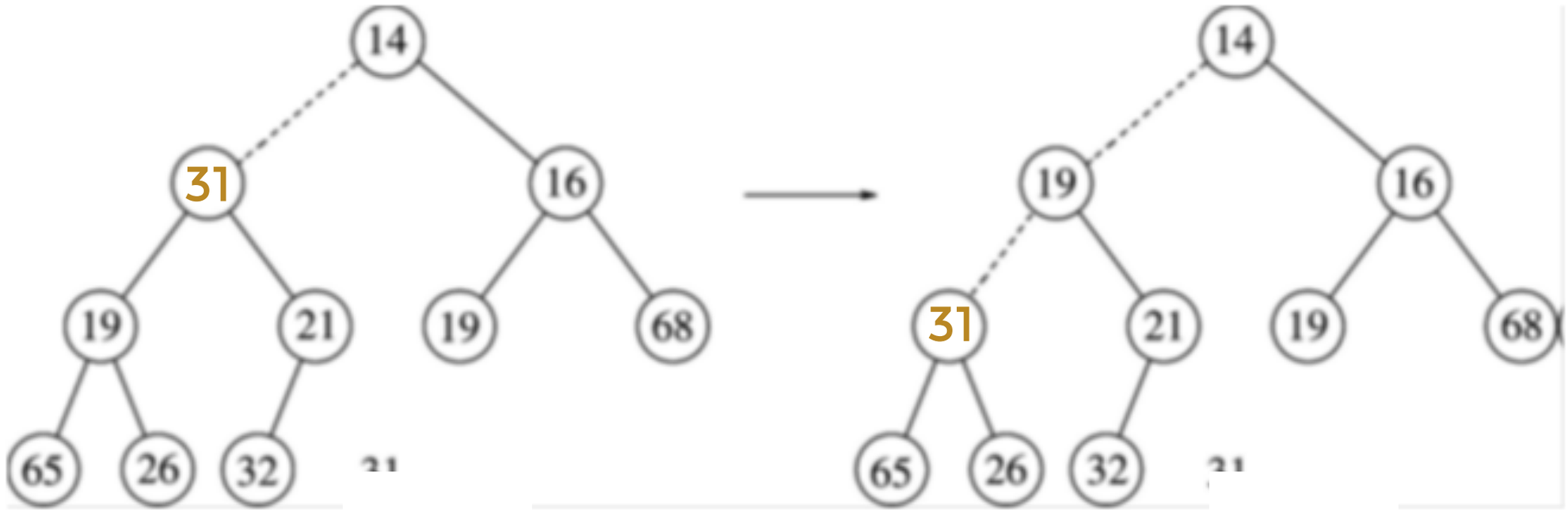

**Is 31 > min(14, 16)?**
**If yes, swap 31 with min(14, 16)**
**If no, leave 31 in hole**

Value in child is always greater than parent, and this is retained.

I assume the initial removal and movement is an atomic action.

Yellow 181 137 0 #b58900

# Percolating down…




Is 31 > min(19, 21)? 1 swap
If yes, swap 31 with min(19, 21)
If no, leave 31 in hole

Is 31 > min(65, 26)? 2 swaps
If yes, swap 31 with min(65, 26)
If no, leave 31 in hole

Yellow 181 137 0 #b58900

# Percolating down…


3 swaps

Heap-order property okay;
Structure okay;
Done.

Magenta 211 54 130 #d33682

# V = 11

# 11 nodes = _ levels?

# LOG V = LOG(2) 11 = 3.4

# WORST CASE =
# 3 SWAPS

Green 133 153 0 #859900

$$(V \times \log V) + E$$

Popping every node in the graph

Violet 108 113 196 #6c71c4

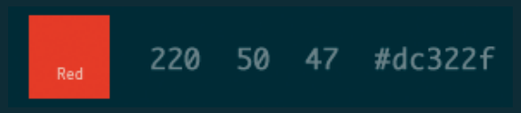
$$(V \times \log V) + E$$

Updating distance estimates for everyone's neighbours

```
(* Dijkstra's Algorithm *)
let val q: queue = new_queue()
  val visited: vertexMap = create_vertexMap()
  fun expand(v: vertex) =
    let val neighbors: vertex list = Graph.outgoing(v)
        val dist: int = valOf(get(visited, v))
        fun handle_edge(v': vertex, weight: int) =
          case get(visited, v') of
            SOME(d') =>
              if dist+weight < d'
              then ( add(visited, v', dist+weight);
                     incr_priority(q, v', dist+weight) )
              else ()
          | NONE => ( add(visited, v', dist+weight);
```

```
// Update the estimate to the neighbour, based on going through this node
distanceEstimate.put(neighbour, getDistanceEstimate(node) + getDistance(node, neighbour));
```

```
    end
in
  add(visited, v0, 0);
  expand(v0);
  while (not (empty_queue(q)) do expand(pop(q))
end
```




# BELLMAN-FORD

# FLOYD-WARSHALL

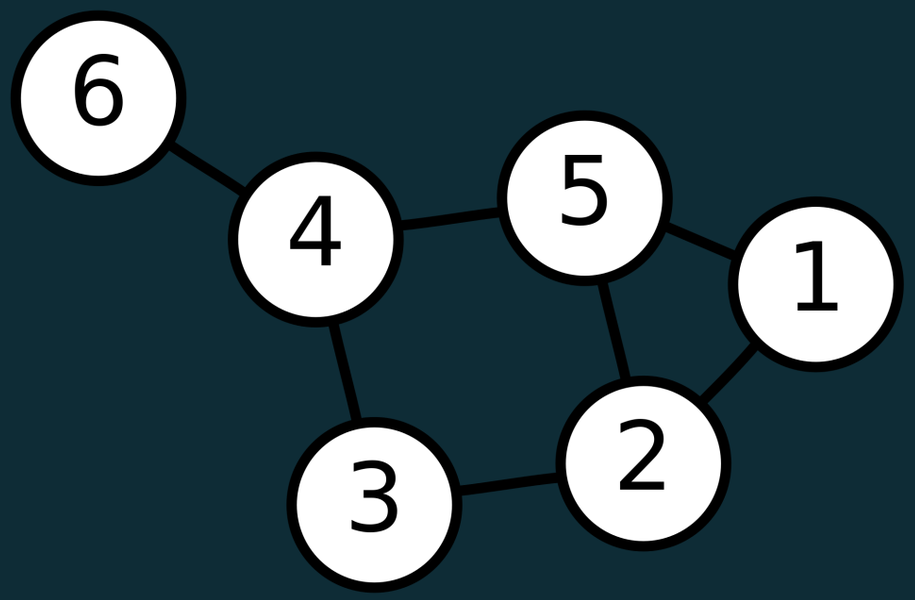
# A*

Orange 203 75 22 #cb4b16

# LOOKING FOR HIDDEN THINGS

Violet 108 113 196 #6c71c4

+




=

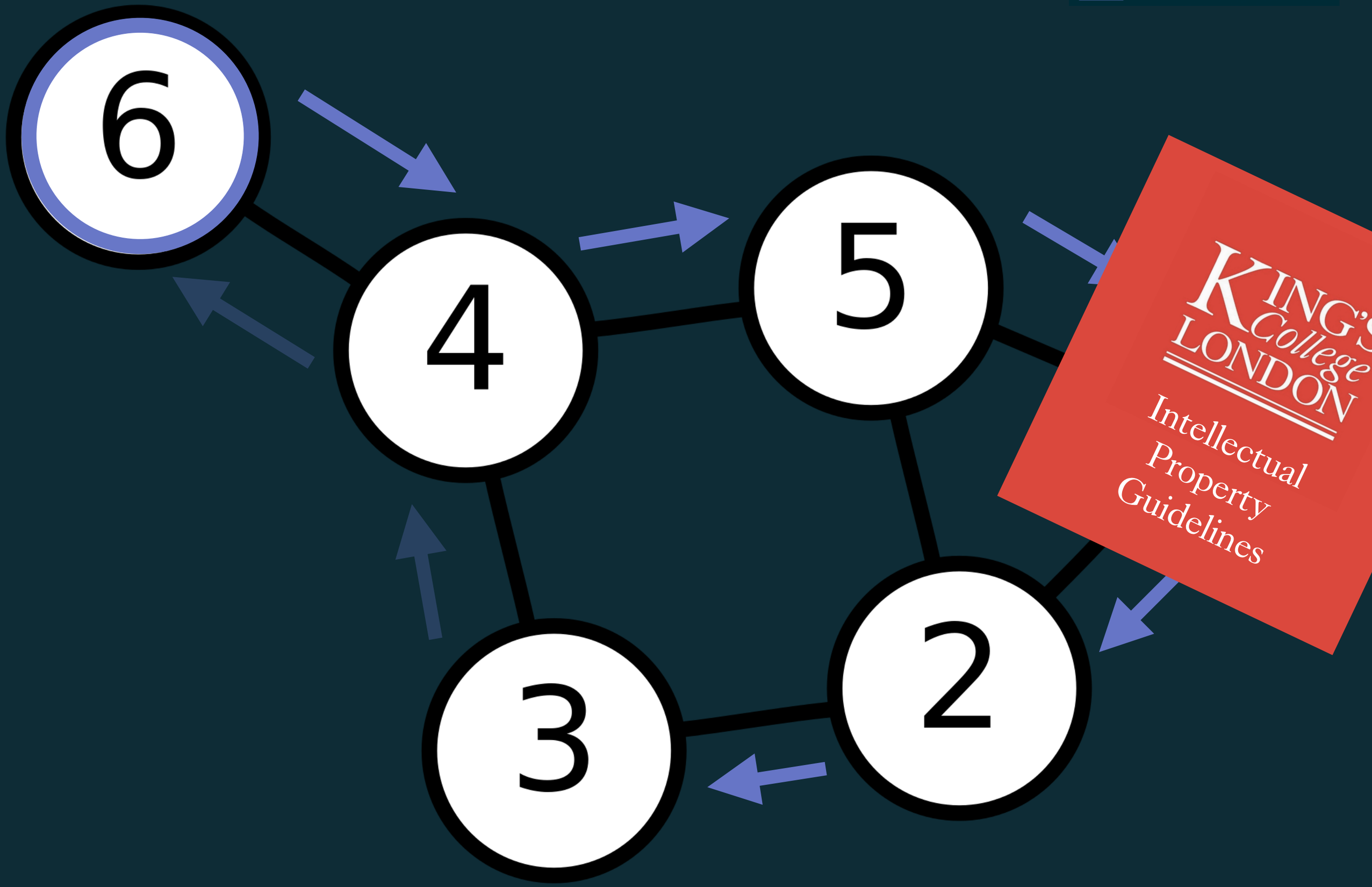
6

1

# KING'S College LONDON

# Intellectual Property Guidelines

Violet 108 113 196 #6c71c4



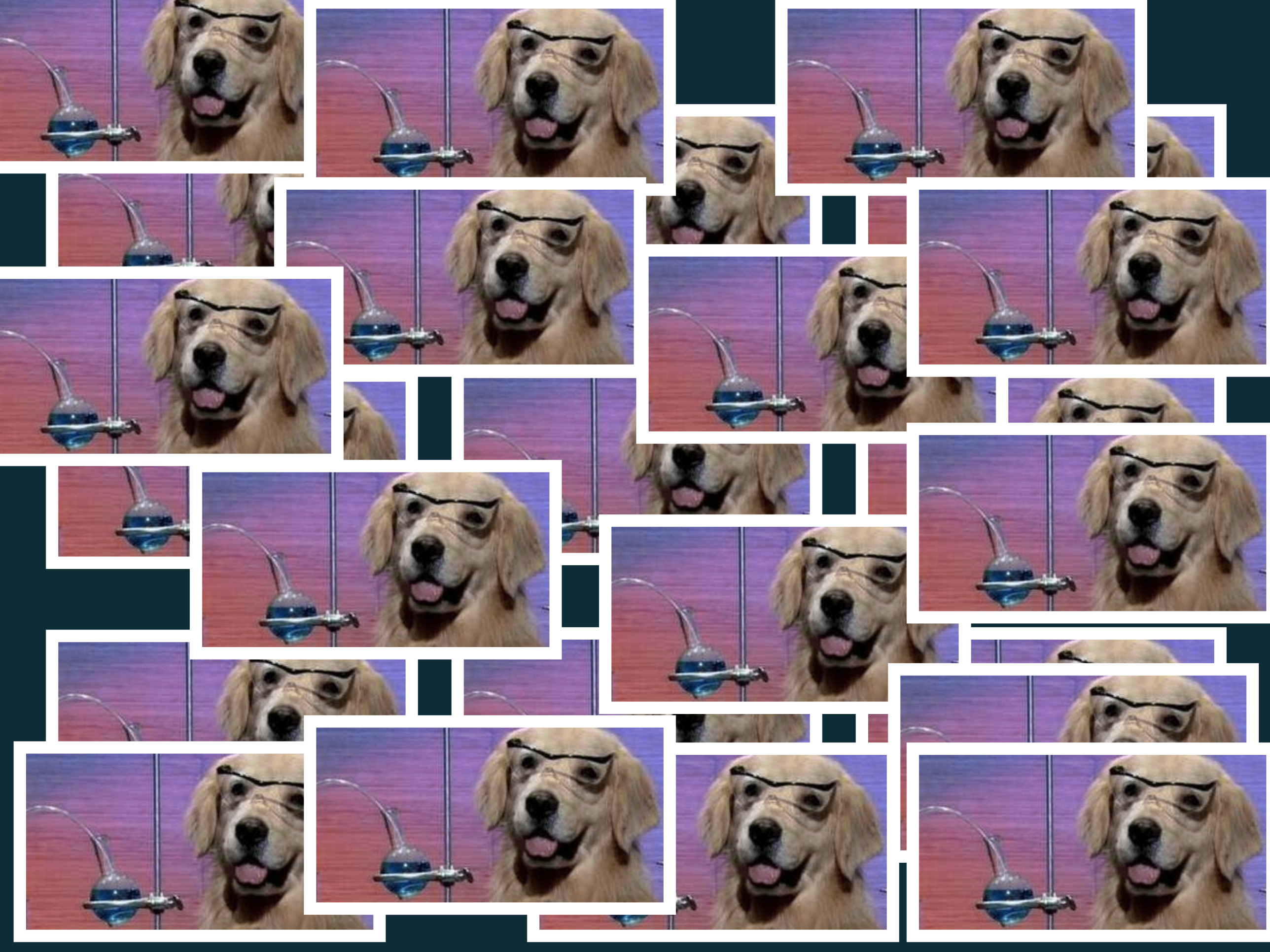

Notice that we've dropped the weights

Blue 38 139 210 #268bd2

# HAMILTONIAN PATH (CYCLE)

Violet 108 113 196 #6c71c4

# COMPLEXITY



Green 133 153 0 #859900

# QUADRATIC

# $N^2$

Seems a bit optimistic. Many different permutations on this depending on the implementation.

Cyan 42 161 152 #2aa198

| Notation | Name | Example |
|---|---|---|
| $O(1)$ | constant | Determining if a binary number is even or odd; Calculating $(-1)^n$; Using a constant-size lookup table |
| $O(\log \log n)$ | double logarithmic | Number of comparisons spent finding an item using interpolation search in a sorted array of uniformly distributed values |
| $O(\log n)$ | logarithmic | Finding an item in a sorted array with a binary search or a balanced search tree as well as all operations in a Binomial heap |
| $O((\log n)^c), c > 1$ | polylogarithmic | Matrix chain ordering; Parallel Random Access Machine |
| $O(n^c), 0 < c < 1$ | fractional power | Searching in a kd-tree |
| $O(n)$ | linear | Finding an item in an unsorted list or a malformed tree (worst case) or in an unsorted array; adding two *n*-bit integers by ripple carry |
| $O(n \log^* n)$ | n log-star n | Performing triangulation of a simple polygon using Seidel's algorithm, or the union–find algorithm. Note that $\log^*(n) = \begin{cases} 0, & \text{if } n \leq 1 \\ 1 + \log^*(\log n), & \text{if } n > 1 \end{cases}$ |
| $O(n \log n) = O(\log n!)$ | linearithmic, loglinear, or quasilinear | Performing a fast Fourier transform; heapsort, quicksort (best and average case), or merge sort |
| $O(n^2)$ | quadratic | Multiplying two *n*-digit numbers by a simple algorithm; bubble sort (worst case or naive implementation), Shell sort, quicksort (worst case), selection sort or insertion sort |
| $O(n^c), c > 1$ | polynomial or algebraic | Tree-adjoining grammar parsing; maximum matching for bipartite graphs |
| $L_n[\alpha, c], 0 < \alpha < 1 = e^{(c+o(1))(\ln n)^\alpha (\ln \ln n)^{1-\alpha}}$ | L-notation or sub-exponential | Factoring a number using the quadratic sieve or number field sieve |
| $O(c^n), c > 1$ | exponential | Finding the (exact) solution to the travelling salesman problem using dynamic programming; determining if two logical statements are equivalent using brute-force search |
| $O(n!)$ | factorial | Solving the traveling salesman problem via brute-force search; generating all unrestricted permutations of a poset; finding the determinant with expansion by minors; enumerating all partitions of a set |

Red 220 50 47 #dc322f

# NP-Complete

1

## What is NP?

NP is the set of all decision problems (questions with a yes-or-no answer) for which the 'yes'-answers can be ***verified*** in polynomial time ($O(n^k)$ where *n* is the problem size, and *k* is a constant) by a deterministic Turing machine. Polynomial time is sometimes used as the definition of *fast* or *quickly*.

## What is P?

P is the set of all decision problems which can be **solved** in *polynomial time* by a *deterministic Turing machine*. Since they can be solved in polynomial time, they can also be verified in polynomial time. Therefore P is a subset of NP.

2

## What is NP-Complete?

A problem x that is in NP is also in NP-Complete *if and only if* every other problem in NP can be quickly (ie. in polynomial time) transformed into x.

In other words:

1. x is in NP, and
2. Every problem in NP is *reducible* to x

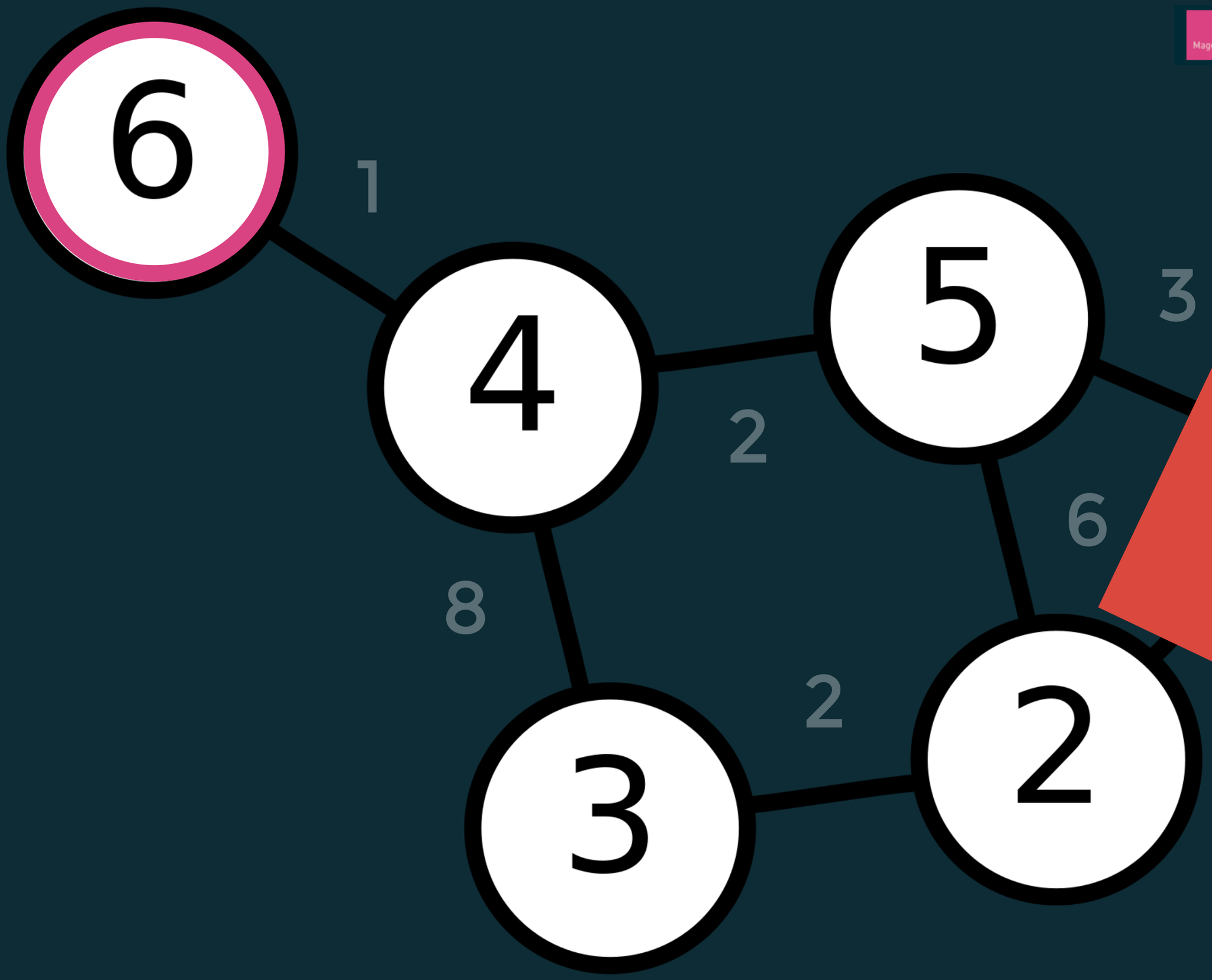
So, what makes *NP-Complete* so interesting is that if any one of the NP-Complete problems was to be solved quickly, then all *NP* problems can be solved quickly.

See also the post What's "P=NP?", and why is it such a famous question?

## What is NP-Hard?

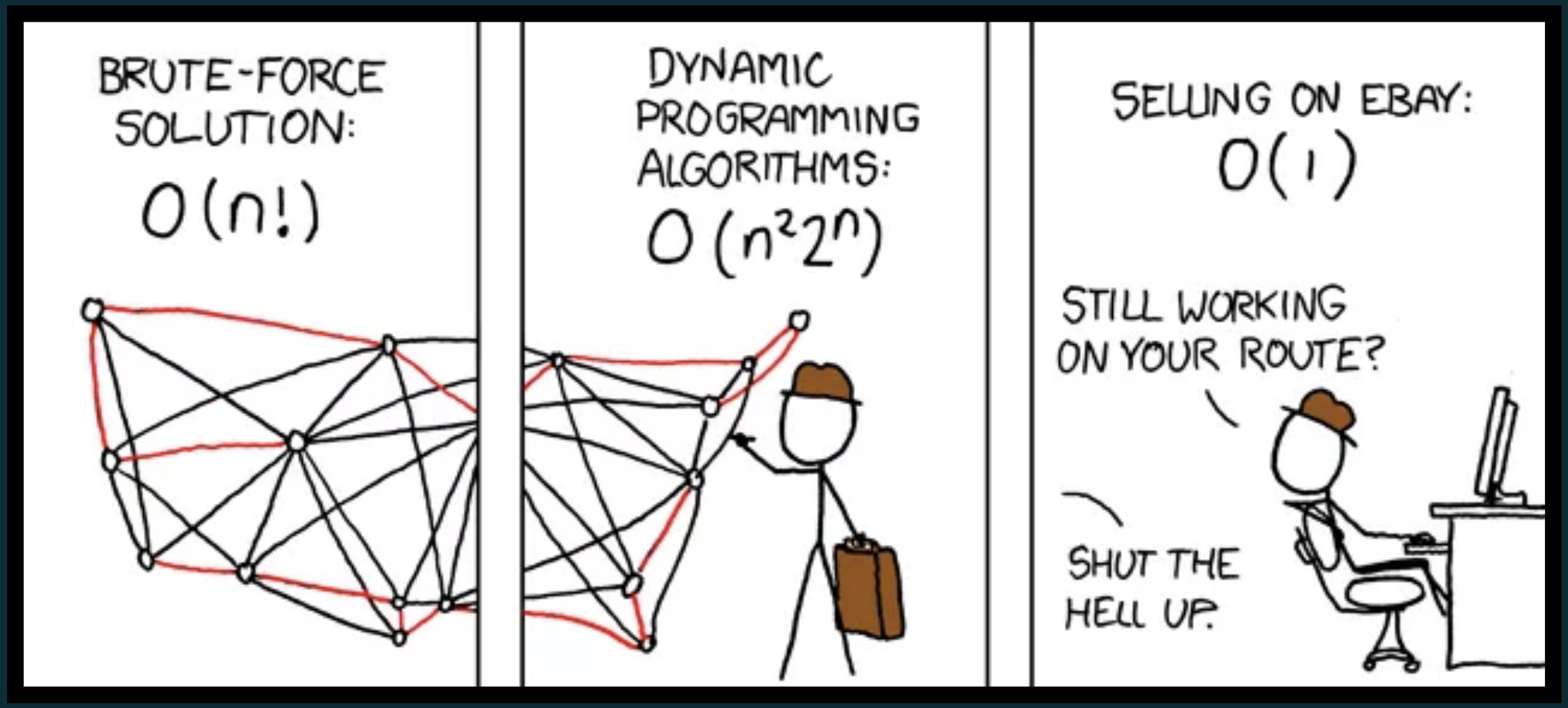
NP-Hard are problems that are at least as hard as the hardest problems in NP. Note that NP-Complete problems are also NP-hard. However not all NP-hard problems are NP (or even a decision problem), despite having `NP` as a prefix. That is the NP in NP-hard does not mean *non-deterministic polynomial time*. Yes, this is confusing, but its usage is entrenched and unlikely to change.

Magenta 211 54 130 #d33682



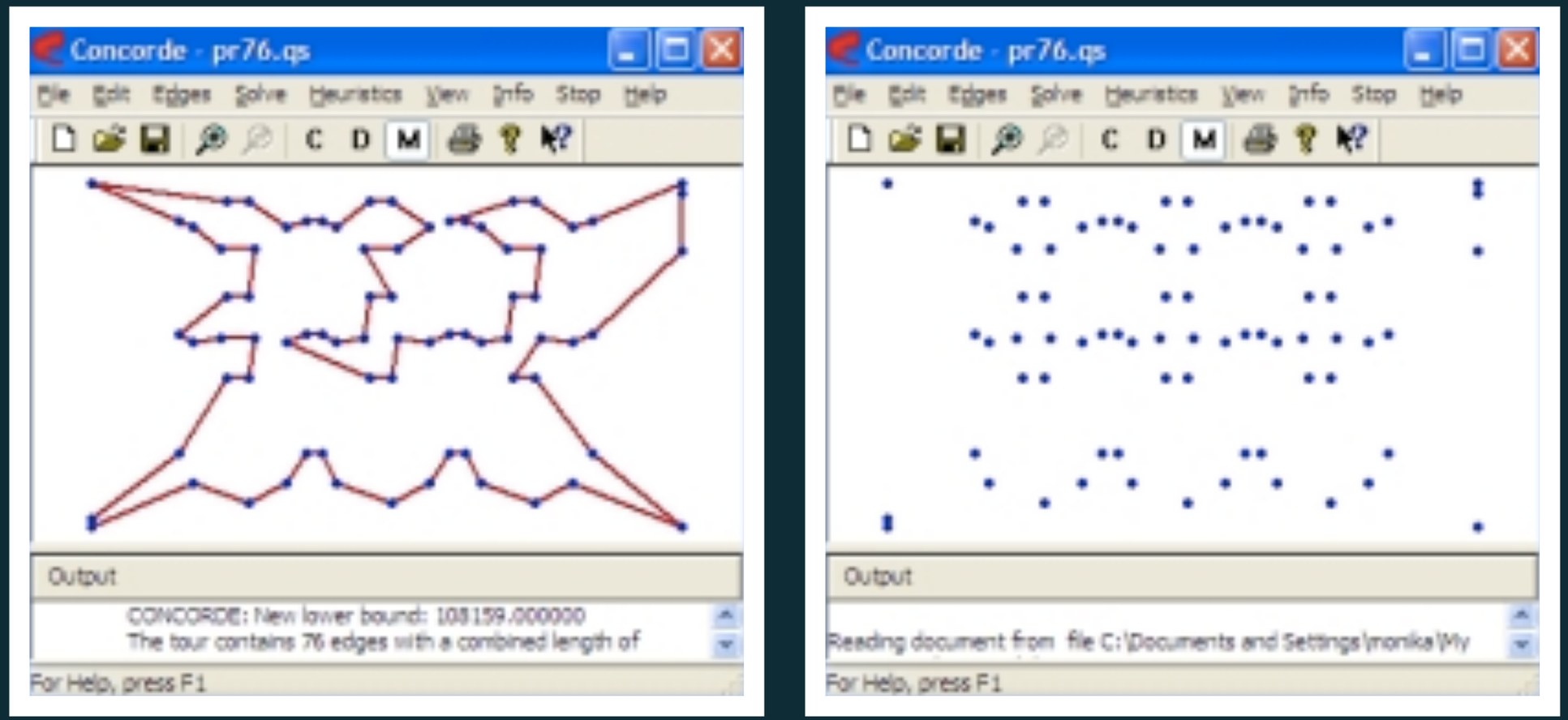

Weights are back, need to think about efficiency: Exponential and NP-hard

Violet 108 113 196 #6c71c4




An intuitive analogy for the relationship between running times (complexity)

Green 133 153 0 #859900




David Applegate, Ribert Bixby, Vasek Chvatal, and William Cook. Concorde TSP Solver. Available at http://www.math.uwaterloo.ca/tsp/concorde/, 2006.

Violet 108 113 196 #6c71c4

6

1

4

5

KING'S College LONDON

Intellectual Property Guidelines

3

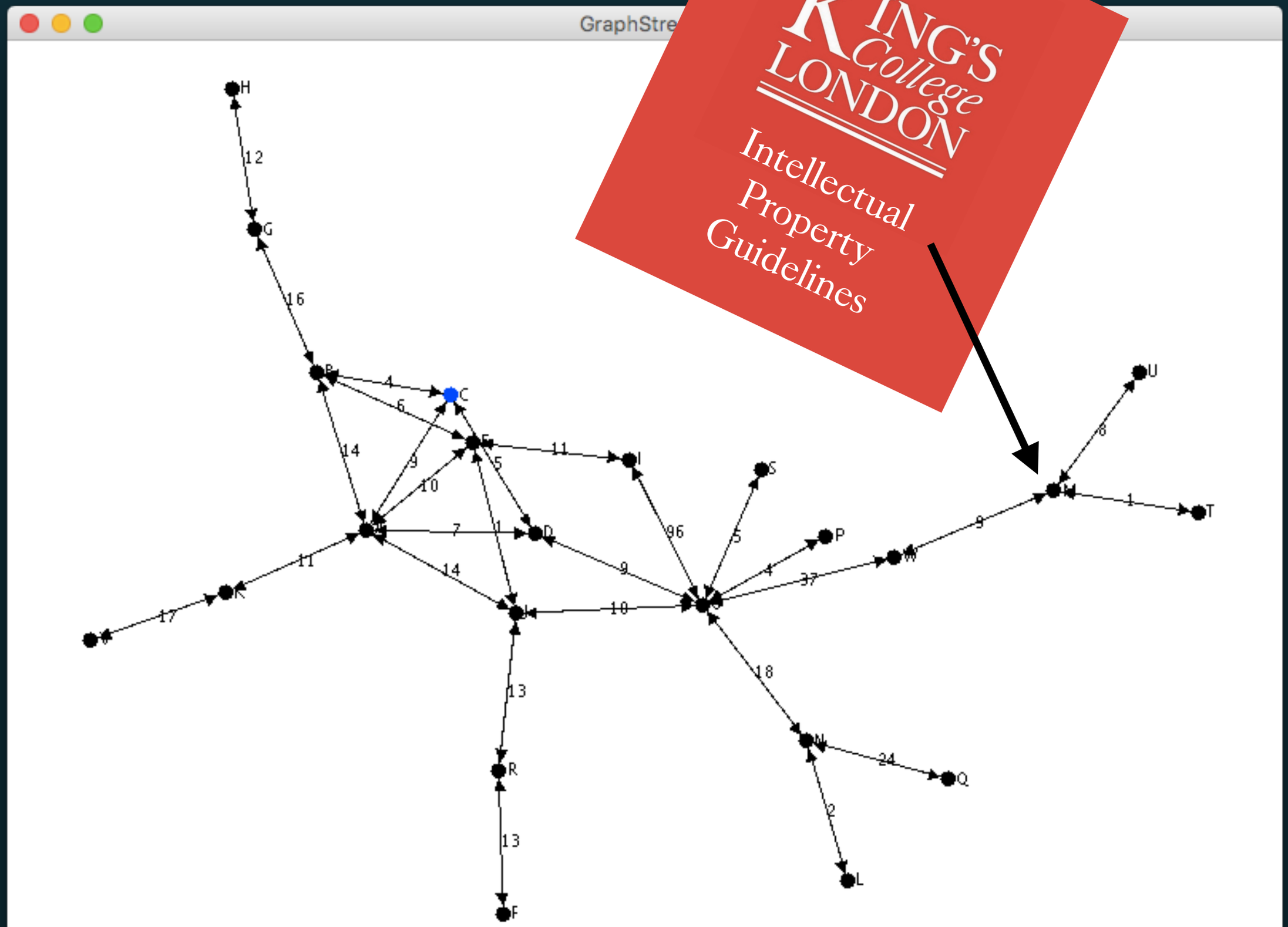
2

Orange 203 75 22 #cb4b16

# MORE GENERAL EXPLORATION STRATEGIES ARE NEEDED

210 #268bd2

Experimentation is needed.




KING'S College LONDON

Intellectual Property Guidelines

## Constructor Summary

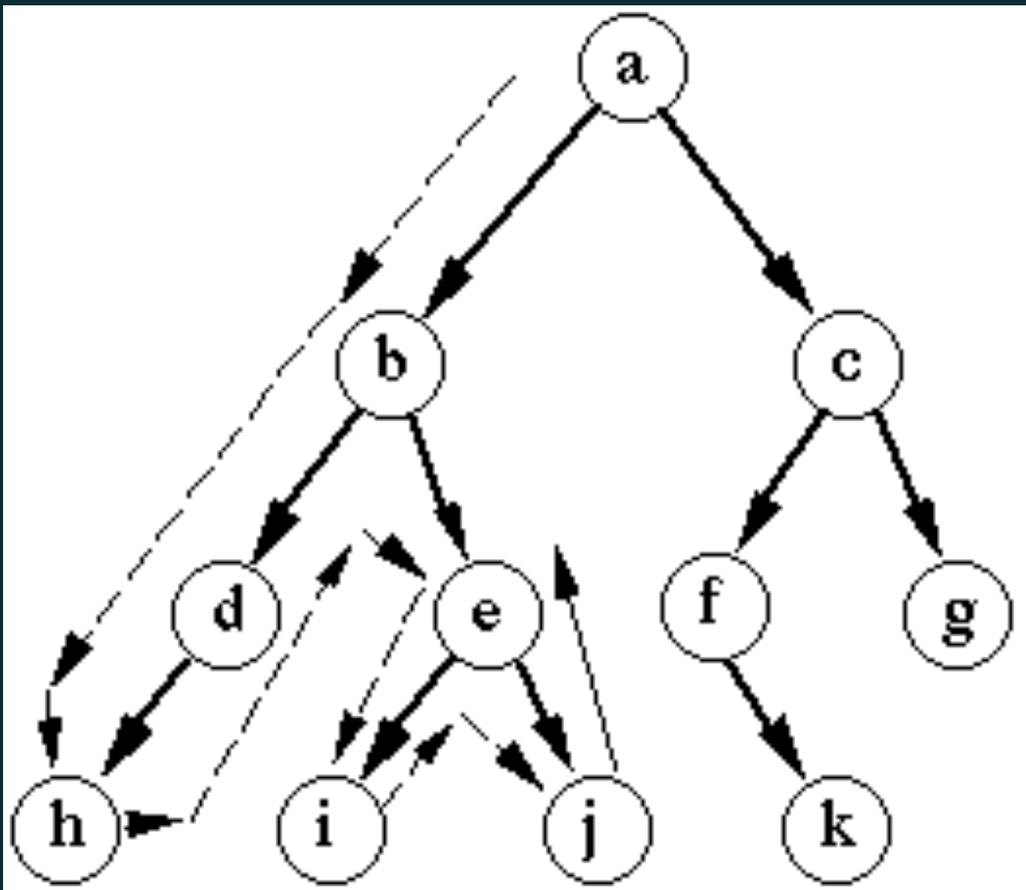
**Constructors**

| Constructor and Description |
| --- |
| **`EncapsulatedGraph(java.lang.String yourName)`**<br>Create a new graph that will communicate with my server. |

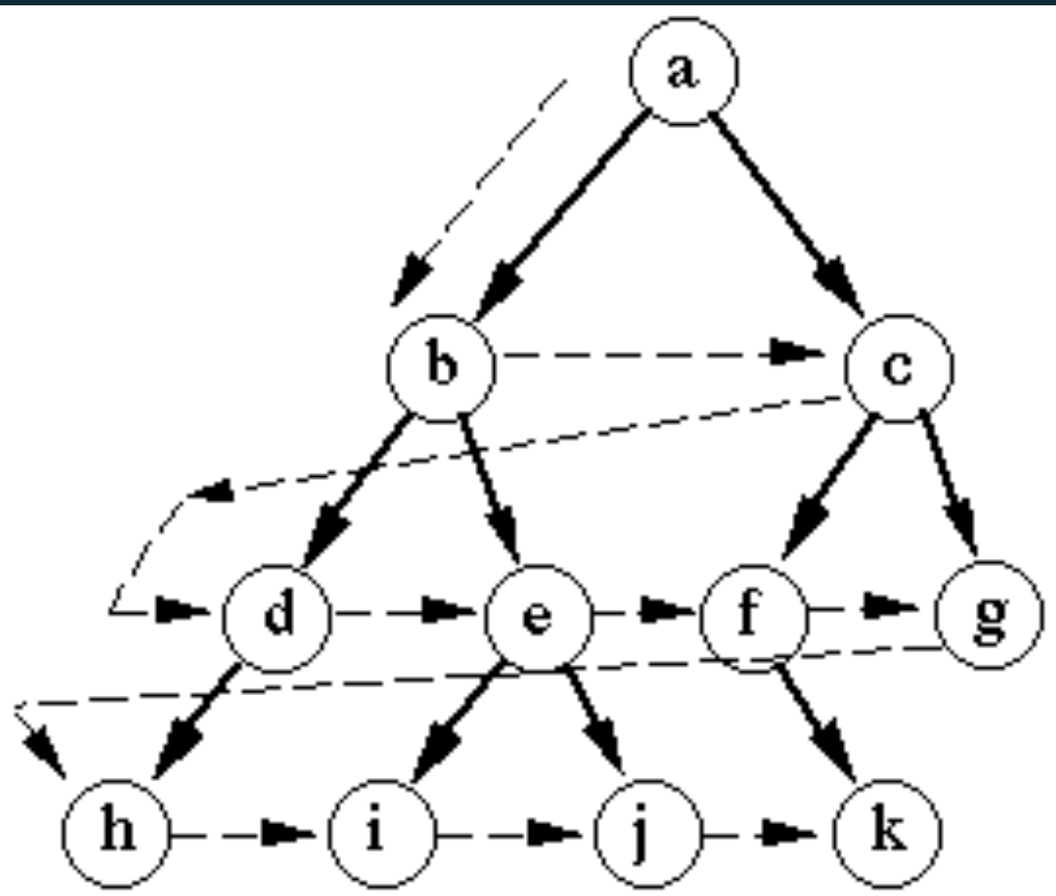
## Method Summary

**All Methods** | **Instance Methods** | **Concrete Methods**

| Modifier and Type | Method and Description |
| --- | --- |
| `java.util.ArrayList<org.graphstream.graph.Edge>` | **`edgesOfCurrentNode()`**<br>Gets the edges associated with the current node |
| `boolean` | **`found()`**<br>Whether the desired object, hidden within this graph, has been found. |
| `org.graphstream.graph.Node` | **`getCurrentNode()`**<br>The most important piece of encapsulated graph state is the current node, upon which a `searcher' currently exists. |
| `java.util.ArrayList<org.graphstream.graph.Node>` | **`getPath()`**<br>Get the path you have created so far with your search. |
| `boolean` | **`moveToNewNode(org.graphstream.graph.Node node)`**<br>Move the `pointer' within the encapsulated graph to a new node. |
| `void` | **`sendPath()`**<br>Upload your path to the server for scoring! |




Depth-first search




Breadth-first search

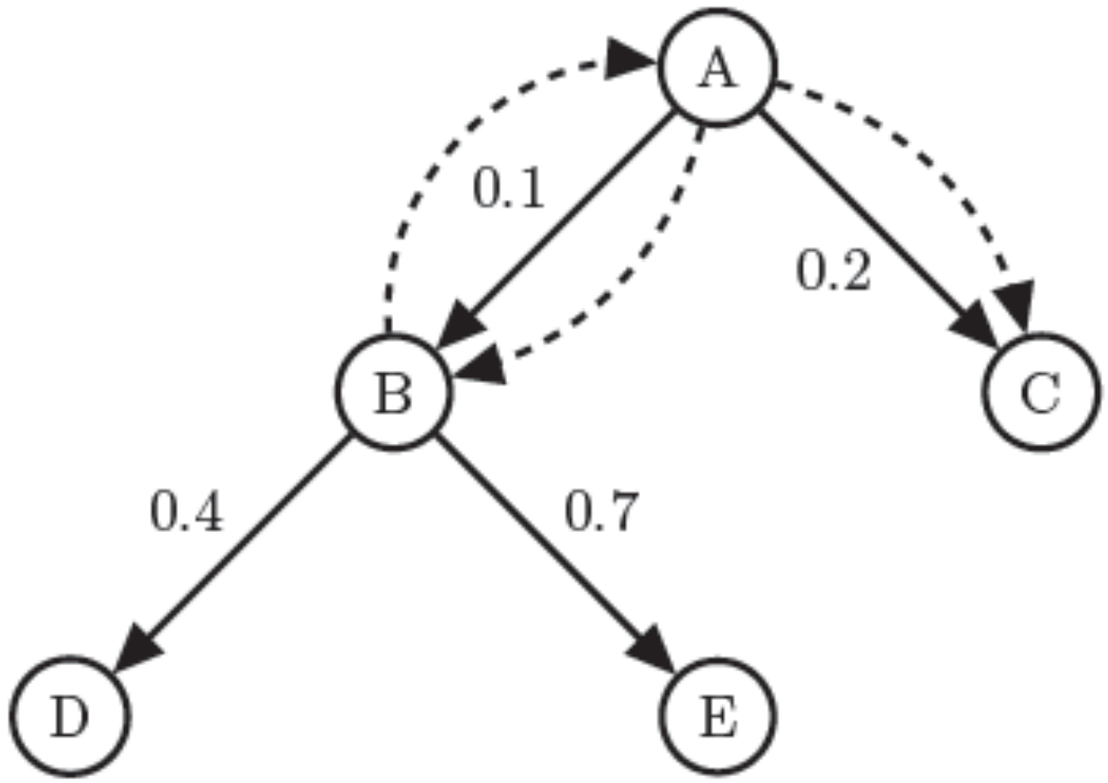



Figure 4.1: An example of how sBacktrackGreedy may backtrack to save cost: this strategy starts at node A, and then proceeds to node B, after recording the distance to node C. At vertex B, sBacktrackGreedy calculates that it will cost strictly less to move to node C *via* node A, where it has already been, than to take the single hop to nodes D or E, so it makes this move.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Base03 | 0 | 43 | 54 | #002b36 | Yellow | 181 | 137 | 0 | #b58900 |
| Base02 | 7 | 54 | 66 | #073642 | Orange | 203 | 75 | 22 | #cb4b16 |
| Base01 | 88 | 110 | 117 | #586e75 | Red | 220 | 50 | 47 | #dc322f |
| Base00 | 101 | 123 | 131 | #657b83 | Magenta | 211 | 54 | 130 | #d33682 |
| Base0 | 131 | 148 | 150 | #839496 | Violet | 108 | 113 | 196 | #6c71c4 |
| Base1 | 147 | 161 | 161 | #93a1a1 | Blue | 38 | 139 | 210 | #268bd2 |
| Base2 | 238 | 232 | 213 | #eee8d5 | Cyan | 42 | 161 | 152 | #2aa198 |
| Base3 | 253 | 246 | 227 | #fdf6e3 | Green | 133 | 153 | 0 | #859900 |