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1. a)
$$x_1 + 2x_2 - x_3 = -1$$
$$2x_1 + 2x_2 + x_3 = 1$$
$$3x_1 + 5x_2 - 2x_3 = -1$$

$$\xrightarrow{Cs} \begin{pmatrix} 1 & 2 & -1 & -1 \\ 2 & 2 & 1 & 1 \\ 3 & 5 & -2 & -1 \end{pmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - 3R_1} \begin{pmatrix} 1 & 2 & -1 & -1 \\ 0 & -2 & 3 & 3 \\ 0 & -1 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow{R_2 \to -\frac{1}{2}R_2} \begin{pmatrix} 1 & 2 & -1 & -1 \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & -1 & 1 & 2 \end{pmatrix} \xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & 2 & -1 & -1 \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\xrightarrow[R_2 \to R_2 - 3R_3]{R_1 \to R_1 - 2R_3} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \xrightarrow[R_3 \to -2R_3]{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow{Cs} \begin{aligned} x_1 &= 4 \\ x_2 &= -3 \\ x_3 &= -1 \end{aligned}$$

b)
$$x_1 + 2x_2 + 2x_4 = 6$$
$$3x_1 + 5x_2 - x_3 + 6x_4 = 17$$
$$2x_1 + 4x_2 + x_3 + 2x_4 = 12$$
$$2x_1 - 7x_3 + 11x_4 = 7$$

$$\xrightarrow{Cs} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 3 & 5 & -1 & 6 & 17 \\ 2 & 4 & 1 & 2 & 12 \\ 2 & 0 & -7 & 11 & 7 \end{pmatrix}$$

$$\xrightarrow[\substack{R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - 2R_1}]{R_2 \to R_2 - 3R_1} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -4 & -7 & 7 & -5 \end{pmatrix} \xrightarrow{R_2 \to -R_2} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -4 & -7 & 7 & -5 \end{pmatrix}$$

$$\xrightarrow{R_4 \to R_4 + 4R_2} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 7 & -1 \end{pmatrix} \xrightarrow{R_4 \to R_4 + 3R_3} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 + 2R_4]{R_1 \to R_1 - 2R_4} \begin{pmatrix} 1 & 2 & 0 & 0 & 8 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{R_2 \to R_2 - R_3} \begin{pmatrix} 1 & 2 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{Cs} \begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= -2 \\ x_4 &= -1 \end{aligned}$$

c) $$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 2 \\ 2x_1 + 4x_2 - x_3 + 6x_4 &= 5 \\ x_2 \quad + 2x_4 &= 3 \end{aligned} \xrightarrow{CS} \begin{pmatrix} 1 & 2 & -1 & 3 & 2 \\ 2 & 4 & -1 & 6 & 5 \\ 0 & 1 & 0 & 2 & 3 \end{pmatrix}$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & -1 & 3 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 & 3 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & -1 & 3 & 2 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \to R_1 + R_3} \begin{pmatrix} 1 & 2 & 0 & 3 & 3 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \xrightarrow{CS}$$

Let $x_4 = t$, then:

$$x_1 = -3 + t$$
$$x_2 = 3 - 2t$$
$$x_3 = 1$$
$$x_4 = t$$

So, the system's set of solutions is $\{(-3+t, 3-2t, 1, t) \mid t \in \mathbb{R}\}$

2. $$A = \begin{pmatrix} 1 & 0 & ? & 1 & ? \\ -1 & -1 & ? & -2 & ? \\ 3 & 1 & ? & 0 & ? \end{pmatrix} \text{ and } A \text{ in REF} = \begin{pmatrix} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & -5 & 0 & -3 \\ 0 & 0 & 0 & 1 & 6 \end{pmatrix}$$

Attempt Gauss Elimination to determine the unknown columns:

Set each position of unknown entry to a specific variable

$$\begin{pmatrix} 1 & 0 & x_1 & 1 & y_1 \\ -1 & -1 & x_2 & -2 & y_2 \\ 3 & 1 & x_3 & 0 & y_3 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 + R_1} \begin{pmatrix} 1 & 0 & x_1 & 1 & y_1 \\ 0 & -1 & x_1 + x_2 & -1 & y_1 + y_2 \\ 0 & 1 & -3x_1 + x_3 & -3 & -3y_1 + y_3 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & 0 & x_1 & 1 & y_1 \\ 0 & -1 & x_1 + x_2 & -1 & y_1 + y_2 \\ 0 & 0 & -2x_1 + x_2 + x_3 & -4 & -2y_1 + y_2 + y_3 \end{pmatrix}$$

$$\xrightarrow[R_2 \to -R_2]{R_3 \to -\frac{1}{4}R_3} \begin{pmatrix} 1 & 0 & x_1 & 1 & y_1 \\ 0 & 1 & -x_1 - x_2 & 1 & -y_1 - y_2 \\ 0 & 0 & \frac{1}{2}x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 & 1 & \frac{1}{2}y_1 - \frac{1}{4}y_2 - \frac{1}{4}y_3 \end{pmatrix}$$

$$\xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_3} \left(\begin{array}{ccccc} 1 & 0 & \frac{1}{2}x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_3 & 0 & \frac{1}{2}y_1 + \frac{1}{4}y_2 + \frac{1}{4}y_3 \\ 0 & 1 & -\frac{3}{2}x_1 - \frac{3}{4}x_2 + \frac{1}{4}x_3 & 0 & -\frac{3}{2}y_1 - \frac{3}{4}y_2 + \frac{1}{4}y_3 \\ 0 & 0 & \frac{1}{2}x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 & 1 & \frac{1}{2}y_1 - \frac{1}{4}y_2 - \frac{1}{4}y_3 \end{array}\right)$$

Since the matrix is in RREF now, set variables equal to the actual entries, creating a new lin. system of equations:

$\frac{1}{2}x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_3 = 2$

$-\frac{3}{2}x_1 - \frac{3}{4}x_2 + \frac{1}{4}x_3 = -5$

$\frac{1}{2}x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 = 0$

Multiply by 4 each row and convert into augmented matrix:

$$\begin{pmatrix} 2 & 1 & 1 & 8 \\ -6 & -3 & 1 & -20 \\ 2 & -1 & -1 & 0 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 + 3R_1} \begin{pmatrix} 2 & 1 & 1 & 8 \\ 0 & 0 & 4 & 4 \\ 0 & -2 & -2 & -8 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 1 & 1 & 8 \\ 0 & -2 & -2 & -8 \\ 0 & 0 & 4 & 4 \end{pmatrix}$$

$$\xrightarrow[R_3 \to \frac{1}{4}R_3]{R_1 \to \frac{1}{2}R_1,\ R_2 \to -\frac{1}{2}R_2} \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 4 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_1 \to 2R_1} \begin{pmatrix} 2 & 1 & 1 & 8 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_3} \begin{pmatrix} 2 & 1 & 0 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \begin{pmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{C_S} \quad x_1 = 2 \quad x_3 = 1$$

$x_2 = 3$

$\frac{1}{2}y_1 + \frac{1}{4}y_2 + \frac{1}{4}y_3 = -2$

$-\frac{3}{2}y_1 - \frac{3}{4}y_2 + \frac{1}{4}y_3 = -3$

$\frac{1}{2}y_1 - \frac{1}{4}y_2 - \frac{1}{4}y_3 = 6$

Multiply each row by 4 and convert into augmented matrix

$$\begin{pmatrix} 2 & 1 & 1 & -8 \\ -6 & -3 & 1 & -12 \\ 2 & -1 & -1 & 24 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 + 3R_1} \begin{pmatrix} 2 & 1 & 1 & -8 \\ 0 & 0 & 4 & -36 \\ 0 & -2 & -2 & 32 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 1 & 1 & -8 \\ 0 & -2 & -2 & 32 \\ 0 & 0 & 4 & -36 \end{pmatrix}$$

$$\xrightarrow[R_3 \to \frac{1}{4}R_3]{R_2 \to -\frac{1}{2}R_2} \begin{pmatrix} 2 & 1 & 1 & -8 \\ 0 & 1 & 1 & -16 \\ 0 & 0 & 1 & -9 \end{pmatrix} \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_3} \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -9 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \begin{pmatrix} 2 & 0 & 0 & 8 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -9 \end{pmatrix} \xrightarrow{C_S} \quad y_1 = 4, \quad y_2 = -7, \quad y_3 = -9$$

Thus:

$$A = \begin{pmatrix} 1 & 0 & 2 & 1 & 4 \\ -1 & -1 & 3 & -2 & -7 \\ 3 & 1 & 1 & 0 & -9 \end{pmatrix}$$

3. a) True, by VS3

b) False, $\bar{0}$ is unique since by VS3, if x and y are 0 vectors, then $x = x + y = y$

c) False, let x be zero vector, then $2x = 4x$, but $2 \neq 4$

d) False, let $a = 0$ and $x = \bar{2}$, $y = \bar{3}$, then $0x = 0y$, but $x \neq y$

e) True, column vector with n entries is of the same form as matrices in $M_{n \times 1}$ over same field

f) False, by definition m is rows, n is columns

g) False, ~~the~~ resulting polynomial of two of different degrees is still in same vector space

h) False, the difference of equivalent values of the leading coefficients would cancel out the leading coefficient and thus the highest degree, resulting in polynomial of a lesser degree.

i) True, closed under multiplication, keeping all coefficients in place.

j) True, by definition a non-zero scalar is a polynomial of degree 0, thus in $P(\mathbb{F})$

k) True by definition, the two functions have the same domain and codomain, thus same value at each element of domain

4. Proof:

Let V be the set of all even functions, so $V = \{ f \mid f: \mathbb{R} \to \mathbb{R},\ f(-t) = f(t),\ \forall t \in \mathbb{R} \}$.

Consider function $h(t)$ s.t. $h(t) = f(t) + g(t)$. To show $h(t)$ ~~is~~ ~~even~~ ~~and~~ is in V, we need to show that it is even.

$f, g \in V \Rightarrow f(-t) + g(-t) = h(-t) = f(t) + g(t)$. Thus, h is even and V is closed under addition.

Let $f \in V$ and $a \in \mathbb{R}$. $af(t)$ defines some function $g(t)$ s.t. $af(t) = g(t)$. Need to check $g(t)$ is even:

$g(-t) = af(-t)$. Since f is even, $g(-t) = af(t)$. Thus, g is even and in V.

Since V is closed under addition and scalar multiplication, it's defined on the real line and thus a valid subspace of v.s. of all real-valued functions on $\mathbb{R}$, thus V is a vector space. $\blacksquare$

5. Let $V = \mathbb{R}^2$, equipped with following addition and scalar multiplication:
$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$

$c(a_1, a_2) = (ca_1, ca_2)$

Let $x, y, z \in V$, then by VS 2, $(x+y)+z = x+(y+z)$

$x = (x_1, x_2)$, $y = (y_1, y_2)$, ~~&~~ $z = (z_1, z_2)$
Check left side of VS_2 equation:
$((x_1, x_2) + (y_1, y_2)) + (z_1, z_2)$.

$= (x_1 + 2y_1, x_2 + 3y_2) + (z_1, z_2)$

$= (x_1 + 2y_1 + 2z_1, x_2 + 3y_2 + 3z_2)$
Check right side of VS 2 equation:

$(x_1, x_2) + ((y_1, y_2) + (z_1, z_2))$

$= (x_1, x_2) + (y_1 + 2z_1, y_2 + 3z_2)$

$= (x_1 + 2y_1 + 4z_1, x_2 + 3y_2 + 9z_2)$

Since $(x+y)+z \neq x+(y+z)$, V is not a vector space since VS 2 axiom doesn't hold.

6. a) False, consider $V = \mathbb{R}$ and $W = \mathbb{Q}$, both are vector spaces, but W is not a vector space over $\mathbb{R}$ and thus isn't ~~a~~ ~~its~~ subspace of V since ~~the~~ two must have same operations.

b) False, $\bar{0} \notin \emptyset$, which is required by defn. of subspace

c) True, let $W = \{0\}$, which $\neq V$ and is its subspace.

d) False, $W_1 = \{0, 1\}$, $W_2 = \{1\}$ are subsets of $V = \mathbb{R}$, however $W_1 \cap W_2 = \{1\}$ and doesn't contain zero vector, thus not a subspace.

e) True, since only entries on diagonal can be non-zero, n of them

f) Trace of square matrix is the sum of its diagonal entries, so False

g) False, elements of W have ~~atleast~~ 1 more coordinate, thus they are not the same.

7. a) $W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = 3a_2,\ a_3 = -a_2\}$

$(0,0,0) \Rightarrow (3\cdot 0, 0, -0) = (0,0,0) \in W_1$ for $a_2 = 0$ ✓

Let $a = (a_1, a_2, a_3),\ b = (b_1, b_2, b_3) \in W_1$, then:

$a = (3a_2, a_2, -a_2),\ b = (3b_2, b_2, -b_2)$.

$$a + b = (3a_2 + 3b_2,\ a_2 + b_2,\ -a_2 - b_2)$$
$$= (3(a_2+b_2),\ a_2+b_2,\ -(a_2+b_2)) \in W_1 \checkmark$$

Let $c \in \mathbb{R}$

$ca \Rightarrow c(3a_2, a_2, -a_2) = (3ca_2, ca_2, -ca_2) \in W_1$ ✓

W_1 is a subspace of $\mathbb{R}^3$ by definition

b) $W_2 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = a_3 + 2\}$

$\bar{0} \in \mathbb{R}^3 \Rightarrow (0,0,0)$. $a_1 = 0$ if $a_3 = -2$. Thus, $(0,0,-2) \neq (0,0,0)$, hence $\bar{0} \notin W_2$ and W_2 is not a subspace of $\mathbb{R}^3$

c) $W_3 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 - 7a_2 + a_3 = 0\}$

$\bar{0} \in \mathbb{R}^3 \Rightarrow (0,0,0) \Rightarrow a_1, a_2, a_3 = 0$, so $(0,0,0) \in W_3$ ✓

Let $a, b \in W_3$, then $a = (a_1, a_2, a_3)$ s.t. $2a_1 - 7a_2 + a_3 = 0$, and $b = (b_1, b_2, b_3)$ s.t. $2b_1 - 7b_2 + b_3$

$a + b \Rightarrow 2a_1 + 2b_1 - 7a_1 - 7b_1 + a_3 + b_3 = 0$. $\Longrightarrow$

$2(a_1+b_1) - 7(a_2+b_2) + (a_3+b_3) = 0$, thus, $a+b \in W_3$ ✓

Let $c \in \mathbb{R}$, then

$ca \Rightarrow 2ca_1 - 7ca_2 + ca_3$, so $ca \in W_3$ ✓.

Thus, W_3 is a subspace of $\mathbb{R}^3$ by definition

d) $W_4 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 - 4a_2 - a_3 = 0\}$

$\bar{0} \in \mathbb{R}^3 \Rightarrow (0,0,0) \Rightarrow a_1, a_2, a_3 = 0 \Rightarrow (0,0,0) \in W_4$ ✓

Let $a, b \in W_4$, so $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$ s.t.
$a_1 - 4a_2 - a_3 = 0$ and $b_1 - 4b_2 - b_3 = 0$.

$a+b \Rightarrow a_1 + b_1 - 4a_2 - 4b_2 - a_3 - b_3 = 0$

$\Rightarrow a_1 + b_1 - 4(a_2+b_2) - (a_3+b_3) = 0$, thus
$a+b \in W_4$ ✓

Let $c \in \mathbb{R}$, then $ca \Rightarrow ca_1 - 4ca_2 - ca_3$, so $ca \in W_4$ ✓

Thus, W_4 is a subspace of $\mathbb{R}^3$ by definition

e) $W_5 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 + 2a_2 - 3a_3 = 1\}$

$\bar{0} \in \mathbb{R}^3 \Rightarrow (0,0,0) \Rightarrow a_1, a_2, a_3 = 0$, however
$0 + 0 + 0 \neq 1$, thus $\bar{0} \notin W_5$, hence W_5 is not a subspace of $\mathbb{R}^3$

f) $W_6 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : 5a_1^2 - 3a_2^2 + 6a_3^2 = 0\}$

$\bar{0} \in \mathbb{R}^3 \Rightarrow (0,0,0) \Rightarrow a_1, a_2, a_3 = 0 \Rightarrow \bar{0} \in W_6$ ✓

Let $a, b \in W_6$, so $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$ s.t.
$5a_1^2 - 3a_2^2 + 6a_3^2 = 0$ and $5b_1^2 - 3b_2^2 + 6b_3^2 = 0$.

$a+b \Rightarrow 5(a_1^2 + b_1^2) - 3(a_2^2 + b_2^2) + 6(a_3^2 + b_3^2) = 0$, thus
$a+b \in W_6$ ✓

Let $c \in \mathbb{R}$, then $ca \Rightarrow 5ca_1^2 - 3ca_2^2 + 6ca_3^2 = 0$, thus $ca \in W_6$ ✓

Therefore, W_6 is a subspace of $\mathbb{R}^3$

8. $$\begin{aligned} x_1 + cx_2 \quad &= 0 \\ x_1 + 2x_2 - x_3 &= 0 \\ x_2 + x_3 &= 2 \end{aligned} \xrightarrow{E_s} \begin{pmatrix} 1 & c & 0 & 0 \\ 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & c & 0 & 0 \\ 0 & 2-c & -1 & 0 \\ 0 & 1 & 1 & 2 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & c & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 2-c & -1 & 0 \end{pmatrix} \xrightarrow{R_3 \to R_3 - (2-c)R_2} \begin{pmatrix} 1 & c & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -3+c & 2c-4 \end{pmatrix}$$

~~[illegible]~~ the system is inconsistent if $c - 3 = 0$ and $2c - 4 \neq 0$ and this is when $c = 3$

9. Let A and B be matrices with same RREF, denoted as R. We need to show that A can be transformed into B by a finite sequence of elementary row operations.

Since R is RREF of A, there exists a finite sequence of elementary row operations $S_1, S_2, \ldots, S_n$ that transform A into R.

The same applies for B, let's denote the operations as $T_1, T_2, \ldots, T_m$.

Since both A and B have R as their RREF, R can be converted back into B by a finite sequence of elementary row operations which are the inversion of $T_1, T_2, \ldots, T_m$.

~~We know that the~~ this is possible because each elementary row operation is reversible:

Row swapping can be undone by swapping same rows again.

Multiplying a row by scalar k can be undone by multiplying by its reciprocal, $1/k$.

Adding a multiple k of one row to another can be undone by adding $-k$ of that row.

Thus, inverse sequence $T_1, T_2, \ldots, T_m$ transforms R back to B.

Since A can be transformed into R and R can be transformed into B by finite elementary row sequences, the combined sequence of row operations allows to transform A into B by finite sequence of elementary row operations