CS 577: Introduction to Algorithms Fall 2025

Homework 0

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This short assignment is intended as a review of recursion, program correctness, and asymptotic notation. All problems will be discussed during the first discussion session. You do not need to turn in anything. The optional programming problem is intended to familiarize yourself with the system. Please see the guidelines on Canvas for detailed instructions.

Review problems

1. Rank the following functions in ascending order of growth rate. That is, if function $f(n)$ precedes function $g(n)$ in the list, it must be the case that $f(n) = O(g(n))$:

   $f_1(n) = n^{2.5}$
   $f_2(n) = n + 10$
   $f_3(n) = \sqrt{2n}$
   $f_4(n) = 10^n$
   $f_5(n) = 5 \log n$
   $f_6(n) = (\log n)^4$
   $f_7(n) = n \log n$
   $f_8(n) = (\log n)^{n-2}$

2. Recall that an integer d is a divisor of an integer a if there exists an integer q such that $a = d \cdot q$. The greatest common divisor of two integers a and b, denoted $\gcd(a, b)$, is the largest positive integer d such that d is a divisor of both a and b. The notion is well-defined as long as not both a and b are zero, but we will only care about values of a and b that are positive.

   The following are two well-known properties of gcd: For all integers a and b, $\gcd(a, b) = \gcd(b, a)$ and $\gcd(a, b) = \gcd(a, b - a)$. You can take those properties for granted but it would be good to convince yourself of their validity.

   For each of the following programs, analyze whether they correctly compute the greatest common divisor of positive integers a and b.

Algorithm 1

```
1: procedure GCD1(a, b)
2:     if a = b then
3:         return a
4:     if a < b then
5:         return GCD1(a, b − a)
6:     else
7:         return GCD1(a − b, b)
```

Algorithm 2

```
1: procedure GCD2(a, b)
2:     if a > b then
3:         a ← a − b
4:     else
5:         b ← b − a
6:     if a = b then
7:         return a
8:     else
9:         return GCD2(a, b)
```

Algorithm 3

```
1: procedure GCD3(a, b)
2:     if a = b then
3:         return a
4:     if a < b then
5:         return GCD3(b − a, b)
6:     else
7:         return GCD3(a, a − b)
```

Algorithm 4

```
1: procedure GCD4(a, b)
2:     if a = b then
3:         return a
4:     if a < b then
5:         return GCD4(a, b − a)
6:     else
7:         return GCD4(b, ab)
```

Algorithm 5

```
1: procedure GCD5(a, b)
2:     if a = b then
3:         return a
4:     if a < b then
5:         return GCD5(a, b − a)
6:     else
7:         return GCD5(b, a(b + 1))
```

Challenge problem

3. For each of the programs in problem 2, determine exactly for which pairs (a, b) of positive integers the program correctly returns $\gcd(a, b)$.

Programming problem

4. SPOJ problem TEST (problem code TEST).