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1.

a) True, by definition elementary matrix is identity matrix which is square, with elementary row op. applied to it.

b) False, after el. row op. is applied to I_n, it need not to have only 0 and 1 entries.

c) True if it I_n multiplied by 1.

d) ~~True, this is equivalent to applying elementary row operations twice to the same identity matrix~~

False $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix}$, but there is no way to obtain such matrix from In by a single elementary operation

e) True inverse of an elementary matrix that corresponds to inverse elementary operation.

f) False, $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$ which is not possible to get from Identity matrix by a single em. operation.

g) True, for transpose same elementary operations on rows correspond to same operations on columns.

h) False, consider $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$ which can be obtained from $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, but there is no way to obtain same matrix by performing an elementary column operations on A since columns will remain multiples of $(1,1)$

i) True, A can be obtained from B by an inverse elementary row operation $\Rightarrow$ ~~$E(R)\cdot B = A \Rightarrow E(R)A = B$~~

$E(R) A = B \Rightarrow$ ~~(...)~~ $E^{-1}(R) B = A$

2. Suppose that Q can be obtained from a matrix P by an elementary row operation. Consider all 3 types of them such that:

$(R_i \leftrightarrow R_j)$ row interchange

$(R_i \to \lambda R_i)$ row multiplication by λ

$(R_i \to R_i + \lambda R_j)$ row add of λ multiple of the other

Then for above row operations there exist inverse row operations as follows:

$(R_j \leftrightarrow R_i)$

$(R_i \to \frac{1}{\lambda} R_i)$

$(R_i \to R_i - \lambda R_j)$

Which undo all of the related elementary row operations. Thus, obtaining P from Q is a matter of applying the inverse of the corresponding elementary row operation of the same type.

3. Rank of a matrix = # of leading entries in its REF.

a) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{pmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix} \Rightarrow \text{rank} = 2$

b) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{rank} = 1$

c) $\begin{pmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 0 & 2 & -3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[R_1 \leftrightarrow R_4]{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 1 & 2 & 3 & 1 & 1 \end{pmatrix}$

$\xrightarrow[R_4 \to R_4 - R_1]{R_3 \to R_3 - R_1} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 \\ 0 & 4 & 0 & 1 & 2 \\ 0 & 2 & 3 & 1 & 1 \end{pmatrix} \xrightarrow[R_4 \to R_4 - R_2]{R_3 \to R_3 - 2R_2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 \\ 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 6 & 1 & 0 \end{pmatrix} \xrightarrow{R_4 - R_3} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 \\ 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow \text{rank} = 3$

4) If RREF of this matrix is I_n, then $(\text{matrix} \mid I_n) \Rightarrow (\text{RREF}(\text{matrix}) \mid$ ~~mat~~ inverse of matrix$)$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 + R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{3} R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 2 & -\frac{1}{3} & \frac{2}{3} & 1 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{2} R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{2}{6} & \frac{1}{2} \end{array}\right)$$

$$\xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{7}{6} & -\frac{2}{6} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{3}{6} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{2}{6} & \frac{1}{2} \end{array}\right) \xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{6} & -\frac{2}{6} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{6} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{2}{6} & \frac{1}{2} \end{array}\right)$$

Thus the rank of this matrix is 3 and inverse since 3 leading entries

is $\begin{pmatrix} \frac{1}{6} & -\frac{2}{6} & \frac{1}{2} \\ \frac{1}{6} & 0 & -\frac{1}{2} \\ -\frac{1}{6} & \frac{2}{6} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & -\frac{1}{3} & \frac{1}{2} \\ \frac{1}{6} & 0 & -\frac{1}{2} \\ -\frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix}$

5. ~~Linear transformation is invertible if it is 1-1 and onto.~~ T is invertible iff its matrix is.

Let β denote standard basis of $P_2(\mathbb{R})$, then the listed transformation can be expressed as a matrix:

$$[T(1)]_\beta = [-1]_\beta = -1 \cdot 1 + 0 \cdot x + 0 \cdot x^2$$

$$[T(x)]_\beta = [(2-x)]_\beta = 2 \cdot 1 - 1 \cdot x + 0 \cdot x^2$$

$$[T(x^2)]_\beta = 2 + 4x - x^2 = 2 \cdot 1 + 4 \cdot x - 1 x^2$$

$$\Rightarrow [T]_\beta = \begin{pmatrix} -1 & 2 & 2 \\ 0 & -1 & 4 \\ 0 & 0 & -1 \end{pmatrix}$$

To find its inverse we create augmented matrix: $([T]_\beta \mid I_n)$ and convert it into RREF

$$\left(\begin{array}{ccc|ccc} -1 & 2 & 2 & 1 & 0 & 0 \\ 0 & -1 & 4 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to -R_3]{\substack{R_2 \to -R_2 \\ R_1 \to -R_1}} \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & -1 & 0 & 0 \\ 0 & 1 & -4 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right)$$

$$\xrightarrow{\substack{R_1 \to R_1 + 2R_3 \\ R_2 \to R_2 + 4R_3}} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -1 & -4 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right) \xrightarrow{R_1 \to R_1 + 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & -10 \\ 0 & 1 & 0 & 0 & -1 & -4 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right)$$

Thus, $[T]_\beta$ is invertible and its inverse $([T]_\beta)^{-1}$ is $\begin{pmatrix} -1 & -2 & -10 \\ 0 & -1 & -4 \\ 0 & 0 & -1 \end{pmatrix} = [T^{-1}]_\beta$.

Then we have $[T^{-1}(a_0 + a_1 x + a_2 x^2)]_\beta$

$$= \begin{pmatrix} -1 & -2 & -10 \\ 0 & -1 & -4 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -a_0 - 2a_1 - 10a_2 \\ -a_1 - 4a_2 \\ -a_2 \end{pmatrix},$$

Hence T^{-1} is given by:

$T^{-1}(a_0 + a_1 x + a_2 x^2) = -a_0 - 2a_1 - 10a_2 + (-a_1 - 4a_2)x + (-a_2)x^2$

6. For $E_1, \ldots, E_k$ elementary row operations, we have

For a general invertible matrix:

$A^{-1} = E_k \cdot \ldots \cdot E_1$ and $A = E_1^{-1} \cdot \ldots \cdot E_k^{-1}$

Thus we find the inverse of A first and record the elementary row operations as matrices to then find their inverses and create the above product ordering to represent the matrix as a product of elementary matrices.

Create augmented matrix as follows and reduce it to RREF to find its inverse.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\mathrm{I}]{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right) \xrightarrow[\mathrm{II}]{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\text{III}]{R_2 \to -R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right) \xrightarrow[\text{IV}]{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 1 \end{array}\right)$$

$$\xrightarrow[\text{V}]{R_1 \to R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{3}{2} & +\frac{1}{2} & -1 \\ 0 & 1 & 0 & +\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & +\frac{1}{2} & -\frac{1}{2} & 1 \end{array}\right) \xrightarrow[\text{VI}]{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{3}{2} & -1 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 1 \end{array}\right)$$

Each of the elementary row operations can then be expressed as elementary matrices and RREF $(A) = I$ as the products of these elementary matrices applied on A.

I as elementary matrix $= E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

II $= E_2 \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

III $= E_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

IV $= E_4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

V $E_5 = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

VI $E_6 = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Then it follows $E_6 E_5 E_4 E_3 E_2 E_1 A = I$ and since A is invertible, we have
$A = I E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} E_5^{-1} E_6^{-1}$, thus we expressed invertible matrix as product of elementary matrices.

7. Area of parallelogram determined by u and v is given by $\left|\det \begin{pmatrix} u \\ v \end{pmatrix}\right|$, thus:
for $u = (4, -1)$, $v = (-6, -2)$, we have

$$\left|\det \begin{pmatrix} 4 & -1 \\ -6 & -2 \end{pmatrix}\right| = |-8 - 6| = 14$$

8. Let $A \in M_{2\times2}(F)$ arbitrary, then

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad \text{and} \quad A^t = \begin{pmatrix} a_1 & a_3 \\ a_2 & a_4 \end{pmatrix}.$$

$\det(A) = a_1 a_4 - a_2 a_3 = \det(A^t)$, thus $\det(A^t) = \det(A)$

9. Since determinant is linear in its rows, we have left matrix obtained from right by multiplying each row by 3, thus

$$\det\begin{pmatrix} 3a_1 & 3a_2 & 3a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{pmatrix} = 3\det\begin{pmatrix} a_1 & a_2 & a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{pmatrix}$$

$$= 3\cdot 3\cdot \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{pmatrix}$$

$$= 27\cdot \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

Thus, $k = 27$.

10. Again, by linearity of A in its rows, we have left matrix obtained from right by multiplying first row by 2, then multiplying second row by 3, then adding 5 times the third row to 2nd, and finally multiplying 3rd row by 7.

As a shortcut, we can use the relation fact between determinant and row operations: ~~for row~~ let B be matrix obtained after elementary row operation and A initial matrix, then:

$\det B = -\det(A)$ for row swapping,

$\det B = c\det(A)$ for row multiplication

$\det B = \det(A)$ for row addition of k-multiple of another.

Thus, with respect to row operations described above, $K = 2\cdot 3\cdot 7 = 42$.

11. ~~Also~~ In accordance to det relation to elem. row operations described in (0), we have ~~gave~~ left matrix be obtained from right by the following:

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 + R_2} \begin{pmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \xrightarrow{R_1 \to R_1 + R_3} \begin{pmatrix} a_1+b_1+c_1 & a_2+b_2+c_2 & a_3+b_3+c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$\xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} a_1+b_1+c_1 & a_2+b_2+c_2 & a_3+b_3+c_3 \\ -a_1-c_1 & -a_2-c_2 & -a_3-c_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \xrightarrow{R_3 \to R_3 - R_1} \begin{pmatrix} a_1+b_1+c_1 & a_2+b_2+c_3 & a_3+b_3+c_3 \\ -a_1-c_1 & -a_2-c_2 & -a_3-c_3 \\ -a_1-b_1 & -a_2-b_2 & -a_3-b_3 \end{pmatrix}$$

$R_3 \to -R_3$
$R_2 \to -R_2$

$$\longrightarrow \begin{pmatrix} a_1+b_1+c_1 & a_2+b_2+c_2 & a_3+b_3+c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix}$$

$$\xrightarrow{R_1 \to 2R_1} \begin{pmatrix} 2(a_1+b_1+c_1) & \cdots & \cdots \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \begin{pmatrix} a_1+2b_1+c_1 & a_2+2b_2+c_2 & a_3+2b_3+c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - R_3} \begin{pmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix}$$

Thus, $\det\left(\quad \right) = 2\det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$

So $k = 2$.

12. det of a matrix with cofactor expansion along any row is given by:

$$\det(A) = \sum_{j=1}^{n} (-1)^{r+j} A_{rj} \det(\tilde{A}_{rj})$$, thus having

$r = 3$ as specified in the problem, we have

$$\det\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ -1 & 3 & 0 \end{pmatrix} = -1^{4} \cdot -1 \cdot \det(\tilde{A}_{31}) + (-1)^{5} \cdot 3 \cdot \det(\tilde{A}_{32}) + (-1)^{6} \cdot 0 \cdot \det(\tilde{A}_{32})$$ ← equates to 0

$\tilde{A}_{31} = \begin{pmatrix} 0 & 2 \\ 1 & 5 \end{pmatrix}$, then $\det(\tilde{A}_{31}) = (0 \cdot 5) - 2 \cdot 1 = -2$

$\hat{A}_{32} = \begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix}$, then $\det(\tilde{A}_{32}) = 1 \cdot 5 - 2 \cdot 0 = 5$,

Thus, $\det\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ -1 & 3 & 0 \end{pmatrix} = -1 \cdot -2 + (-1) \cdot 3 \cdot 5$
$= -13$

13. We will compute det of specified matrix ~~using~~ by converting it to REF and using the fact that, if A is the matrix, then

$$\det A = \frac{\det(REF(A))}{\mathcal{E}(R_1) \dots \mathcal{E}(R_k)}$$

where $\det(REF(A))$ is given by product of its diagonal entries since REF is an upper triangular matrix and $\mathcal{E}(R_1) \dots \mathcal{E}(R_k)$ is the product of row operation relation coefficients where

$\mathcal{E}(R) = -1$ for row swap
c for row mult. by scalar c.
$+1$ for row add ~~k~~ k times other row

Thus, we first find REF of the matrix

$$\begin{pmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \to R_2 + 3R_1} \begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -5 & 11 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix} \xrightarrow{R_4 \to R_4 - 2R_1} \begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -5 & 11 \\ 0 & 4 & -1 & 1 \\ 0 & 3 & 4 & -5 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -5 & 11 \\ 0 & 0 & 19 & -43 \\ 0 & 3 & 4 & -5 \end{pmatrix} \xrightarrow{R_4 \to R_4 - 3R_2} \begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -5 & 11 \\ 0 & 0 & 19 & -43 \\ 0 & 0 & 19 & -38 \end{pmatrix}$$

$$\xrightarrow{R_4 \to R_4 - R_3} \begin{pmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -5 & 11 \\ 0 & 0 & 19 & -43 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

So we have REF, but since we didn't use row swapping or row multiplication by a scalar, we have det equal to simply the product of diagonal entries $\Rightarrow 1 \cdot 1 \cdot 19 \cdot 5 = 95$.