Hierarchical clustering-Max linkage:

Starting matrix:

- matrix of relative distances among the points

	0	1	2	3	4
0	**0**	2.7	0.99	2.29	0.51
1	-	**0**	3.1	0.47	3.21
2	-	-	**0**	2.63	1.08
3	-	-	-	**0**	2.08
4	-	-	-	-	**0**

First round:

1.1 choose the lowest value between two points **0.47** and merge them into **a new cluster 5** (1+3)

	0	1	2	3	4
0	**0**	2.7	0.99	2.29	0.51
1	-	**0**	3.1	**0.47**	3.21
2	-	-	**0**	2.63	1.08
3	-	-	-	**0**	2.08
4	-	-	-	-	**0**

1.2 choose max values between new cluster and remaining points

	0	2	4	5(1+3)
0	**0**	0.99	0.51	**2.7**
2	-	**0**	1.08	**3.1**
4	-	-	**0**	**3.21**
5(1+3)	-	-	-	**0**

$$d(C,X) = \max\left(d(A,X), d(B,X)\right)$$

C=5, A=1, B=3:

X=0:

$$d(5,0) = \max(d(1,0), d(3,0)) = \max(2.7, 2.29) = \mathbf{2.7}$$

X=2:

$$d(5,2) = \max(d(1,2), d(3,2)) = \max(3.1, 2.63) = \mathbf{3.1}$$

X=4:

$$d(5,4) = \max(d(1,4), d(3,4)) = \max(3.21, 2.08) = \mathbf{3.21}$$

Second round:

2.1. choose the lowest value between two points **0.51** and merge them into **a new cluster 6 (0+4)**

	0	2	4	5(1+3)
0	0	0.99	**0.51**	2.7
2	-	0	1.08	3.1
4	-	-	0	3.21
5(1+3)				0

2.2. choose max values between new cluster and remaining points

	2	5(1+3)	6(0+4)
2	-	3.1	**1.08**
5(1+3)	-	-	**3.21**
6(0+4)	-	-	-

$$d(C,X) = \max\left(d(A,X), d(B,X)\right)$$

C=6, A=0, B=4:

X=2:

$$d(6,2) = \max(d(0,2), d(4,2)) = \max(0.99, 1.08) = \mathbf{1.08}$$

X=5:

$$d(6,5) = \max(d(0,5), d(4,5)) = \max(2.7, 3.21) = \mathbf{3.21}$$

Third round:

3.1. choose the lowest value between two points **1.08** and merge them into **a new cluster 7**

- new cluster 7 has 3 clusters in total 6(0 + 4) + 2

	2	5(1+3)	6(0+4)
2	-	3.1	**1.08**
5(1+3)	-	-	3.21
6(0+4)	-	-	-

……..

Final table with steps:

Merge Step	Cluster 1 (Index)	Cluster 2 (Index)	Distance	Cluster Size	New Cluster Index
1	1	3	0.47725916	2	5
2	0	4	0.51783804	2	6
3	2	6	1.08380521	3	7
4	5	7	3.21646751	5	8