Soft Thresholding Function:
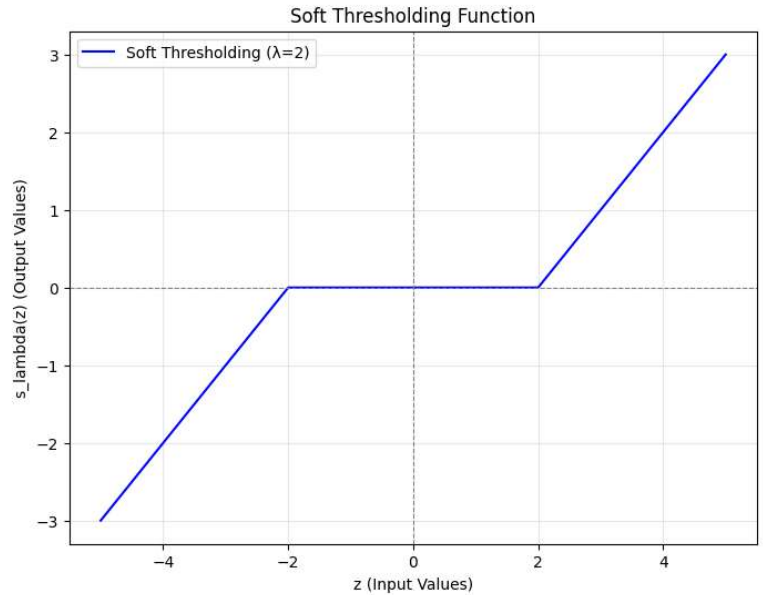
Goal:

- examine behavior of soft thresholding function for different values

Input data:

$$\lambda = 2$$

$$z = [-3, -1, 0, 1, 3] \qquad (1)$$

Soft Thresholding Function

Calculations:

$$S_\lambda(z) = sign(z).max(|z| - \lambda, 0)$$

for z=-3:

$$S_\lambda(-3) = sign(-3).max(|-3| - 2, 0) = -1.max(3 - 2, 0) = -1.max(1, 0) = \mathbf{-1}$$

for z=-1:

$$S_\lambda(-1) = sign(-1).max(|-1| - 2, 0) = -1.max(1 - 2, 0) = -1.max(-1, 0) = \mathbf{0}$$

$$(2)$$

for z=0:

$$S_\lambda(0) = sign(0).max(|0| - 2, 0) = 0.max(0 - 2, 0) = 1.max(-2, 0) = \mathbf{0}$$

for z=1:

$$S_\lambda(1) = sign(1).max(|1| - 2, 0) = 1.max(1 - 2, 0) = 1.max(-1, 0) = \mathbf{0}$$

for z=3:

$$S_\lambda(3) = sign(3).max(|3| - 2, 0) = 1.max(3 - 2, 0) = 1.max(1, 0) = \mathbf{1}$$

Results:

- from the results we can see that soft thresholding function shrinks the values lower than threshold lambda to zero

$$z < -\lambda \ \rightarrow \ S_\lambda(z) = z + \lambda$$
$$|z| \leq -\lambda \ \rightarrow \ S_\lambda(z) = 0 \qquad (3)$$
$$z > \lambda \ \rightarrow \ S_\lambda(z) = z - \lambda$$

Soft-Thresholding Function $S_\lambda(z)$ with $\lambda = 2$