Module 10: Liability-Driven & Index-based Strat.

More commonly-used terms:

Asset-Liability Management (ALM)

- consider assets in relation to liab.
- appropriate when asset & liabilities change with int. rate. change

Liability-driven Investing (LDI)

- manages the assets to meet the liabilities

  e.g. defined benefit pension plan

Asset-Driven Investing (ADI)

- takes assets as a given & matches liabilities in relation

  e.g. leasing company

4 types of liabilities:

1. Type I: KNOWN future amts, KNOWN payout dates
   ↖ suitable for macaulay & modified duration
2. Type II: KNOWN future amts, UNKNOWN payout dates
3. Type III: UNKNOWN future amts, KNOWN payout dates
4. Type IV: UNKNOWN future amts, UNKNOWN payout dates

Type I, III, IV require effective duration. (modeled to reflect initial shape, assumed upward & downward sh.ft. in YC to estimated amt & timing of payouts)

Immunizing Single Liability

Immunization: process to minimize the variability of rate of return earned over a specific time period. (i.e. FV of a fund can be confidently predicted)

CF Matching

- simplest but least flexible
- to immunize a single liab, BUY (DEFAULT-FREE) 0-cpn bond.

Macaulay Duration (0-cpn bond) = Time to payment

↑ weighted average time until CFs of an inst. are received

Macaulay Duration = bal. point b/w price & reinvestment risk.

price risk: uncertainty of proceeds. if SOLD B4 MATURITY

reinvestment risk: uncertainty FV of CF received & reinvested before end of holding period

unreasonable assumption ↓

if rates chg ONCE, BY A LIL BIT, IN PARALLEL, &,
goes up ⇒ higher reinvestments offset lower proceeds.
goes down ⇒ lower reinvestments offset higher proceeds.

- for some immunization techniques, macaulay duration is the more appropriate measure of time.

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{(1 + \text{YTM})}$$

- more accurate measure of immediate price chg. of the instrument.

Portfolio statistic should be used for ALM (over weighted avg. calc. on each bond):

YC.	Diff b/w ALM & Weighted Avg.
Flat.	No diff
Upward-sloping	Port. duration & IRR will be higher-than-avg. duration & YTM of bonds

(downward-sloping are unusal & would need spreadsheets to model it)

Goal of Immunized Portfolio = EARN THE INITIAL IRR (NOT YTM!)

Dispersion causes convexity:

$$\text{Convexity} = \frac{\text{Mocaulay Dur.}^2 + \text{Mocaulay Dur.} + \text{dispersion}}{(1 + \text{periodic IRR})^2}$$

Dispersion & immunization: convexity indicates risk exposure of the strategy to shifts & twists in the YC.

Effects of YC Changes

Given a portfolio used to immunize a single liab in 5 year:

Portfolio & Liability D matched at 6.0 medium duration (M)
Initial value of portfolio = PV of liab. (@IRR)
Convexity (Portfolio) > Convexity (liability)
Portfolio assets = bond w/ shorter duration (S)
+ bond w/ longer duration (L)

if YC shifts up/down in parallel,
return (portfolio) > amt required to pay the future liab.
(b/c of positive convexity)

if large parallel increase in curve:
PVA > PVL (b/c of positive convexity)
portfolio IRR will increase by the same amount as the increase in discount rate for PVL.
Future rate of increase in A & L are the same

if large parallel decrease in curve:
PVA > PVL (b/c of positive convexity)
portfolio IRR will decrease by the same amount as the increase in discount rate for PVL.
Future rate of increase in A & L are the same

Parallel Shift Analysis indicate STRUCTURAL RISK. (due to port. duration formed by S & L vs. liab. duration of M)

Parallel Shift Analysis indicates:

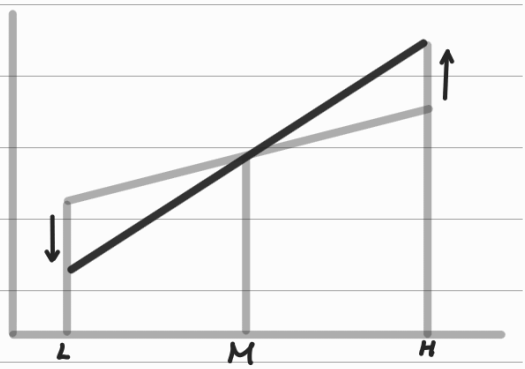
- immunization can be described as zero-replication (chg in portfolio value & IRR will replicate or outperform chg in yield & value of 0-cpn bond)

- the strategy IS NOT ALWAYS RISK FREE. (hurt by other types of shifts in YC)

- the strategy is SUFFICIENT BUT NOT NECESSARY to lead to success.

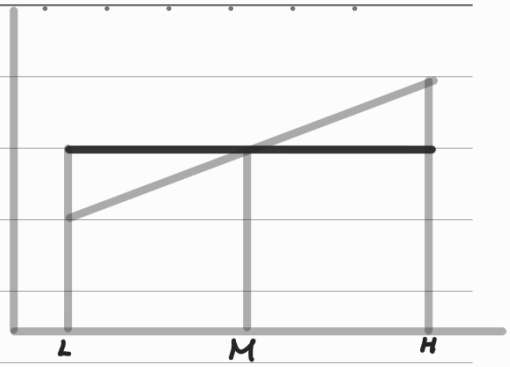
- the strategy isn't always BUY-AND-HOLD b/c cpn-bearing cpn duration declines more slowly than maturity & would need rebalancing

If YC steepens, Yield L decreases, H increases relative to M.



Portfolio MV will decrease b/c.
loss in value (H) > gain in value (L)
No change on PVL.
Can still required FV if portfolio IRR increases sufficiently

If YC flattens, Yield H decreases, L increases relative to M.

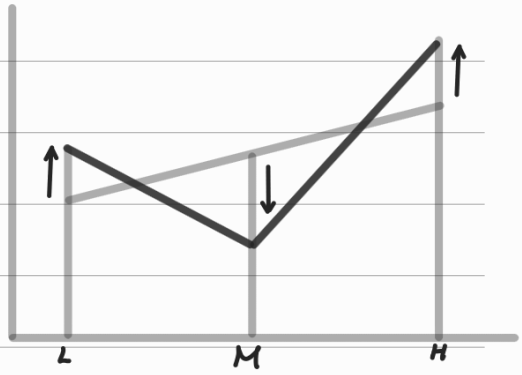


Portfolio MV will increase b/c.
loss in value (L) < gain in value (H)
No change on PVL.
Can still fail to meet required FV if portfolio IRR drops sufficiently.

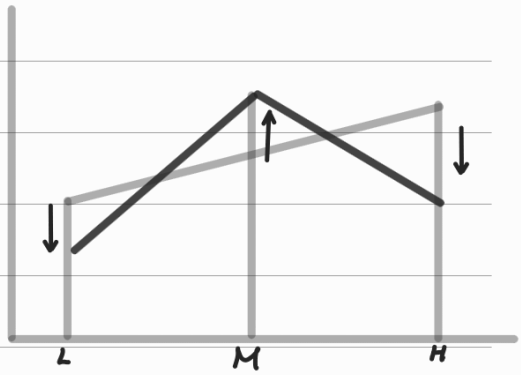
Both flattening & steepening may create structural risk.

Butterfly Twists.

Positive Twist: Yield (L) & Yield (H) ↑,
Yield (M) ↓
Value (Port) ↓, PVA < PVL



Negative Twist: Yield (L) & Yield (H) ↓
Yield (M) ↑
Value (Port) ↑, PVA > PVL



Risk of Immunization is higher when chg in port. IRR doesn't match chg in yield of the replicating zero.

Rules of immunizing single liability:

1. PVA ≥ PVL @ $t = 0$.
2. Macaulay Duration (Port) = Macaulay Duration (Liab.)
3. Minimize convexity
4. Regularly rebalance

1 & 2 can be satisfied by matching the money duration of assets & liab:

$$BPV = MD \times V \times 0.0001$$

↑ modified duration

10.c,d,e: Immunizing Multiple Liabilities

CF Matching

Laddered Portfolios

- a portfolio w/ roughly par amounts purchased a/c diff maturities
- acheive the target duration via a bullet portfolio (concentreting in the middle) or a barbell portfolio

Advantages of a laddered portfolio:

1. Regular liquidity / low bid-ask spread to generate CF.
2. Diversification a/c time & YC so lower concentrated exposure to non-parallel shifts
3. Diversification b/w price risk & reinvestment risk. so. overall duration will be more stable over time.
4. More convexity than a bullet portfolio, a benefit if there's large parallel shifts. (less convexity than a barbell portfolio)

- alternative to laddered bond portfolios: laddered portfolio of target-date (fixed-maturity) bond ETFs.
- disadvantages (comp. to ongoing / no target date ETF): less diversatification, larger, more liquid

Duration Matching

Money duration is more useful when initial amounts & discount rates of assets & liabilities differ.

1. Initial PVA = Initial PVL
2. $BPV_A = BPV_L$.
3. Asset dispersion & convexity > liabilities (by not too much otherwise exposed to reshaping risk)
4. Regularly rebalance

EXAMPLE: U.K. bond company

A U.K.-based company has several option-free bond issues (liabilities) outstanding. The company would like to retire the bonds early, and has more than sufficient funds to do so. The company considers a bond tender offer (offer to repurchase the bonds from the public), but the bonds are widely distributed among buy-and-hold investors. The prices that would have to be paid are too high to make the tender desirable to the company.

The company could also establish a dedicated cash flow matched portfolio of U.K. government bonds and legally defease the bonds. In that case, both the company's bond liabilities and assets (the cash flow matching bond portfolio) could be removed from the company's balance sheet. The cost of the portfolio would be GBP475 million, and the company considers this too high.

The third alternative is to establish a duration matching portfolio; using high-quality corporate bonds, the cost will be less than for the government bond portfolio. While the portfolio will not qualify for defeasance, the company believes it will improve its credit rating and is the better choice.

The portfolio statistics for the company's liabilities and three proposed corporate bond portfolios are shown in the following table. All calculations are annualized and based on aggregate portfolio cash flows. Each portfolio is considered sufficient to pay the liabilities. Monetary amounts are in GBP:

Statistics	Company's Liabilities	Proposed Portfolios		
		A	B	C
Market value	457,780,900	Approximately 460,000,000		
Modified D	7.52	7.51	7.53	7.37
BPV	344,250	343,100	345,400	339,120
Convexity	45.12	35.14	46.29	65.97

1. **Select** the most appropriate portfolio (A, B, or C) to immunize the liabilities and **justify** your selection with *two* reasons.
2. If the company expects high volatility and the potential for very large parallel shifts in the yield curve, **select** the *one* other portfolio (A, B, or C) it would most likely consider and **explain** why.

1. B b/c A doesn't have more convexity than liab.
   C doesn't have more BPV
2. C b/c of the much higher convexity

Module 10.2: Managing Duration Gap.

Derivatives overlay used to adjust the portfolio & maintain duration matching w/o the expenses (b/c. fowards are used)

$$\text{futures BPV} \approx \frac{BPV_{CTD}}{CF_{CTD.}}$$

$$N_f = \frac{\text{BPV of liab.} - \text{BPV of portfolio}}{\text{BPV of future}}$$

EXAMPLE: U.K. bond company revised

Suppose the U.K.-based company seeking to immunize its bond liabilities had instead chosen to immunize only a portion of its debt. The company selected an asset portfolio with a BPV of 217,525 to immunize a portion of the liabilities with a BPV of 217,512—a duration gap of only 13. After a modest increase in rates with a significant positive butterfly twist, the asset and liability BPVs are now 203,456 and 218,517 for a duration gap of 15,061.

Assume that there are government bond–based futures contracts based on 5- and 10-year notes, with features similar to U.S. Treasury–based futures contracts. Each contract specifies a range of deliverable government notes that the contract seller may select for delivery at contract expiration. Each deliverable note requires the seller to deliver 100,000 par and the buyer to pay initial contract price multiplied by the conversion factor for that bond (CF_{CTD}).

Figure 10.7: Characteristics of the CTD Note for the 5- and 10-Year Contracts; Per 100,000 Par

CTD Characteristics	5-Year Contract	10-Year Contract
YTM	1.71%	2.51%
Modified D	4.75	8.67
BPV of CTD	48.1650	86.7001
Conversion factor	0.9237	0.9169

1. **Determine** and **justify** if contracts will be bought or sold, assuming:
   i. the 5-year contract is used.
   ii. the 10-year contract is used.
2. **Calculate** contracts to use, assuming:
   i. the 5-year contract is used.
   ii. the 10-year contract is used.
3. If the liability duration is 8.99, **state** which contract is most likely to minimize structural risk based only on the information provided. You must choose either the 5 or 10 year, and not a combination of both. **Justify** your answer.

1. To increase asset BPV to match liab. BPV, ** buy contract regardless of contracts used

2. i. 15061 ÷ 48.165 = X

Contract BPV = 48.1650 ÷ 0.9237 = 52.1436.

No. of contracts = 15061 ÷ 52.1436 ≈ 289. contracts

ii. Contract BPV = 86.7001 ÷ 0.9169 = 94.5579

No. of contracts = 15061 ÷ 94.5579 ≈ 159. contracts

3. 10-year is a better duration match

In practice PVL = PVA is not practical. usually it's more common to overfund the assets.

Contingent immunization (CI):

- hybrid active/passive strategy & requires a surplus.
- as long as surplus is of a sufficient size, the portfolio can be actively managed

CI approaches are vulnerable to liq. risk:
- if all assets are actively managed & surplus declines, the assets must be quickly liquidated & converted to immunizing portfolio. bf surplus is negative.
- Even if only the surplus is actively managed, if short options are used, the downside potential is unlimited. & can exceed portfolio surplus.

MODULE QUIZ 10.1, 10.2

1. A bond issuer is reviewing the four main categories of liabilities. He is interested in issuing a liability that can be managed easily and has known future amounts and payout dates. Which of the following types of liabilities *most likely* meets the issuer's demands?
   A. Callable bonds.
   B. Option-free fixed-rate bonds.
   C. Treasury inflation-protected securities.
2. An investor is looking to immunize a single liability, but is concerned with the impact from yield curve shifts and twists. When attempting to immunize this liability, which of the following rules should the investor apply?
   A. The portfolio Macaulay duration should match the due date of the liability.
   B. The dispersion of asset cash flows around the liability should be maximized.
   C. The present value of liabilities should exceed the initial portfolio market value.
3. Which of the following statements correctly describes contingent immunization?
   A. It uses Treasury futures contracts to adjust the portfolio and maintain duration matching.
   B. It constructs a portfolio of zero-coupon bonds that provide enough cash inflows to meet liabilities.
   C. It uses active bond portfolio management, as long as the present value of assets exceeds the present value of liabilities.
4. A client at RBI Funds would like to build a laddered bond portfolio. In terms of the construction and advantages of a laddered portfolio, which of the following statements is *most correct*?
   A. With a laddered portfolio, the investor is diversified between price and reinvestment risk.
   B. A laddered portfolio has more reinvestment risk in any single year compared to a barbell portfolio.
   C. The more distributed cash flows of a ladder portfolio compared to a bullet portfolio will provide less convexity.

1. B ✓
2. B (A) C should be PVA > PVL.
   A should be <u>minimize</u> port convexity / dispersion of asset CFs around the liability.
3. C ✓
4. C (A) B is wrong b/c laddered port has <u>less</u> reinvestment risk in single yr vs. the barbell / bullet. More distributed CF of laddered portfolio ⇒ <u>GREATER CONVEXITY</u>

Module 10.3: Advanced Strategies.

2 Types of liability projections for pension plans:

1) **Accumulated Benefit Obligation (ABO):**
generally lower, reps the legal obligation if the plan were now

2) **Projected Benefit Obligation (PBO):**
more realistic, reps. the ongoing payment.

e.g. PBO = 2.57 Bln, Eff. dur = 9.35.
Plan assets = 3.07 Bln, 60% Asset, 40% Liab.
duration of bonds = 2.85:
Futures of 100,000 PAR, CTD bond BPV = 128.98,
duration of 13.53 & conv. factor of 0.9436

BPV_L = 2.57 Bln × 9.35 × 0.0001 = 2.402 MM
BPV_A = 3.07 Bln × 40% × 2.85 × 0.0001 = 0.349 MM

Duration Gap = $BPV_A - BPV_L$ = 2,052,970.
BPV_{Future} = 128.98 ÷ 0.9436 = 136.6893

N_f (100% hedge) = 2,052,970 ÷ 136.6893
= 15,019 contracts.
N_f (110% hedge) = 15,019 × 110% = 16,521 to buy

Adjust duration gap w/ swaps:

e.g. duration gap = 2,052,970 (from last example)
duration (fixed leg) = 9.18.
duration (floating leg) = 0.25

swap BPV = fixed side BPV − floating BPV

= (100 · 9.18 · 0.0001) − (100 · 0.25 · 0.0001)
= 0.0893

Notional Swap Principal (NP) required to close the duration gap w/ a 100% hedge:

$$NP = \frac{2,052,970}{(0.0893/100)} = 2.3 \text{ Billion}$$

↑ BPVs are per 100 NP so must divide BPV by 100

Use Receive-Fixed to increase portfolio duration b/c. when rate declines, the value of fixed payment will go up w/ the liab.

Other alternatives: swaption (i.e right to enter a receive-fixed swap @ a prespecified swap fixed rate (SFR))

compare $SFR_{t=0}$ & $SFR_{t=n}$ to determine if the swaption has value:

$SFR_{t=n} < SFR_{t=0}$ ⇒ swaption has value

maybe think of a call??

Payer Swaption ⟺ LONG pay-fixed

Receiver Swaption ⟺ LONG receive-fixed

EXAMPLE: Swap vs. swaption hedges

A U.S. pension plan has a 450,000 BPV duration gap with BPV of assets less than of liabilities. The plan uses a swap with a BPV per 100 notional of 0.2571 to construct a 50% hedge ratio. After setting up the 50% hedge, the manager forms the opinion that rates will increase, and would like to benefit if his view is correct, but be unaffected if he is wrong. The manager would be willing to adjust the hedge position by 15% to a 35% or 65% hedge. He checks and finds that both payer and receiver swaptions are available with a strike rate of 2.7%. The premiums for the payer and receiver swaptions are 55 and 75 basis points, respectively.

1. **State** the terms and **calculate** the notional principal of the 50% hedge ratio swap the manager would use.
2. **State** the terms and **calculate** the initial cost of the swaption the manager would buy or sell to adjust his hedge to a 35% hedge.
3. **Determine** the rate on new swaps and **state** whether new rates will have to be higher or lower than that rate to make exercising the swaption profitable.

1. $450{,}000 \times 50\% \div (0.2571 \div 100)$
   $= 87.515$ MM
2. sell received-fixed swaption to adjust to 35% hedge ratio:

   15% of full hedge $= 450{,}000 \times 0.15 \div (0.2571 \div 100)$

   $= 26.254$ MM

   aka pay-fixed

   $\Rightarrow 26.254 \times 0.55\% = 144{,}397$

3. if new SFR rate is greater than 2.75%, payer swaption will have positive value.

One more alternatives: swaption collar
(i.e. buying receiver swaption, selling payer swaption)

Choosing the Optimal Strategy

So far the 3 strategies covered:

1. Enter receive-fixed swap
2. Buy a receiver swaption
3. Build a zero-cost collar
   (i.e. buying receiver swaption, selling payer swaption)

All 3 strategies work but it requires some ability to predict the direction/magnitude of int rate changes.

e.g.

	Cost
2.5 fixed-rate swap	0
2.3% receiver swaption	75 bp.
3.3% payer swaption	75 bp.

if the manager expects the rates to be:

SFR < 2.5% ⇒ swap.

2.3 < SFR < 3.3% ⇒ collar

SFR > 3.3% ⇒ receiver swaption

Swap (if SFR < 2.5%):

- same as receive 2.5%, paying floating SFR/MRR, will benefit from the rate decline.
- swaption is less optimal b/c of the cost of the premium.

Collar (if $2.3\% < r < 3.3\%$):
- has no intrinsic value which is the best choice
- both the short call / pay-fixed & long put / receive-fixed has 0 value

Swaption (if $r > 3.3\%$):
- loses initial premium (a sunk cost anyway) but no intrinsic value
- swap will bound you to pay floating ($> 3.3\%$)

Risks

- Hedge is based on assumed duration but ignore convexity which matters for large-rate movements
- Duration assumes parallel shift.
- Twists in yc can create structural risk.
- Model risk can be significant.
- Futures BPV is based on CTD bond which can change & affect BPV & duration.
an more accurate estimate of futures BPV should include accrued interest as well.

- Portfolio Yield might not be the same as. Liability discount rate. which Creates spread risk:

- OTC derivatives have counterparty risk.

- Asset Liquidity risk may occur if positions can't be quickly liquidated

MODULE QUIZ 10.3, 10.4

1. Assume that the BPV duration gap of a defined benefit pension plan is equal to $300,000. The pension fund manager would like to hedge 100% of this duration gap with a 10-year swap. The manager finds that the duration of the fixed side of the swap is 8.25 and the duration of the floating side of the swap is 0.5. What is the notional swap principal required to fully close the duration gap?
   A. $125 million.
   B. $387 million.
   C. $495 million.

2. A risk analyst is discussing the risks associated with managing a bond portfolio against a liability structure. She makes three statements regarding liability-driven investment risks. Which of her statements is *most likely* an indicator of spread risk?
   A. "Bond positions cannot be adjusted with reasonable transaction costs in a timely manner."
   B. "Hedge calculations are approximated based on only the duration of the assets and liabilities."
   C. "The liability discount rate may reflect corporate debt rates, and the asset discount rate may reflect government bond yields."

1. $300,000 ÷ 1bps ÷ (8.25 - 0.5) = 387 MM (B) ✓
2. C ✓

Module 10.5: Index-based Investing

Indexing for bonds is more difficult than for equity

Matching risk characteristics of bond index is more practical than full replication. Primary risk factors include:

- Modified Duration / Effective Duration (for bond w/ embedded options): $\%\Delta \text{ value} = -MD\,\Delta y$

- Key rate duration: $\%\Delta \text{ value} = -MD_{\text{key rate } n}\,\Delta Y_n$

- Weighting exposure to various bond sector & quality ratings of the index. For non-gov securities, it is important to distinguish price change due to general chg in rates from spread change

MD = Mccaulay Duration / (1 + YTM)

%Δ value = − MD Δy
% Δ relative value = $-D_s \Delta s$.
Spread = $Y_{\text{higher yield}} - Y_{\text{government}}$

- matching sector/coupon/maturity cell weights of the index.

- matching issuer exposure weights to control for specific event risk

Minimize YC Risk: PV distribution of CF

e.g.

t			w=PV%	(t)(w)
0.5	2	1.9608	0.0196	0.0098
1	2	1.9223	0.0192	0.0192
1.5	2	1.8846	0.0188	0.0283
2.	2	1.8477	0.0185	0.0370
2.5	2	1.8115	0.0181	0.0453
3	102	90.5731	0.9057	2.7172.
		100	1.000	2.8567

Alternative Methods of Obtaining Passive Bond Market Exposure

Stratified sampling (cell matching) can be used to implement matching index

Alternative Methods for establishing bond market exposure passively

Bond index mutual funds can be well suited for smaller investors.

Open-ended funds can be redeemed & purchased @ NAV once a day.

ETFs provide some advantages where shares trade continuously on exchanges

Total Return Swaps (TRS)

- receives bond market index total return, pay MRR + spread
- equals buying the index & borrowing funds needed for purchase @ MRR + spread.
- negative return
- if not fully collateralized, TRS = leverage investment in index.

Bond BM Selection

Smart Beta: identifying relatively simple, defineable rules that can be followed to add value

Credit Barbell:
e.g. 50% in 5-10 Yr Treasuries, 50% 1-3 yr investment-grade corporates as BM

MODULE QUIZ 10.5

1. Using a full replication approach for bond indexing may be impractical, given the large size of the bond market and the varying characteristics of individual bonds. Instead, it may be easier to match the primary risk characteristics of the selected bond index. When reviewing specific risk characteristics, the risk factor that minimizes tracking error due to nonparallel shifts in the yield curve is associated with matching:
   A. modified duration.
   B. key rate durations.
   C. issuer exposure weights.
2. A bond portfolio manager is looking to gain passive exposure to the bond market. Which of the following approaches would allow the manager to receive the return from a desired bond market index in exchange for paying MRR plus a spread?
   A. Total return swap.
   B. Exchange-traded funds (ETFs).
   C. Exchange-traded derivatives.
3. Which of the following statements regarding fixed-income benchmarks is *most likely* false?
   A. If a static bond index is used as a benchmark, the duration will remain the same as the bonds age.
   B. New bond issuance may cause the characteristics of the selected benchmark to change over time.
   C. Issuers may shift to shorter or longer security issuances, making a given benchmark no longer appropriate for a given investor.

1. B ✓ Matching modified duration = parallel shift.
2. C (A)
3. B (A) A is wrong b/c duration will <u>decline</u>

/ /