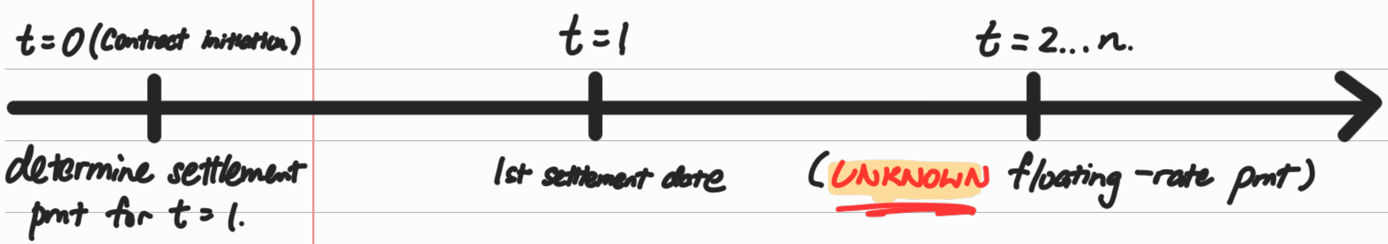
Module 7 Swaps, Forwards & Futures Strategies

7.1. Interest Rate Swap (IRS)

Payer Swap : pays fixed-rate,
receives floating-rate.

Receiver Swap : receives fixed-rate,
pays floating-rate.



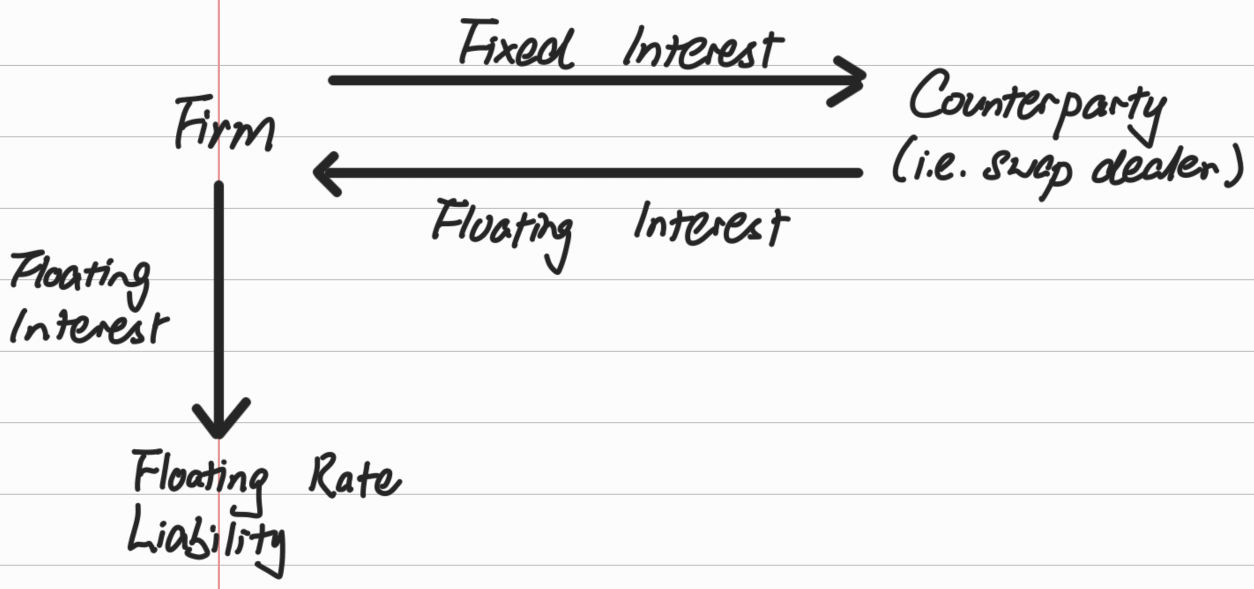
Floating-Rate Exposure ⇒ Fixed-Rate Exposure

(using payer swap) NOT Receiver SWAP!!

has a liability on floating-rate.

Textbook:

> A company with a floating-rate exposure can use a payer swap (pay fixed, receive floating) to change it into a fixed-rate exposure. The swap must have settlement dates that match the payment dates on the floating-rate liability.



EXAMPLE: Converting a floating-rate liability to a fixed-rate liability

ABC, Inc., has issued $30 million four-year, semiannual-pay, floating-rate notes (FRNs) with coupons equal to the six-month market reference rate (MRR) plus a 25 basis point margin. After one year, the firm expects interest rates to rise. ABC enters into a three-year payer swap with a fixed rate of 2%, semiannual payments, and a notional principal of $30 million. At initiation of the swap, the six-month reference rate was 1.5%. The FRN and the swap have the same settlement dates.

@ $t=1$, 1st settlement date,

Swap: Fixed rate $= 30MM \cdot 2\% / 2$

$= 0.3 MM$

Floating rate $= 30MM \cdot (1.5\%)/2$

$= 0.225MM$

Net Swap P&L $= -0.3 + 0.225 = -0.075 MM$

FRN: $30MM \times (1.5\% + 0.25\%)/2$

$= 262,500$

Total P/L $= -262,500 - 75,000$

$= -337,500$

@ 2nd settlement date, (given 6-m ref rate at t=1. was 3%)

Swap: Fixed $= 0.3MM$

Floating $= 30MM \times (3\%)/2$

$= 0.45 MM$

Swap P&L $= 0.45MM - 0.3MM$

$= 0.15MM$

FRN: $30 MM \times (3\% + 0.25\%)/2$

$= 487,500$

Total PMT $= -487,500 + 150,000$

$= 337,500$

Using IR Swap to alter portfolio DURATION

Duration (Fixed-rate notes) > Duration (FRN)
=> Duration (Payer swap) < 0

(i.e. increase in value when ir increases)

Adding a payer swap to a FI Port. will increase portfolio duration

$$NP_s = \left(\frac{MD_T - MD_P}{MD_S}\right)(MV_P)$$

NP_s = Notional Swap Principal
MD_T = target mod. duration
MD_P = current port mod. duration
MD_S = swap mod. duration
MV_P = MV of portfolio

EXAMPLE: Using an interest rate swap to alter portfolio duration

A fund manager has a portfolio of £120 million fixed-rate U.K. bonds. The fund manager believes that interest rates will fall over the next three years and wishes to increase the portfolio's modified duration (exposure to changes in interest rates). She decides to increase portfolio duration from 4.5 to 6. A payer swap has a modified duration of –2 and a receiver swap has a modified duration of +2.

What type of swap will achieve the desired portfolio duration, and what is the required notional principal of the swap?

Receiver swap.

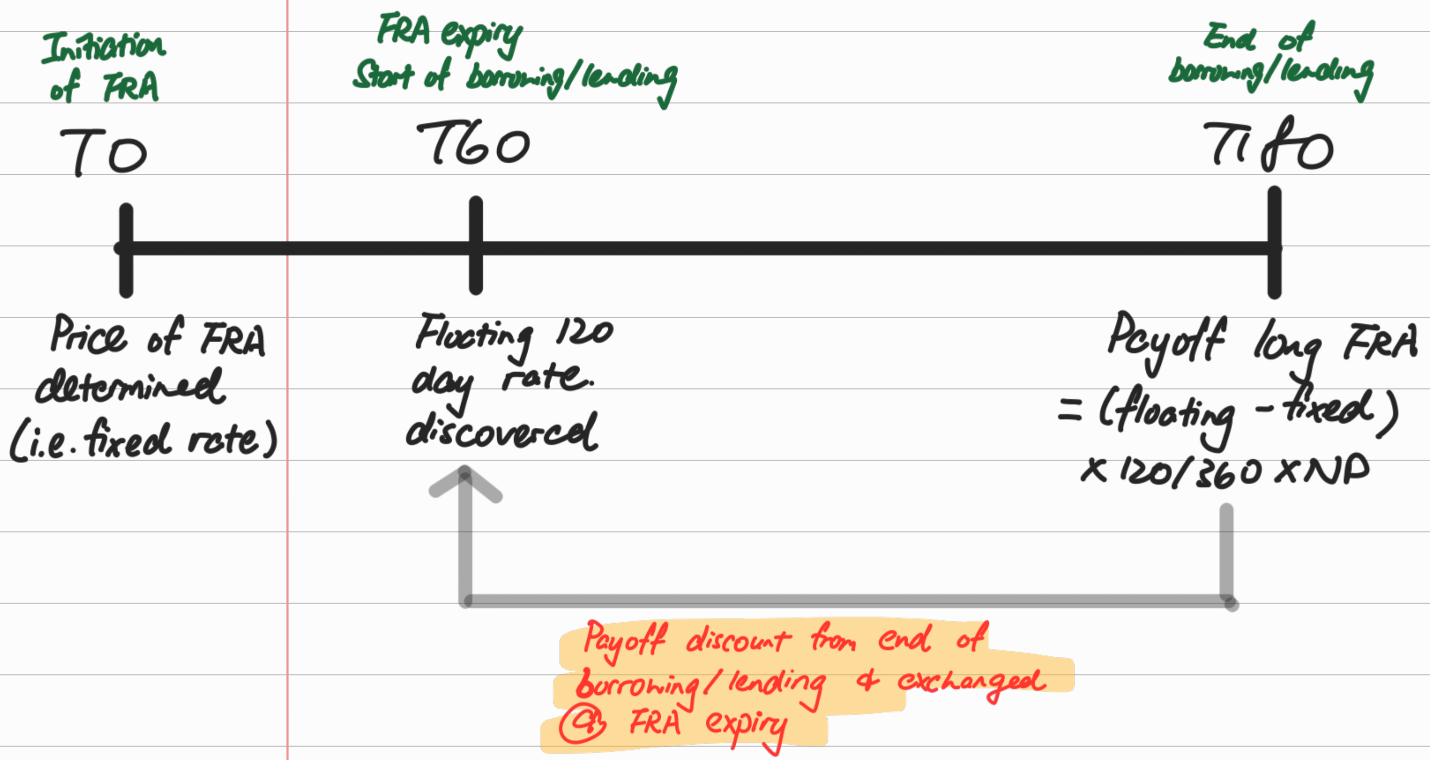
$NP = (6 - 4.5) / 2 \times 120MM$
$= 90MM$

Interest Rate Forward & Futures

Forward Rate Agreements (FRA)

- usually used to hedge uncertainty about future short-term borrowing / lending rate.
- Long FRA : receive payment if market rate > forward rate. (essentially long FRA <=> payer swap, profits if floating rate goes up)
- Notional Amount never exchanges hand

e.g. 2×6 FRA



needs to borrow @ floating rate. => enter Long FRA

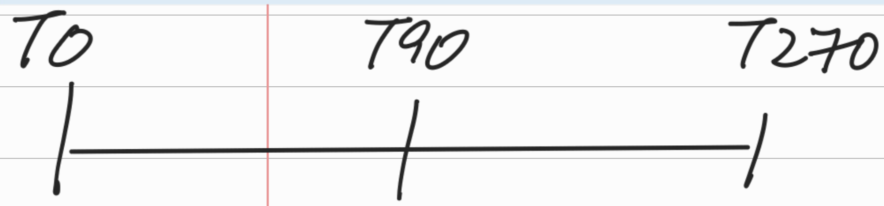
↑ receive.

EXAMPLE: Using an FRA to hedge short-term future borrowing

Smithies Plc needs to borrow £1,000,000 for 180 days, 90 days from now, at the market reference rate (MRR) + 50 basis points (bp). The company is concerned that interest rates will rise over the 90-day period, increasing the cost of the 180-day loan. Smithies takes a long position in an FRA on the six-month reference rate, 90 days in the future with a forward rate of 5% (annualized) and a notional principal of £1,000,000.

Calculate the firm's borrowing costs on the loan net of the FRA payment for realized six-month reference rate values, 90 days from now of 7% and 3%.

180d



7% : FV of FRA $= (7\% - 5\%) \times \frac{180}{360} \times NP$

$= 1\% \times 1MM$

$= 10{,}000$

Cost of loan $= (7.50\%) \times \frac{180}{360} \times NP$

$= -37{,}500$

Net Borrowing Cost $= -37{,}500 + 10{,}000$

$= -27{,}500$

3% : FV of FRA : $(3\% - 5\%) \times \frac{180}{360} \times 1MM$

$= -10{,}000$

Cost of loan $= 3.50\% \times \frac{180}{360} \times 1MM$

$= -17{,}500$

Net $= -27{,}500$

Short-Term Interest Rate (STIR) Futures

- Conceptually very similar to FRA
- future prices = forward interest rate on deposit starting @ expiry of the future & lasting for 90 days.
- Eurodollar futures ($-based STIR futures) are based on deposits of $1 MM & priced using IMM index convention (i.e. 100 − annualized fwd rate)
  (i.e. future price ↑ when forward rates ↓)
- Forward Interest Rate (from current spot MRR) ⟺ Forward Price of FRA
- Long Eurodollar future value ↑ ⟺ fwd. rate ↓

EXAMPLE: Using Eurodollar futures to hedge short-term future investing

Allan Luard is expecting to receive a $20 million inheritance in 120 days. Allan intends to invest the $20 million in a 90-day deposit account at the market reference rate (MRR) –25 bp. Currently, Eurodollar futures expiring in 120 days are trading at 95. Allan is concerned that short-term rates may fall before he makes his deposit and would like to lock in a guaranteed interest rate today. He takes a long position in 20 Eurodollar futures contracts.

A. After 120 days, the three-month reference rate is quoted at 3.5%. Allan closes out his future position and invests his inheritance in a 90-day deposit account at the market reference rate – 25 bp. What is Allan's $ return from depositing his inheritance combined with his futures position?

Allan lock in STIR @ $95 (fwd price)
100 − 95 = 5% ← fwd interest rate.

a) Allan's return $= (5\% - .25\%) \cdot 20MM \times 90/360$
$= 237{,}500$

OR Interest Income $= (3.5\% - .25\%) \cdot 20MM \cdot 90/360$
$= \$162{,}500$

Profit from future $= [(100 - 3.5) - 95]\% \cdot 20 \cdot 90/360$
$= 75{,}000$

B. If the three-month reference rate is 6.5% in 120 days, what is Allan's $ return from depositing his inheritance combined with his futures position?

B.) same as A).

interest income $= (6.5\% - .25\%) \cdot 20MM \cdot \frac{90}{360}$

$= 312,500$

Loss from future $= [(100 - 6.5) - 95]\% \cdot 20MM \cdot \frac{90}{360}$

$= -1.5\% \cdot 5MM$

$= -75,000$

Total P/L $= 312,500 - 75,000$

$= 237,500$

Similarities b/w Eurodollar future & FRA

- allow both parties to lock in rate for future borrowing / lending.
- FRA forward price = Eurodollar Future fwd. ref rate

Diff. b/w Eurodollar future & FRA

Eurodollar future: standardized, exchange-traded, require margin deposit, mark to market.

FRA: customized, created by dealer, not liquid, subject to counterparty risk

FI Futures

- hedge the int-rate risk of short-maturity bonds (2-3 years)
- requires estimating the sensitivity of the value of each of the bond's CFs to change in corr. forward rate.
- Liquidity of IR futures ↓ for fwd rates further in the future. => Longer-maturity bonds are most often hedged w/ FI Futures (which have very good liquidity)
- Treasury futures available on T-bills, Treasury notes & Treasury bonds & traded on CBOT & CME

Mechanics of FI Futures

- price of treasury bond futures are based on notional gov. bond with the assumed cpn rate of 6%
- Conversion Factor: used in the contract to reflect each deliverable bond's value relative to notional bond

$$\text{principal invoice price} = \left(\frac{\text{future settlement price}}{100}\right) \times 100{,}000 \times CF$$

$$\text{total invoice amount} = \text{principal invoice price} + \text{accrued int.}$$

Since CF disregards term structure & each deliverable bond has diff maturity / cpn / duration, short party will deliver the bond w/ highest gain / smallest loss (i.e. Cheapest-to-deliver (CTD) bond)

P&L on delivery = [(settlement price × CF) + AI_T]
− (CTD clean price + AI_T)

EXAMPLE: Identifying the CTD bond at delivery

	Bond 1	Bond 2
Coupon	5.5%	5.75%
Time to maturity	20 years	19 years
Bond price	$141.13	$145.10
Accrued interest at delivery	$0	$0
CF	0.9422	0.9719
Futures settlement price	$148.75	$148.75

Bond 1: Principal Invoice Amt = 148.75 × 0.9422 = 140.15
Total Invoice Amt = 140.15 + 0 = 140.15.
CTD Clean Price = 141.13
CTD Dirty Price = 141.13 + 0
= 141.13
P/L = 140.15 − 141.13 = −0.98

Bond 2: Principal Invoice Amt = 148.75 × 0.9719 = 144.57
Total Invoice Amt = 144.57 + 0 = 144.57
CTD Clean price = 145.10
CTD Dirty price = 145.10
P/L = 144.57 − 145.10 = −0.53

Hedge IR Risk using Treasury Futures

sell treasury bond futures to hedge IR risk of a long bond portfolio.

if IR ↑, ⇒ Price (bond) ↓
Price (future) ↓
Value (short futures) ↑

$$\Delta \text{ future price} = \frac{\Delta CTD}{CF} \quad ①$$

To hedge against IR change, the change in port. value must be offset by Δ in futures value

$$\Delta P = HR \times \Delta \text{ future price}$$

ΔP = chg. in port value
HR = hedge ratio
= no. of future contract

$$\Delta P = HR \times \frac{\Delta CTD}{CF}$$

Substitute ①.

$$HR = \frac{\Delta P}{\Delta CTD} \times CF \quad ②$$

CTD bond & bond's portfolio is unlikely to be perfect sub.
⇒ Causing mismatch in risk b/w port. & derivative used to hedge. (i.e. basis risk / spread risk)
② will be effective ONLY IF portfolio only contains the CTD bond.

If portfolio doesn't consist solely of CTD Bond, better use duration-based hedge ratio (BPVHR):

$$BPVHR = \frac{-BPV_{port.}}{BPV_{CTD}} \times CF$$

where BPV = basis point value (exp. change in value of security or portfolio given a b.p. chg in yield)

BPVHR = no. of short futures

$BPV_{portfolio} = MD_{portfolio} \times 0.01\% \cdot MV_{portfolio}$

$BPV_{CTD} = MD_{CTD} \times 0.01\% \cdot MV_{CTD}$

MV_{CTD} = CTD price / 100 × 100,000

NOT 1MM !!

To achieve target duration,

$$BPVHR = \frac{BPV_{target} - BPV_{port}}{BPV_{CTD}} \times CF.$$

where $BPV_{Target} = MD_{target} \times 0.0001 \times MV_{portfolio}$

EXAMPLE: Immunizing a bond portfolio from interest rate risk

A fixed-income portfolio manager is holding a portfolio with a market value of £60 million and wants to fully hedge the portfolio value against parallel movements in the yield curve. The portfolio has a modified duration of 10.75. The portfolio manager will sell U.K. Government Long Gilt futures to hedge the portfolio.

U.K. Government Long Gilt Futures Specifications	
Futures price	£130.21
Futures contract size	£100,000
CTD	4.75% coupon, 12 years to redemption
CTD price	£139.56
CTD CF	1.0709
CTD modified duration	9.7

1. **Compute** the number of U.K. Government Long Gilt futures to be sold to immunize the portfolio.
2. **Compute** the number of Gilt futures that need to be sold to achieve a target duration of 8.7.

1) $BPVHR = - BPV_{port} / BPV_{CTD} \times CF$

$BPV_{port} = 60MM \cdot 0.0001 \cdot 10.75 = 64,500$

$BPV_{CTD} = (139.56 \times 100,0000/100) \cdot 0.0001 \cdot 9.7$

$= 135.37$

$BPVHR = - 64,500 / 135.37 \times 1.0709$

$\approx 510.$

need to short 510 futures to hedge.

2) $MD_{target} = 8.7.$

$BPV_{target} = 8.7 \cdot 0.0001 \cdot 60MM$

$= 52,200$

$BPVHR = \dfrac{52,200 - 64500}{135.37} \times 1.0709 \approx 97.$

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Hedging Results are not perfect b/c :

1) constructed using CTD bonds which is prone to change of slope of term structure.

2) Constructed using duration (prone to convexity).

3) Mod. duration only captures parallel movements (prone to shaping risk)

MODULE QUIZ 7.1

1. Ben Root holds an interest rate swap with a tenor of one year and quarterly settlement dates. The variable reference rate is the floating market reference rate (MRR). The variable payment/receipt on day 270 will be determined by:
   A. 270-day MRR at initiation.
   B. 90-day MRR at day 270.
   C. 90-day MRR at day 180.
2. Virat Sharma, a high-net-worth individual, holds a four-year floating-rate note (FRN) with semiannual coupons at the six-month reference rate + 40 bp and a par value of $4 million. Sharma is concerned about falling interest rates and would like to hedge this risk.

   Four-year semiannual swaps are quoted at a swap rate of 4%; the six-month reference rate at initiation is 3.5% and the six-month reference rate at the first settlement date is 2.5%.

   What type of swap should Sharma use? **Compute** the net cash flow on the swap at the first and second settlement dates. **Compute** the net return on Sharma's combined positions.
3. Carlos Hendricks is a fixed-income fund manager. Hendricks is running a fund with a modified duration of 12 and a market value of $200 million. He is concerned that interest rates will increase and wants to reduce the duration of his portfolio to 8 using U.S. Treasury futures.

U.S. Treasury Future	
Futures price	$164.20
Contract size	$100,000
CTD	4%, 20 years to maturity
CTD price	$126.39
CF	0.7689
CTD modified duration	14.54

   What Treasury future position is required to achieve a portfolio duration of eight?
   A. Sell 335 contracts.
   B. Sell 435 contracts.
   C. Sell 1,004 contracts.

1. C

2. Wants fixed-rate $\Leftrightarrow$ receiver swap

$4MM Par, pays MRR + 40 bps.

Swap rate = 4%

6-month reference rate @ init = 3.5%

@ 6M = 2.5%

No CF @ init,

CF @ 1st settlement date : $4MM \cdot 4\% \div 2 = +80{,}000$

$- 4MM \cdot (3.5\% \quad \%) \div 2 = -70{,}000$

CF @ 2nd settlement date : $4MM \cdot 4\% \div 2 = +80{,}000$

$- 4MM \cdot (2.5\% + .4\%) \div 2 = -50{,}000$

Net Return : FRN + receipt on swap

@ 1st. date $= 4MM \cdot (3.5\% + 0.4\%) \div 2 = 78{,}000$

$+ 10{,}000 = 88{,}000$

\$ return $= 88{,}000 / 4MM = 2.2\%$

annualized = 4.4%

@ 1st. date $= 4MM \cdot (2.5\% + 0.4\%) \div 2 = 58{,}000$

$+ 30{,}000 = 88{,}000$

\$ return $= 88{,}000 / 4MM = 2.2\%$

annualized = 4.4%

3. $BPV_{target} = 8 \times 1Bp \times 200MM = 160{,}000$

$BPV_{port} = 12 \times 1Bp \times 200MM = 240{,}000$

$BPV_{CTD} = 14.54 \times 1bp \times \frac{126.39}{100} \times 100{,}000 = 183.77.$

$BPV_{HR} = \frac{160{,}000 - 240{,}000}{183.77} \times 0.7689 = -334.72.$

7.2 Currency Swap.

currency risk = chg in value of assets & liabilities denominated in overseas currency when converted to domestic currency & exchange rate fluctuations

Cross-currency basis swap : Swap interest payments & principal currency

> **EXAMPLE: Cross-currency basis swap**
>
> A euro-based company requires USD but does not have access to direct USD borrowing or finds it prohibitively expensive. The company decides to borrow in euros at the three-month euro reference rate and enter into a cross-currency basis swap to USD based on the three-month USD reference rate (a floating-for-floating swap). The swap has a tenor of two years with quarterly settlement. The principal on the euro loan is €50 million and the \$/€ exchange rate at initiation of the swap is \$1.1236.

@ init,

EUR Company ← 56.18MM USD — Swap Dealer

EUR Company — 50MM EUR → Swap Dealer

Bank — 50MM EUR. → EUR Company

$50 \cdot 1.1236 = 56.18$ MM USD

@ qtr. st date.

EUR Company — 56.18MM USD × qtr. \$ ref. rate → Swap Dealer

EUR Company ← 50MM EUR × qtr. ref eur rate — Swap Dealer

EUR Company — 50MM EUR. × qtr. ref eur rate → Bank

Covered Interest Rate Parity (CIRP)

borrowing cost (USD) = borrowing cost (other currencies) + hedging FX risk via currency swaps

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In frictionless market, CIRP holds & cross-currency basis doesn't exist

Since financial crisis, CIRP doesn't hold b/c. frictions in arbritage market. (i.e. basis has been non-zero)

Cross-currency basis rep. the additional cost of borrowing dollars synthetically (typically from the foreign-market perspective)

IF foreign currency has a NEGATIVE BASIS
⇒ borrowing USD directly IS CHEAPER
SYNTHETIC borrowing MORE EXPENSIVE THAN DIRECT borrowing

EXAMPLE: Cross-currency basis swap

Boivis Patisseries Sarl is a French chain of patisseries that has an extensive network of shops in continental Europe. As part of their expansion strategy, they are looking to set up shops in the United States. Boivis estimates that it will initially require $50 million to set up shops and cover working capital requirements. The finance directors at Boivis have looked at directly borrowing in USD but have found that costs would be the U.S. dollar reference floating rate + 100 bp. The decision is made to borrow for four years in euros at a rate of the euro reference floating rate + 60 bp with interest paid quarterly and enter a currency swap to exchange euros for dollars. Basis on the Eurodollar swap is being quoted at –20 basis points (–20 bp). The swap pays variable interest on both legs on a quarterly settlement basis. The current $/€ exchange rate is $1.1815.

The three-month euro reference rate is 1.5% and U.S. dollar reference rate is 2.0% at swap initiation. Three months later at the first settlement date, the three-month euro reference rate is 1.6% and the U.S. dollar reference rate is 1.9%.

Compute the principal flows exchanged at the start and end of the swap's tenor. **Compute** the interest payments at the first and second settlement dates on the swap and the cost to Boivis for its synthetic dollar loan.

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Strategy : Borrow EUR (floating rate + 60 bp)
+ enter swap

Principal Amt = 50 MM ÷ 1.1815
= €42.32 MM

=> Borrow €42.32 MM from bank
Lend €42.32 MM to swap dealer for $50 ~~MM~~ ~~at~~ basis??

First Settlement Date,

Swap dealer pays $42.319 \times (1.5\% + 60\text{ bp}) \times 90/360$
$= 0.222\text{ MM}$
$= 222{,}175$ $42.319 \times (1.5\% - 20\text{bps}) \times 90/360$

Sarl pays bank = 222,000 $\times$ = €137,537

Sarl pays swap dealer = $50\text{MM} \times (2.0\% + 60\text{ bp} - 20\text{bp}) \cdot 90/360$
$= 50\text{MM} \cdot 0.6\%$
$= 300{,}000$ X
$= 50\text{MM} \cdot 2\% \times 90/360$
$= \$250{,}000$

Cost of borrowing USD directly $= 50\text{MM}(2\% + 100\text{bp}) \cdot \frac{90}{360}$
$= 875{,}000$

Cost of swap borrowing $= 50\text{MM}(2\% + 60\text{ bps} + 20\text{bp}) \times 90/360$
$= 350{,}000$

EUR swap leg

Currency Forwards & Future

- allow exchange of a specfic amt of one currency for another @ a time
- Currency futures are standardized & range of currency pairs might be limited

$$HR = \frac{\text{Amt of currency to be exchanged}}{\text{future contract size.}}$$

EXAMPLE: Hedging exchange rate risk using futures

A U.S. firm is due to receive €20 million in 90 days for goods they sold. The firm is seeking to hedge this risk by selling EUR futures contracts maturing closest to date the euros will be received. The EUR-USD FX future contract size is €125,000. The futures price is 1.3150 USD/EUR. The firm will sell futures contracts (promising to deliver euros at the rate of 1.3150 USD per EUR).

Calculate the number of futures contracts required to hedge the asset and the amount of USD to be received at contract settlement.

No. of contract $= 20\text{MM} / 0.125$

$= 160$ contracts

\$ received $= 160 \cdot 125{,}000 \times 1.3150$

$= 26{,}300{,}000$

MODULE QUIZ 7.2

1. The New Zealand dollar (NZD) is trading at a positive cross-currency basis against the U.S. dollar (USD). Sterling (GBP) is trading at a negative cross-currency basis to the USD. Which of the following strategies would generate the greatest return?
   A. Swapping USD for NZD and investing in New Zealand government securities.
   B. Swapping NZD for USD and investing in U.S. government securities.
   C. Swapping GBP for NZD and investing in New Zealand government securities.
2. Barney Wood imports goods to the U.K. from the Eurozone. He is due to make a payment in 30 days of €20 million. Wood is concerned that the pound will depreciate against the euro over this period and would like to hedge his currency risk with futures. The current spot rate is £/€ = 0.8929.

Cross-Currency EUR-GBP Future	
Futures price* £/€	0.8989
Contract size	€125,000

*Expires in 40 days

Calculate the futures position needed to hedge Wood's liability.

In 30 days, the exchange rate is £/€ = 0.9034 and the futures price is £/€ 0.9054. **Calculate** the cost to Wood if he leaves his euro liability unhedged and if he hedges the position using the future.

3. ABC Robotics, Inc., a U.S. firm, will borrow £30 million to set up a subsidiary in the U.K. ABC can borrow GBP directly at a cost of the pound reference rate + 50 bp. ABC Robotics can borrow in USD at the dollar reference rate + 40 bp. A GBP-USD swap is quoted at −15 bp. The spot exchange rate at swap initiation is quoted as \$/£ = \$1.2000. The swap is a four-year semiannual swap where both the USD and GBP reference rates are based on six-month floating reference rates. The six-month U.S. dollar reference rate is 2.5% and pound reference rate is 1.5% at initiation of the swap. At the first settlement date, the six-month U.S. dollar reference rate is 2.25% and pound reference rate is 1%. **Calculate** the interest payments at the first and second settlement dates. How much better off is ABC Robotics from using the cross-currency swap rather than directly borrowing GBP?

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1. C × B

NZD has positive cross-currency basis against USD
⇒ investors can earn superior returns by lending NZD + enter a currency swap. & investing in U.S. gov. bond

2. The future positions needed = €20MM ÷ 0.125
= 160 positions

Wood need € ⇒ Buy futures

£/€ = 0.9034 (€ appreciate)

cost if left unhedged = €20MM · (0.9034 − 0.8929)
= £210,000 Loss ①

cost if using future = €20MM · (0.8989 − 0.9034) ×
= £210,000 loss (from ①) + profit on hedge
= + (0.9054 − 0.8989) · 20MM
= −210,000 + 130,000
= −80,000

loss on the heedge = result of Δ in basis

	Spot	Future	Basis
@ init	0.8929	0.8989	+60 bps
@ close	0.9034	0.9054	+20 bps

chg in pips. = −40 bps

3. Borrow £30MM., lend 30MM · 1.2 = $36MM USD
@ 1st settlement date, pay 30MM·(1.5% − 15bp) ÷ 2 = 202,500
receive 36MM · (2.5%) ÷ 2 = 450,000

pay USD loan = 36MM · (2.5% + 0.40%) ÷ 2 = 522,000
Total pmt = £202,500, $72,000 TWO, DIFF CURRENCIES

Cost of direct borrowing = 30MM · (1.5% + 0.50%) ÷ 2
= £0.3MM

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Cost of synthetic borrowing $= 30MM \cdot (1.5\% + 40bps - 15bps)$
$= £262,500$

(a) 2nd settlement date.

$(2.25\% + 40bps) \div 2$

Cost of borrowing USD direct $= 36MM \cdot (2.25\% \div 2 + 40bps)$
$= \$549,000$ x 477,000

x $(1\% - 15bps) \div 2$.

Cost of paying GBP swap $= 30MM\ (1\% \div 2 - 15bps)$
$= £\ 44,999.995$ x £127,500

receiving $= \$36MM \cdot 2.25\% \div 2$
$= \$405,000$ ✓

Total cost $= \$72,000,\ £127,000$

Cost of direct borrowing $= 30MM \cdot (1\% + 50bps) \div 2$
$= £225,000$

benefit of swap $= 225,000 - 127,500 - 72,000 \div 1.2$
$= 37,500$

7.3. Managing Equity Risk

Equity Swap types:

1) Pay Fixed, receive equity return
2) Pay Floating, receive equity return
3) Pay another equity return, receive equity return

Equity Return can be based on

- Index
- Single stock
- basket.

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Pros	Cons.
- gain exposure to eq. mkt when participation is restricted	- Liquidity
- Avoid tax & custody fees	- usually require collateral
- Avoid cost of monitoring physical conditions which may increase due to Corp. action	- don't convey voting right

EXAMPLE: Changing equity exposure using a swap

A German pension fund manager holds a €200 million portfolio of domestic stocks passively tracking the DAX 30 stock market index. The pension fund expects the index to rise in the next year and wishes to increase its exposure by 40%. The fund manager enters an equity swap with a notional principal of €80 million, agreeing to pay a floating interest rate + 30 bp and receive the return on the equity index. The swap has a tenor of one year and semiannual settlements. The DAX 30 at the time of swap initiation is 12,400 points.

Scenario 1:

The six-month floating reference rate relating to the first settlement date is = 6%.

The DAX 30 at the first settlement date = 13,020.

Scenario 2:

The six-month floating reference rate relating to the first settlement date is = 6%.

The DAX 30 at the first settlement date = 11,780.

Compute the gain or loss on the portfolio, the cash flows on the swap, and the net position for the fund manager in both scenarios.

Scenario 1 : $(13{,}020 \div 12{,}400 - 1) \times 80MM = 4MM$

$- 80MM \cdot (6\% + 30bp) \div 2 = 2.52MM$

$P/L = 1.48MM$

2 : $(1{,}178 \div 1{,}240 - 1) \times 80MM = -4MM$

$- 80MM \cdot (6\% + 30bp) \div 2 = 2.52MM$

$P/L = -6.52MM$

Equity Futures & Forwards.

Equity Futures usage:

- Implement TAA:
  selling futures (short position) to lower equity exposure
  buying futures (long position) to gain equity exposure
- diversify
- gain exposure to int'l market
- make directional bets.

Actual future price = quoted future price × multiplier

EXAMPLE: Using index futures to hedge equity market exposure

A fund manager holds a £200 million equity portfolio, which is passively tracking the FTSE 100 Index. The fund manager wishes to hedge 30% of the portfolio against equity market risk.

Contract Details for FTSE 100 Index Futures	
Quotation	Index points
Multiplier	£10 per point
Delivery dates	March, June, September, December
Settlement price	FTSE 100 cash price on last day of trading
Futures price – September delivery	7,300

Compute the number of contracts required to hedge 30% of the portfolio's equity position. **Compute** the profit or loss if the FTSE 100 increases by 5% and the futures price is 7,665. **Compute** the profit or loss if the FTSE 100 falls by 5% and the futures price changes to 6,935.

200 MM / (7,300 × 10) × 30% = 821.9 ≈ 822. Contracts
(short)

@ 7,665, & FTSE ↑ 5%,

MV (portfolio) = 200MM × 1.05 = 210MM

Loss on future = (7300 − 7665) × 10 · 822
= −3,000,300

Net Position = 210MM − 3,000,300 60MM · 5%
= 206,999,700

Impact of Hedge = −3,000,300 + 3,000,000
= −300

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if. FTSE falls by 5%,

Gain on equity future $= (7{,}300 - 6{,}935) \cdot 822 \cdot 10$
$= 3{,}000{,}300$

Loss on mkt $= 200 \cdot -5\%.$
$= -10MM$

Impact on Hedge $= 3{,}000{,}300 - 3{,}000{,}000$
$= 300$

Achieving Portfolio Beta

no. of futures req. $= \left(\frac{B_T - B_P}{B_F}\right)\left(\frac{MV_P}{F}\right)$

T = target

F = futures value
(= contract prize × multipler)

B_F = futures beta
= beta of stock index

EXAMPLE: Achieving a target portfolio beta using index futures

A fund manager has a $60 million portfolio of aggressive stocks with a portfolio beta of 1.2 relative to the S&P 500. The fund manager believes the market will decline over the next six months and wishes to reduce the beta of the portfolio to 0.8 using S&P 500 futures. S&P 500 futures currently have a contract price of 2,984 and a multiplier of $250. At the end of the six-month period, the S&P 500 Index has decreased by 1.5%. By definition, beta of the S&P 500 Index equals 1.

Calculate the number of futures contracts and **determine** whether they should be bought or sold to achieve the target portfolio beta. **Compute** the effectiveness of the strategy at the end of the six-month period.

$B_P = 1.2. \quad B_T = 0.8 \quad B_F = 1$

no. of contracts required $= (1.2 - 0.8)\dfrac{60MM}{2{,}984 \cdot 250}$

$= \dfrac{24MM}{746{,}000} = -32.17$

P&L after 6M: $60MM \cdot (-1.5\%) \cdot 1.2 + 746{,}000 \times (1.5\%)$
$= -0.9MM + 0.$

Cash Equitization

- purchasing index futures to replicate the returns that would have been earned by investing in an index with risk & return characteristics similar to the portfolio.

EXAMPLE: Cash equitization

A U.S. fund manager runs a passive fund, which tracks the S&P 500. Cash balances have built up in the portfolio and the fund manager is concerned that the cash drag will lead to portfolio underperformance relative to the S&P 500. The fund currently holds $8 million in cash. S&P 500 futures currently have a contract price of 2,780, a multiplier of $250, and a beta of 1.

Calculate the number of stock index futures needed to equitize the portfolio's excess cash.

No. of contracts $= 8{,}000{,}000 / (2{,}780 \times 250)$

$= 11.51$

MODULE QUIZ 7.3

1. A U.K. fund manager has a defensive equity portfolio with a market value of £20 million and a beta equals 0.8 relative to the FTSE 100. The manager believes that U.K. equity will perform strongly over the next year and wishes to increase the portfolio beta to 1.4. FTSE 100 futures are currently trading at 7,425 points with a multiplier of £10. How many futures contracts are required to achieve the desired beta?
   A. 162 FTSE 100 futures.
   B. 377 FTSE 100 futures.
   C. 1,620 FTSE 100 futures.
2. Hideko Kobayashi is an active fund manager running an equity portfolio benchmarked against the Nikkei 225. The funds market value is ¥4,350,000,000. Due to recent sales, Hideko is worried that cash balances have built up to 5% of the fund's value. Hideko is worried about the cash drag affecting her performance fees and wishes to temporarily invest the surplus cash in the Nikkei 225. The Nikkei future she is considering has a price of 21,624 and a multiplier of ¥1,000. How many Nikkei 225 futures will she require for this cash overlay?
   A. 10 futures.
   B. 100 futures.
   C. 201 futures.

1. No. of futures req $= \left(\frac{B_T - B_P}{B_F}\right)\left(\frac{MV_P}{F}\right)$

$= ((1.4 - 0.8) \div 1)\,(20MM \div (7{,}425 \cdot 10))$

$= 161.42$ A ✓

2. $4{,}350{,}000 \times 5\% \div (21{,}624 \times 1{,}000) = 10.06$ A ✓

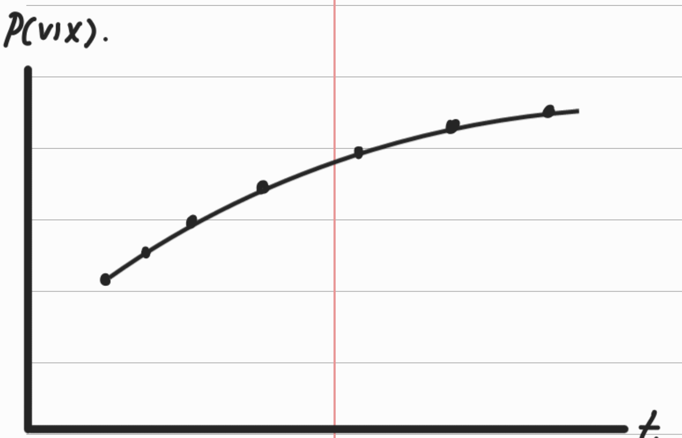
7.4. Derivatives on Volatility.

Best known measure : CBOE Volatility Index (VIX)
(note : NOT MEASURE OF ACTUAL VOL but expected vol that is priced into options, aka. FEAR INDEX)

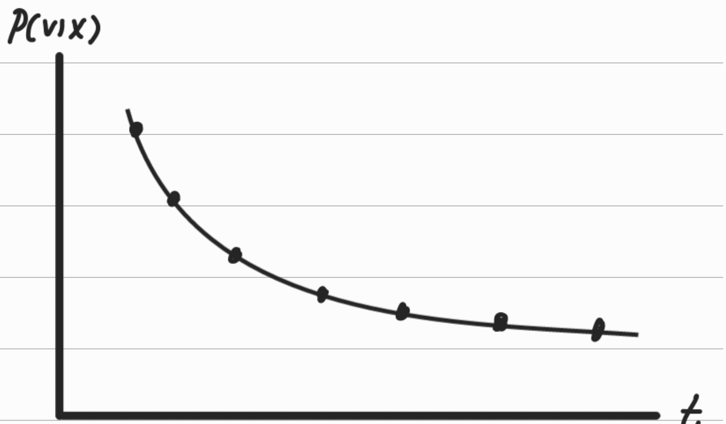
VIX index value = annualized S.D. of expected +/- % moves in S&P 500 over the following 30 days.
e.g. VIX = 20 => +/- 20% change w/ 68% lv. of confidence over the next year

VIX Futures.

- Can protect extreme downturn (i.e. tail risk) w/ VIX futures.
- Term structure of VIX futures can provide insights into mkt expectation of vol. over time.



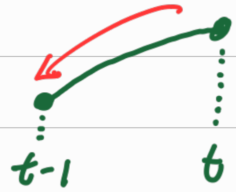
Contango



Backwardation

Futures Position	Term Structure	Roll Yield.
Long.	Contango	Negative.
Short.	Contango	Positive
Long.	Backwardation	Negative.
Short.	Backwardation	Positive

Roll Yield.



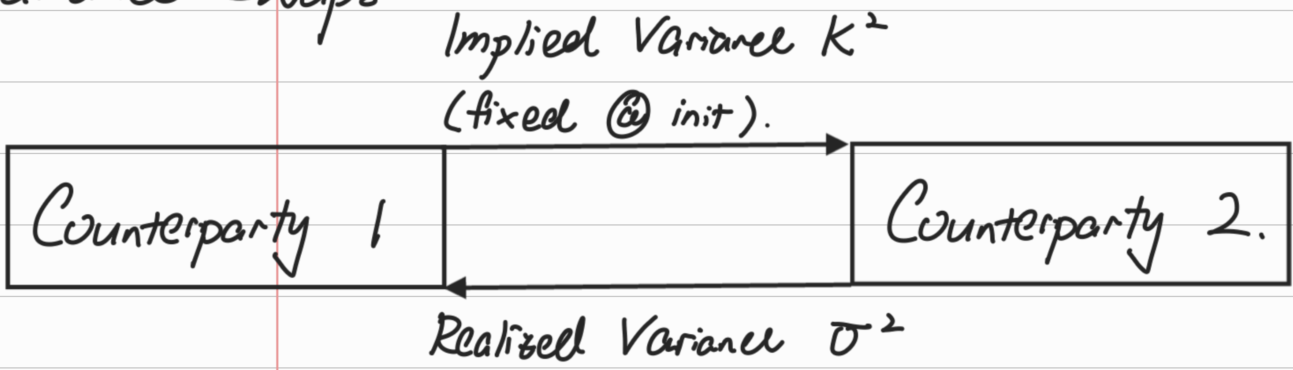
move backward on the graph

VIX Options.

- cash-settled EUR-style options.

Other volatility indexes

Variance Swaps.



settlement amt_T = (var notional) (realized var − var strike)

Long Var swap <=> Receive floating / realized var

Realized Var is calc. by

$R_i = \ln(P_t / P_{t-1})$.

daily variance $= \dfrac{\sum R_i^2}{(N-1)}$

using this insted of. $(R_i - \bar{R})$ b/c we dc. abt directions.

annualized var = daily var. × 252.

/ /

Variance Notional : P/L per point difference b/w implied variance (strike^2) & realized variance (σ^2).

Market Convention on Var Notional : quote as vega notional (N_{vega}) &

$$\boxed{N_{var} = \frac{N_{vega}}{2 \cdot K}} \Rightarrow P/L = \boxed{N_{vega} \times \frac{\sigma^2 - K^2}{2K}}$$

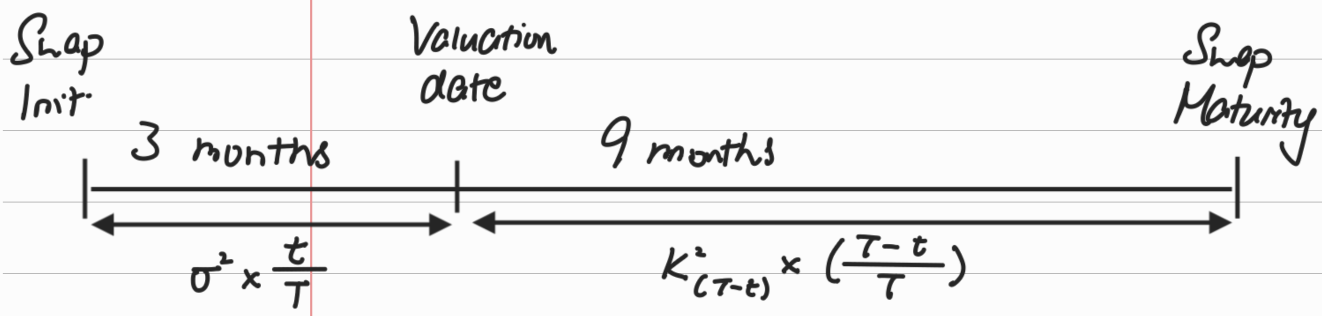
Convexity : Payoffs is based on variances but strike price is based on volatility

$\sigma > K \Rightarrow (\sigma^2 - K^2) > \sigma - K.$

$\sigma < K \Rightarrow (\sigma^2 - K^2) < \sigma - K.$

(i.e payoff increasing at a faster rate but. decreasing at a slower rate).

Mark to Market.



expected var. to maturity =

$$\sigma^2 \times \frac{t}{T} + K^2_{(T-t)} \times \left(\frac{T-t}{T}\right)$$

$FV(\text{payoff}) = N_{var} \times$ (expected var to mat − orig. strike)

EXAMPLE: Valuing a variance swap during its life

Luke Amos, an equity fund manager, has purchased a one-year variance swap on the S&P 500 with vega notional of $100,000 and a strike of 20%.

Nine months have passed and the S&P has realized a volatility of 21%. The strike price for a three-month variance swap at this time is quoted at 22%, and the annual interest rate is 2%.

Compute the current value of the swap.

$T = 12,\ t = 9.$

exp. var. to mat. $= 21\%^2 \cdot \frac{9}{12} + 22\% \cdot \frac{3}{12}.$

$= 451.75\%$

FV (payoff) $= N_{var} \times (451.75\% - 20\%^2).$

$= \frac{N_{vega}}{2 \cdot K} \times (451.75\% - 400\%).$

$= \frac{100{,}000}{2 \cdot 20} \cdot 51.76\%$ ✗ No Need to keep %

$= 2{,}500 \times 51.75$

$= \$129{,}375.$

payoff $= 129{,}378 \cdot (1+2\%)^{-3/12}.$ ✗.

$= 129{,}375 \cdot (1 + 0.005)$

$= 128{,}731.$

MODULE QUIZ 7.4

1. Which of the following comments is *least* accurate regarding the VIX Index?
   A. Empirically, the VIX Index and equity returns are negatively correlated.
   B. VIX measures realized volatility over a 30-day period on the S&P 500.
   C. The VIX Index level is the annualized standard deviation of implied volatility on the S&P 500.
2. Which of the following comments is *most* accurate? The payoff on a variance swap can be calculated by multiplying the difference between actual variance and implied variance by:
   A. notional vega.
   B. notional variance.
   C. the expected return to volatility.
3. Quark Dealers sold a one-year FTSE MIB variance swap with a strike of 15 three months ago. Quark set the vega notional at €150,000. As part of their swap agreements, Quark requires the contract to be marked to market and subject to margining every three months.

   At the end of the first three-month period, realized volatility is 28. The strike on a nine-month variance swap is 32, and the annual interest rate is 0.6%.

   Compute the mark-to-market value of the swap.

1. B / VIX measures the expected volatility over the forthcoming 30 days.

2. A B payoff = notional variance $(\sigma^2 - K^2)$

3 Step 1: Compute the expected variance

$$= \left(\sigma^2 \times \frac{t}{T}\right) + \left(K^2_{(T-t)} \times \frac{T-t}{T}\right)$$

$$= (28^2 \times 3 \div 12) + (32^2 \times 9/12)$$

$$= 964$$

Step 2: expected the expected payoff @ maturity:

$$\text{variance notional} = \frac{\text{vega notional}}{2 \times K}$$

$$= 150{,}000 \div (2 \times 15)$$

$$= €5{,}000$$

expected payoff @ maturity

$$= (964 - 225) \times €5000 = €3{,}695{,}000$$

Step 3: Discount expected payoff from maturity to valuation date

$$= €3{,}695{,}000 \div \left(1 + 0.6 \times \frac{9}{12}\right)$$

$$= €3{,}678{,}447$$

Inferring Market Expectations

Market expectations are current expectation derived from market prices. In event of market shocks, market expectations are changing rapidly.

Application	Derivative
Inferring expectations of FOMC moves	Feds Fund Rate.
Inferring expectations of inflations.	CPI swaps.
Inferring expectations of future volatility.	VIX futures & options

Using Funds Future to infer the expected Average Federal Funds Rate

Federal funds rate: interest rate deposit institutions charge each other for overnight loans.

Federal funds effective (FFE) rate: weighted avg. of interest rates charged on overnight loans.

Federal fund target rate:

- set by governor of Federal Reserve in Federal Open Mkt Committee (FOMC) meetings.
- influenced by inflation rate & GDP growth rate considerations.
- when market participant refer to feds changing rate, they usually refer to this rate

- usually set a range.
- Feds doesn't control FFE rate, but influence it via monetary policy tools.

Fed Fund Futures:
- future price reflect the market expectation of FFE rate @ time of contract maturity
- future price will reflect market expectation of fed fund target rate

percent probability of rate change

$$= \frac{\text{effective rate implied by future} - \text{current fed funds target rate}}{\text{Fed funds rate assuming a rate chg} - \text{current fed funds target rate}}$$

or,

$$= \frac{\text{implied Fed Funds effective rate} - \text{current target rate}}{\text{expected size of rate change}}$$

EXAMPLE: **Determining the markets expectation of a target rate increase**

Joe Stokes works at a bank where the interest received on loans made is linked to the FFE rate. Stokes has been asked to compute the likelihood that the FOMC will increase the rate by 25 bp at the next FOMC meeting.

Stokes has collected the following market data:

Fed funds future price*	98.1625
Current Fed funds target rate	1.50%–1.75%

*Nearest futures contract after the date of the next FOMC meeting

Calculate the following:

1. The expected average FFE rate at the futures contract maturity
2. The probability of a 25 bp increase in target rate at the next FOMC meeting

1. Expected avg. FFE rate = 100 − 98.1625 = 1.8375%.

2. current midpoint = (1.50% + 1.75%) ÷ 2 = 1.625%

midpt after 25 bp hike = 1.625% + .25% = 1.875%

$$\% = \frac{1.8375\% - 1.625\%}{1.875\% - 1.625\%}$$

$$= 85\%$$

MODULE QUIZ 7.5

1. Elise Schwarz manages a €400 million fund that invests in German and Spanish equities and government bond futures. Currently, her portfolio has a 60% exposure to equity and a 40% exposure to government securities. Schwarz believes that the monetary policy of the ECB will provide a significant stimulus to European equity markets and would like to increase her equity exposure to 70% of the portfolio's value.

   Additionally, the current portfolio is 50% invested in Spanish stocks and 50% in German stocks. Schwarz would like to change the proportions to 60% Spanish stocks and 40% German stocks. She also wishes to change the asset allocation of her fixed-income portfolio—which is currently 50% invested in German Bunds and 50% in Spanish Obligaciones del Estado—to 70% German government debt and 30% Spanish government debt.

   Her current portfolios have the following details:

	Beta		Modified Duration
Spanish equity	$\beta = 1.2$	Spanish fixed income	7.34
German equity	$\beta = 0.9$	German fixed income	10.25

   Elise intends to use futures to achieve her new asset allocation, to avoid transaction costs involved with liquidating positions, and reinvesting.

   Elise has gathered the following futures information:

Equity Futures	German	Spanish
Index	DAX 30	IBEX 35
Futures price	12,500	9,200
Multiplier	€25	€10
Futures beta	1	1

Government Bond Futures	German Bund	Spanish Obligaciones del Estado
Contract size	€100,000	€100,000
CTD price	€105.44	€149.94
CTD CF	0.6095	0.9628
CTD modified duration	9.67	8.26

   Calculate and **describe** the future positions that would achieve Elise's new target asset allocation.

1. 400MM fund

	Orig.	New
Total Equity	240	280
Total FI	160	120
Total Spansh Eq.	120	280 · 60% = 168.
Total German Eq.	120	280 · 40% = 112
Total Spanish FI	80	120 · 30% = 36
Total German FI	80	120 · 70% = 84.

Spanish Eq. $= (168 - 120) \times 1.2 \div 9{,}200 \div 10$

$= 626$ · contract

German Eq $= (112 - 120) \times 0.9 \div 12500 \div 25$

$= -23$ contract

Spanish FI: $(36MM - 80MM) \div (149.94 \cdot 0.9628 \cdot 8.26)$
$\div 100{,}000$ ✗

$$BPVHR = \frac{BPV_{target} - BPV_{port.}}{BPV_{CTD}} \times CF$$

$BPV_{port} = MD_{port} \times 0.0001 \times MV_{port}$
$= 7.34 \times 0.0001 \times (36MM - 80MM)$
$= -32{,}296$

$BPV_{CTD} = MD_{CTD} \times 0.0001 \times$ [price / 100 × Contract size]
$= 8.26 \times 0.0001 \times €149.94/100 \times €100{,}000$
$= €123.88$

$$BPVHR = \frac{-32{,}296}{123.88} \times 0.9628 = -251.$$

Germany FI:

$$BPVHR = \frac{BPV_{Target} - BPV_{port}}{BPV_{CTD}} \times CF$$

$BPV_{target} = 10.25 \cdot 0.0001 \times (84 - 80)$
$= 4{,}100$

$BPV_{CTD} = 9.67 \cdot 0.0001 \times 105.44 \div 100 \times 100{,}000$
$= €101.96$

$$BPVHR = \frac{4100}{101.96} \times 0.6095 \doteq 24.51$$

2. Stuart Zackaman has been announced as the new chair of the Federal Reserve. In his inaugural speech, he mentions that it is time that the United States stopped punishing savers to bail out irresponsible lenders.

Joe Bear works at a bank where the interest paid on deposits is linked to the Fed funds rate. Bear observes that Zackaman's speech caused Fed funds futures prices to fall.

Joe has collected the following market data:

Fed funds future price*	97.925
Current Fed funds target rate	1.75%–2.00%

*Nearest futures contract after the date of the next FOMC meeting

What is the probability of a 50 bp increase in target rate at the next FOMC meeting implied from the current Fed funds future price?

A. 20%.
B. 40%.
C. 80%.

1. A

Expected avg. effective rate = 100 − 97.925
= 2.075%

Current target midpoint = (1.75% + 2%) ÷ 2
= 1.875%

Target rate assuming 50 bp rise = 1.875% + 0.50%
= 2.375%

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