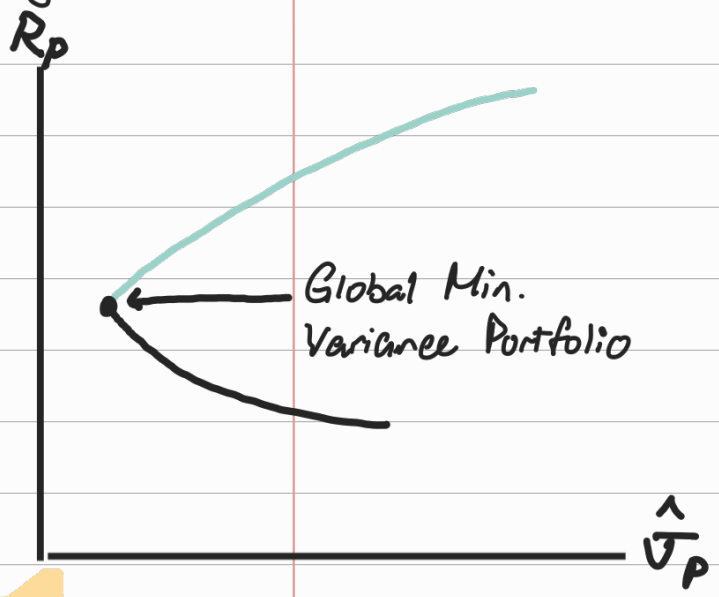
Module 4 : Principle of Asset Allocation

4.1 Basic Mean-Variance Optimitization

Figure 4.1 Mean-Variance Efficient Optimitization



Utility Maximization

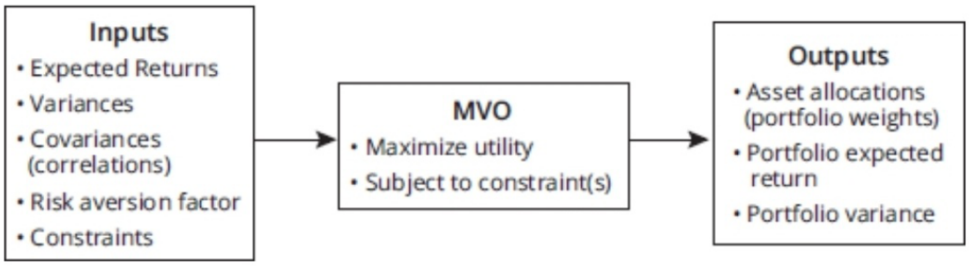
certainty-equivalent return

$$U_m = E(R_m) - 0.005 \times \lambda \times Var_m$$

λ = risk aversion coefficient

assuming $E(R)$ & var_m are shown in percentage. terms (e.g. 8% is input as 8)

Figure 4.2: MVO Process



Previous Assumptions of MVO (Lv I & II):

- All assets are tradeable

  X no longer assume. all assets are. used.

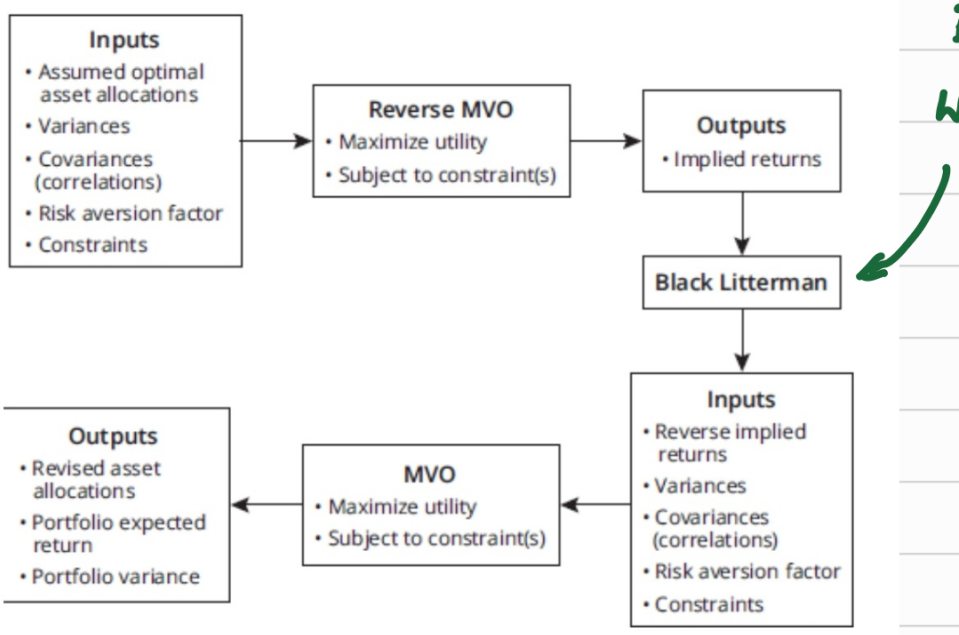
- A true risk-free asset exists

  X no longer assume risk-free asset exists.

Criticisms of MVO:

- Garbage In, Garbage out
- Concentrated asset class allocation
- Skewness & Kurtosis
- Risk diversification
- Ignore liabilities
- Single-period framework.

4.2. Reverse Optimization & Black-Litterman



Ext. of reverse optimization where implied return

EXAMPLE: Recommending and justifying an asset allocation using MVO

Marsha Bronsten has come to her investment adviser for help to determine an appropriate asset allocation. The adviser discerns in conversations with Bronsten that her risk tolerance is average ($\lambda = 4$) and that she would also like to minimize the chance of earning less than 3%. The available asset allocations are as follows:

	Expected Return	Variance
Allocation 1	8%	0.0225
Allocation 2	6%	0.0144
Allocation 3	4%	0.0025

Recommend which of the three strategic asset allocations is appropriate for Bronsten.

Answer:

$$U_{A1} = E(R_{A1}) - 0.005 \cdot \lambda \cdot Var(R_1)$$
$$= 8\% - 0.5 \cdot 4 \cdot 0.0225$$
$$= \mathbf{3.5\%}$$

$$U_{A2} = 6\% - 0.5 \cdot 4 \cdot 0.0144$$
$$= 3.12\%$$

$$U_{A3} = 4\% - 0.5 \cdot 4 \cdot 0.0025$$
$$= \mathbf{3.6\%}$$

Roy's safety-first criterion: $(R_p - R_L) / \sigma_p$.

Allocation 1: $(0.08 - 0.03) / \sqrt{0.0225} = 0.33$ ✓

Allocation 3: $(0.04 - 0.03) / \sqrt{0.0025} = 0.2$

⇒ Allocation 1 is the most appropriate

EXAMPLE: Recommending and justifying an asset allocation using MVO

The Plowshare University endowment fund has an annual return objective of 9%, which is sufficient to cover its spending rate, expected inflation, and cost of earning investment returns. Its risk objective is to minimize risk (as measured by standard deviation of returns) while meeting its minimum expected return objective. The table provides the output from an MVO with a budget constraint and a nonnegativity constraint. Allocations AA, BB, CC, and DD can also be referred to as corner portfolios.[1]

	Expected Return	Standard Deviation of Returns
Allocation AA	15%	24%
Allocation BB	18%	27%
Allocation CC	12%	20%
Allocation DD	10%	14%

The risk-free rate is 3%. If the client and manager believe a true risk-free asset exists and can be used to construct the SAA, **identify** the appropriate asset allocation for Plowshare and **calculate** the risk of the optimal allocation.

Sharpe Ratio: ← calc. this first. b/c. the mix of this & Rf. ⇒ less risky

AA : (15% − 3%) / 24% = .5

BB : (18% − 3%) / 27% = .56 ← use this asset.

CC : (12% − 3%) / 20% = .45

DD : (10% − 3%) / 14% = .5

$\Rightarrow 9\% = 18\% w + (1-w) \cdot 3\%$

$6\% = 15\% w$

$w = 0.4$

$\sigma_P^2 = \sigma_{BB}^2 w_{BB}^2$ ← don't forget the square.

$\sigma_P = \sigma_{BB} w_{BB}$

$= .27 (.4)$

$= .108$ //

MODULE QUIZ 4.1, 4.2, 4.3

1. Which of the following methods is *most appropriate* for addressing highly concentrated allocations in portfolios?
   A. Reverse optimization.
   B. Monte Carlo simulation (MCS).
   C. Liability-relative MVO.

2. Jane Cullis is considering three potential asset allocations. She wishes to earn a nominal return of no less than 4%, and she has a high risk tolerance with a lambda of 2.
   The following asset allocations are available:

	Expected return	Variance
Allocation 1	6%	0.02
Allocation 2	8%	0.03
Allocation 3	10%	0.04

   Based on the information provided, which of the following allocations should Cullis choose?
   A. Allocation 1.
   B. Allocation 2.
   C. Allocation 3.

1. C (A) Reverse optimization is most likely to produce a more diversified portfolio b/c it starts w/ weights of all assets in the global market portfolio & solves for the consensus exp. returns consistent w/ that highly diversified portfolio. MCS does not address this issue @ all b/c it is used to model behavior over time of any one specific asset allocation. Liability-relative MVO is focused on the change in value of the surplus (PVA - PVL). Resampled MVO is another way to address the concentration issue, but that was not a choice.

2. 2. $0.005 \cdot 4$ = Allocation (1)

Allocation 1: $0.06 - (0.5 \times 2 \times 0.02) = 0.04$
Allocation 2: $0.08 - (0.5 \times 2 \times 0.03) = 0.05$
Allocation 3: $0.10 - (0.5 \times 2 \times 0.04) = \underline{0.06}$
Allocation 3 has the highest certainty-equivalent return.

3. Melody Chan is considering three potential asset allocations. She wishes to earn a nominal return of no less than 4% and maximize her chances of exceeding a 4% return.

The following asset allocations are available:

	Expected return	Variance
Allocation 1	6%	0.02
Allocation 2	8%	0.09
Allocation 3	10%	0.16

Based on the information provided, which of the following allocations would Chan *most likely* choose?

A. Allocation 1.
B. Allocation 2.
C. Allocation 3.

3 A (C)

Allocation 1 : $(0.06 - 0.04) / (0.02)^{1/2} = 0.141$

Allocation 2 : $(0.08 - 0.04) / (0.09)^{1/2} = 0.133$

Allocation 3 : $(0.10 - 0.04) / (0.16)^{1/2} = 0.150$

Allocation 3 has the highest prob. of exceeding the threshold return.

Module 4.4 : Issues for Individuals

MVO doesn't incorporate assets like human capital into the framework.

→ For investors w/ stable jobs / wages, can model CF like inflation-linked bonds.

→ For investors w/ less stable CF, model the human capital as a mix of equities, bond (corporate / inflation linked).

Residential real estate should be treated in similar fashion

4.e. Use of Monte-Carlo Simulation (MCS)

- address the weakness of MVO
  1. single-period constraint
  2. tax implication of rebalancing (which doesn't happen in MVO)

- Guide the investor to identify the risk tolerance by illustrating the range of outcomes.

4.f. Asset Class Liquidity Consideration

- assets like real estate & infrastructure require a HIGHER LIQUIDITY PREMIUM.
- Harder to model these asset classes b/c:
  - few index to track.
  - not investible indices to be an proxy
- Risk / Return of one specific asset IS NOT R/R of the asset class.

Solution:

- Exclude illiquid asset in MVO but use them to meet separately set target allocation.

- Include illiquid asset in MVO but model the (systematic/idiosyncratic) risk of the specific asset

- Include illiquid asset in MVO using highly diversified asset class inputs, recognizing the actual investment will have diff characteristics.

/ /

MODULE QUIZ 4.4

1. Louise Davey is 26 years old and has just paid off her student loans from college and hopes to begin saving to accumulate a portfolio. She is currently set to begin a career as a licensed insurance salesperson, working primarily on a commission-based pay structure. Louise is an only child and will be the sole beneficiary of her parents' substantial estate, which she estimates she will receive in about 30 years.

   Based on the information provided, which of the following statements is *most accurate*?

   A. Louise's human capital could likely be modeled as an inflation-linked bond.
   B. Louise's capacity to bear investment risk is relatively high.
   C. Louise's human capital is a small component of her total economic worth.

2. When dealing with illiquid assets, such as the investor's personal residence or a private company he owns, it is *most accurate* to say:

   A. a liquidity premium reduces the assets' expected return.
   B. the positions can be excluded when running mean variance optimization for the client.
   C. use data on publicly available funds to model the specific characteristics of the client's holdings when running mean variance optimization for the client.

3. Which of the following methods is the *least appropriate* way of incorporating client risk preferences into asset allocations?

   A. Specify additional constraints.
   B. Specify a maximum return.
   C. Use MCS.

4. Which of the following statements regarding MCS is *most accurate*?

   A. MCS replaces mean variance optimization (MVO) by addressing the limitations of MVO.
   B. There is a high level of consistency between the MCS tools available in the marketplace.
   C. MCS is not necessary to model taxes and portfolio rebalancing in a single period.

1. A (B) A high ability to take risk is plausible. She appears to have no financial capital so human capital appears to be her only capital. Her HC is somewhat riskier & uncertain so not at all like an inflation-linked bond

2. B / Ignoring positions & running MVO for the rest of the portfolio essentially recognizes the client will not sell the position.

3. C (B) 3 methods: specifying additional constraints, specify risk aversion factor & using MCS.

4. C / In a single-period model, taxes are easy to incorporate into the analysis, & rebalancing is irrelevant. MVO can address taxes in a single-period model & MCS is not necessary.

4.5. Risk Budgeting Approach

Marginal Contribution to total risk. = $\beta_i \cdot$ total port. risk (σ).
(MCTR)

Absolute Contribution to total risk = $w_i \cdot MCTR_i$
(ACTR)

Allocation of Asset Class is OPTIMAL if MCTR for all asset classes are the same. & equal to portfolio sharpe ratio.

$$\frac{\text{Excess return}_i}{MCTR_i} = \text{portfolio sharpe ratio.}$$

Figure 4.5: Optimal Risk Allocation Example

	Weight	Excess Return	Beta	MCTR	ACTR	% Contribution to Risk	Ratio of Excess Return to MCTR
U.S. equities	60%	6.50%	1.300	15.60%	9.36%	78.03%	0.417
U.S. bonds	30%	3.66%	0.732	8.78%	2.64%	21.97%	0.417
Cash	10%	0.00%	0.000	0.00%	0.00%	0.00%	
Total portfolio	100%	5.00%	1.000		12.00%	100.00%	

Portfolio sharpe ratio = 5% / 12% = 0.417.
$MCTR_{equity}$ = 12% · 1.3 = 15.6%
$ACTR_{equity}$ = 15.6% · 60% = 9.36%
% of contribution = 9.36% ÷ 12% = 78.03%.
excess return ÷ MCTR =

4.h. Client needs & prefcrenees incorporations
- Specify additional constraints
- speeify risk aversion factor
- use MCS

4.i. Use of Investment factors
- use. opportunity set as factors
(e.g. market exposure, size, valuation, momentum, liquidity, duration, credit, volatility).
- e.g. Fama-French model.

- Factors are zero-dollar investment portfolios.
(long better-performing attributes, short underperforming attributes)
e.g. 0-dollar portfolio that long VALUE. (high book-to-market) & short GROWTH (short book-to-market)

Factors-formed portfolios are not highly correlatcel w/ each other or market portfolio
=> improves risk-return tradeoff. & expands efficient frontier

MODULE QUIZ 4.5

1. The following portfolio is being analyzed. The data is incomplete and does not show other asset classes held in the portfolio.

	Weight	Excess Return	Beta
Canadian equities	50%	8.25%	1.19
Canadian bonds	15%	2.75%	0.88

The portfolio standard deviation is 10%.

Which of the following amounts is *closest* to the ratio of excess returns to marginal contribution to total risk (MCTR) for the Canadian equities within the portfolio?

A. 0.313.
B. 0.595.
C. 0.693.

2. Regarding the use of investment factors in forming an asset allocation, it is *most accurate* to say:
A. this method is superior to asset class–based allocations for institutional portfolios.
B. this method will replicate the allocations produced by asset class–based allocations for institutional portfolios.
C. factor exposures may be investable by forming a series of zero-dollar long/short portfolios.

1. C $MCTR_{equities} = 1.19 \times 10\% = 11.9\%$

Ratio of excess return to $MCTR_{equities}$

$= 8.2\% / 11.9\% = 0.693$

2. B © Not all factors are replicable, but the ones that are can be obtained with a position that is long the desired factor (e.g. growth) & short the undesired factor (e.g. value). Neither asset class-based allocation nor factor-based alloc is superior.

Module 46. ALM Approach

Liability-Relative Asset Allocation

Plan Surplus = MV(Investment Portfolio assets) − PV(Pension Liabilities)

↑ 0 ⇔ fully funded

Funding Ratio = MV(Assets) / PV(Liabilities)

↑ 1 (=> Fully funded
<1 (=> underfunded
>1 (=> overfunded

4.j. Chareeterfics of Liability
- Fixed vs. contingent.
- Legal vs. quasi-legal.
- Duration & convexity
- Liability val vs. size of Sponsoring org.
- Factors that affect future CF.
- Timing consideration
- Regulations

Approaches to Liability-Relative. Asset Alloc.

(1) Surplus Optimization

extension of MVO to determine an efficient frontier based on surplus

$R_{s,m}$ = Surplus Return = $\frac{\Delta \text{Asset Val} - \Delta \text{lias. val.}}{\text{inital asset value}}$

$$U_m = E(R_{s,m}) - 0.005 \times \lambda \times Var_{s,m}$$

(2) Two-Portfolio Approach.

- create an asset portfolio that hedges liabilities (+) reminder managed independently using MVO (outlined in FI readings).

- most used in insurance companies & overfunded companies wanting to minimize risk of underfunding

Limitations:

- if funding ratio < 1 ⇒ difficult to create hedging portfolio

- Not all kinds of risk can be captured in a hedging portfolio

(1) & (2) both feature composition of liab. that is already in place. when asset allocation decisions are made.
(i.e. asset & liab asset allocation decisions are made INDEPENDENTLY).
⇒ not always the case.

(3) Integrated Asset - Liability Approach

- continuous feedback loops.
- over multiple period

MODULE QUIZ 4.6

1. Which of the following items is *best* described as a contingent liability?
   A. A company's fixed-coupon debt.
   B. An insurance company's obligations to pay policyholders.
   C. Planned distributions by a foundation.
2. A bank is *most likely* to use which of the following approaches to liability-relative asset allocation?
   A. Surplus efficient frontier approach.
   B. Integrated asset-liability approach.
   C. Two-portfolio approach.

1. B ✓ Contingent = uncertain in timing, amount or both

2. C (B) A bank can typically vary the nature of both its assets & liabilities. A joint optimization is common. The other approaches take the liabilities as a given & only focus on managing the assets.

Module 4.7 Goals-based & Misc. Approach

Goal-based approach:

- break down each investment goal into one subportfolio.

- Taxable & Tax-exempt investments are part of the opportunity set.

- Instead of expected avg. return on portfolio, identify/document "min. expectation" ??

EXAMPLE: Goals-based approach

An investor has a goal of having $500,000 to fund his daughter's undergraduate and graduate education beginning in 10 years with a 90% required probability of success. He also has a goal of transferring $6,000,000 to his daughter in 30 years with a required probability of success of 75%.

The modules in the table are available to the adviser to implement each of these goals.

	Module A	Module B	Module C
Expected return	5%	6%	8%
Expected volatility	4%	7%	14%

The annual minimum expectation returns at various probabilities of success over the 10-year time horizon are as follows:

Required Success	Module A	Module B	Module C
90%	3.0%	2.4%	–2.2%
75%	3.6%	3.8%	1.7%

The annual minimum expectation returns at various probabilities of success over the 30-year time horizon are as follows:

Required Success	Module A	Module B	Module C
90%	4.0%	4.3%	4.7%
75%	4.1%	4.8%	5.2%

Determine the module to use and **calculate** the amount to invest in that module to meet each goal. Treat each goal separately.

For the 1st goal, Module A has the highest return given a 90% required prob. of success of 3.0% over the 10-year time horizon.

$\Rightarrow$ investment today = $500,000 \times (1+3.0\%)^{-10} = \$372,047$

For the 2nd goal, Module C has the highest return given a 75% required prob. of success of 5.2% over the 30-year time horizon

$\Rightarrow$ investment today = $6,000,000 \times (1+5.2\%)^{-30} = \$1,311,231.$

4.n. Other approaches to allocate assets

- 120 MINUS YOUR AGE
  (i.e. 120 - your age = % allocation to eq.).

- 60/40 Split.

popular w/ uni endowment fund

- Endowment Model / Yale Model
  (allocate larger % to alt. investment than typical MVO)
  aiming to profit via. information asymmetry & lack of liquidity

- Risk Parity
  ensures each asset class contributes THE SAME OF RISK

- 1/N
  n being no. of asset classes.
  (i.e. equally weighted)

  Performs better than expected

Module 4.8. Rebalancing Policy

purpose (CFA): adjusting asset allocation to move towards the original allocation goal.

Optimal Corridor width / Rebalancing Range is based on:

- risk tolerance
- transaction cost
- correlation w/ rest of portfolio
- volatility of rest of portfolio

MODULE QUIZ 4.7, 4.8

1. Which of the following statements regarding subportfolios within the context of the goals-based approach to asset allocation is *most accurate*?
   A. The most significant difference between the subportfolios is the return-risk tradeoff.
   B. Higher priority goals require higher return assets.
   C. The size of the investment in a particular subportfolio is the present value of the future goal discounted by the risk-free rate.
2. Which of the following statements regarding the "120 minus your age" heuristic is *most correct*?
   A. A 70-year-old individual should have 50% of the investment portfolio invested in equity securities.
   B. A 60-year-old individual should have 60% of the investment portfolio invested in fixed-income securities.
   C. The approach generally does a poor job of mimicking the allocations of target-date retirement funds.
3. A portfolio has invested in Asset Class Z and the manager is setting the optimal rebalancing corridor. The corridor will be wider if:
   A. the rest of the portfolio is highly volatile.
   B. the correlation of Z with the rest of the portfolio is highly positive.
   C. transaction costs are low.

1. C (A) C is wrong b/c it's discounted by expected rate of return, not risk-free rate. The priority of the goal determines the amt of the risk taken. You expect significant diff in risk & return b/w subportfolios.

2. B (A) 120 - your age = % in equity
3. C (B) Narrower corridor = more windows
   High correlation allows wider corridors b/c if the assets move in sync, divergence b/w them is less likely.

/ /