

Deadline: 11/10/2017.

### Objective of the practice

To use a stack to do numerical coding in different bases.

### Background

A *stack* is a *linear data structure* of the LIFO type (*Last-In, First-Out*) in which the data is added and removed by just one side. You must use a stack for integer numbers to solve the *number coding problem* in base  $r$ ,  $2 \leq r \leq 16$ .

### Number codification

In a *positional coding system* (PCS) a number is encoded as a sequence of symbols called *digits*. The sequence of digits is called a *numeral*. The value represented by each digit depends not only on itself but also on its position relative to other digits. A PCS is based on a positive integer *radix* (or *base*)  $r \geq 2$  and the digit set  $\{0, 1, \dots, r-1\}$ . An unsigned integer is represented by a vector of length  $k$  where:

$$x_k x_{k-1} \dots x_2 x_1 = \sum_{i=1}^k x_i \cdot r^{i-1}$$

The term  $x_i \cdot r^{i-1}$  is the corresponding to the  $i$ -th position where:

- $x_i$  is the  $i$ -th digit (from right to left).
- $r^{i-1}$  is the *positional value*.
- $x_i \cdot r^{i-1}$  is the *actual value* of the  $i$ -th digit.

For example, let  $n = 100111011$  be a *binary* numeral (i.e., with  $r = 2$ ; we can also write  $100111011_2$  to denote it). Then:

$$\begin{aligned} n &= 1 \cdot 2^8 + 0 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \\ &= 256 + 32 + 16 + 8 + 2 + 1 \\ &= 315 \end{aligned}$$

Now we say that  $100111011_2 = 315$ . We call the leftmost digit ( $x_k$ ) the *most significant digit* (MSD) due to its position corresponds to the greater positional value.

### Number coding in radix $r$

Assume an integer  $n \geq 0$ . To encode  $n$  in radix  $r$  you must implement the *algorithm of remainders*, which can be stated as follows:

1. Calculate  $x \leftarrow n \bmod r$  ( $n \bmod r$  is the *remainder* of the euclidean division (*integer division*) of  $n$  by  $r$ , also called *the modulo  $r$  of  $n$* ). Reserve  $x$  as the MSD of your numeric encoding.
2. Calculate  $n \leftarrow n \div r$  ( $n \div r$  is the integer division of  $n$  by  $r$ ).
3. Repeat from step 1 while  $n > 0$ .
4. The algorithm ends and provides as a result the sequence of reminders that was generated.

For example, suppose you want to code 315 to the radix 2 ( $r = 2$ ). Then you have the following run:

	$n$	$x$	
Beginning →	315	1	
	157	1	
	78	0	
	39	1	
	19	1	
	9	1	
	4	0	
	2	0	
	1	1	← The last digit is the MSD
End →	0		

Therefore,  $315 = 100111011_2$ .

### Implementation

You must program the algorithm of remainders to code a nonnegative integer to any radix  $2 \leq r \leq 16$ . Note that the encoding algorithm generates an inverted sequence. You must reverse the sequence using stack. For example, for  $n = 315$  we have the following:

<i>Inverted sequence of digits (correct order)</i>	<i>Stack</i>	<i>Sequence of digits as generated by the algorithm</i>
100111011 ←	<div style="border: 1px solid black; padding: 5px; display: inline-block; text-align: center;"> 1 0 0 1 1 1 1 0 1 1 </div>	← 110111001

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You must implement your stack *dynamically*,  
i.e., on a linked list. Do not limit its capacity.

Let  $x$  be a digit. Then  $0 \leq x < r$ . E.g., if  $r = 2$  then  $0 \leq x < 2$  (i.e.,  $x = 0$  or  $x = 1$ ) and if  $r = 10$ , then  $0 \leq x < 10$ . For a radix greather than 10 ( $11 \leq r \leq 16$ ), we use the following additional symbols:

$x$	Symbol	$x$	Symbol
10	$A$	13	$D$
11	$B$	14	$E$
12	$C$	15	$F$

E.g., for the hexadecimal encoding of 43006 ( $r = 16$ ), the algorithm of remainders generates the succession 14, 15, 7 and 10. According to the suggested symbology, this sequence is  $E$ ,  $F$ , 7 and  $A$ . Then, the hexadecimal coding of 43006 is  $A7FE$ .

**Operation of your program:** Your program must ask for a nonnegative number  $n$  and a radix  $r$  in the range of 2 to 16. In response it must show on screen the codification of  $n$  with radix  $r$ .

### Examples

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1. A run of your program should look like:

```
Number:  2017
Radix:   7
```

```
Encoding: 5611
```

2. Assume  $n = 7791$ . Below is the coding to each of the possible radices:

$r$	Encoding	$r$	Encoding
		9	11616
2	1111001101111	10	7791
3	101200120	11	5943
4	1321233	12	4613
5	222131	13	3714
6	100023	14	2BA7
7	31500	15	2496
8	17157	16	1E6F

### Delivery

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Send your code in a .zip, .rar or .7z file named `encoding.<name>.zip`, where *<name>* is your name. For example: `encoding.tyron.lannister.zip`. Send your file to [aaguilar.itszapopan@gmail.com](mailto:aaguilar.itszapopan@gmail.com) no later than the deadline with the subject *Encoding*. They will not be reviewed consignments that fail to comply with any of these requirements or if they do not compile. The deadline is on Friday, November 10th.