Computer Systems

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1 Data Representation and Manipulation

1.1 Binary

Computers consist of a collection of switches called transistors, which can either be on or off. This leads to binary representation, where off is 0 and on is 1. All data and instructions are stored in binary. Binary digits are known as 'bits'. It is possible to build computers that use other number systems, but it is less expensive and more reliable to use basic components with only two states.

1.2 Characters

Characters are represented as binary values using text encoding schemes such as ASCII and Unicode. Modern schemes support internationalisation and tend to use 8 bit, 16 bit or 32 bit to encode characters. Strings are sequences of characters.

1.3 Number Bases

1.3.1 Decimal to Binary, Octal and Hexadecimal

234_{10} :

$$\left.\begin{array}{r|l} 2\lfloor 234 & 0 \\ 2\lfloor 117 & 1 \\ 2\lfloor 58 & 0 \\ 2\lfloor 29 & 1 \\ 2\lfloor 14 & 0 \\ 2\lfloor 7 & 1 \\ 2\lfloor 3 & 1 \\ 2\lfloor 1 & 1 \end{array}\right\} = 11101010_2$$

234_{10} :

$$\left.\begin{array}{r|l} 8\lfloor 234 & 2 \\ 8\lfloor 29 & 5 \\ 8\lfloor 3 & 3 \end{array}\right\} = 352_8$$

234_{10} :

$$\left.\begin{array}{r|l} 16\lfloor 234 & 10 \\ 16\lfloor 14 & 14 \end{array}\right\} = EA_{16}$$

1.3.2 Binary, Octal and Hexadecimal to Decimal

101011_2 :

$$\begin{aligned} 1 \times 2^0 &= 1 \\ 1 \times 2^1 &= 2 \\ 0 \times 2^2 &= 0 \\ 1 \times 2^3 &= 8 \\ 0 \times 2^4 &= 0 \\ 1 \times 2^5 &= \underline{32} \\ &= 43_{10} \end{aligned}$$

724_8 :

$$\begin{aligned} 4 \times 8^0 &= 4 \\ 2 \times 8^1 &= 16 \\ 7 \times 8^2 &= \underline{448} \\ &= 468_{10} \end{aligned}$$

ABC_{16} :

$$\begin{aligned} C \times 16^0 &= 12 \\ B \times 16^1 &= 176 \\ A \times 16^2 &= \underline{2560} \\ &= 2748_{10} \end{aligned}$$

1.3.3 Binary to Octal and Octal to Binary

$1011010111_2 = 1327_8$:

1 011 010 111

1 3 2 7

$705_8 = 111000101_2$:

7 0 5

111 000 101

1.3.4 Binary to Hexadecimal and Hexadecimal to Binary

$1010111011_2 = 2BB_{16}$:

10 1011 1011

2 B B

$10AF_{16} = 1000010101111_2$:

1 0 A F

0001 0000 1010 1111

1.3.5 Octal to Hexadecimal and Hexadecimal to Octal

Conversions between octal and hexadecimal can be performed by first converting to binary.

1.4 Integers in Binary

Counting in powers of 2.

Prefix	Power of 2	Value
kibi	2^{10}	1024
mebi	2^{20}	1048576
gibi	2^{30}	1073741824
tebi	2^{40}	1099511628000

1.4.1 Overflow

In a computer, an integer is represented by a fixed number of bits. The maximum value that can be stored in an unsigned integer of n bits is $2^n - 1$. It is possible that the addition of two n bit numbers yields a result that requires $n + 1$ bits.

In Java, the 'overflow' bits are lost with no error. The remaining bits give the wrong answer. It is important to make sure the data type used is big enough for the values it will represent.

1.4.2 Two's Complement

In 8 bit arithmetic, $255 + 1$ appears to be 0 due to overflow. In this case, 255 is behaving like -1. This leads to the 'two's complement' representation of negative integers in binary.

The two's complement representation of $-x$ is equivalent to the unsigned binary representation of $2^n - x$. Another method to find the representation is to flip all the bits of the binary representation of x and add 1 to the least significant bit.

Using two's complement, a signed binary integer of n bits can hold any value from -2^{n-1} to $2^{n-1}-1$, inclusive. The most significant bit represents -2^{n-1}. If the number of bits in a signed number is increased, the new bits are given the same value as the most significant bit. This is known as sign extension.

In Java, all integers are signed.

1.5 Real Numbers in Binary

1.5.1 Fixed Point Decimal to Binary

The integral part is converted as usual. The fractional part is converted by doubling as follows.

$$
\begin{aligned}
0.537_{10} = 0.100010_2 :& \\
0.537 \times 2 &= \underline{1}.074 \\
0.074 \times 2 &= \underline{0}.148 \\
0.148 \times 2 &= \underline{0}.296 \\
0.296 \times 2 &= \underline{0}.592 \\
0.592 \times 2 &= \underline{1}.184 \\
0.184 \times 2 &= \underline{0}.368
\end{aligned}
$$

1.5.2 Fixed Point Binary to Decimal

$$
\begin{aligned}
1101.0101_2 :& \\
1 \times 2^{-4} &= \ 0.0625 \\
0 \times 2^{-3} &= \ 0 \\
1 \times 2^{-2} &= \ 0.25 \\
0 \times 2^{-1} &= \ 0 \\
1 \times 2^{0} &= \ 1 \\
0 \times 2^{1} &= \ 0 \\
1 \times 2^{2} &= \ 4 \\
1 \times 2^{3} &= \ \underline{8} \\
&= 13.3125_{10}
\end{aligned}
$$

1.5.3 Floating Point Numbers

Fixed point is convenient and intuitive, but has two major problems.

1. Numerical precision — only values that are multiples of the smallest used power of two can be represented.
2. Numerical range — fractional precision comes at the expense of numerical range.

Floating point representation in binary is similar to scientific notation in decimal. In binary, real numbers can be represented in the form $\pm m \times 2^e$. Floating point numbers consist of a sign bit ($\pm$, 0 for positive or 1 for negative), a mantissa (m, the significant bits) and an exponent (e, two's complement signed binary).

S	Offset exponent, e'	Normalised mantissa, m'

The stored representation of a floating point number.

Since the mantissa is normalised (unless the value is small enough to represent without an exponent), the leading bit is almost always 1. It is therefore omitted from the stored representation. The exponent has a bias (offset) of $2^{n-1} - 1$ added to it for engineering purposes.

Floating point data types in Java.

Type	Bits				Bytes	Exponent bias
	Sign	Mantissa	Exponent	Total		
`float`	1	23	8	32	4	127
`double`	1	52	11	64	8	1023

With the 52 bit mantissa of a `double` (and the additional hidden bit), $2^{53} \approx 8 \times 10^{15}$. Hence, the `double` data type can be used to represent 15 significant decimal digits.

In Java, special values are returned for floating point overflow and other unusual circumstances. For example, if the result is too large for the `double` data type, `Double.POSITIVE_INFINITY` or `Double.NEGATIVE_INFINITY` are returned. If the result is indistinguishable from zero but known to be negative, `-0.0` is returned. If the result is not a real number, `Double.NaN` is returned.

1.5.4 Numerical Precision

Floating point arithmetic loses accuracy in its less significant digits. This is an issue even in values of type `double`.

It is a bad idea to use floating point representation for money — i.e. with integral pounds and fractional pence — as this will result in rounding errors. Taking £0.10 as an example, $0.10_{10} = 0.0\overline{0011}_2$.

Since currency is inherently integral, integer types with suitable range, such as `int`, `long` or `java.math.BigInteger`, should be used to represent multiples of the smallest denomination.

2 Introduction to Computer Architecture

2.1 Anatomy of a Computer System

A computer is a complex system (machine) that can be instructed to carry out sequences of arithmetic or logical operations automatically via computer programming.

A computing device must be able to

- load a program (input interface),
- process instructions in the correct order (track progress, storage and decoding of instructions),
- access data according to its instructions (local storage),
- perform computations (calculation 'engine'),
- make decisions according to its computations (control mechanism), and
- send results to an external device (output interface).

Thus, all computers, regardless of their implementing technology, have five basic subsystems.

1. Memory
2. Control unit
3. Arithmetic logic unit (ALU)
4. Input unit
5. Output unit

2.1.1 Memory

Memory locations have finite capacity. Data may not fit in a memory location. Blocks of four or eight bytes are used so often as a unit that they are known as memory 'words'.

Computer memory is known as random access memory (RAM). This simply means that the computer can refer to memory locations in any order.

2.1.2 Control unit

The control unit of a computer is where the fetch/execute cycle occurs. It fetches a machine instruction from memory and performs other operations of the fetch/execute cycle accordingly.

2.1.3 Arithmetic Logic Unit (ALU)

The arithmetic logic unit carries out each machine instruction with a separate circuit. It contains various circuits for arithmetic and logic, such as addition, subtraction, multiplication, comparison and logic gates.

2.1.4 Input and Output Units

These components are the wires and circuits through which information travels into and out of a computer. Without input or output, a computer is useless.

Peripherals connect to the computer through input/output (I/O) ports. They are not considered parts of the computer.

2.2 The Fetch-Execute Cycle

A program is loaded into memory and the address of the first instruction is placed in the program counter (PC).

The fetch-execute cycle proceeds as follows.

1. Instruction fetch (IF)
2. Instruction decode (ID)
3. Data fetch (DF) / operand fetch (OF)
4. Instruction execution (EX)
5. Result return (RR) / store (ST)

2.2.1 Instruction Fetch (IF)

The instruction at the memory address given by the PC is copied to the instruction register of the control unit. The PC is incremented to point at the next instruction to be fetched.

2.2.2 Instruction Decode (ID)

The ALU is prepared for the operation specified by the instruction. The decoder finds the addresses of the instruction operands. These addresses are passed to the ALU circuit that fetches the operands from memory in the next stage. The decoder also finds the destination address for the result. This is passed to the RR circuit.

2.2.3 Data fetch (DF)

The operands are copied from the specified memory addresses into the ALU circuits. The data remains in memory and is not destroyed.

2.2.4 Instruction Execute (EX)

The ALU performs the operation on its operands. The result is held in its circuitry.

2.2.5 Result Return (RR)

The result of the EX stage is stored at the specified destination memory address. The cycle begins again.

2.3 Machine Instructions

A computer "knows" very few instructions. The decoder hardware in the control unit recognises, and the ALU performs, of the order of 100 instructions. Everything that a computer does must be

reduced to some combination of these primitive instructions. Computers can carry out millions of these instructions per second.

3 Instructions, Assembly Language and Machine Code

3.1 The von Neumann Architecture

3.1.1 Central Processing Unit (CPU)

The central processing unit (CPU) consists of a control unit, ALU and registers. These registers include

- the program counter (PC) (holds the address of the next instruction),
- the instruction register (holds the instruction currently being decoded or executed),
- the address register (holds a memory address from which data will be fetched, or to which data will be returned), and
- the accumulator (holds temporarily the results of ALU computations).

3.1.2 System Bus

The system bus includes

- a control bus (carries commands from the CPU and returns status signals from devices),
- an address bus (carries information about the device with which the CPU is communicating, namely physical memory addresses), and
- a data bus (carries the data being processed).

3.1.3 Memory

In the von Neumann architecture, the memory contains both program instructions and data.

3.1.4 Clock

A clock cycle is a signal that oscillates between high and low. It is used to coordinate actions. The rate of the fetch-execute cycle is determined by the clock. Instructions begin execution on the rising edge of the signal. The time between rising edges is known as the 'clock period'. The time between the rising and falling edges of the signal is known as the 'clock width'. The number of clock cycles per second is used to determine the speed of a CPU.

Modern computers attempt to start an instruction on each clock tick. Since each stage of the fetch-execute cycle is handled by separate circuitry, the fetch unit is freed to start the next instruction before the previous instruction is complete. This process is known as 'pipelining'. When the pipeline is filled, five instructions are in process at a time, each at a different stage of the fetch-execute cycle Additionally, one instruction is finished on each clock tick, making it appear as though the computer is running one instruction per tick.

3.2 The Harvard Architecture

The main difference between the von Neumann and Harvard architectures is their storage of instructions and data in memory.

A von Neumann (or 'stored-program') machine stores its instructions and data in a shared memory. This means that an instruction fetch and a data operation cannot occur at the same time because they share a common bus. This is known as the 'von Neumann bottleneck' and can impinge on the performance of the system. However, the architecture allows for self-modifying code that can tune its performance at runtime.

A Harvard machine stores its instructions and data in separate memories. This makes the machine more complex, but allows instruction fetch and data operation to occur at the same time.

3.3 MIPS R4000

MIPS R4000 uses the modified Harvard architecture. Instructions and data are stored in the same memory (as in von Neumann), but they have separate interfaces (as in Harvard).

MIPS R4000 was one of the first 64 bit microprocessors. It features integrated caches (primary on-chip and secondary off-chip), an integrated floating point unit and a deep pipeline. It uses MIPS III — a compact instruction set with a regular format — which makes it easy to convert from high-level code to machine code.

3.3.1 Registers

The CPU contains 32 registers denoted `$0`–`$31` (or `r0`–`r31`). The registers can be either 32 bit or 64 bit wide, depending on the mode of operation. `$0` (`r0`) always stores zero. `$31` (`r31`) is the link register used by jump and link instructions. It should not be used by other instructions.

The ALU takes input from up to two registers and sends output to registers only.

3.3.2 Deep Pipeline

The typical MIPS pipeline consists of the following five stages.

1. Instruction fetch (IF)
2. Instruction decode (ID) and register fetch (RF)
3. Execute (EX)
4. Memory access (MEM)
5. Write back (WB)

MIPS R4000 is super-pipelined; its pipeline contains eight stages rather than the regular five. The additional stages exist due to the extension of the IF and MEM stages to account for cache overheads. When the pipeline is full, eight instructions are in process at a time — i.e. the pipeline is eight-deep.

3.4 Assembly Language

Binary is the only form in which a computer can be given instructions. Computers can be programmed to translate instructions expressed in other forms into binary code. This is known as 'assembly'.

Assembly language is a human-readable alternative to machine language. A computer can scan assembly code and convert words to binary. The binary is assembled into instructions.

High-level languages are compiled to assembly language, which is then assembled into binary.

3.5 Instruction Sets

Every machine has a unique instruction set architecture (ISA). This is the set of primitive instructions that the CPU can execute. MIPS R4000 uses the MIPS III ISA. Each MIPS III instruction has a 32 bit representation. It can be represented as human-readable assembly code, or binary machine code. There are three types of MIPS III instruction.

I-type requires one or two registers and an operand.
R-type requires three registers.
J-type requires an operand.

4 Compilation, Interpretation and an Overview of the Java Virtual Machine (JVM)

4.1 Levels of Programming Languages

High-level programming languages, such as Java, C, C++ and C# are close to the English language and, therefore, easy for humans to read and write. Code written in these languages is more concerned with problem-solving than implementation. Programmers are less likely to make errors when programming in these languages and do not need to worry about memory management and other complexities.

Low-level languages, such as machine code, use instructions stored in memory (opcodes) and refer to specific locations in memory. Code written in these languages reflects the processes being used rather than the problem being solved. Machine language is difficult for humans to read and write. Since it runs directly on hardware, it is also hardware-specific.

Assembly language is slightly higher than machine code. It uses mnemonics so that it can be more easily read or written by humans. This is used to translate high-level languages to machine code.

4.2 Language Processing Systems

To run on a computer, a program must be supplied in machine code. Programs written in high-level languages are converted to binary through a series of phases.

1. Preprocessor (prepares high-level code for the compiler)
2. Compiler/interpreter (produces assembly/intermediate code)
3. Assembler (produces relocatable machine code)
4. Linker/loader (produces object code)

The preprocessor organises and prepares the source code for the compiler. This includes importing packages, macro processing and file inclusion.

The compiler reads the entire source code in one go and creates tokens. It checks the meaning of these tokens and generates intermediate assembly code. Alternatively, the interpreter reads the source statement by statement, converting each statement to intermediate code that is executed immediately.

The assembler calculates relative addresses for jumps and produces relocatable machine code.

The linker combines assembled parts into a whole. Alternatively, the loader places the code directly into memory.

4.3 High-Level Program Execution

A program written in a high-level language can be run in two different ways.

1. Compiled into a program in the native machine language of the target machine
2. Directly interpreted and executed via simulation on the target machine

4.4 Compilation

A compiler converts source code into assembly, object or machine code that does the same thing as the original. Object code is usually relocatable, so it can later be linked or loaded.

This is done just once for each program. Hardware features can be exploited to optimise object code so that it will run more quickly.

However, this is more difficult than interpretation, as the compiler must understand the entire program in order to convert it. Compilers are also hardware-dependent, so cannot run on different platforms.

A compiler runs on the same platform as the target machine. A cross-compiler can produce code that will run only on a separate platform.

A compiler can be divided into a front-end and a back-end. Both perform their operations in a sequence of phases that each generate a data structure to be used by the following phase.

Front-end:

1. Lexical analysis
2. Syntax analysis
3. Semantic analysis
4. Intermediate code generation

Back-end:

5. Optimisation
6. Code generation

4.4.1 Lexical Analysis

During lexical analysis, the compiler breaks the source code into meaningful words known as 'lexemes' and generates a sequence of tokens from the lexemes. A lexeme is a word recognised by the compiler according to the language specification. This includes keywords, identifiers, operators etc. A token is an object describing a lexeme. Along with the value of the lexeme, it includes information about the type of the lexeme (keyword, identifier, operator etc.).

4.4.2 Syntax Analysis (Parsing)

During syntax analysis, the compiler interprets the structure of the source code. This is known as 'parsing' and involves grouping tokens into higher-level constructs. This phase produces an abstract syntax tree (AST).

This is also the phase where syntax errors are detected. If no error is found, the compiler continues to the semantic analysis phase.

4.4.3 Semantic Analysis

During semantic analysis, the compiler interprets the meaning of the AST. The compiler checks that the program is consistent with the rules of the source language.

The compiler can deal with ambiguity in a number of ways.

- Type inference (the compiler annotates nodes in the AST with inferred type information)
- Type checking (the compiler checks that all values assigned to variables are of the correct type)
- Symbol management (the compiler uses a symbol table to determine whether variables have been declared or whether multiple variables with the same name exist in the same scope)

Semantic analysis produces an annotated AST.

4.4.4 Intermediate Code Generation

During intermediate code generation, the compiler produces code in an intermediate language between that of the source code and machine language.

4.4.5 Optimisation

During optimisation, the compiler simplifies or removes unnecessary code. This allows the program to run more quickly or use fewer resources.

4.4.6 Code Generation

During code generation, the compiler maps the optimised intermediate code to the target machine language.

4.5 Interpretation

An interpreter is a program that follows the source code and performs appropriate actions accordingly. A CPU can be viewed as a hardware implementation of an interpreter for machine code.

Interpreters begin execution immediately and facilitate interactive debugging and testing. A user can read and modify the values of variables and invoke procedures from the command line. Interpreted languages are not hardware-dependent. However, interpreted programs have slower execution than compiled programs.

Compilers are compute-intensive and require more preparation time. Additionally, compiled programs are hardware-dependent. However, they can run very quickly.

4.6 Combined Compilation and Interpretation

Combined compilation and interpretation is a method by which programs are compiled to an intermediate language that can be interpreted efficiently. Execution of programs produced by this method is slower than pure compilation, but quicker than pure interpretation.

Compilation for any platform requires only a single compiler that is independent of the target CPU. Interpretation of the intermediate language is delegated to the target CPU.

Java was conceived as a language that uses combined compilation and interpretation. The `javac` compiler converts `.java` source code to `.class` bytecode, which is interpreted by the Java Runtime Environment (JRE) on the Java Virtual Machine (JVM).

4.7 Compilation and Execution on Virtual Machines

A virtual machine executes an instruction stream using software rather than hardware. Virtual machines emulate hardware using software and are, therefore, hardware-independent. Virtual machines are used in languages such as Java, Pascal and C#.

Java compilers generate bytecode that is interpreted by the JVM. This requires a JVM to exist on the target machine. The JVM may translate portions of bytecode to machine code at runtime via just-in-time (JIT) compilation if it finds those portions are used frequently.

5 Subroutines and Stacks

5.1 How Subroutines Work

5.1.1 Call and Return Instructions

When execution jumps from line n to a subroutine, there needs to be a way to instruct the computer to continue execution on line $n + 1$ once the subroutine is complete.

Two hypothetical instructions that could be used to achieve this are `call`, which jumps to the subroutine and stores the return address (the current PC value) somewhere suitable, and `ret`, which reads the return address from where it was stored and loads it into the PC.

5.1.2 Method 1 — Storing Return Address as First Byte

Using this method, the return address is stored as the first byte of the subroutine. The instruction `call N` would store the return address at location N and begin executing the subroutine at location $N + 1$. The instruction `ret N` would load the return address stored at location N into the PC.

The disadvantage of this method is that only one return address can be stored for each subroutine. Consequently, a subroutine could not call itself, as this would require two return addresses (one for the original call and another for the recursive call).

FORTRAN — the first commercially available high-level programming language — disallowed recursion in order to implement this method.

5.1.3 Introduction to Stacks

A stack is a data structure that can flexibly store a variable number of bytes. Stacks employ 'last in, first out' (LIFO) semantics. Elements are 'pushed' to or 'popped' from the top of the stack. A stack pointer (SP) is a CPU register that points to the top of the stack. It is not necessary to know where the bottom of the stack is, but programmers must make sure they do not attempt to pop from an empty stack.

When a value is pushed to the stack, it is written to memory at the address given by the SP. The SP is then incremented.

When a value is popped from the stack, the SP is decremented, then the value is read from memory at the address given by SP. Popped values remain in memory and are later overwritten by pushed values.

5.1.4 Method 2 — Storing Return Address on a Stack

Using this method, the return address is pushed to a stack when the subroutine is entered. The instruction `call N` would push the return address to the stack and begin executing the subroutine at location *N*. The instruction `ret` would pop the return address from the stack and load it into the PC.

This method allows multiple return addresses to be stored and, therefore, makes recursion possible.

5.1.5 Saving Registers

A subroutine may need to use CPU registers in its calculations. However, the previous register values will need to be restored when execution returns to the caller.

A common solution to this problem is to push all register values to the stack at the beginning of a subroutine (after the return address has been pushed) and to pop the register values from the stack at the end of the subroutine (before the return address is popped). Register values are popped in reverse order.

The return address and saved registers are known as a 'stack frame'.

5.2 Stacks for Calculations

5.2.1 Reverse Polish Notation (RPN)

Reverse Polish notation (RPN) or postfix notation is a method of writing calculations in which operators follow their operands and the order of operations is as written (no brackets are needed). This differs from standard infix notation, in which operators are written between or around their operands and the order of operations is determined by precedence.

RPN is a useful way of implementing calculations with stacks. If the next item in the input stream is a value or variable, it is pushed to the operand stack. If the next item is an operation, the relevant number of operands is popped from the operand stack, the operation is performed, and the result is pushed to the operand stack.

The following infix calculation and postfix calculation are equivalent.

$$(5 + 2) \times \sqrt{x \times x + y \times y} + 8$$

$$5\ 2 + x\ x \times y\ y \times + \sqrt{\ } \times 8 +$$

One notation that shows the effect of an operation, such as subtraction, on a stack is as follows.

$$\ldots, \text{val1}, \text{val2} \quad \rightarrow \quad \ldots, \text{val1} - \text{val2}$$

Java bytecode uses operand stacks for calculations. In Java, each method call has its own operand stack.

5.2.2 Stack Machines

Stack machines use a return stack for subroutine return, and an operand stack for RPN calculations. Registers are not needed for calculations. This allows more space for calculations, and machine instructions do not need to specify registers. However, it is difficult to know where things are on the stack.

The term 'operand' could refer either to the byte(s) after the instruction opcode in memory, or the entries in the operand stack.

5.3 Bitwise Boolean Operations

The XOR operation is an exclusive OR. It returns true only if exactly one operand is true.

Truth tables.

A	B	A AND B	A OR B	A XOR B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

5.4 Conditional and Unconditional Jumps

No stacks are used when an unconditional jump is executed. Conditional jumps are only executed if a condition is true; if the condition is false, execution continues to the next instruction.

Java bytecode contains six comparisons that are used in conditional jump instructions. There are two types of conditional jump instruction. Jump instructions of the form `ifXX` pop one value from the stack and compare it to zero. Jump instructions of the form `if_cmpXX` pop two values from the stack and compare them to each other.

For example, `ifeq N` pops a value from the stack and jumps to location *N* if the value is zero. The instruction `if_cmpeq N` pops two values from the stack and jumps to location *N* if they are equal.

Conditional jump instructions.

Comparison		Instruction	
Symbol	Mnemonic	`ifXX`	`if_cmpXX`
$=$	`eq`	`ifeq`	`if_cmpeq`
$<$	`lt`	`iflt`	`if_cmplt`
$\leq$	`le`	`ifle`	`if_cmple`
$\neq$	`ne`	`ifne`	`if_cmpne`
$>$	`gt`	`ifgt`	`if_cmpgt`
$\geq$	`ge`	`ifge`	`if_cmpge`

6 The Java Virtual Machine and Java Bytecode

6.1 Execution on the Java Virtual Machine

The `javac` compiler compiles Java source code in `.java` files to intermediate-level Java bytecode in `.class` files. As long as there is a JVM on another platform, the bytecode can be run on that platform without recompilation. This feature is known as 'portability'.

The JRE loads Java bytecode files, verifies their internal consistency and executes their methods using the JVM. Different CPUs or operating systems can run the same Java bytecode but use different JREs.

6.2 Stack Frames

Every time a method is called in Java, a stack frame is constructed for it. Each frame has

- a reference to the frame of its calling method,
- space for local variables,
- space for an operand stack
- a PC and SP (for the JVM, not the CPU), and
- other items.

6.2.1 Variable and Operand Storage

All local variables and operand stack entries are stored as 4 bytes. The `long` and `double` data types, which need 8 bytes, are stored as two consecutive entries.

Storage of data types.

Data type	Size		Slots
	/ bit	/ B	
`bool`	1		1
`byte`		1	1
`char`		2	1
`short`		2	1
`int`		4	1
`float`		4	1
reference		4	1
`long`		8	2
`double`		8	2

6.2.2 Local Variables

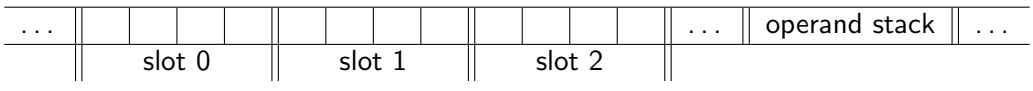
In the context of Java bytecode, local variables include:

1. `this` reference (for non-static methods only)
   - a reference to the object on which the method was called
   - stored as local variable in slot 0
2. Parameters of the method
   - stored from slot 0 onwards for static methods
   - stored from slot 1 onwards for non-static methods
3. Variables declared inside the method
   - stored in slots immediately after parameters

Instance variables and class (static) variables are not stored as local variables since they are defined outside of methods.

6.2.3 Slot Numbers

Java bytecode does not refer to local variables by their identifiers in the source code. Instead, it refers to them by their slot numbers. A local variable of the `long` or `double` data types takes the number of the first of its pair of slots.



Local variable slots in the stack frame.

6.2.4 Stack Overflow

The operand stack can never overflow. The Java compiler calculates exactly how much space is required and the loader checks each method to verify that no operand stack underflow or overflow is possible. Operand stack overflow is different from stack overflow, which occurs when a new frame is needed, but there is not sufficient memory.

6.3 Java Bytecode Instructions

Each Java bytecode instruction takes up at least one byte (for its opcode). It may also have one or more operand bytes. For each opcode, there is a corresponding mnemonic.

A single operand may consist of multiple bytes. In this case, the operand is stored with its most significant byte first (in the smaller address). This order is known as 'big endian'.

6.3.1 Arithmetic

Each arithmetic instruction has a prefix letter that denotes the data type on which it operates. For example, the `iadd` instruction pops two entries from the operand stack, adds them as if they are of the type `int`, and pushes them back onto the stack.

Mnemonic prefixes.

Data type	Prefix	Addition
`byte`	`b`	
`short`	`s`	
`int`	`i`	`iadd`
`long`	`l`	`ladd`
`float`	`f`	`fadd`
`double`	`d`	`dadd`
reference	`a`	

Errors are likely to occur if an instruction is used on the wrong data type. For example, calling `fadd` on `int` entries will result in the wrong answer since the instruction expects entries in floating point representation. If there is only one operand on the stack and the instruction expects two, operand stack underflow will occur. This can occur when `ladd` is called on `int` entries.

No checks are performed during execution. The JRE verifies the bytecode when the class is loaded. This includes checking that types are used consistently.

6.3.2 Pushing

The `bipush <1_byte_operand>` instruction has two prefixes: `b` for `byte` and `i` for `int`. This instruction sign extends its `byte` operand (1 B) to an `int` value (4 B) and pushes the result to the operand stack. Similarly, there exists an `sipush <2_byte_operand>` instruction to push a `short` to the operand stack as an `int`.

There exist seven instructions for pushing common integer constants to the operand stack that do not require an operand. Instructions that take no operand help to save space in the bytecode.

Load instructions of the form `iload <slot_number>` push local variables (indicated by their slot numbers) onto the operand stack. There are special load instructions that take no operand for pushing values stored in common slots to the operand stack (`iload_0`, `iload_1`, `iload_2`, `iload_3`).

Instructions for pushing common integer constants.

Instruction	Value pushed
`iconst_m1`	-1
`iconst_0`	0
`iconst_1`	1
`iconst_2`	2
...	...
`iconst_5`	5

6.3.3 Popping

Store instructions pop entries from the operand stack into local variable slots. For example, `istore <slot_number>` or `istore_2`.

6.3.4 Jumps

In Java bytecode, the source and destination of a jump must be within the same method. It is not possible to jump to another method.

Unconditional jumps are achieved with the `goto <2_byte_offset>` instruction. The operand is a 2 B offset that is added to the address of the instruction to calculate the address of the next instruction to execute. There exists a version of this instruction that takes a 4 B operand to allow for larger jumps: `goto_w <4_byte_offset>`.

Conditional jumps are achieved with instructions of the form `ifeq <offset>` and `if_icmpeq <offset>`.

6.4 Call and Return

6.4.1 The Stack

If method `A` calls a static method `B(p0, p1)` that returns a result:

1. method `A` calculates the arguments on its operand stack,
2. a stack frame is constructed for method `B`,
   - the arguments are popped from the operand stack of method `A`,
   - the values are used to initialise the local variables `p0` and `p1` of method `B`,
3. method `B` calculates the result on its stack frame
4. the result is pushed to the operand stack of method `A`, and
5. execution returns to method `A`, discarding the stack frame of method `B`.

On the operand stack of method `A`, this has the effect of

$$\ldots, \mathsf{p0}, \mathsf{p1} \quad \rightarrow \quad \ldots, \mathsf{result}$$

Each frame has its own PC. While method `B` is under execution, its PC is used. When execution returns to method `A`, it resumes with its old PC value. The local variables of method `A` are unchanged by method `B`.

Linked frames have the effect of a return stack. A chain of linked frames in the JVM is officially known as a stack.

6.4.2 Runtime Constant Pool

The class variables, instance variables and methods of a class may be used by other classes. Since classes are compiled separately, the compiler does not know the address of these items in other classes. Hence, the class, source file and bytecode file must share the same name to create a 'symbolic reference'.

In addition to class variables, instance variables and method definitions, each class in Java has a runtime constant pool that contains read-only constants used in the class. This includes literal constants, symbolic references to methods and their classes, types and more.

Each pool entry has an index. Each stack frame includes a pointer to the constant pool of its class in its space for 'other items'.

6.4.3 Static Method Calls

Static methods are called using the `invokestatic <2_byte_index>` instruction. The operand is an index in the runtime constant pool of the class of the current method. The indexed constant pool entry is a symbolic reference to the requested class and method.

The JVM uses this information to find the address of the class, the size needed for the new frame, and the address of the bytecode for the method.

6.4.4 Method Returns

The return instruction for void methods is `return`. This has no effect on the operand stack of the caller.

For methods that return a result, the instruction is of the form `ireturn`. This pops the result from the operand stack of the current frame and pushes it to the operand stack of the caller.

After either of these return instructions, the current frame is discarded and execution continues on the frame of the caller, resuming from its PC.

6.5 Java Disassembler

The Java 'disassembler' (`javap`) converts Java bytecode to mnemonics with the `-c` option. For example, `MyClass.class` can be viewed with `javap -c MyClass`. Jump instruction operands in

disassembler output show the destination instruction address rather than the offset used in the bytecode.

7 Complexity of Algorithms

7.1 Algorithm Design and Analysis

An algorithm is an effective method for solving a problem expressed as a finite sequence of instructions. Often, multiple algorithms exist for the same task. There are many criteria for choosing a suitable algorithm for a particular task, including efficiency, simplicity, clarity, elegance and proof. It is also necessary to consider whether the algorithm is always correct, always terminates or whether the problem can even be solved with an algorithm.

In general, efficiency is achieved by finding a balance between many conflicting parameters, such as run time, response time, memory usage, bandwidth usage and power consumption. One very important issue is understanding how problems and algorithms grow in complexity. Although a solution may work very well for small problems, it is important to know how its performance may be affected if the problem becomes larger.

It may be possible to improve code so that it runs quickly or uses less memory, but it may be the case that an algorithm will inherently perform worse as the amount of data increases. It may even be the case that the problem itself is inherently difficult and becomes more difficult as the size of the problem grows. Sometimes the problem can be modified so that it still satisfies the overall requirements, but behaves in a more manageable way.

Algorithm efficiency can be considered in terms of time and space.

Space (or bandwidth) complexity is not considered in the same way as computational complexity, since memory and bandwidth are limited by practical constraints. The bandwidth of a system can often be increased if necessary.

Instead, computational complexity is usually considered in terms of 'time complexity' — a measure of how much computation is involved in solving a problem. It is not necessary to know quantitatively exactly how many times harder a problem will grow. Often a qualitative indication of complexity will suffice.

7.2 Time Complexity

There are two ways to determine the run time of an algorithm.

The first method is to measure empirically. This involves benchmarking the algorithm using a representative set of inputs. This does not measure the complexity of the algorithm, rather the complexity of an implementation of the algorithm on a particular piece of hardware.

The second method is to analyse theoretically the worst case scenario, by identifying the operations of the algorithm — its time complexity.

Time complexity is a measure of the number of operations that an algorithm must execute in terms of the size of the input or problem. Since it is the algorithm that is analysed, and not the implementation, time complexity is analysed using pseudocode. The time complexity $\mathrm{T}(n)$ of an algorithm is a function of the size n of its input.

The following algorithm has time complexity $\mathrm{T}(n) = 2n^2$. It is quadratic because it has two nested loops that go through every element. It has a coefficient of 2 because there are two operations in the nested statement.

Matrix-vector multiplication of size n.

```
for i=0...n-1:
    for j=0...n-1:
        x[i] = x[i] + A[i][j] * b[j]
```

7.3 Complexity Class and Big-O Notation

Usually, it is not necessary to know the exact time complexity. It is sufficient to know the complexity class, which ignores constant factors or overheads, and expressions of lower orders. This focusses on performance for large n.

Class complexity is expressed using 'big-O notation'. For example, $\mathrm{O}\left(n^2\right)$ is "of the order of n^2".

Corresponding time complexities and complexity classes.

Time complexity	Complexity class
$\mathrm{T}(n) = n$	$\mathrm{O}(n)$
$\mathrm{T}(n) = n + 2$	$\mathrm{O}(n)$
$\mathrm{T}(n) = 2n^2$	$\mathrm{O}\left(n^2\right)$
$\mathrm{T}(n) = 10n^3$	$\mathrm{O}\left(n^3\right)$
$\mathrm{T}(n) = 5\,(n + 2)$	$\mathrm{O}(n)$
$\mathrm{T}(n) = 1000$	$\mathrm{O}(1)$
$\mathrm{T}(n) = n^2 + n + 1$	$\mathrm{O}\left(n^2\right)$

To determine complexity class, it is often sufficient to count the number of loops and the number of times they are executed.

Polynomial classes are described as 'tractable'. Exponential classes are described as 'intractable' — they can execute with small amounts of data, but cannot with large amounts.

The binary search algorithm (on a sorted array) has class $\mathrm{O}(\log_2 n)$ because the size of the loop is halved on each iteration.

Common complexity classes.

Complexity class	Name
$O(1)$	Constant
$O(\log_2 n)$	Logarithmic
$O(n)$	Linear
$O(n\log_2 n)$	Log Linear
$O(n^2)$	Quadratic
$O(n^3)$	Cubic
$O(2^n)$	Exponential

Binary search algorithm.

```
left = 0; right = n-1
while left < right:
    mid = (left + right) / 2
    if x[mid] < v:
        left = mid + 1
    else:
        right = mid
if x[left] == v:
    return left
else:
    return -1
```

The total algorithm complexity is determined by the complexities of its components. Sequential algorithm phases are summed, and the maximum term is considered. Function or method calls are multiplied.

For example, this following altered matrix-vector multiplication algorithm has two sequential phases and a total complexity of $O(n) + O(n^2) \Rightarrow O(n^2)$.

Initialisation and matrix-vector multiplication of size n.

```
for i=0...n-1:
    x[i] = 0
for i=0...n-1:
    for j=0...n-1:
        x[i] = x[i] + A[i][j] * b[j]
```

The following iterative binary search algorithm uses a call to a method of logarithmic complexity within a fixed loop. It has complexity $O(n) \times O(\log_2 n) \Rightarrow O(n\log_2 n)$.

7.4 Algorithm and Problem Complexity

Algorithm complexity is the worst-case run time of an algorithm. This an upper bound and is the most informative complexity.

Iterative binary search.

```
for i=0...n-1:
    binary_search(x, v[i])
```

A problem has a complexity class of O(f(n)) if there exists an algorithm of complexity class O(f(n)) to solve it. Sometimes lower bounds are considered.

Problems solvable in polynomial time (PTIME or P) are assumed to be tractable. If a solution can be guessed and then checked in polynomial time, the problem is solvable in non-deterministic polynomial time (NP).