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BLACK–SCHOLES VERSUS ARTIFICIAL NEURAL NETWORKS IN PRICING FTSE 100 OPTIONS

JULIA BENNELL AND CHARLES SUTCLIFFE*
School of Management, University of Southampton, UK

SUMMARY

This paper compares the performance of Black–Scholes with an artificial neural network (ANN) in pricing European-style call options on the FTSE 100 index. It is the first extensive study of the performance of ANNs in pricing UK options, and the first to allow for dividends in the closed-form model. For out-of-the-money options, the ANN is clearly superior to Black–Scholes. For in-the-money options, if the sample space is restricted by excluding deep in-the-money and long maturity options (3.4% of total volume), then the performance of the ANN is comparable to that of Black–Scholes. The superiority of the ANN is a surprising result, given that European-style equity options are the home ground of Black–Scholes, and suggests that ANNs may have an important role to play in pricing other options for which there is either no closed-form model, or the closed-form model is less successful than is Black–Scholes for equity options. Copyright © 2005 John Wiley & Sons, Ltd.

1. INTRODUCTION

Access to a pricing model that can be solved in real time to give arbitrage-free prices, i.e. prices that do not permit traders to engage in arbitrage activity, is of paramount concern to those trading options. Such a pricing model enables traders to avoid selling underpriced options or buying overpriced options. It also allows traders to identify favourable prices quoted by others, which they can then exploit via arbitrage trades. In addition, such a model can be used to compute hedge ratios.

The conventional approaches to option pricing are based on theory. Closed-form pricing models have been derived for some types of option, while the pricing of other options relies on numerical procedures such as Monte Carlo simulation and the binomial model. These pricing models are derived using theoretical arguments based on assumptions concerning the behaviour of the underlying asset price and the riskless interest rate. In consequence, they are abstractions from reality whose performance depends on their ability to capture the dynamics of the underlying prices. Underlying assets can have different price processes, and the way in which the price of the underlying asset or assets affects the corresponding option price can vary as between types of option, e.g. Asian options, barrier options, exploding options, rainbow options, baseball options, binary options, Bermudan options, etc. The accuracy and solution time of these option pricing models varies. In some cases (e.g. European-style equity options) the performance of the pricing models is good, while for others (e.g. convertible bonds) the problem is more difficult (Philips, 1997; Connolly, 1998), and the performance of the available pricing models is mixed.

* Correspondence to: Professor Charles Sutcliffe, Accounting and Finance Division, School of Management, The University, Southampton SO17 1BJ, UK. E-mail cms@soton.ac.uk

This paper investigates the performance of an alternative way of developing option pricing models, i.e. artificial neural networks (ANNs). ANNs are information-processing tools commonly used for prediction and classification. Their particular strength lies in deriving meaning from complicated or imprecise data by extracting patterns or detecting relationships. They were inspired by the characteristics of the biological nervous system that enable learning by experience and the generalization of lessons to new examples. In consequence, ANNs are designed to model the way in which the brain learns.

The properties of ANNs that make them attractive for problems such as pricing options are that they have the ability to model non-linear relationships and do not rely on the restrictive assumptions implicit in parametric approaches. ANNs model the interaction between input variables by processing past examples. The key to a successful ANN implementation is its ability to generalize the lessons from past examples to new examples. Historical data are the only input, and the network is able to learn the relationships between inputs that characterize the phenomena being modelled. In order for the ANN to learn, data on the possible factors influencing the phenomena is required. In the case of option pricing, these factors may be chosen from the inputs required by a corresponding theory-based option pricing model. An ANN does not rely on assumptions concerning the price process of the underlying asset (e.g. constant-volatility geometric Brownian motion), nor does it depend on the specification of theory that connects the price of the underlying asset to the price of the option. Therefore, the strength of ANNs lies in modelling those relationships between the input and output variables that may be complex and difficult to capture in a convenient mathematical formulation. Finally, ANNs are flexible and can be used to generate pricing models for a wide variety of options, including options that are difficult to price using the conventional theory-based approach.

Theory-based models require numerical values for the parameters that appear in the model. The specification of these parameters may require past options prices, but usually does not; so, theory-based models can price previously untraded options, whereas ANNs cannot. The pricing model produced by an ANN may be valid only for data falling within the range covered by the data on which the network was trained. Since they are not trained, theory-based models are not susceptible to this problem. Given the validity of the theoretical model underlying the option pricing model, theory-based models can identify persistent biases in pricing behaviour. If past market prices are biased in some way, creating arbitrage opportunities, then an ANN will incorporate these biases into the pricing model and fail to indicate such opportunities. However, since liquid financial markets are generally arbitrage-free, ANNs will usually generate arbitrage-free option pricing models, and so are capable of identifying mispricings.

The first and most famous option pricing model is that proposed by Black and Scholes (1973), which is designed to price European-style equity options. This closed-form model does a good job in using five inputs to price European-style equity options in the absence of dividends and is widely used by option traders (Rubinstein, 1985).

However, empirical research has documented a number of biases in Black–Scholes prices for equity index options (Bates, 1996). The volatility smile (which is a 'moneyness' bias) has been found, the size of which changes over time. The presence of a volatility smile means that Black–Scholes prices are lower than the actual prices for options that are deep-in-the-money and deep-out-of-the-money. There is also a non-flat term structure, which means that the bias in Black–Scholes prices varies with option maturity; and a put-call skew (which is a bias that depends on whether a put or a call option is being priced) and varies with the relative volume of put and call trading. These biases suggest that, for a given type of option (put or call), 'moneyness' and maturity affect option prices in ways not allowed for by Black–Scholes.

The existence of these biases reveals that the Black–Scholes model is not perfect, and Bakshi *et al.* (1997) found that more complicated closed-form models can improve on Black–Scholes prices for European-style S&P500 options. Therefore, improving on the pricing performance of Black–Scholes on its home ground (European-style equity options) presents a tough, but not impossible challenge for ANNs.

This paper compares the performance of Black–Scholes with an ANN in pricing European-style call options on the FTSE 100 index. Its contribution lies in being the first extensive study of the performance of ANNs in pricing UK options, the first to allow for dividends in the closed-form model, and one of a small number of studies to apply ANNs to pricing European-style options. It is also the first paper to provide a review of the diffuse and fragmented literature, which is scattered over a wide range of disciplines. Section 2 summarizes the previous literature, and Section 3 describes the rival pricing models used in this study. Section 4 documents the data used, and Section 5 explains how the models were fitted to the data. Section 6 has the results, and Section 7 concludes.

2. PREVIOUS LITERATURE

A number of previous studies have examined the relative performance of ANNs in pricing equity options in the USA, the UK, Australia, Brazil, France, Germany, Japan and Sweden.[1,2]

USA. Hutchinson *et al.* (1994) compared three ANNs with the Black–Scholes model in pricing American-style call options on S&P500 futures, and found that all three ANNs were superior to Black–Scholes. Geigle and Aronson (1999) also examined the performance of ANNs in pricing American-style options on S&P500 futures, and found they were superior to Black–Scholes. Malliaris and Salchenberger (1993a,b) compared the performance of the Black–Scholes model and an ANN in pricing American-style S&P100 call options. They found that Black–Scholes was preferable for in-the-money options, whereas the ANN performed better for out-of-the-money options. Kitamura and Ebisuda (1998) found that the performance of an ANN in pricing American-style S&P100 call options was poor. However, as well as a very small sample, this result may be due to the use of only two inputs to the ANN. Qi and Maddala (1996) compared the performance of an ANN in pricing European-style call options on the S&P500 index with that of Black–Scholes and concluded that the ANN was superior. A similar conclusion was reached by Garcia and Gençay (1998, 2000), Gençay and Qi (2001), Gençay and Salih (2001), Ghaziri *et al.* (2000), Liu (1996) and Saito and Jun (2000). Dugas *et al.* (2002) found that constraining the ANN produced better prices for European-style call options on the S&P500 index than those by an unconstrained ANN. Kelly (1994) priced American-style put options on four US firms using an ANN and the binomial option pricing model. He found that the ANN was clearly more accurate than the binomial model.

UK. Niranjan (1996) used daily data from February to December 1994 for call and put FTSE 100 options. He compared the pricing errors for an ANN and Black–Scholes, and for a sample of 100 days found no clear dominance in pricing accuracy. Using the Niranjan (1996) data, De Freitas

[1] Genetic programming imitates the evolutionary process to generate new pricing formulas that perform better than the initial model, and a few studies have used genetic programming, rather than ANNs (Chen and Lee, 1997a,b; Chen *et al.*, 1998, 1999; Chidambaran *et al.*, 1998a,b, 2000; Keber, 1999, 2000, 2002; Trigueros, 1997).

[2] Using data simulated from a Black–Scholes model, Galindo-Flores (2000) found that ANNs are superior to regression, decision trees and the *k*-nearest neighbour technique in modelling out-of-sample European-style call option prices.

et al. (2000) applied ANNs and Black–Scholes to price FTSE 100 call and put options, and found that all the ANNs considered were superior to the Black–Scholes model.[3] Healy *et al.* (2002) used closing prices for FTSE 100 call options for 1992–1997 and found that their ANN fitted the data well (there was no direct comparison with the Black–Scholes prices).

Australia, Brazil, France, Germany, Japan and Sweden. Lajbcygier *et al.* (1996a,b) compared three ANNs with three closed-form models (Black–Scholes, Barone-Adesi and Whaley and modified Black) in pricing American-style call options on Australian Share Price Index futures. They concluded that the ANNs were inferior to the theory-based models; however, for observations that were near-the-money for short-maturity options, the ANNs were superior. Lachtermacher and Rodrigues Gaspar (1996) used ANNs to price options on the shares of the Brazilian company Telebrás,[4] and found the ANNs were superior to Black–Scholes. De Winne *et al.* (2001) employed an ANN to price options on French CAC 40 index options, which are American style. They compared their ANN with the binomial model when both models used dividends, and found that their ANN was almost as good as the binomial model. Anders *et al.* (1998) used data on European-style DAX call options and discovered that the ANN was superior to Black–Scholes, as did Ormoneit (1999) and Krause (1996). Herrmann and Narr (1997) studied both call and put options on the DAX (both European style), and found that all four ANNs outperformed Black–Scholes. Hanke (1999a) applied ANNs and the Black–Scholes model to European-style call options on the DAX index. After optimizing the volatility and interest rate data to suit the Black–Scholes model, the ANN was less accurate than Black–Scholes. Yao *et al.* (2000) used ANNs to price call options on Nikkei 225 futures, which are American style, and found they outperformed Black–Scholes. Amilon (2001) compared the performance of an ANN with Black–Scholes in pricing European-style call options on the OMX index. He controlled for dividends by omitting data for the 2 months when shares go ex-dividend in Sweden. For both historical and implied volatilities, the ANN was generally superior.

These papers support the view that ANNs are capable of outperforming well-regarded closed-form models in pricing call options. Futures contracts do not pay dividends, and so this complication was absent from the studies by Geigle and Aronson (1999), Hutchinson *et al.* (1994), Lajbcygier *et al.* (1996a,b) and Yao *et al.* (2000). However, American-style options on a futures contract may be exercised early, as may warrants, and so being American style may be valuable. Since Black–Scholes is only appropriate for pricing European-style options, the outperformance of Black–Scholes by the ANN found by Geigle and Aronson (1999), Ghaziri *et al.* (2000), Hutchinson *et al.* (1994), Lajbcygier *et al*. (1996a,b), Malliaris and Salchenberger (1993a,b) and Yao *et al.* (2000) may be due to the omission of the early exercise option from the theory-based valuation model.[5]

While many studies have considered options on an underlying asset that pays dividends, the theory-based option pricing models used were often not adjusted to incorporate dividends. This will have biased the theory-based models, leading them to overprice call options and underprice put options. Although the ANNs in these studies were usually not supplied with dividend information, they need not have been biased by the omission of dividends to the same extent as the theory-based models.

[3] It is not clear whether these two studies of the FTSE 100 used European- or American-style options, nor whether they included dividends.
[4] The style and type of option were not specified.
[5] Although Kelly (1998) considered American-style options, his benchmark was the binomial model, and this allows for the options being American style. Therefore, in this case the superior performance of the ANN was not due to the benchmark failing to allow for the possibility of early exercise. Yao *et al.* (2000) also used the binomial model as their benchmark, and found their ANN was almost as good as the benchmark in pricing American-style options.

As well as pricing exchange-traded equity options, ANNs have also been applied to other options. Hanke (1997) used simulated data to investigate the performance of ANNs in pricing Asian-style call options, and White (1998, 2000) used real and simulated data for European-style call and put options on Eurodollar futures. Raberto *et al.* (2000) used an ANN to price options on German treasury bonds (Bunds), and Karaali *et al.* (1997) used an ANN to price options on an index of the volatility of the $–DM exchange rate. Taudes *et al*. (1998) considered using ANNs to value real options, and Carelli *et al.* (2000) applied ANNs to pricing $–DM forex call and put options. ANNs have also been proposed for pricing European-style contingent claims with state-dependent volatility (Barucci *et al.*, 1996, 1997). Provided they are traded on competitive markets for which a price history is available, ANNs have the potential to price a very wide range of financial securities.[6]

3. THE RIVAL PRICING MODELS

3.1. Black–Scholes

The Black–Scholes call prices were computed using the standard formula, but with the Merton (1973) adjustment for dividends:

$$C = MN(d) - K\,\mathrm{e}^{-rt}\,N(d - \sigma\sqrt{t}) \qquad \text{where} \qquad d = \frac{\ln(M/K) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}} \tag{1}$$

$M = S\,\mathrm{e}^{-Dt}$ is the Merton adjustment for dividends, S is the current share price, K is the exercise (or strike) price, r is the annual risk-free rate of interest on a continuously compounded basis (e.g. 0.06), t is the time to expiry in years (e.g. 0.25), σ is the standard deviation of the share's continuously compounded annual rate of return (e.g. 0.30), D is the annual dividend rate (e.g. 0.05), and $N(d)$ is the probability that a standardized normally distributed random variable will be less than or equal to d.

3.2. Artificial Neural Networks

A number of different approaches are classified as members of the ANN family. Our investigation concentrates on the multilayer perceptron (MLP). This is one of the most popular approaches and has been used in the majority of applications to options pricing (e.g. Lajbcygier and Connor, 1997a,b; Anders *et al.*, 1998; White, 1998). It has also been applied successfully to a range of difficult and diverse problems (Brockett *et al.*, 1997; OhnoMachado and Rowland, 1999). Further, Hornik *et al.* (1989, 1990) demonstrated that multilayer feedforward networks are able to approximate a large class of functions and their derivatives accurately with a single hidden layer. A further advantage of feedforward networks is their ability to deal with missing or spurious data.

MLPs consist of connected layers of processing elements, called neurons, that pass information through the network by weighted connections. The input variables are presented to the input layer of processing elements, which sends a signal that propagates through the network layer by layer. The network learns by comparing the resulting output with the desired output and then applying an

[6] ANNs can also compute the hedge ratio and the sensitivity of the option price to a range of factors, and these sensitivities are collectively known as 'the Greeks'.

adjustment to the network weights in accordance with an error correction rule. This is called error back-propagation and is commonly based on the least mean square algorithm.

In order to construct an MLP, various decisions must be made. These are: the number of hidden layers, the number of processing elements in the hidden layer(s), the learning rate and momentum, the set of input variables and the sample period. In addition, preprocessing the inputs before presenting them to the network can reduce the learning required of the network. For example, the ratio of two inputs may be more important in determining an outcome than each input individually. In which case, it is beneficial to generate a new input by dividing these inputs in the preprocessing phase. All the above decisions are key to the success of the MLP.[7]

4. DATA

The analysis used data on European-style FTSE 100 call options traded on LIFFE over the period from 1 January 1998 to 31 March 1999. While American-style options on the FTSE 100 index are also traded on LIFFE, the Black–Scholes model is designed to price only European-style options correctly. The analysis was restricted to call options because, while the appropriate version of the Black–Scholes model is equally applicable to European-style put options, the ANN must be trained separately on calls and puts. Across all strike prices and maturities (of which there were 1226) 83 873 call option closing prices were available from LIFFE for this period. However, most of these prices were for call options that had not traded that day. When these prices were eliminated only 11 036 observations remained. A further 1480 observations were dropped for various reasons,[8] leaving 9556 for use in the analysis.

The Black–Scholes model (with the Merton adjustment for dividends) requires values for six parameters: spot price, strike price, maturity, riskless interest rate, dividend rate and volatility. The daily closing values of the FTSE 100 index were taken from DataStream, the strike price for each observation was available from LIFFE, and the maturity of each observation (in days) was computed using the date of the observation and the expiry month of the option. As the sample data had an average maturity of 70 days, the annualized riskless interest rate was measured using the 3 month Treasury bill rate. The dividend rate was taken from DataStream, which provides daily values of the annual dividend rate over the past year on the FTSE 100 index basket of shares.[9] The daily volatilities were supplied by LIFFE, which uses the Black-I model.[10] These daily volatilities are a linear interpolation of the two implied volatilities generated by the strike prices either side of the daily settlement price. This procedure has been shown to provide virtually unbiased volatility estimates for a wide range of option pricing models (Corrado and Miller, 1996). Daily open interest and volume in each option contract were supplied by LIFFE.

[7] All experiments were run using NeuroSolutions V3.01.

[8] Observations were dropped if (a) the lower boundary condition for European-style call options, (i.e. $C \geq [S - (K + D)/(1 + r)^t]$), was violated, (b) LIFFE's estimate of spot volatility was outside the range 1 to 40%, (c) the closing price of the option was zero or exceeded 1600 index points, or (d) the maturity of the option was zero days.

[9] Ideally, what is wanted is the forecast of the annualized dividend rate on the shares in the index basket over the life of the option. Dividend rates tend to be stable from year to year, and so the historic annual dividend rate is a reasonable forecast of the dividend rate over the next 12 months.

[10] The Black-I model uses the Black (1976) option pricing model, and is mathematically equivalent to the Merton model in equation (1), with the current index value adjusted for subsequent dividend payments.

There are two possible problems with the spot and options closing prices: options trading ceased at a slightly different time from stock market trading for the first 7 months of the period, so that the prices were non-synchronous; and the index and the options prices are subject to a stale price problem.[11] Until 20 July 1998, the London Stock Exchange ceased trading at 4:30 p.m., whereas FTSE 100 options stopped floor trading on LIFFE at 4:10 p.m., and so there was a difference of 20 min between the spot and options closing times during this period. However, an analysis using the spot prices at 4:10 p.m. implied by the futures market indicates there is no problem.[12] As from 20 July 1998 until the end of the data period, the close of trading for the London Stock Exchange and FTSE 100 options (open outcry) were identical (4:30 p.m.) and so there is no non-synchronicity problem for these 8 months.

European-style FTSE 100 call options are frequently traded[13] and so will generally have a low level of staleness. Until 20 October 1997, the FTSE 100 index was computed using current quotes from SEAQ, rather than trade prices, and so prices were not stale. From 20 October 1997 the FTSE 100 index was computed using the last trade price from the SETS screen-based trading system.[14] Therefore, stale prices are a potential problem from October 1997. However, companies that are heavily weighted in the FTSE 100 index are traded frequently, so that most prices used in computing the index are very recent and the stale price effect is of low importance.

The tick size for European style FTSE 100 options is half an index point, and so the actual call prices (which are in index points) can only be integers or half integers. Since the Black–Scholes or ANN models are not constrained in this way, there will inevitably be small differences between the actual prices and those produced by the two pricing models. The proportionate size of this rounding effect increases as the call price decreases; and for calls priced at (say) one index point, this will be very large. This has implications for the choice of performance measure (see Section 5).

Since the same data are used for both Black–Scholes and the ANN, it is relative not absolute pricing accuracy that is being studied, and the playing field for these two pricing methods is fairly level. Hanke (1999a) has argued that the presence of biases in the data will have a greater adverse effect on the Black–Scholes prices than on those produced by ANNs. This is because ANNs can

[11] Provided the index and options prices are equally stale, the results will be unaffected.

[12] For the period when the spot market closed 20 min later than the futures and options markets (23 March 1992 to 20 July 1998), the closing prices of FTSE 100 index futures (F) were used to compute the implied spot prices as at 4:10 p.m. using the no-arbitrage pricing formula $S = F/(1 + r) + D$, where D is the present value of dividends on the index until delivery, and r is the riskless rate until delivery. For the assumptions underlying this formula, see Sutcliffe (1997). The resulting Black–Scholes European-style FTSE 100 call option prices were very similar to those obtained using the spot price at 4:30 p.m., with a correlation of 0.999, and the mean, standard deviation, maximum and minimum values listed in the table below.

Spot price	Black–Scholes call prices			
	Mean	Std. Dev.	Max	Min
Actual at 4:30	135.5	168.4	1471.6	0.003
Implied at 4:10	136.2	168.4	1458.1	0.004

[13] The volume of European-style FTSE 100 call options traded on LIFFE in 1998 was 1 754 528, and between 1 January and 31 March 1999 it was 429 960.

[14] As from 14 December 1998 the calculation of the closing value of the FTSE 100 index was changed to the volume-weighted average of the prices of SETS trades in the last 10 min of trading. (On 30 May 2000 the computation of the closing price changed again to the price determined by a closing auction, provided there is sufficient volume involved in the auction process.)

learn to allow for such biases, whereas the Black–Scholes model cannot. In real-world situations, estimates of the input variables (e.g. volatility) must be used, and this paper compares the performance of the two models using the quality of input data that is likely to be available in a trading situation.

5. FITTING THE MODELS

The Black–Scholes call price was computed for each observation using equation (1). For the entire sample, while the average Black–Scholes price was within 2.6% of the actual average price, there was considerable offsetting inaccuracy for individual Black–Scholes prices. This left scope for the ANN, in our case an MLP, to outperform Black–Scholes.

The aim of our approach is to develop a strategy that could have been used at the time for pricing options. This approach still leaves sufficient potential input variables to make the selection of the number and combination of inputs non-trivial. The complexity of the problem is further increased by the variety of potential network designs, learning parameters and sample periods. These factors prohibit exhaustive experimentation of all possibilities. Therefore, fitting the MLP began with a period of 'brainstorming' experiments that drew on the expertise of the authors and the experience of the literature to establish a suitable base network structure that provided a lower bound against which further developments could be benchmarked. This facilitated setting many of the parameters while investigating others, beginning with the selection of the input variables. This was followed by preprocessing to generate new variables, and refining the network parameters.

5.1. Inputs

The inputs considered for use by the MLP were the six inputs of the Black–Scholes model with the Merton dividend adjustment, together with open interest (which Healy *et al.* (2002), Qi and Maddala (1996), and Ghaziri *et al.* (2000) found to be useful) and daily volume (which Healy *et al.* (2002) did not find helpful) for the particular option contract being priced. In addition, new variables were generated from these basic inputs, and a key input was constructed using the *homogeneity hint.* Provided that returns on the underlying asset are distributed independently of its price, the Black–Scholes pricing model (equation (1)) is homogeneous of degree one in S and K, and both sides can be divided through by K, so that C/K is a function of S/K (Merton, 1973). In which case, the two inputs S and K can be combined into a single input, S/K, which can be interpreted as a measure of 'moneyness'. Most previous studies have made use of this homogeneity property.[15] This has the added advantage of explicitly including a measure of 'moneyness', one of the dimensions along which the Black–Scholes model is biased. Anders *et al.* (1998) included both S/K and S as separate inputs into an MLP, and concluded that the addition of S did not improve matters. Similarly, Garcia and Gençay (1998, 2000) found that an ANN using S/K was superior to an ANN that used S and K separately. The present study also found that using the homogeneity property significantly

[15] Amilon (2001), Anders *et al.* (1998), Boek *et al.* (1995), De Winne *et al.*, (2001), Dugas *et al.* (2002), Garcia and Gençay (1998, 2000), Gençay and Qi (2001), Gençay and Salih (2001), Geigle and Aronson (1999), Hanke (1997, 1999a,b), Hutchinson *et al.* (1994), Lajbcygier *et al.* (1996a,b, 1997), Lajbcygier and Flitman (1996), Lajbcygier and Connor (1997a,b), Niranjan (1996) and Raberto *et al.* (2000).

Table I. Out-of-the-money European-style FTSE 100 call options

		PEs	MD	MAD	MPD	R^2	MSD
1	MLP – Inputs *S*, *K*, *t*, *D*, *r*, σ Output *C*	4	−41.15	66.7	−0.480	0.85	8323
2	MLP – Inputs *S*, *K*, *t*, σ Output *C*	4	−45.11	68.7	0.12	0.82	9630
3	MLP – Inputs *S*, *K*, *S/K*, *t*, *D*, *r*, σ Output *C/K*	5	6.65	17	−0.006	0.99	2293
4	MLP – Inputs *S/K*, *t*, *D*, *r*, σ Output *C/K*	4	5.18	12.7	0.02	0.99	321.9
5	MLP – Inputs *S/K*, *t*, σ Output *C/K*	4	18.96	19.6	0.171	1	677.1
	Black–Scholes	–	−21.66	22.7	−0.445	0.99	872

Table II. In-the-money European-style FTSE 100 call options

		PEs	MD	MAD	MPD	R^2	MSD
1	MLP – Inputs *S*, *K*, *t*, *D*, *r*, σ Output *C*	4	83.07	121.6	0.247	0.87	24 313
2	MLP – Inputs *S*, *K*, *t*, σ Output *C*	3	−91.80	119.7	−0.275	0.89	20 215
3	MLP – Inputs *S*, *K*, *S/K*, *t*, *D*, *r*, σ Output *C/K*	5	54	99.91	0.196	0.92	14 709
4	MLP – Inputs *S/K*, *t*, *D*, *r*, σ Output *C/K*	4	−29.48	51.28	−0.096	0.98	4172.9
5	MLP – Inputs *S/K*, *t*, σ Output *C/K*	3	−10.27	42.27	−0.028	0.97	3241.1
6	MLP – Inputs *S/K*, *S/K*, *r*, *t*, σ Output *C/K*	4	−25.63	42.77	−0.066	0.98	3943.3
7	MLP – Inputs *S/K*, *t*, σ, $(\sigma\sqrt{t})$ Output *C/K*	3	−27.94	37.7	−0.059	0.99	3082.1
	Black–Scholes	–	13.7	17.25	0.03	0.99	609.8

improved the capabilities of the ANN in comparison with using *S* and *K* as independent inputs. This can be attributed to the reduction in complexity of the network and the dangers of overfitting, so improving the network's ability to generalize. The homogeneity hint is a preprocessing measure that arms the network with fuller information, so reducing the learning required.[16]

Initial experiments eliminated open interest and volume, leaving the following input variables: spot price *S*, strike price *K*, moneyness *S/K*, time to expiry *t*, annual dividend rate *D*, risk-free interest rate *r*, and standard deviation σ. A number of models were generated using different combinations of the input variables. In each case there is a single output, either the closing price *C* or *C/K*. The input and output variables of each model are detailed in Tables I–III.

[16] Gençay and Qi (2001) showed that, if the homogeneity hint is not used, there are other ways of avoiding overfitting ANNs. They applied three alternative methods, i.e. Bayesian regularization, early stopping and bagging, to data on S&P500 call options. They found that ANNs using these three methods are superior to Black–Scholes, a baseline ANN and linear regression in option pricing.

Table III. In-the-money European-style FTSE 100 call options where $1 < S/K < 1.15$ and $t < 200$

		PEs	MD	MAD	MPD	R^2	MSD
5	MLP – Inputs S/K, t, σ Output C/K	3	6.65	22.9	–0.0018	0.98	898
	Black – Scholes	–	12.2	15.6	0.025	0.99	471

5.2. Training Period and Partitioning of the Data

An investigation of the training period found that more data does not necessarily lead to better results. Training the network on 1 year of data appeared to represent the phenomena well. A longer sample period significantly increased the training period required to achieve convergence of the network weights, and did not improve on the results. On average, the classification error increased with the larger sample and longer training period, and this could be due to overfitting. Since the data are presented to the network in a randomized order to reduce the possibility of cycling through network weights, the recency of the data is not taken into account when training the network, and the operating environment is assumed to be stable. The most recent data available to the study terminated on 31 March 1999. Hence, the training period begins on 1 January 1998 and ends on 31 December 1998, and the test period begins on 1 January 1999 and ends on 31 March 1999. This testing set is not involved in either parameter estimation or selection of network architecture. The training data are further divided, where the first two-thirds is used for training and the final third for cross-validation. The cross-validation data set can be considered as a second testing data set. It provides a means of testing the network during training, but it does not influence the parameter estimation. Training is terminated when either the mean squared error for the cross-validation data set increases, or when the maximum number of epochs is exceeded. Using cross-validation as a termination criterion should improve generalization.

A number of implementations of ANNs to price options have removed particular types of observation from their input data. For example Anders *et al.* (1998) excluded options if they were traded at less than 10 index points, if they had less than 15 days to maturity, if moneyness was above 1.15, or if moneyness was below 0.85. Malliaris and Salchenberger (1993a,b) and White (1998) restricted the days to expiry to be between 30 and 60 days and between 30 and 270 days respectively. Our investigation took a different approach. A common categorization of options is by moneyness, where if the ratio S/K is greater then 1.0 the option is 'in-the-money'; otherwise it is 'out-of-the-money'. Instead of excluding observations from the input data, thereby reducing the scope of application, the data were partitioned according to this moneyness criteria, and separate MLPs trained for each group.

5.3. Network Structure and Parameters

Investigation of the number of hidden layers using the FTSE 100 data concurs with the majority of research in this area, finding that more than one hidden layer increases the complexity of the network, but does not enhance its capability to generalize. Since a number of different ANNs were built using different sets of input variables, it is quite possible that each ANN requires a different number of processing elements (PE), or neurons, in the hidden layer. In order to determine the number of PE for each ANN, experiments were initially run with just two PE, and incrementally increased until no further improvement in out-of-sample testing was found. The number of PE for each combination of the output and input variables are detailed in Tables I–III.

An investigation of the network parameters found that sequential or on-line training was preferable to batch training.[17] This feature was also recognized by Hutchinson *et al.* (1994). As a result of the on-line updating of weights, the values for the momentum and learning rates that produced the best results were found to be smaller than those normally used with batch training. The learning rate (sometimes called the step size) controls the magnitude of change to the network weights from one iteration to the next. This was set to 0.5 for the hidden layer and to 0.1 for the output layer. The momentum, which was set to 0.4, smooths the changes in network weights, so reducing the influence of one iteration, and provides a means of breaking away from local optima.

5.4. Performance Measurement

As there is no agreement on the appropriate loss function, a single generally accepted measure of the pricing accuracy of Black–Scholes and the MLP in generating the actual call prices is not available. Therefore, five alternative summary measures of performance were used: (a) squared correlation between the actual and computed prices (R^2); (b) mean deviation (MD); (c) mean absolute deviation (MAD); (d) mean proportionate deviation (MPD); mean squared deviation (MSD). The measures used were defined as follow:

$$R^2 = \frac{(n\,\Sigma\,xy - (\Sigma\,x)(\Sigma\,y))^2}{(n\,\Sigma\,x^2 - (\Sigma\,x)^2)(n\,\Sigma\,y^2 - (\Sigma\,y)^2)} \tag{2}$$

$$\text{MD} = (\Sigma(y - x))/n \tag{3}$$

$$\text{MAD} = (\Sigma|y - x|)/n \tag{4}$$

$$\text{MPD} = (\Sigma(y - x)/x)/n \tag{5}$$

$$\text{MSD} = \Sigma(y - x)^2/n \tag{6}$$

where x is the actual value of the dependent variable (e.g. C/K), y is the estimated value of the dependent variable, and n is the number of observations.

6. RESULTS

The network was trained on-line for 1000 epochs.[18] The training sample period was the calendar year 1998 (2469 in-the-money and 4713 out-of-the-money observations), where one-third of the

[17] On-line training updates the network weights after each training example according to the error, as opposed to batch training, which updates the weights after all the training examples have been presented (i.e. after an epoch) according to the average squared error. The advantages of on-line training for our implementation are that the back-propagation algorithm is less likely to be trapped in a local optimum, and more efficient use is made of data that exhibit exactly the same pattern.

[18] One epoch is a complete presentation of the entire training data set to the network. The network learns by repeated presentation of the training data. After each training example or each epoch the network weights are updated according to the back-propagation algorithm in order to minimize the local error. The learning process continues on an epoch-by-epoch basis until the network weights stabilize.

input data were used for cross-validation, and the optimal network weights were saved at the minimum mean squared cross-validation error. In order to reduce the dependence of network performance on the initial network weights, the network was trained 100 times. The weights that produced the minimum mean squared cross-validation error over all training runs were retained for testing. A variety of combinations of inputs were selected for extensive investigation. For each combination of inputs, the optimal number of hidden-layer PE was determined. A selection of the results for out-of-the-money and in-the-money options are detailed in Tables I and II respectively. Both tables give results for the out-of-sample performance of the MLP and Black–Scholes models, where the out-of-sample period was 1 January 1999 to 31 March 1999 (846 in-the-money and 1528 out-of-the-money observations). In the case of MLPs that utilize the homogeneity hint, the output was multiplied by K in order to compare the Black–Scholes error with the MLP error.

For both in- and out-of-the-money options data, a comparison of the results for MLPs 1 and 4 and 2 and 5 shows that the homogeneity hint clearly improves on the MLP's performance relative to when S and K are used as separate inputs. This preprocessing of the data is based on the result from finance theory that S and K can be combined into the variable S/K, which is moneyness. The results presented are typical of all experiments run, with respect to the use of the homogeneity hint. They also accord with previous studies by other researchers who have used the homogeneity hint and moneyness. The use of moneyness (not S and K) as an input variable, and C/K (not C) as the output variable, is the key to ANNs outperforming Black–Scholes. Using the most successful combination of inputs and network design, but training on all data with no moneyness restrictions, the MLP performance was MD = –68.70, MAD = 72.28, MPD = –0.472, R^2 = 0.958 and MSD = 9656.2. Therefore, partitioning the data using moneyness dramatically increased the performance of the MLP.

Table 1 shows that, on all criteria, the three MLPs that utilized the homogeneity hint (MLPs 3, 4 and 5) produced a clearly better performance than Black–Scholes for out-of-the-money options.[19] The MLP which was best on three criteria (MLP 4) used the six Black–Scholes inputs (along with the homogeneity hint). Figure 1 plots the results from this MLP as the deviation from the closing price against moneyness, and there is no sign of any bias in the MLP prices. This can be compared with the deviations of the Black–Scholes price from the closing price in Figure 2. This shows that the Black–Scholes price is biased, with the size of this bias increasing as moneyness decreases.

For in-the-money options Black–Scholes is superior to all the MLPs on four of the five criteria. This is because Black–Scholes fits the data better for in-the-money options than it does for out-of-the-money options, whereas the reverse is generally the case for the MLPs. Malliaris and Salchenberger (1993a,b) and Healy *et al.* (2002) also found that the performance of ANNs was better for out-of-the-money options. The MLP that was best on two of the measures (MLP 5) used moneyness (S/K), maturity (t) and volatility (σ), while the MLP that was best on the other three measures used these inputs plus the product of volatility and maturity ($\sigma\sqrt{t}$).

Further analysis of the results produced by MLP 5 shows that the neural network is having difficulty pricing options that are deep in the money, and those with a long expiry date. Figures 3 and 4 illustrate this relationship. Figure 3 plots the deviations of the MLP 5 price from the actual closing price against moneyness, and the network is overpricing options that are deep in the money. Figure 4 plots the same deviations against time to expiry, and this indicates that the network is underpricing options with a long expiry date. This motivated us to remove options with a moneyness

[19] Black–Scholes is superior only for the R^2 and MSD criteria for MLP 3.

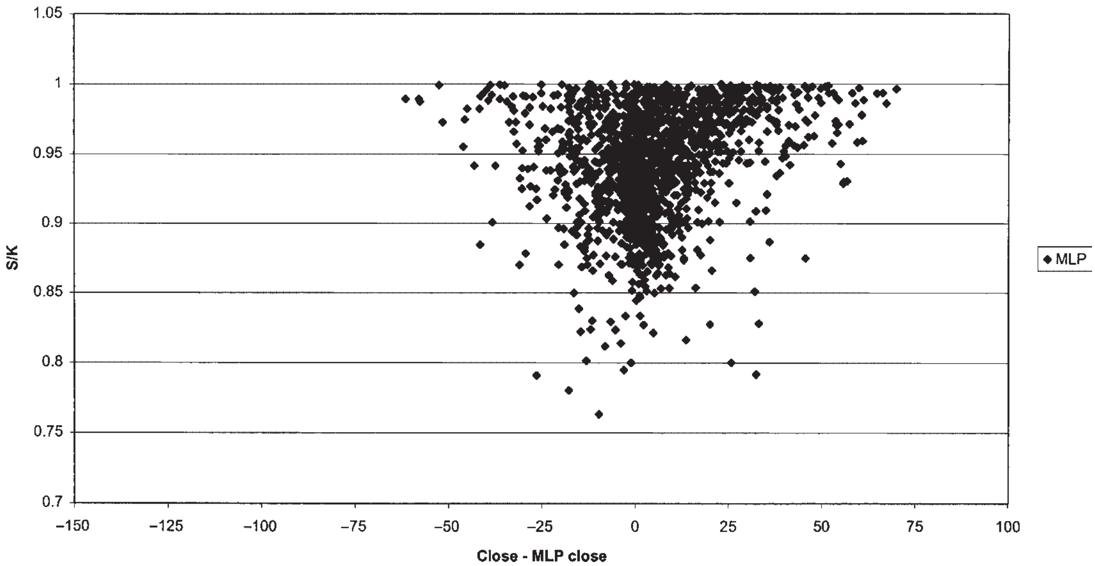

Figure 1. Scatter plot of the deviation of the MLP price from the actual closing price versus moneyness for out-of-the-money options

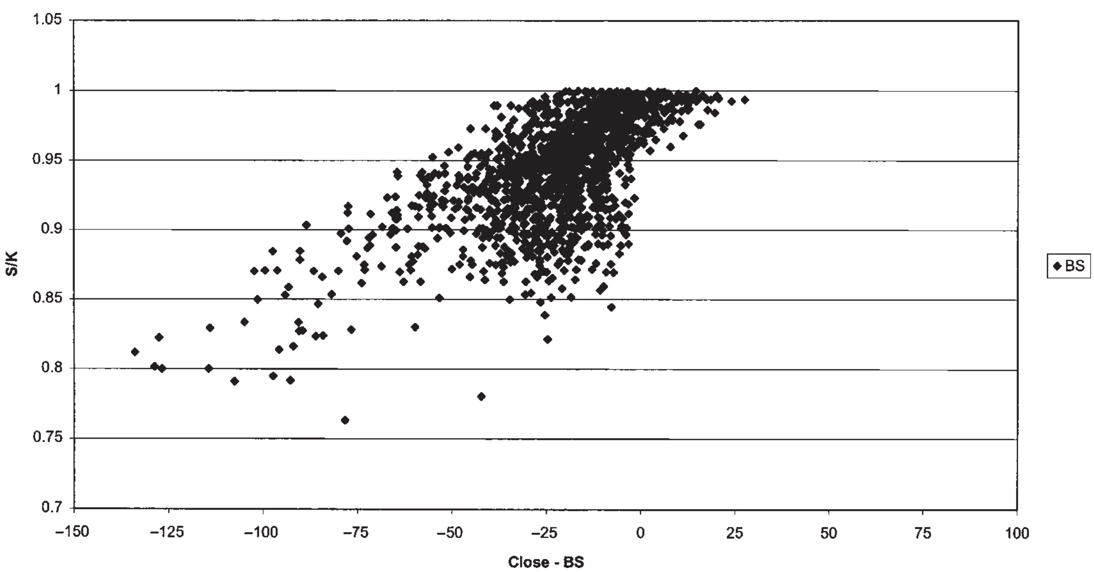

Figure 2. Scatter plot of the deviation of the Black–Scholes price from the actual closing price versus moneyness for out-of-the-money options

greater than 1.15 or a maturity longer than 200 days, and train MLP 5 on this reduced data set. The results are detailed in Table III.

For the restricted data set the MLP produces significantly better results than for the unrestricted data set, and outperforms Black–Scholes on two of the five performance measures (i.e. mean

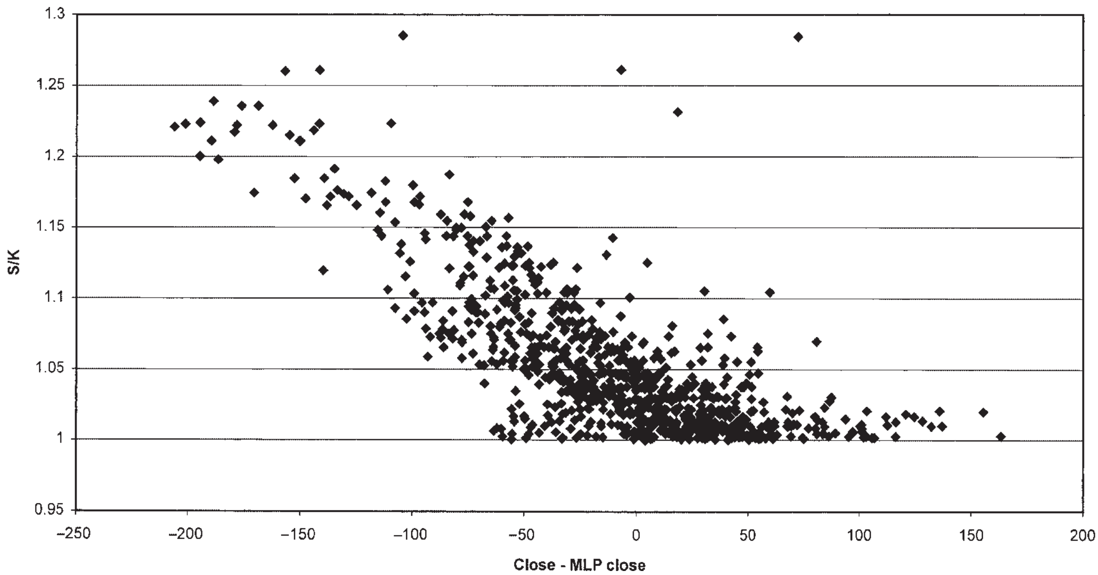

Figure 3. Scatter plot of the deviation of the MLP price from the actual closing price versus moneyness for in-the-money options

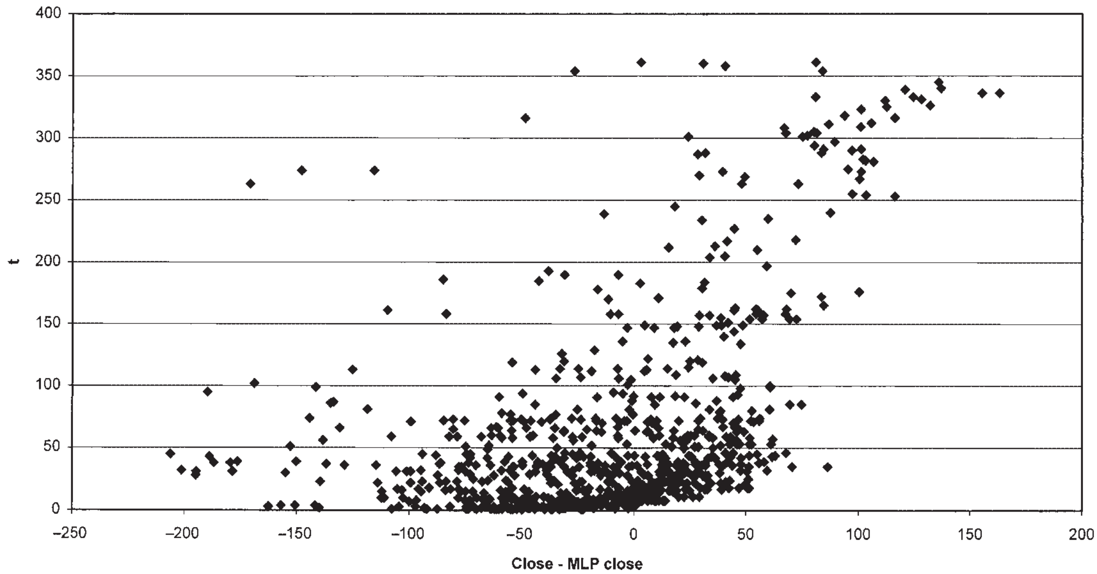

Figure 4. Scatter plot of the deviation of the MLP price from the actual closing price versus time to maturity for in-the-money options

deviation and mean percentage deviation). The MLP is also much closer to Black–Scholes than previously on the three remaining performance measures. Lajbcygier *et al.* (1996a,b) have also found that ANN performance is relatively better for near-the-money short-maturity options. Since the vast majority of option trading is in short-maturity options that are close to the money, this MLP will be able to price most options, and the restriction on the domain is of limited practical importance.[20]

7. CONCLUSIONS

The aim of the paper was to investigate the use of ANNs as a tool for pricing options. In order to evaluate the performance of this approach, European-style equity index options were selected as a case study. The particular advantage in choosing this type of option is that there exists a widely used and highly respected closed-form model (Black and Scholes, 1973) that can be used to benchmark the performance of the ANN. This paper shows that the use of the homogeneity hint and moneyness is of key importance to outperforming Black–Scholes. For out-of the-money options, the ANN is clearly superior to Black–Scholes. Switching to in-the-money options, the performance of Black–Scholes improves; that of the ANN worsens, though, leading to Black–Scholes superiority. However, if the sample space is restricted by excluding options with a moneyness greater than 1.15 or a maturity longer than 200 days, then the performance of the ANN becomes comparable to that of Black–Scholes. Since this restriction excludes only a small number of options trades (3.4% of volume), it is concluded that the ANN approach is generally superior to Black–Scholes in pricing European-style FTSE 100 call options. This is a surprising result, given that European-style equity options are the home ground of Black–Scholes, and suggests that ANNs may have an important role to play in pricing other options for which there is either no closed-form model or the closed-form model is less successful than Black–Scholes for equity options.

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[20] For in-the-money options, 14% of the volume is traded using options with either 200 or more days to expiry, or a moneyness greater than 1.15, or both. These excluded trades correspond to 3.4% of the total volume of in- and out-of-the-money options. Thus, neural networks can successfully price options corresponding to 96.6% of volume.

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