Hi, the goal of this informal document is to provide a presentation of my completed works. In a nutshell, I like working at the interface between percolation theory and group theory (geometric or ergodic) or number theory.

In percolation, we like to take a graph $\mathscr{G}$ and a probability measure $\mathbb{P}$ on the space of subgraphs of $\mathscr{G}$, and then study the resulting connected components — the so-called clusters. This lacks structure so it is a good idea to consider a group G acting on $\mathscr{G}$ and to assume that the action preserves $\mathbb{P}$. Then, by simply assuming the action on the vertices to be transitive, many nice results and questions can be stated. By taking $\mathbb{P}$ to be arbitrary or to satisfy a few soft properties, we can go a long way. But it is also worthwhile to understand in full depth some archetypal examples, such as Bernoulli percolation: in bond (resp. site) Bernoulli percolation, each edge (resp. vertex) is independently kept with probability p, and you know that there is p_c, etc.

Bernoulli percolation as a function of the graph

The critical point $p_c(\mathscr{G})$ can be defined by letting p move and observe when a dramatic change occurs. But what happens to $p_c(\mathscr{G})$ when the graph $\mathscr{G}$ is moving? This leads to several questions. Notably:

1. Is $\mathscr{G} \longmapsto p_c(\mathscr{G})$ continuous? *The class of transitive graphs is endowed with the local topology, and we toss away graphs with $p_c = 1$. Indeed, they produce silly counterexamples such as finite tori or cylinders of large radius converging to the square grid.*
2. Is p_c weakly monotone with respect to covering maps on the class of transitive graphs? strictly monotone?
3. What can be said about the set K of all values of $p_c(\mathscr{G})$ when $\mathscr{G}$ ranges over all transitive graphs?

Regarding the first question, during our PhD, Vincent Tassion and myself treated in [9] the case of Cayley graphs of abelian groups. The chalenge was to prove a weak version of the celebrated slab result of Grimmett-Marstrand [GM90] holding also in the anisotropic case, meaning when we have few symmetries. Years later and together with Daniel Contreras, we used a completely different approach to deal with all transitive graphs of polynomial growth [4, 5]. On purpose, we stated our intermediate results in their natural generality, hoping this could lead to a full affirmative resolution of question 1 above, known as Schramm's Locality Conjecture. This conjecture was then solved by Easo and Hutchcroft [EH], which was an event within the field of "percolation beyond $\mathbb{Z}^d$". Our work [4] was one of the key inputs leading to their breakthrough.

In [BS96], Benjamini and Schramm initiated the systematic study of percolation at the generality of Cayley graphs and transitive graphs. Their first proposition was that p_c is monotone under covering maps, and they asked whether it is strictly monotone under the additional assumption that the graphs are transitive and have $p_c < 1$. Franco Severo and myself proved in [8] that it is indeed the case, and proved a similar statement for the uniqueness threshold p_u under the additional assumption that fibers are finite — if one makes no assumption at all, even weak monotonicity fails. Our proof relied on a careful dynamical essential enhancement [AG91, BBR].

In a more recent work, Paul Rax, Rémy Poudevigne--Auboiron *(yes, his name does contain a double dash)* and myself revisited these monotonicity results in two different ways [3]. To explain the first way, let me state a result.

Take a countable array. Independently, for each column, pick a Bernoulli random variable with parameter p. Whenever you see 1, write a 1 somewhere in the column; when you see 0, do nothing. We prove that, however you make your choices, it is always possible to fill in the remaining spots with suitable 0 and 1 so that the resulting array consists in independent Bernoulli random variables with parameter p. Actually, we prove much more general statements, a variation, and disprove overoptimistic generalisations.

This connects to monotonicity of p_c under covering maps as follows: if $\mathscr{G}$ covers $\mathscr{H}$, one can declare our array to be given by the vertex-set of $\mathscr{G}$ and state that two vertices are in the same column if they lie in the same fibre, i.e. if they sit above the same vertex of $\mathscr{H}$. With our lemma at hand, the following argument becomes sound: if a parameter is supercritical in $\mathscr{H}$, we can find an infinite open path, which we can lift in $\mathscr{G}$, entailing that the same parameter is supercritical (or critical) in $\mathscr{G}$. This is very different from the proof of Benjamini–Schramm, which relied on the fact that natural exploration algorithms enjoy a Markov property. Our argument only requires to be able to lift all infinite paths anywhere, while the classical argument requires that each edge (u, v) in $\mathscr{H}$ can be lifted to put its head at any point in the fiber of u. Interestingly, our array-lifting result also entails the BK inequality [vdBK85].

We also revisit the proof of strict monotonicity. We do not use these array-techniques: instead, we continue to use exploration algorithms. The novelty is that we bypass essential enhancements and differential inequalities, working with couplings all along. To be honest, this is the proof I was looking for when Vincent Tassion and I started thinking about this question around 2010–11. The idea that was missing can be summarised in a nutshell. In order to prove that $p_c(\mathscr{G}) < p_c(\mathscr{H})$, we introduce a model of percolation with some "bonus" on $\mathscr{G}$ and we prove that $p_c(\mathscr{G}) < p_c^{\mathrm{bonus}}(\mathscr{G}) \leqslant p_c(\mathscr{H})$. The key observation is that the bonus process can be explored in two different ways, each of them leading to one of the two inequalities we need to compound. However, this revisited proof currently only works for bond percolation, while that of [8] covers both bond and site percolation.

At last, let K be the set of all $p_c(\mathscr{G})$ when $\mathscr{G}$ ranges over transitive graphs: what can be said about K? The two main questions are: is 1 an isolated point? is the set K large in some sense? These questions admit variations where $\mathscr{G}$ ranges over a smaller set, such as transitive graphs of fixed degree d, or Cayley graphs of groups of a specific type. Much progress has been made in the past few years regarding the fact that 1 should be isolated; see notably [EST25, HT24, LMTT23, PS23]. In the other direction, a natural and folklore open question goes as follows: does K have nonempty interior? This is still wide open but it is known that K contains a Cantor set: this results from Strict Monotonicity and Schramm's Locality Conjecture. Essentially, start with a sufficiently rich group, pick a relation and mod out, or not, by it. By strict monotonicity, this gives two values in K. Then, do the same starting from both groups, and mod out by a large enough relation: then by strict monotinicity and locality, you get 2 values that cannot be the previous ones. Iterating this k times produces 2^k values, and it then suffices to handle suitably the case "$k = \mathbb{N}$".

Before having the general results of strict monotonicity and locality, Gady Kozma had an unpublished argument relying on groups H that are free products of the form $G \star G$. Unknowing of that, I observed that strict monotonicity and locality was known in rather large generality for another quantity of statistical mechanics, the connective constant μ. In [10], I implemented the argument above but with 3-solvable groups, thus proving that the set of all $\mu(\mathscr{G})$ where $\mathscr{G}$ ranges over 3-solvable groups contains a Cantor set. This is optimal in the sense that there are countably many 2-solvable groups, hence Cayley graphs thereof. With [8], this result became available for p_c as well.

More recently, with Christoforos Panagiotis, we investigate locality beyond p_c; given any interesting quantity or function, one can try to determine whether it can be well approximated by knowing only a ball of large radius of our graph $\mathscr{G}$. Likewise, if one has crucial estimates (such as the so-called supercritical sharpness), one can try to prove a bound by knowing a large ball only[1]. In [1], we prove such things for the class of transitive graphs of superlinear polynomial growth. Superlinear is natural here: it avoids $p_c = 1$ and silly counterexamples. More precisely, Georgakopoulos and Panagiotis proved in [GP23] that the cluster density function $\theta : p \mapsto \mathbb{P}(o \leftrightarrow \infty)$ and other natural functions are analytic in the supercritical regime. We prove that this holds for all graphs we consider, and that away from p_c, the analytic extension can be well approximated by knowing a large ball only. We also provide local (a.k.a. finitary) versions of supercritical sharpness and of the Kesten–Zhang estimate $\mathbb{P}(n < |\text{Cluster}| < \infty) \leqslant \exp(-cn^\alpha)$, where $\alpha = \frac{d-1}{d}$ and d is the growth-dimension of the

[1]This is related to finitary structure theorem à la [ST10, BGT12, TT21, TT].

graph. Our proof relies on [4, GP23, TT] but we also need a novel ingredient. In [Tim07], Timár proves in particular that for every transitive graph $\mathscr{G}$ of superlinear polynomial growth, there is a constant c such that every minimal cutset between any two vertices is connected if we allow to walk with legs of length c. We prove that it is possible to pick such a constant so that this condition holds not only for $\mathscr{G}$ but also for any graph close enough to $\mathscr{G}$ for the local topology.

Robust techniques in percolation theory

Several of my works go in the direction of developing robust tools to study percolation processes. In [9], we study percolation beyond the isotropic case: if a box is symmetric, then all its faces touch the infinite cluster with the same probability; in particular, if this probability is large for at least one of them, then it is large for all. In the anisotropic case, probability and geometry cannot be treated separately anymore. We construct a possibly distorted box so that all its faces touch the infinite cluster with the same probability. This makes this part of the argument work, and we need to prove that other parts of the argument are not badly distorted.

A much stronger result was then achieved in [4]. The goal is to prove supercritical sharpness for transitive graphs of superlinear polynomial growth, meaning that for $p > p_c$, the probability that "two vertices belong to the same cluster and that this cluster is finite" decays exponentially in the distance between these vertices. This improves drastically the understanding of the supercritical regime for Bernoulli percolation on this class of graphs. We obtain it by using simultaneously a wide array of recent techniques, which we had to hone and combine in a suitable way. The key new ingredient is the use of a Hamming Inequality to prove that, above p_c, if vertices have a poor probability to be connected without getting too far, then (with high probability) there are many disjoint open paths connecting the r-ball around the origin to the $10r$-sphere. This results in a win-win dichotomy: either one can find two vertices that have a fair probability to be connected without getting too far, or we are able to get many disjoint paths from r to $10r$. Another novel ingredient is a way to perform coarse-graining without dezooming: instead of working with the graph of boxes, we stay on the original graph and study a dependent percolation model defined by declaring a vertex v to be active if the box of center v satisfies whatever good property we need. Our techniques are indeed robust. This is testified by the fact that they could be use very fruitfully by Easo and Hutchcroft as ingredients for the final resolution of the Locality Conjecture.

The work [3] has already been discussed above, and we pointed out that we do not need the map connecting $\mathscr{G}$ to $\mathscr{H}$ to be as rigid as a covering map or even a fibration: as long as every infinite path can be lifted in at least one way, we are good.

There is no need to be able to lift it from *every* point in the fiber of the starting point; being able to do so for *some* point suffices. Our result has the flavour of a versatile lemma. In particular, it was conceived to revisit monotonicity of p_c but, a posteriori, seems taylored for the BK inequality.

At last, with Bernardo de Lima, Humberto Sanna and Daniel Valesin, we investigated percolation on $\mathbb{Z}^d$ with a sublattice of defect. This means that the percolation parameter is taken equal to p everywhere, except that on $\mathbb{Z}^s \times \{0\}^{d-s}$, it is taken equal to some other value q. We can now do better but in the published version of this work [7], we prove the Grimmett-Marstrand Theorem in this setup when $s \geqslant 2$ and uniqueness of the infinite cluster when $s \geqslant d-1$ or $p \neq p_c(\mathbb{Z}^d)$. The idea is to investigate a natural percolation model when the natural symmetry group does not act quasi-transitively but still has infinite orbits.

Percolation and pseudoperiodicity

Let us consider a dynamical system, in the sense of a probability measure preserving action of $\mathbb{Z}$ on a standard probability space. It is said to be ergodic if it cannot be decomposed further with respect to unions, and any system can be decomposed into ergodic parts. We are then left with the task of understanding ergodic systems. A nice class of examples of probabilistic nature is that of weakly mixing systems, where the archetype is Bernoulli shift. Another class, with an algebraic flavour, is that of pseudoperiodic systems, where the archetype is $\mathbb{Z}$ acting by aperiodic rotations on the circle. A celebrated result of Furstenberg–Zimmer [Fur77, Zim76] states that any ergodic system can be decomposed as a transfinite tower over the trivial one-point system, in such a way that all extensions are pseudoperiodic relative to what is below, except for the last one, which is weakly mixing *(it may be that the number of pseudoperiodic extensions is zero or that the weakly mixing extension is trivial).* Decomposing an ergodic system makes sense because even though ergodic systems are indecomposable with respect to unions, the Furstenberg decomposition is in the sense of factors.

This result paved a new way for understanding general systems: in some sense, it suffices to understand perfectly both the weakly mixing case and the pseudoperiodic case. A similar discussion can be done on $\mathbb{Z}^d$, with additional care.

In percolation, all results are either general or focus on non-pseudoperiodic examples. Regarding non-pseudoperiodic examples, almost all of them are weakly mixing, meaning that they have no pseudoperiodic content at all, but a few of them turn out to have both pseudoperiodic and weakly mixing content; see [JMP00, Hof05, Pet08, DCHKS18, HS19, KSV22, HSST23]. For the sake of exploring percolation, it seems worthwhile to have an archetypal example of pseudoperiodic percolation and to try to understand it well. This is what I investigated in [6, 2], where the second paper is in collaboration with Samuel Le Fourn and Mike Liu.

Consider a lattice Γ in $\mathbb{R}^d$, for $d \geqslant 2$. Given $u, v \in \Gamma$, say that u is visible from v if they are distinct and the straight line segment $[u, v]$ intersects Γ at no other point than u and v. Now take a point X "uniformly at random in Γ", and try to describe the structure of the following colouring: a vertex is white if it is visible from X, and black otherwise. Of course, this question makes no sense, as there is no uniform probability measure on $\mathbb{Z}^d$. The point of [6] is to make sense[2] of this question and its answer — and deal with diverse generalisations of it. The answer is: independently, for every prime p, pick a coset B_p of $p\Gamma$ uniformly at random; then declare $\bigcup_p B_p$ to be black and its complement to be white. Independence comes from the Chinese Remainder Theorem. A routine argument using continuity abstract nonsense only cannot work, so an additional little twist is needed *(we need either tightness or agreement of marginals)*. Results are stated for graphons, Benjamini–Schramm limits, and both simultaneously.

I also used results of Vardi [Var99] to prove that, if $\Gamma = \mathbb{Z}^d$ is endowed with its usual nearest-neighbour graph structure, then there is almost surely exactly one[3] infinite white cluster and zero infinite black cluster. Then, in [2], we got a much more direct proof of this result. We also generalised the result about white clusters to nearest-neighbour graphs structures on some other interesting lattices, including the triangular lattice, D_d, E_8, and the Leech lattice. We do not work directly on these examples: we get a conceptual theorem that covers in particular these examples. We also have inexistence of black infinite clusters for the triangular lattice and D_d.

Other works

I have a few other works. In [14], following an idea of Gaboriau, I introduced a percolation-counterpart to the ergodic notion of strong ergodicity. By combining results from [CI10, GL09, LS99], I got that strong ergodicity holds for Bernoulli percolation on nonamenable Cayley graphs. I also proved that a natural percolation model on $\mathbb{F}_2$ defined by coalescing directed random walks enjoys indistinguishability

[2]Here are two approaches. One way is to pick a point uniformly at random in a big Følner set F_n and let $n \to \infty$. A second way is to pick a Haar-distributed point in G^d, where the group G is a suitable compactification of $\mathbb{Z}$: one may pick $G = \prod_{p \text{ prime}} \mathbb{Z}/p\mathbb{Z}$ or $\hat{\mathbb{Z}} = \varprojlim_n \mathbb{Z}/n\mathbb{Z}$. Both compactifications work very well and give the same satisfactory answer. Følner sequences do not always produce the same (good) answer: on top of the "additive condition" of being Følner, there is a radial condition, of multiplicative nature. We use both addition and multiplication: OK we're doing number theory. The compactification approach gives pseudoperiodicity for free, and bears ressemblance with the archetype of aperiodic rotations on the circle — which indeed corresponds to representing $\mathbb{Z}$ as a dense subgroup of the compact group $\mathbb{R}/\mathbb{Z}$.

[3]Uniqueness is a nontrivial result there because the model has absolutely no insertion-tolerance at all.

and that it does not enjoy strong indistinguishability.

I also got interested in the geometry of graphs. In [12], I investigated the geometry of transitive locally infinite connected graphs. For instance, I produced Cayley graphs of $\bigoplus_n \mathbb{Z}/n\mathbb{Z}$ and of $\mathbb{Z}$ with infinite generating sets such that these graphs have infinite diameter but contain no one-sided infinite geodesic ray. I also introduced a notion of transfinite radius of a graph: for example, a graph has transfinite diameter at least $\omega \cdot 2$ if and only if there are arbitrarily long geodesic paths starting from the origin that can be extended to arbitrarily long geodesic paths. I proved that there are Cayley graphs of abelian groups of all transfinite radius. Another pecularity of locally infinite connected graphs is that the local topology is not Hausdorff anymore. This led me to ask several natural questions when we spice things up with transitivity. Some of these questions are answered in [12] and some are left unanswered.

A connected locally finite graph is Cayley if and only if it can be endowed with a free transitive action of some group by graph automorphisms. In a nutshell, Cayley graphs are transitive due to translations. Are there graphs transitive due to rotations, meaning via some action where every element has a fixed point? There is no such *finite* graph with at least 2 vertices. By using special Tarski monsters built by Ivanov, I prove in [12] that there are rotarily transitive graphs. But are there strongly rotarily transitive graphs, meaning that every element of the full automorphism group of the graph acts with at least one fixed point. By using the elliptic plane (i.e. the projective plane seen as a Riemannian space), I prove that there are if one drops the condition of local finiteness. By working with suitable coefficients, such examples can further be taken to be countable.

At last, in [11], I initiated the study of a model suggested by Vincent Beffara, called Directed Diffusion-Limited Aggregation. This model is a midpoint between the well-understood model of ballistic deposition and the notoriously hard DLA model.

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