Problem 01. Suppose two gaussian distributions associated each one to a class such as

- $p(x|c_0) \sim N(\mu_0, \sigma_0)$
- $p(x|c_1) \sim N(\mu_1, \sigma_1)$

where both have same variance, different mean, same prior distribution and 1 feature:

- $P(c_0) = P(c_1) = 1/2$
- $\sigma_0^2 = \sigma_1^2 = 1$
- $\mu_0 = 6$ and $\mu_1 = 12$

Find analytically the decision function. How is the obtained function?

Solution:

$$p(x \mid c_0) > p(x \mid w_1)$$

then

$$\mathbf{exp}\left[\frac{-1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2\right] > \mathbf{exp}\left[\frac{-1}{2}\left(\frac{x-\mu_1}{\sigma}\right)^2\right]$$

applying $\mathbf{ln(x)}$ on each side of the inequality

$$\frac{-1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2 > \frac{-1}{2}\left(\frac{x-\mu_1}{\sigma}\right)^2$$

replacing with the known parameters

$$(x-6)^2 < (x-12)^2$$

$$x^2 - 12x + 36 < x^2 - 24x + 144$$

$$12x < 108$$

then final solution is

$$x < 9$$

Problem 02. Now suppose the same input parameters of the problem 01 but with different prior distributions such as

- $P(c_0) = 3/4$
- $P(c_1) = 1/4$

Then from the bayes rule the Likelihood ratio will be expressed as

$$\frac{p(x \mid c_0)}{p(x \mid c_1)} > \frac{P(c_1)}{P(c_0)}$$

$\Rightarrow$

$$\frac{p(x \mid c_0)}{p(x \mid c_1)} > \frac{0{,}25}{0{,}75} = \frac{1}{3}$$

$\Rightarrow$

$$3.p(x \mid c_0) > p(x \mid c_1)$$

$\Rightarrow$

$$3.\mathbf{exp}\left[\frac{-1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2\right] > \mathbf{exp}\left[\frac{-1}{2}\left(\frac{x-\mu_1}{\sigma}\right)^2\right]$$

$\Rightarrow$

$$\mathbf{ln}(3) - \frac{1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2 > \frac{-1}{2}\left(\frac{x-\mu_1}{\sigma}\right)^2$$

$\Rightarrow$

$$\mathbf{ln}(3) - \frac{1}{2}(x-6)^2 > -\frac{1}{2}(x-12)^2$$

$\Rightarrow$

$$\mathbf{ln}(3) - \frac{1}{2}(x^2 - 12x + 36) > -\frac{1}{2}(x^2 - 24x + 144)$$

$\Rightarrow$

$$\mathbf{ln}(3) - 0{,}5x^2 + 6x - 18 > -0{,}5x^2 + 12x - 72$$

$\Rightarrow$

$$-6x > -72 + 18 - \mathbf{ln}(3) = -55$$

$\Rightarrow$

$$x < \frac{55}{6} = 9{,}16$$

Problem 07. Suppose a binary classification problem where each class is generated by a gaussian distribution such as they have different variance, different mean, same prior and 1 feature:

- $P(c_0) = P(c_1) = 1/2$
- $\sigma_0^2 = 3$

- $\sigma_1^2 = 1$
- $\mu_0 = 6$ and $\mu_1 = 12$

$\Rightarrow$

$$\frac{1}{\sqrt{2\pi\sigma_0^2}} exp \left[-\frac{1}{2} (\frac{x-\mu_0}{\sigma_1}) \right] > \frac{1}{\sqrt{2\pi\sigma_1^2}} exp \left[-\frac{1}{2} (\frac{x-\mu_1}{\sigma_1}) \right]$$

$\Rightarrow$

$$ln(1) - ln(\sqrt{2\pi\sigma_0^2}) - \frac{1}{2} \left(\frac{x-\mu_0}{\sigma_0} \right)^2 > ln(1) - ln(\sqrt{2\pi\sigma_1^2}) - \frac{1}{2} \left(\frac{x-\mu_1}{\sigma_1} \right)^2$$

using the input parameters then

$$-ln\left(\sqrt{2\pi 3}\right) - \frac{1}{2}\frac{(x-6)^2}{3} > -ln\left(\sqrt{2\pi}\right) - \frac{1}{2}\frac{(x-12)^2}{1}$$

where $-ln\left(\sqrt{2\pi 3}\right) + ln\left(\sqrt{2\pi}\right) = -0{,}55$

$$-0{,}55 - \frac{1}{6}(x^2 - 12x + 36) < -\frac{1}{2}(x^2 - 24x + 144)$$

$\Rightarrow$????