Machine Learning and Pattern Recognition Practice Session IV

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Overview

Support Vector Machines - Binary Case

Support Vector Machines

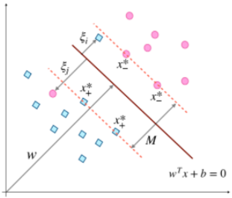
- Support Vector Classification [1] can build a nonlinear rule by constructing a linear boundary in a transformed and high dimensional version of the feature space.
- In binary classification the goal is to estimate a function $f : \mathbb{R} \rightarrow \{+1, -1\}$ from training data samples x_i with label Y_i. SVC aims to estimate a hyperplane

$$x : f(x) = x^T \omega + \omega 0 = 0 \tag{1}$$

corresponding to the decision function:

$$D\left(x\right) = sign\left[x^T \omega + \omega_0\right] \tag{2}$$

Support Vector Machines



$$\min \quad \frac{1}{2}\left\|w\right\|^2$$

$$\text{s.t} \quad y_i(wx_i + b) \geq 1$$

$$\mathcal{X} \in \mathbb{R}^d \quad y \in \{-1, 1\}$$

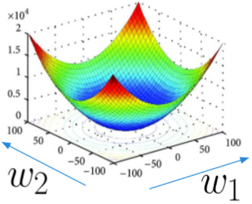


Figure: The SVM boundary is the optimal hyperplane obtained after solving a convex quadratic optimization with a global minima.

Soft Margin

The decision function D obtained corresponds to the hyperplane which maximizes the separating margin M between the two classes where $M = 1/\left\|\omega\right\|$. Now supose that both classes overlap and are not linearly separable. A set of slack variables $\xi = (\xi_1, ..., \xi_m)$ are defined to allow some miss classifications when a sample fall on the wrong side of the margin [1].
Then a convex optimization problem is expressed in equation 10 where the cost C parameter penalizes every miss classification.

$$min \frac{1}{2}\left\|\omega\right\|^2 + C\sum_{i=1}^{m}\xi_i \tag{3}$$

$$s.t. \xi_i \geq 0; y_i\left(x^T\omega + \omega 0\right) \geq 1 - \xi_i \tag{4}$$

After a quadratic programming solution applying Lagrange Multipliers the solution of ω is expressed as

$$\hat{\omega} = \sum_{1}^{m} \hat{\alpha}_i y_i x_i \tag{5}$$

with non zero coefficient $\hat{\alpha}_i$ only for the samples lying on the edge of the margin or for the miss classified ones. These samples are known as support vectors [1].

SVM and the kernel trick

Support Vector Machines and the Kernel trick

The core idea is to apply a transformation/mapping to the input feature vector X and then use linear models in the new space [1]. The transformation is denoted as ϕ where ϕ_m corresponds to the m_{th} transformation of X and $m = 1...M$. Then the decision function can be written as

$$f(x) = \phi(x)^T w + w_0 = \sum_{i=1}^{M} \alpha_i y_i \langle \phi(x), \phi(x') \rangle + w_0 \qquad (6)$$

where $\phi(x)$ is used only for inner products. For this reason it is not necessary to determine the transformation $\phi(x)$ but it is required to know the positive and semi-definite kernel function $K(x_i, x_j) = \langle \phi(x_i), \phi(x_j) \rangle$ responsible to compute the inner products in the transformed space.

Support Vector Machines and the Kernel trick

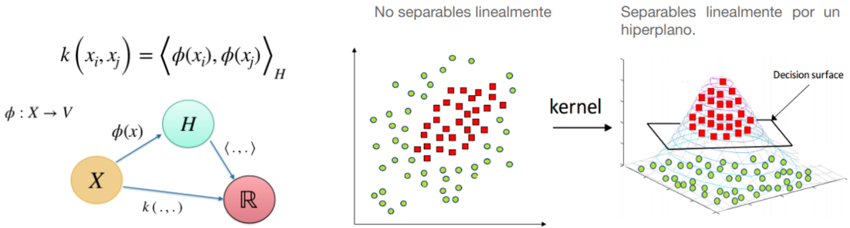


Figure: By using non-linear kernel functions it is possible to non-linear map the samples to a high dimensional hilbert space where the linear SVM is solved.

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