Rules of type theory

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$$\frac{\Gamma \vdash}{1 : \Gamma \to \Gamma} \qquad \frac{\sigma : \Delta \to \Gamma \quad \delta : \Theta \to \Delta}{\sigma\delta : \Theta \to \Gamma}$$

$$\frac{\Gamma \vdash A \quad \sigma : \Delta \to \Gamma}{\Delta \vdash A\sigma} \qquad \frac{\Gamma \vdash t : A \quad \sigma : \Delta \to \Gamma}{\Delta \vdash t\sigma : A\sigma}$$

$$\frac{}{() \vdash} \qquad \frac{\Gamma \vdash \quad \Gamma \vdash A}{\Gamma.A \vdash} \qquad \frac{\Gamma \vdash A}{\mathsf{p} : \Gamma.A \to \Gamma} \qquad \frac{\Gamma \vdash A}{\Gamma.A \vdash \mathsf{q} : A\mathsf{p}}$$

$$\frac{\sigma : \Delta \to \Gamma \quad \Gamma \vdash A \quad \Delta \vdash u : A\sigma}{(\sigma, u) : \Delta \to \Gamma.A}$$

$$\frac{\Gamma.A \vdash B}{\Gamma \vdash \Pi\ A\ B} \qquad \frac{\Gamma.A \vdash B \quad \Gamma.A \vdash b : B}{\Gamma \vdash \lambda b : \Pi\ A\ B}$$

$$\frac{\Gamma.A \vdash B}{\Gamma \vdash \Sigma\ A\ B} \qquad \frac{\Gamma.A \vdash B \quad \Gamma \vdash u : A \quad \Gamma \vdash v : B[u]}{\Gamma \vdash (u, v) : \Sigma\ A\ B}$$

$$\frac{\Gamma \vdash w : \Pi\ A\ B \quad \Gamma \vdash u : A}{\Gamma \vdash \mathsf{app}(w, u) : B[u]}$$

$$\frac{\Gamma \vdash w : \Sigma\ A\ B}{\Gamma \vdash w.1 : A} \qquad \frac{\Gamma \vdash w : \Sigma\ A\ B}{\Gamma \vdash w.2 : B[w.1]}$$

$$1\sigma = \sigma 1 = \sigma \qquad (\sigma\delta)\nu = \sigma(\delta\nu) \qquad [u] = (1, u)$$

$$A1 = A \qquad (A\sigma)\delta = A(\sigma\delta)$$

$$u1 = u \qquad (u\sigma)\delta = u(\sigma\delta)$$

$$(\sigma, u)\delta = (\sigma\delta, u\delta) \qquad \mathsf{p}(\sigma, u) = \sigma \qquad \mathsf{q}(\sigma, u) = u$$

$$\mathsf{app}(w, u)\delta = \mathsf{app}(w\delta, u\delta) \qquad \mathsf{app}(\lambda b, u) = b[u]$$

$$(\Pi\ A\ B)\sigma = \Pi\ (A\sigma)\ (B(\sigma\mathsf{p}, \mathsf{q})) \qquad (\Sigma\ A\ B)\sigma = \Sigma\ (A\sigma)\ (B(\sigma\mathsf{p}, \mathsf{q}))$$

$$(t_0, t_1)\sigma = (t_0\sigma, t_1\sigma) \qquad (u, v).1 = u \qquad (u, v).2 = v \qquad (t.1)\sigma = t\sigma.1 \qquad (t.2)\sigma = t\sigma.2$$

References

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[3] P. Dybjer. Internal Type Theory. in *Types for Programs and Proofs*, Springer, 1996.