The base category

$$\frac{}{[] \vdash} \qquad \frac{\Gamma \vdash}{1 : \Gamma \to \Gamma} \qquad \frac{\sigma : \Delta \to \Gamma \qquad \tau : \Theta \to \Delta}{\sigma \circ \tau : \Theta \to \Gamma}$$

$$1 \circ \sigma = \sigma = \sigma \circ 1 \qquad (\sigma \circ \tau) \circ \nu = \sigma \circ (\tau \circ \nu)$$

Substitutions

$$\frac{\Gamma \vdash A\ \mathrm{Type}_\ell \qquad \sigma : \Delta \to \Gamma}{\Delta \vdash A[\sigma]\ \mathrm{Type}_\ell} \qquad \frac{\Gamma \vdash t : A \qquad \sigma : \Delta \to \Gamma}{\Delta \vdash t[\sigma] : A[\sigma]}$$

$$A[1] = A \qquad A[\sigma][\tau] = A[\sigma \circ \tau] \qquad t[1] = t \qquad t[\sigma][\tau] = t[\sigma \circ \tau]$$

Context comprehension

$$\frac{\Gamma \vdash A}{\Gamma.A \vdash} \qquad \frac{\Gamma \vdash A}{\mathrm{p} : \Gamma.A \to \Gamma} \qquad \frac{\Gamma \vdash A}{\Gamma.A \vdash \mathrm{q} : A[\mathrm{p}]} \qquad \frac{\sigma : \Delta \to \Gamma \qquad \Gamma \vdash A \qquad \Delta \vdash u : A[\sigma]}{(\sigma, u) : \Delta \to \Gamma.A}$$

$$\mathrm{p} \circ (\sigma, u) = \sigma \qquad \mathrm{q}[(\sigma, u)] = u \qquad (\mathrm{p}, \mathrm{q}) = 1 \qquad (\sigma, u) \circ \tau = (\sigma \circ \tau, u[\tau])$$

Σ-types

$$\frac{\Gamma \vdash A\ \mathrm{Type}_i \qquad \Gamma.A \vdash B\ \mathrm{Type}_j}{\Gamma \vdash \Sigma AB\ \mathrm{Type}_{\max(i,j)}}$$

$$\frac{\Gamma \vdash a : A \qquad \Gamma \vdash b : B[(1, a)]}{\Gamma \vdash (a, b) : \Sigma AB} \qquad \frac{\Gamma \vdash w : \Sigma AB}{\Gamma \vdash \mathrm{pr}_1(w) : A} \qquad \frac{\Gamma \vdash w : \Sigma AB}{\Gamma \vdash \mathrm{pr}_2(w) : B[(1, \mathrm{pr}_1(w))]}$$

$$\mathrm{pr}_1(a, b) = a \qquad \mathrm{pr}_2(a, b) = b$$

$$(\Sigma AB)[\sigma] = \Sigma(A[\sigma])(B[(\sigma \circ \mathrm{p}, \mathrm{q})])$$

$$(a, b)[\sigma] = (a[\sigma], b[\sigma]) \qquad \mathrm{pr}_1(w)[\sigma] = \mathrm{pr}_1(w[\sigma]) \qquad \mathrm{pr}_2(w)[\sigma] = \mathrm{pr}_2(w[\sigma])$$

Π-types

$$\frac{\Gamma \vdash A\ \mathrm{Type}_i \qquad \Gamma.A \vdash B\ \mathrm{Type}_j}{\Gamma \vdash \Pi AB\ \mathrm{Type}_{\max(i,j)}}$$

$$\frac{\Gamma.A \vdash b : B}{\Gamma \vdash \lambda b : \Pi AB} \qquad \frac{\Gamma \vdash f : \Pi AB \qquad \Gamma \vdash a : A}{\Gamma \vdash \mathrm{app}(f, a) : B[(1, a)]}$$

$$\mathrm{app}(\lambda b, a) = b[(1, a)] \qquad \lambda(\mathrm{app}(v[\mathrm{p}], \mathrm{q})) = v$$

$$(\Pi AB)[\sigma] = \Pi(A[\sigma])(B[(\sigma \circ \mathrm{p}, \mathrm{q})])$$

$$(\lambda b)[\sigma] = \lambda(b[(\sigma \circ \mathrm{p}, \mathrm{q}])) \qquad (\mathrm{app}(f, a))[\sigma] = \mathrm{app}(f[\sigma], a[\sigma])$$

Universes

$$\frac{\Gamma \vdash}{\Gamma \vdash \mathrm{U}_\ell\ \mathrm{Type}_{\ell+1}} \qquad \frac{\Gamma \vdash A\ \mathrm{Type}_\ell}{\Gamma \vdash |A| : \mathrm{U}_\ell} \qquad \frac{\Gamma \vdash T : \mathrm{U}_\ell}{\Gamma \vdash \mathrm{El}(T)\ \mathrm{Type}_\ell}$$

$$\mathrm{El}(|A|) = A \qquad |\mathrm{El}(T)| = T$$

$$\mathrm{U}_\ell[\sigma] = \mathrm{U}_\ell \qquad |A|[\sigma] = |A[\sigma]| \qquad (\mathrm{El}(T))[\sigma] = \mathrm{El}(T[\sigma])$$