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Anti-control of Chaos in Mechanical Centrifugal Governor System ⋆

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Abstract

The dynamical equation and the state equation of mechanical centrifugal governor system are derived through Lagrange equations. The system of periodic motion is analyzed by using the phase portrait, the time course diagram and the Poincaré map of the system. The chaos anti-control method by addition of linear feedback is dissertated and analyzed numerically by numerical methods. Numerical simulation shows the effectiveness and feasibility of the chaos anti-control strategy for controlling the related stable periodic orbit of the non-autonomous system to chaotic orbit.

Keywords: Chaos; Poincaré Map; Chaos Anti-control; Numerical Simulation

1 Introduction

Chaos exists widely in physics [1], mathematics [2], engineering systems [3], networks [4] and so on. Chaos research interest is increasingly strong. Chaos is not only harmful, but also can be used. Anti-control of chaos is making a non-chaotic dynamical system chaotic, which implies that the regular behavior will be destroyed and replaced by chaotic behavior. In the real word, chaotic behavior is important. Examples include liquid mixing, human heartbeat regulations, resonance prevention in mechanical systems and secure communication [5]. Due to the chaos anti-control has significant research value and very attractive application foreground in engineering technology, experts in the international nonlinear dynamical systems and engineering controls have paid attention to it. Chaos anti-control has become one of the research focus in nonlinear science. In recent years, chaos anti-control has made great progress [6-16].

In the paper, the mechanical centrifugal governor system is studied. It plays an important role in many rotational machines such as steam engine, diesel engine and so on. A linear controller with certain feedback gain is proposed to anti-control chaotic. We achieved anticipated target.

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It confirms that the approach is effective and practical, which provides a theoretical basis for chaotic sequence.

2 Dynamic Model and Differential Equations of Motion

The mechanical centrifugal governor system is shown in Fig. 1 [17]. The engine drives the flywheel to rotate. Angular velocity is ω. The flywheel and shaft coupled by gear box, shaft rotation speed is $n\omega$. Shaft end is hinged with two rods in length of l. The other end of the rods are connected to two rigid sphere with mass m, and then two rigid sphere connecting with a sleeve by two rods. The shaft is sheathed with a spring with stiffness k. One end of the spring is at the top of the end face of the sleeve. Mechanical centrifugal governor system can adjust the fuel flow Q of internal combustion engine or the steam flow Q of steam engine, enabling the flywheel to rotate at the constant speed ω_0. When $\Delta\omega = \omega - \omega_0 \neq 0$, the rod will move up and down, θ as control variable.

In order to analyze the system easily, the following basic assumptions are proposed.

(1) Ignoring the mass of the rods and the spring.

(2) The viscous friction coefficient of the axis is c.

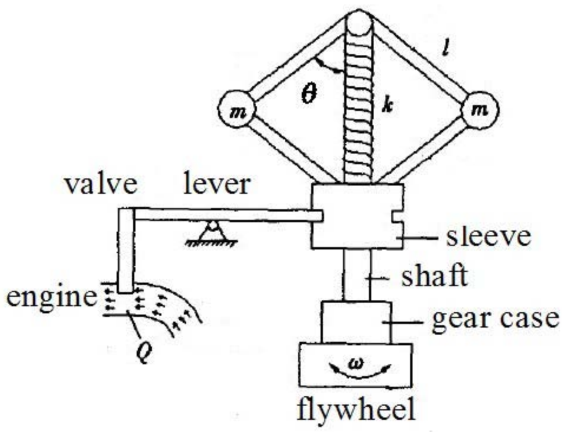

Fig. 1: Physical model of the system

According to the Lagrange equation, the dynamic equations of the system are given by

$$\begin{cases} \ddot{\theta} = (n\omega)^2 \sin\theta\cos\theta + \dfrac{2k}{m}\sin\theta\cos\theta - \dfrac{2kl+mg}{ml}\sin\theta - \dfrac{c}{2ml^2}\dot{\theta}, \\ J\dot{\omega} = \lambda\cos\theta - F. \end{cases} \tag{1}$$

Where J is the flywheel moment of inertia, λ is a scaling factor, F is the external force, g is the gravity acceleration. Let $x_1 = \theta, x_2 = \dot{\theta}, x_3 = \omega$. Then Eq. 1 is written in the standard form of three-dimensional autonomous system

$$\begin{cases} \dot{x_1} = x_2, \\ \dot{x_2} = a + n^2 x_3^2 \sin x_1 \cos x_1 - (a + g/l)\sin x_1 - bx_2, \\ \dot{x_3} = (\lambda\cos x_1 - F)/J. \end{cases} \tag{2}$$

Where $a = 2k/m, b = c/2ml^2, x_3 = \omega$. Make Eq. 2 equal to zero, it can obtain the system equilibrium point for the $(\arccos(F/\lambda), 0, \pm\sqrt{\gamma}/n)$, in which $\gamma = g\lambda + al(\lambda - F)/Fl$.

3 Numerical Simulation

In the discussion of equilibrium points of the system and its stability, in order to facilitate the initial system state research, the system is often imposed by a non-periodic force $F = (P + \lambda\cos\theta_0) - P_1{}'$. However, the mechanical centrifugal flywheel governor is imposed by external forces, commonly used in the form of $F = (P + \lambda\cos\theta_0) - P_1{}' + J\lambda\sin\omega t$. Where $P_1{}'$ is the torque due to steam or fuel effect, P is the torque generated by the load change. System state equation can be transformed into the standard three dimensional non-autonomous equations

$$\begin{cases} \dot{x_1} = x_2, \\ \dot{x_2} = a + n^2x_3^2\sin x_1\cos x_1 - (a + g/l)\sin x_1 - bx_2, \\ dotx_3 = (\lambda\cos x_1 - F)/J - c\sin\omega t. \end{cases} \tag{3}$$

The mechanical centrifugal flywheel governor system, with system parameters $n = 4.0, l = 0.9, a = 0.3, \omega = 1.2, b = 0.4, F = 0.25, J = 1.3, c = 0.8$, has been chosen to be analyzed. The system parameter λ is taken as the bifurcation parameter. Supposing the initial point of the system is $x_1(0) = 0.1, x_2(0) = 0.1, x_3(0) = 0.1$. We applied fourth order Runge-Kutta method to the numerical simulation of the mechanical centrifugal flywheel governor system, the local bifurcation diagram is illustrated in Fig. 2. As $\lambda = 0.55$, the phase portrait, the time course diagram and the Poincaré map of the system are shown in Fig. 3 to 5. It is shown, by numerical results, that the system does periodic motion.

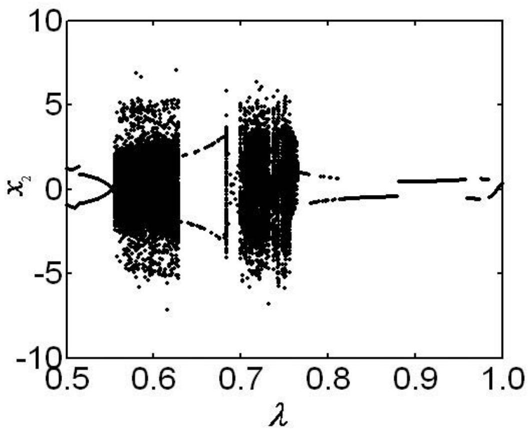

Fig. 2: Local bifurcation diagram of the system

4 Chaos Anti-control

In control theory, based on state feedback control of the general form as follow

$$x = f(x) - u(x). \tag{4}$$

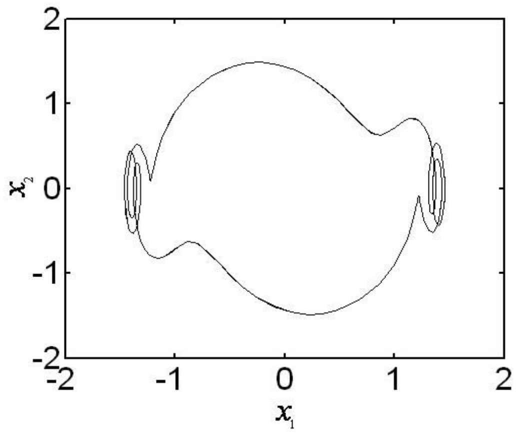

Fig. 3: Phase portrait of the system

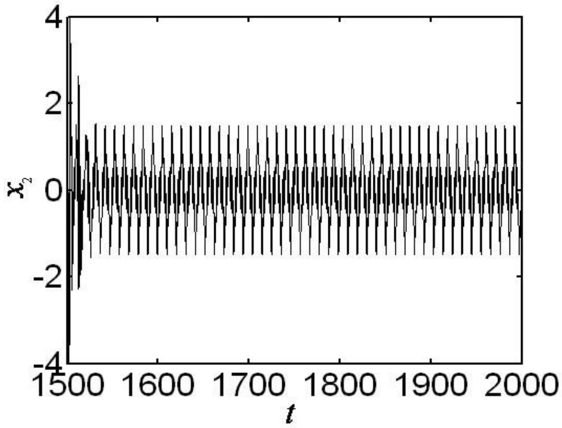

Fig. 4: Time course diagram of the system

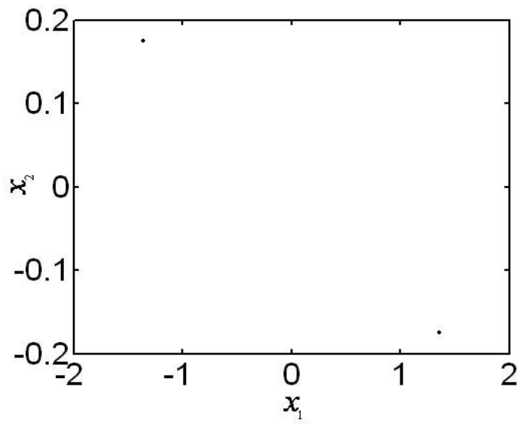

Fig. 5: Poincar map of the system

Where $x \in R^n$, f is a n-dimensional vector field, $u(x)$ is a n-dimensional vector control. In the paper, linear feedback method is used to achieve mechanical governor system bifurcation and chaos control, which n-dimensional vector $u(x)$ has the following form

$$u(x_1, x_2, x_3) = \begin{bmatrix} k(x_1 - x_2) \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

In which, k is the control parameter. Then the control vector is added to the Eq. 1, the equation of the controlled system are given by

$$\begin{cases} \dot{x_1} = x_2 - k(x_1 - x_2), \\ \dot{x_2} = a + n^2 x_3^2 \sin x_1 \cos x_1 - (a + g/l) \sin x_1 - b x_2, \\ dot x_3 = (\lambda \cos x_1 - F)/J - c \sin \omega t. \end{cases} \tag{6}$$

As $\lambda = 0.55$, the system does periodic 2 motion. After linear controller is introducted to the system, numerical analysis is carried out to unfold dynamic evolution of the mechanical centrifugal flywheel governor system by fourth order Runge-Kutta method, the local bifurcation diagram is showed in Fig. 6. From the figure, we can see, as long as selecting the appropriate parameters, the system is controlled from the periodic motion to the chaotic motion. As $k = 0.15$, the phase portrait, the time course diagram and the Poincaré map of the controlled system are obtained, as shown in Fig. 7 to 9. From these figures, we can conclude that the system is chaotic.

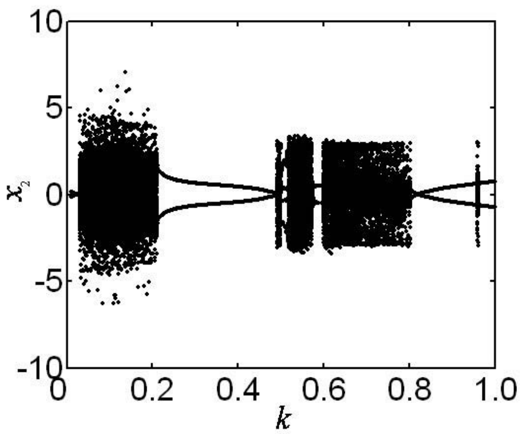

Fig. 6: Local bifurcation diagram of the system

5 Conclusion

Nonlinear system can be varied by changing the system parameters to make periodic motion to chaos motion, this spontaneous chaotic motion for non autonomous system is easy to get. In the paper, the linear feedback anti-control of control method in a given system parameters can make the system obtain chaotic orbit. This is very useful for getting some of the specific nature of the chaotic sequence, and taking advantage of this chaotic sequence.

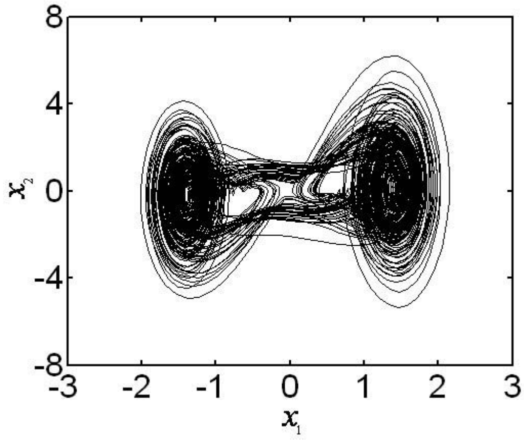

Fig. 7: Phase portrait of the system

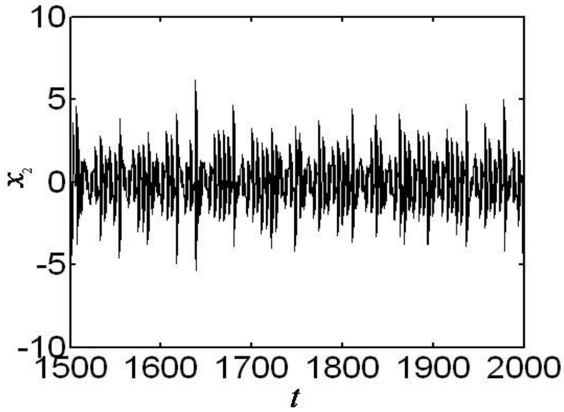

Fig. 8: Time course diagram of the system

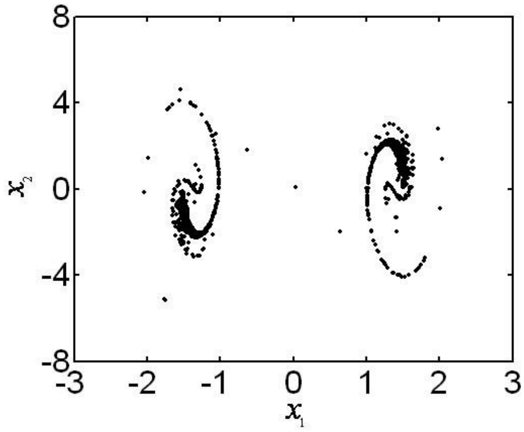

Fig. 9: Poincar map of the system

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