

We present the argument here just assuming tradeable options on X just to keep the eqs shorter but you can add the options on Y and Z yourself.

Let $\bar{\mu} \in \mathcal{P}(\mathbb{R}_+ \times \mathbb{R}_+)$ denote the reference measure¹, $p_{b/a}^j$ the bid/ask prices for call options with strike K_j and $q_{b/a}^j$ the upper and low bounds available to buy/sell. Consider the minmax problem:

$$\inf_{\mu \in \mathcal{P}(\mathbb{R}_+ \times \mathbb{R}_+)} \sup_{x \in \mathbb{R}^n: q_b^j \leq x_j \leq q_a^j} [H(\mu|\bar{\mu}) + \sum_{j=0}^n x_j (\mathbb{E}^\mu((X_T - K_j)^+) - p_a^j x_j 1_{x_j > 0} - p_b^j x_j 1_{x_j < 0})] \quad (1)$$

where $p_{a/b}^j$ denotes the bid/ask price for the j 'th option, $q_{a/b}^j$ denotes the upper and lower bounds for the amount x^j of option j that we buy (note $q_b^j < 0$ using our convention here if there is liquidity on the bid side), and $H(\mu, \bar{\mu}) := \mathbb{E}^\mu(\log \frac{d\mu}{d\bar{\mu}}) = \mathbb{E}^{\bar{\mu}}(\frac{d\mu}{d\bar{\mu}} \log \frac{d\mu}{d\bar{\mu}})$ is the entropy of μ with respect to $\bar{\mu}$. If we can interchange the inf and sup here (which we can do if we replace $\mathbb{R}_+ \times \mathbb{R}_+$ with a finite rectangle here), we can re-write (??) as

$$\sup_{x \in \mathbb{R}^n: q_b^j \leq x_j \leq q_a^j} \inf_{\mu \in \mathcal{P}(\mathbb{R}_+ \times \mathbb{R}_+)} [H(\mu|\bar{\mu}) + \sum_{j=0}^n x_j (\mathbb{E}^\mu((X_T - K_j)^+) - p_a^j x_j 1_{x_j > 0} - p_b^j x_j 1_{x_j < 0})].$$

The inner inf and the optimal μ can be computed explicitly as described on pg 8 in my article, so we can further re-write the sup inf as

$$\sup_{x \in \mathbb{R}^n: q_b^j \leq x_j \leq q_a^j} -\log \mathbb{E}^{\bar{\mu}}(e^{-\sum_{j=0}^n (x_j (X_T - K_j)^+ - p_a^j x_j 1_{x_j > 0} - p_b^j x_j 1_{x_j < 0})}) \quad (2)$$

where the optimal μ (for each $x = (x^1, \dots, x_j)$) is

$$\mu(dz) = \frac{e^{-\sum_{j=0}^n x_j (z - K_j)^+}}{\mathbb{E}^{\bar{\mu}}(e^{-\sum_{j=0}^n x_j (X_T - K_j)^+})} \bar{\mu}(dz).$$

Note we have previously always been assuming infinite liquidity i.e. $q_{b/a}^j = +\infty$, in which case we can re-write as

$$\begin{aligned} & \sup_{x \in \mathbb{R}^n} -\log \mathbb{E}^{\bar{\mu}}(e^{-\sum_{j=0}^n (x_j (X_T - K_j)^+ - p_a^j x_j 1_{x_j > 0} - p_b^j x_j 1_{x_j < 0})}) \\ &= \sup_{x \in \mathbb{R}^n} -\log \mathbb{E}^{\bar{\mu}}(e^{\sum_{j=0}^n (x_j (X_T - K_j)^+ + p_a^j x_j 1_{x_j > 0} + p_b^j x_j 1_{x_j < 0})}) \end{aligned}$$

since in this case we can flip the sign of the x_j 's without changing the answer. This is then our familiar **concave maximization** problem (see Eq 10 in my paper), but now incorporates bid/ask spreads. If $q_{a/b}^j$ are finite, then this is essentially the same problem as Project 3.

Since the sup in (??) doesn't affect $H(\mu, \bar{\mu})$, we can easily evaluate the inner sup to re-write (??) as

$$\inf_{\mu \in \mathbb{R}_+ \times \mathbb{R}_+} [H(\mu|\bar{\mu}) + \sum_{j=0}^n q_a^j (\mathbb{E}^\mu((X_T - K_j)^+) - p_a^j)^+ + \sum_{j=0}^n |q_b^j| (p_b^j - \mathbb{E}^\mu((X_T - K_j)^+))^+] \quad (3)$$

i.e. **we minimize entropy over models which fall within the bid-offer spread, and models which don't (but these model incurs an additional finite penalty for each option which falls outside).** bid-offer spread i.e. there is an infinite penalty for non-calibrated models.

You can try and compute Eq 2 and Eq 3 in Mosek and see if they give the same answer

¹where $\mathcal{P}(\mathbb{R}_+)$ denotes the collection of all probability measures on \mathbb{R}_+