

$A_t = \int_0^t V_s ds$ satisfies

$$A_t = G_0(t) + \int_0^t \kappa(t-s)(-\kappa A_s + W_{A_s} ds)$$

for some $\kappa \in L^1$, and let $A^{(n)}$ satisfy

$$A_t^{(n)} = G_0(t) + \int_0^t \kappa(t-s)(-\kappa A_{[ns]/n}^{(n)} + W_{A_{[ns]/n}^{(n)}} ds.$$

Now let $\Delta_t^{(n)} = A_t - A_t^{(n)}$. Then

$$\Delta_t^{(n)} = \int_0^t \kappa(t-s)(-\kappa \Delta_s^{(n)} + W_{A_s} - W_{A^{(n)}}) ds$$

and from the Hölder continuity of BM we know that

$$|\Delta_t^{(n)}| \leq \int_0^t \kappa(t-s)(\kappa |\Delta_s^{(n)}| + c_1 |\Delta_s^{(n)}|^{\frac{1}{2}-\varepsilon}) ds$$

for some (random) constant $c_1 = c_1(\omega) > 0$.