Consider an integrated variance process $A_t = \int_0^t V_s ds$ which satisfies an eq of the form:

$$A_t \;\; = \;\; G_0(t) \, + \int_0^t \kappa(t-s) W_{A_s} ds$$

for some $\kappa \in L^1$. Consider a discrete-time Monte Carlo scheme for A:

$$A_{j\Delta t} \;\; = \;\; G_0(j\Delta t) \, + \sum_{k=1}^{j} b_{j,k} W_{A_{k\Delta t}} \tag{1}$$

where $b_{j,k} = \int_{t_{k-1}}^{t_k} K(t_j - s) ds$ and $t_k = k\Delta t$. Setting $G_0(t) = V_0 t$, we have

$$\begin{aligned} \Delta A_{j\Delta t} \;\; &= \;\; V_0 \Delta t \, + \sum_{k=1}^{j-1} (b_{j,k} - b_{j-1,k}) W_{A_{k\Delta t}} \, + \, b_{j,j} W_{A_{j\Delta t}} \\ &= \;\; V_0 \Delta t \, + \, Z_j \, + \, b_{j,j}(W_{A_{j\Delta t}} - W_{A_{(j-1)\Delta t}}) \end{aligned}$$

for $j = 1, 2, ...$, where $\tilde{W}_t = W_{A_{j\Delta t}} - W_{A_{(j-1)\Delta t}}$, where $Z_j = \sum_{k=1}^{j-1} (b_{j,k} - b_{j-1,k}) W_{A_{k\Delta t}} + b_{j,j} W_{A_{(j-1)\Delta t}}$.

If $H = \frac{1}{2}$ then

$$A_{j\Delta t} \;\; = \;\; V_0 j \Delta t \, + \, \nu \sum_{k=1}^{j} W_{A_{k\Delta t}} \Delta t$$

so

$$\Delta A_{j\Delta t} \;\; = \;\; V_0 \Delta t \, + \, \nu W_{A_{j\Delta t}} \Delta t \;\; = \;\; V_0 \Delta t \, + \, \nu W_{A_{(j-1)\Delta t}} \Delta t \, + \, \nu (W_{A_{j\Delta t}} \Delta t - W_{A_{(j-1)\Delta t}} \Delta t)$$

Then using the Appendix E argument below, ΔA_j has an Inverse Gaussian distribution.

Appendix E

$$A_t \;\; = \;\; G_0(t) \, + \, \sigma W_{A_t} \, . \tag{2}$$

Now let

$$Y_t \;\; = \;\; -t + \sigma W_t \tag{3}$$

and set $\tilde{A}_t = H_{-G_0(t)}$, where $H_b^Y = \inf\{t : Y_t = b\}$. Then setting $t \mapsto \tilde{A}_t$ in (3), we see that

$$\begin{aligned} -G_0(t) \;\; &= \;\; -\tilde{A}_t \, + \, \sigma W_{\tilde{A}_t} \\ &= \;\; -\tilde{A}_t \, + \, Y_{\tilde{A}_t} + \tilde{A}_t \;\; = \;\; Y_{\tilde{A}_t} \end{aligned}$$

i.e. $\tilde{A}$ satisfies the same equation as A_t in (2), and H_b has an Inverse Gaussian distribution (first hitting time to a barrier for a Brownian motion with drift), and note in particular that

$$\sigma W_{\tilde{A}_t} \;\; = \;\; Y_{\tilde{A}_t} + \tilde{A}_t \;\; = \;\; -G_0(t) + \tilde{A}_t$$

For us $\sigma = b_{j,j}$ for each j.