We present the argument here just assuming tradeable options on X just to keep the eqs shorter but you can add the options on Y and Z yourself.

Let $\bar{\mu} \in \mathcal{P}(\mathbb{R}_+ \times \mathbb{R}_+)$ denote the reference measure[1], $p^j_{b/a}$ the bid/ask prices for call options with strike K_j and $q^j_{b/a}$ the upper and low bounds available to buy/sell. Consider the minmax problem:

$$\inf_{\mu \in \mathcal{P}(\mathbb{R}_+ \times \mathbb{R}_+)} \sup_{x \in \mathbb{R}^n : q^j_b \leq x_j \leq q^j_a} [H(\mu|\bar{\mu}) + \sum_{j=0}^{n} x_j (\mathbb{E}^{\mu}((X_T - K_j)^+) - p^j_a x_j 1_{x_j>0} - p^j_b x_j 1_{x_j<0})] \tag{1}$$

where $p^j_{a/b}$ denotes the bid/ask price for the j'th option, $q^j_{a/b}$ denotes the upper and lower bounds for the amount x^j of option j that we buy (note $q^j_b < 0$ using our convention here if there is liquidity on the bid side), and $H(\mu, \bar{\mu}) := \mathbb{E}^{\mu}(\log \frac{d\mu}{d\bar{\mu}}) = \mathbb{E}^{\bar{\mu}}(\frac{d\mu}{d\bar{\mu}} \log \frac{d\mu}{d\bar{\mu}})$ is the entropy of μ with respect to $\bar{\mu}$. If we can interchange the inf and sup here (which we can do if we replace $\mathbb{R}_+ \times \mathbb{R}_+$ with a finite rectangle here), we can re-write (1) as

$$\sup_{x \in \mathbb{R}^n : q^j_b \leq x_j \leq q^j_a} \inf_{\mu \in \mathcal{P}(\mathbb{R}_+ \times \mathbb{R}_+)} [H(\mu|\bar{\mu}) + \sum_{j=0}^{n} x_j (\mathbb{E}^{\mu}((X_T - K_j)^+) - p^j_a x_j 1_{x_j>0} - p^j_b x_j 1_{x_j<0})] \,.$$

The inner inf and the optimal μ can be computed explicitly as described on pg 8 in my article, so we can further re-write the sup inf as

$$\sup_{x \in \mathbb{R}^n : q^j_b \leq x_j \leq q^j_a} -\log \mathbb{E}^{\bar{\mu}}(e^{-\sum_{j=0}^{n}(x_j (X_T - K_j)^+} - p^j_a x_j 1_{x_j>0} - p^j_b x_j 1_{x_j<0} \tag{2}$$

where the optimal μ (for each $x = (x^1, .., x_j)$) is

$$\mu(dz) \;=\; \frac{e^{-\sum_{j=0}^{n} x_j (z-K_j)^+}}{\mathbb{E}^{\bar{\mu}}(e^{-\sum_{j=0}^{n} x_j (X_T - K_j)^+}} \,\bar{\mu}(dz) \,.$$

Note we have previously always been assuming infinite liquidity i.e. $q^j_{b/a} = +\infty$, in which case we can re-write as

$$\begin{aligned} &\sup_{x \in \mathbb{R}^n} -\log \mathbb{E}^{\bar{\mu}}(e^{-\sum_{j=0}^{n}(x_j (X_T - K_j)^+} - p^j_a x_j 1_{x_j>0} - p^j_b x_j 1_{x_j<0} \\ = \;&\sup_{x \in \mathbb{R}^n} -\log \mathbb{E}^{\bar{\mu}}(e^{\sum_{j=0}^{n}(x_j (X_T - K_j)^+} + p^j_a x_j 1_{x_j>0} + p^j_b x_j 1_{x_j<0} \end{aligned}$$

since in this case we can flip the sign of the x_j's without changing the answer. This is then our familiar **concave maximization** problem (see Eq 10 in my paper), but now incorporates bid/ask spreads. If $q^j_{a/b}$ are finite, then this is essentially the same problem as Project 3.

Since the sup in (1) doesn't affect $H(\mu, \bar{\mu})$, we can easily evaluate the inner sup to re-write (1) as

$$\inf_{\mu \in \mathbb{R}_+ \times \mathbb{R}_+} [H(\mu|\bar{\mu}) + \sum_{j=0}^{n} q^j_a (\mathbb{E}^{\mu}((X_T - K_j)^+) - p^j_a)^+ + \sum_{j=0}^{n} |q^j_b| \, (p^j_b - \mathbb{E}^{\mu}((X_T - K_j)^+))^+]) \tag{3}$$

i.e. **we minimize entropy over models which fall within the bid-offer spread, and models which don't (but these model incurs an additional finite penalty for each option which falls outside)**. bid-offer spread i.e. there is an infinite penalty for non-calibrated models.

You can try and compute Eq 2 and Eq 3 in Mosek and see if they give the same answer

[1] where $\mathcal{P}(\mathbb{R}_+)$ denotes the collection of all probability measures on $\mathbb{R}_+$