Revision quiz

1. The price of a European call option under the Black-Scholes model does not depend on

- The volatility σ
- The strike price
- The interest rate r
- The drift μ

2. Let $X \sim N(0,1)$ where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$ is the standard Normal cumulative distribution function. Then the density of $\Phi(X)$ is

- x
- 1 for $x \in [0,1]$ and zero otherwise since $F_X(X) \sim U[0,1]$, see first lecture
- 1
- $\Phi'(x)$
- $1 - x$

3. Let $dS_t = S_t(\mu dt + \sigma dW_t)$ denote the Black-Scholes model. The price at time zero of an option which pays $\log(S_0/S_T)$ at time T under the Black-Scholes model is

- $e^{-rT}\mu T$
- $e^{-rT}(\frac{1}{2}\sigma^2 - \mu)T$
- $e^{-rT}(\frac{1}{2}\sigma^2 - r)T$, since $\log S_T \sim N(\log S_0 + (r - \frac{1}{2}\sigma^2)T, \sigma^2 T)$ under $\mathbb{Q}$
- $e^{r(T-t)}(\frac{1}{2}\sigma^2 - r)T$

4. The distribution function of a Uniform random variable on the interval $[-1,1]$ is

- $\frac{1}{2}$
- 1
- $\frac{1}{2}x$
- $\frac{1}{2}(x+1)$, since this is the only function here which is zero at -1 and 1 at 1

5. The price of a European put option under the Black-Scholes model with $r = 0$ is

- increasing as a function of maturity due to Jensen's inequality, see Black-Scholes chapter
- decreasing as a function of maturity due to Jensen's inequality
- decreasing as a function of volatility
- Concave as a function of the current stock price

6. Let $dS_t = S_t(\mu dt + \sigma dW_t)$ denote the Black-Scholes model and assume that $S_0 = 1$. Then $\mathbb{E}(S_t^2)$ is

- $e^{2(\mu - \frac{1}{2}\sigma^2)t + 2\sigma^2 t}$, see Hwk q3 with $p = 2$
- $e^{(\mu - \frac{1}{2}\sigma^2)t + 4\sigma^2 t}$
- $e^{2(\mu - \frac{1}{2}\sigma^2)t + 2\frac{1}{2}\sigma^2 t}$
- $e^{\mu t + \frac{1}{2}\sigma^2 t}$

7. Let $U \sim U[0,1]$. Then $\log(1-U)$ is

- An exponential random variable multiplied by -1, since $\mathbb{P}(\log(1-U) \leq x) = \mathbb{P}(1-U \leq e^x) = \mathbb{P}(1-e^x \leq U) = e^x$, which is an exponential density on $(-\infty, 0)$.
- 1 minus an exponential random variable
- An exponential random variable with parameter 1
- An exponential random variable with parameter -1
- A lognormal random variable

8. For the Black-Scholes model under the physical measure $\mathbb{P}$

- S can hit zero in finite time
- S tends to zero a.s. as $t \to \infty$ if and only if $\mu < 0$ because this is the drift of the stock price
- S will always tend to ∞ a.s. as $t \to \infty$ if $\mu > 0$
- S can be absorbed at zero
- S tends to zero a.s. as $t \to \infty$ if and only if $\mu - \frac{1}{2}\sigma^2 < 0$ because (from Ito's lemma) $\mu - \frac{1}{2}\sigma^2 < 0$ is the drift of the log stock price $X_t = \log S_t$.

9. The price of a call option under the Black-Scholes model is

- The discounted expected call option payoff under the real world measure
- The discounted expected call option payoff under the risk-neutral measure
- The expected call option payoff under the risk-neutral measure
- The discounted expected call option payoff with r replaced by μ

10. Let X be a random variable with continuous strictly increasing distribution function $F_X(x)$. What is the density of the random variable $F_X^{-1}(U)$ where $U \sim U[0,1]$?

- $F'_X(x)$, because $F_X^{-1}(U) \sim X$, see Applied Probability Revision chapter
- Cannot say
- -$F'_X(x)$
- 1

11. For the Black-Scholes model $\mathbb{E}^{\mathbb{P}}(S_t) > S_0$ if

- $\mu > 0$, since $\mathbb{E}^{\mathbb{P}}(S_t) = S_0 e^{\mu t}$, see Hwk 1 q3 with $p = 1$
- $r - \frac{1}{2}\sigma^2 > 0$
- $\mu - \frac{1}{2}\sigma^2 > 0$
- $r - \frac{1}{2}\sigma^2 > 0$

12. Consider a self-financing trading strategy under the Black-Scholes model which involves holding $\phi_t = C_S(S_t, t)$ units of stock at time t and placing all remaining wealth in the risk-free bank account, where $C(S,t)$ is the Black-Scholes call option price with Strike K and maturity T at time t. Let X_t denote the total wealth at time t and assume $X_0 = C(S_0, 0)$. What is the Profit/Loss (PnL) at time T?

- $\int_0^T \phi_t dS_t$
- $\max(S_T - K, 0) - C(S_0, 0)$ - this is the PnL from buying a call at $t = 0$ and holding till expiry, and the strategy in the question is just the replication strategy for this
- $\phi_t S_T - \Delta(0) S_0$
- Zero

13. The vega and the gamma of a European call option under the Black-Scholes model are

- Both positive, see Black-Scholes chapter subsection on the Greeks
- Positive and Negative
- Negative and positive
- Both negative

14. Under the Black-Scholes model, the price of a European call option at time t only depends on $T - t$ because

- Because the volatility does not jump
- S_t has stationary increments
- $X_t = \log S_t$ has stationary increments, i.e. $X_t - X_s \sim X_{t-s}$
- Because the interest rate is constant