Some things you ideally should know for the project (+particularly the oral):

1. Difference between fBM and the **RL process**: $Z_t^H = \sqrt{2H}\int_0^t (t-s)^{H-\frac{1}{2}}\,dW_s$; fBM has **stationary increments** i.e. the distribution of $B_t^H - B_s^H$ only depends on $t-s$, which is not true for the RL process.

2. The **covariance** $R(s,t)$ for the RL process is $2H\int_0^s (s-u)^{H-\frac{1}{2}}(t-u)^{H-\frac{1}{2}}\,du$ for $0 \le s \le t$ (which comes from the Ito isometry). This integral can be computed in terms of the confluent hypergeometric function but is more complicated and slower to compute than the covariance function for fBM.

3. A **scale-invariant estimator** for H is an estimator which remains unchanged if we multiply the sample path by a constant λ.

4. A **consistent estimator** $\hat{H}_N$ for H means that $\hat{H}_N \to H$ in probability as N (the number of steps) tends to ∞ (this is just for 1 path).

5. The [HS21] SSE is **scale-invariant**; the basic [HS21] estimator is not (the basic HS21 estimator works well if $X = \sigma B^H$ for $\sigma = 1$, but does not work well for $\sigma \neq 1$. The basic and the SSE estimators tend to the true H as $N = 2^n \to \infty$ and the SSE estimator is model-independent.

6. We can simulate both fBM and the RL process using the **Cholesky** method as in Part 2, which is exact, but very slow for N large because the time to compute is $O(N^3)$ (where N is number of steps), even if we use the LU decomposition

7. For the RL process, we can also simulate Z^H using the **FFT** method via discrete convolution which is $O(N \log N)$ (i.e. much quicker) (or the more involved hybrid method of Pakkanen et al which involves FFT and some Cholesky), but these two methods are bad for simulating the sum of squared increments of the process, which I often denote by SS_q with $q = 2$.

8. The RL process is used for the **rough Bergomi model**, fBM is used for the RFSV model. The rough Bergomi model is more commonly used because the speed of the FFT/**hybrid** method is better for option pricing.

9. The [HS21] SSE estimator simplifies to $1 - \frac{1}{2}\log_2(s_n^2/s_{n-1}^2)$ when their m parameter $= 0$, where s_n is defined in Eq 2.6 in [HS21], so this is much easier to remember+understand than the general SSE with equivalent definitions in Definition 8.2 and Prop 8.3 of HS21.

10. The $m(q,\Delta)$ estimator is what we call a **GMM** (Generalized Method of Moments) estimator for H and σ, which uses **linear regression** over multiple **time-scales**.

11. Many estimators for H are **asymptotically Normal** for N large; typically $\sqrt{N}(\hat{H}_N - H)$ tends to a Normal random variable with mean zero and variance σ_H^2 as $N \to \infty$, where σ_H does not depend on N; you can test this numerically by drawing histograms and performing goodness of fit tests like Kolmogorov–Smirnov, Shapiro–Wilk or Jarque–Bera which Python can do for you.