Consider an integrated variance process $A_t = \int_0^t V_s ds$ which satisfies an eq of the form:

$$A_t \quad = \quad G_0(t) + \int_0^t \kappa(t-s)(-\kappa A_s + W_{A_s} ds)$$

for some $\kappa \in L^1$, and let $A^{(n)}$ satisfy

$$A_t^{(n)} \quad = \quad G_0(t) + \int_0^t \kappa(t-s)(-\kappa A^{(n)}_{[ns]/n} + W_{A^{(n)}_{[ns]/n}} ds\,.$$

Now let $\Delta_t^{(n)} = A_t - A_t^{(n)}$. Then

$$\Delta_t^{(n)} \quad = \quad \int_0^t \kappa(t-s)(-\kappa\Delta_s^{(n)} + W_{A_s} - W_{A^{(n)}})ds$$

and from the Hölder continuity of BM we know that

$$|\Delta_t^{(n)}| \quad \leq \quad \int_0^t \kappa(t-s)(\kappa|\Delta_s^{(n)}| + c_1|\Delta_s^{(n)}|^{\frac{1}{2}-\varepsilon})ds$$

for some (random) constant $c_1 = c_1(\omega) > 0$.