1. Difference between fBM and the **RL process**: $Z_t^H = \sqrt{2H}\int_0^t (t-s)^{H-\frac{1}{2}}\,dW_s$; fBM has **stationary increments** i.e. the distribution of $B_t^H - B_s^H$ only depends on $t-s$, which is not true for the RL process Z^H.

2. The **covariance** for the RL process is $R(s,t) = 2H\int_0^s (s-u)^{H-\frac{1}{2}}(t-u)^{H-\frac{1}{2}}\,du$ for $0 \le s \le t$ (which comes from the **Ito isometry**). This integral can be computed in terms of the confluent hypergeometric function but is more complicated and slower to compute than the covariance function for fBM.

3. A **scale-invariant estimator** for H is an estimator which remains unchanged if we multiply the sample path by a constant λ.

4. A **consistent estimator** $\hat{H}_N$ for H means that $\hat{H}_N \to H$ in probability as N (the number of steps) tends to ∞ (this is just for 1 path).

5. The [HS21] SSE is **scale-invariant**; the basic [HS21] estimator is not (the basic HS21 estimator works well if $X = \sigma B^H$ for $\sigma = 1$, but does not work well for $\sigma \neq 1$. The basic and the SSE estimators tend to the true H as $N = 2^n \to \infty$ and the SSE estimator is model-independent.

6. We can simulate both fBM and the RL process using the **Cholesky** method as in Part 2, which is exact, but very slow for N large because the time to compute is $O(N^3)$ (where N is number of steps), even if we use the LU decomposition

7. For the RL process, we can also simulate Z^H using the **FFT** method via discrete convolution which is $O(N\log N)$ (i.e. much quicker) (or the more involved **hybrid scheme** of Pakkanen et al which involves FFT and some Cholesky), but these two methods are bad for simulating the sum of squared increments of the process, which I often denote by SS_q with $q = 2$.

8. The RL process is used for the **rough Bergomi model**, fBM is used for the **RFSV model**. The rough Bergomi model is more commonly used because the speed of the FFT/hybrid method is more efficient for option pricing.

9. The [HS21] SSE estimator simplifies to $1 - \frac{1}{2}\log_2(s_n^2/s_{n-1}^2)$ when their m parameter $= 0$, where s_n is defined in Eq 2.6 in [HS21], so this is much easier to remember+understand than the general SSE with equivalent definitions in Definition 8.2 and Prop 8.3 of HS21. This estimator was originally suggested much earlier by **Coeurjolly**.

10. The $m(q,\Delta)$ estimator is what we call a **GMM** (Generalized Method of Moments) estimator for H and σ, which uses **linear regression** over multiple **time-scales**.

11. Many estimators for H are **asymptotically Normal** for N large; typically $\sqrt{N}(\hat{H}_N - H)$ tends to a Normal random variable with mean zero and variance σ_H^2 as $N \to \infty$, where σ_H does not depend on N; you can test this numerically by drawing histograms and performing goodness of fit tests like Kolmogorov–Smirnov, Shapiro–Wilk or Jarque–Bera which Python can do for you.