Sinus forkurs matte oppgaver

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1.10

a)

$$-5 \in \mathbb{Z}$$

b)

$$-5 \notin \mathbb{N}$$

c)

$$\frac{2}{3} \notin \mathbb{Z}$$

d)

$$\frac{2}{3} \in \mathbb{Q}$$

e)

$$\sqrt{5} \in \mathbb{R}$$

f)

$$\sqrt{5} \notin \mathbb{Q}$$

1.11

$$2 \in \{1, 2, 3, 4\}$$

$$3 \notin \{0, 1, 2, 4\}$$

$$1, 5 \notin \{1, 2, 3, 4\}$$

$$-1 \notin \{-2, 1, 0, 1\}$$

1.12

$$\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

$$\{21, 23, 25, 27, 29, 31, 33, 35\}$$

$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

1.13

$$\{1, 2, 3, 4\} \backslash \{4\} = \{1, 2, 3\}$$

$$\{1, 2, 3, 4\} \backslash \{2, 4\} = \{1, 3\}$$

$$\{1, 2, 3, 4\} \backslash \{1, 5\} = \{2, 3, 4\}$$

$$\mathbb{Z} \backslash \mathbb{N} = \{..., -4, -3, -2, -1, 0\}$$

1.14

a)

$$\begin{aligned} 4 * 2^2 &= 4 * 4 \\ &= 16 \end{aligned}$$

b)

$$\begin{aligned} 4 * (-2)^2 &= 4 * 4 \\ &= 16 \end{aligned}$$

c)

$$\begin{aligned} 5 - 3^2 &= 5 - 9 \\ &= -4 \end{aligned}$$

d)

$$\begin{aligned} (5 - 3)^2 &= 2^2 \\ &= 4 \end{aligned}$$

e)

$$\begin{aligned} -2^2 + 3^2 - 2 * (-2) &= -4 + 9 + 4 \\ &= 9 \end{aligned}$$

f)

$$\begin{aligned} -(-2)^2 + (-3)^2 - 2^2 &= -(4) + 9 - 4 \\ &= 1 \end{aligned}$$

g)

$$\begin{aligned} (-3)^2 + 5 * (-3) + 6 &= 9 - 15 + 6 \\ &= -5 + 6 \\ &= 0 \end{aligned}$$

Oppgave 1.15

a)

$$\begin{aligned} 2(7-5) + 2 &= 2 * 2 + 2 \\ &= 4 + 2 = 6 \end{aligned}$$

b)

$$\begin{aligned} -3(4-12) + 2 * 3^2 &= -3 * -8 + 2 * 9 \\ &= 24 + 18 = 42 \end{aligned}$$

c)

$$\begin{aligned} -(8-4) - (3)^2 &= -4 - 9 \\ &= -13 \end{aligned}$$

d)

$$\begin{aligned} -2^4 + 3(17 - 3^2) + (3 * 4^2 - 2 * 5^2) &= -2^4 + 3 * 8 + (3 * 4^2 - 2 * 5^2) \\ &= -2^4 + 3(17 - 3^2) - 2 \\ &= -16 + 24 - 2 \\ &= 8 - 2 \\ &= 6 \end{aligned}$$

Oppgave 1.16

a)

$$\begin{aligned} 2(2 * 2 - 2)^2 &= 2(4-2)^2 \\ &= 2 * 4 \\ &= 8 \end{aligned}$$

b)

$$-2^6 + (-2)^6 = -64 + 64 = 0$$

c)

$$\begin{aligned} 4(3-2)^3 - 3(2-3)^3 &= 4 * 1 - 3 * (-1) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

d)

$$\begin{aligned} 4(2^2-3)^5 - 3(2^3-3^2)^5 &= 4 * 1^5 - 3 * (-1)^5 \\ &= 4 * 1 - 3 * -1 \\ &= 4 - (-3) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

Oppgave 1.17

a)

$$\begin{aligned} 2(2 * 2 - 2)^2 &= 2(4-2)^2 \\ &= 2 * 4 \\ &= 8 \end{aligned}$$

b)

$$-2^6 + (-2)^6 = -64 + 64 = 0$$

c)

$$\begin{aligned} 4(3-2)^3 - 3(2-3)^3 &= 4 * 1 - 3 * (-1) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

d)

$$\begin{aligned} 4(2^2-3)^5 - 3(2^3-3^2)^5 &= 4 * 1^5 - 3 * (-1)^5 \\ &= 4 * 1 - 3 * -1 \\ &= 4 - (-3) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

Oppgave 1.20

a)

$$\begin{aligned}\frac{4}{6} &= \frac{4:2}{6:2} \\ &= \frac{2}{3}\end{aligned}$$

b)

$$\begin{aligned}\frac{9}{15} &= \frac{9:3}{15:3} \\ &= \frac{3}{5}\end{aligned}$$

c)

$$\begin{aligned}\frac{18}{21} &= \frac{18:3}{21:3} \\ &= \frac{6}{7}\end{aligned}$$

d)

$$\begin{aligned}\frac{42}{54} &= \frac{42:6}{54:6} \\ &= \frac{7}{9}\end{aligned}$$

Oppgave 1.21

a)

$$\begin{aligned}\frac{72}{120} &= \frac{72:8}{120:8} \\ &= \frac{9}{15} \\ &= \frac{9:3}{15:3} \\ &= \frac{3}{5}\end{aligned}$$

b)

$$\begin{aligned}\frac{126}{294} &= \frac{126:7}{294:7}\\ &= \frac{18:2}{42:2}\\ &= \frac{9:3}{21:3}\\ &= \frac{3}{7}\end{aligned}$$

c)

$$\begin{aligned}\frac{132}{198} &= \frac{132:2}{198:2}\\ &= \frac{66:3}{99:3}\\ &= \frac{22:11}{33:11}\\ &= \frac{2}{3}\end{aligned}$$

d)

$$\begin{aligned}\frac{153}{51} &= \frac{153:3}{51:3}\\ &= \frac{51}{17}\end{aligned}$$

exmaples

$$\begin{aligned}3 * \frac{2}{3} &= \frac{3}{1} * \frac{3}{3} - \frac{2}{3}\\ &= \frac{7}{3} = \frac{3+3+1}{3}\\ &= \frac{3}{3} + \frac{3}{3} + \frac{1}{3}\\ &= 1 + 1 + \frac{1}{3} = 2\frac{1}{3}\end{aligned}$$

1.22

$$\begin{aligned}\frac{1}{12}+\frac{4}{9} &= \frac{9}{9}*\frac{1}{12}+\frac{4}{9}*\frac{12}{12}\\ &= \frac{9}{108}+\frac{48}{108}\\ &= \frac{57:3}{108:3}\\ &= \frac{19}{36}\end{aligned}$$

$$\begin{aligned}\frac{1}{12}*\frac{4}{9} &= \frac{4:2}{108:2}\\ &= \frac{2:2}{54:2}\\ &= \frac{1}{27}\end{aligned}$$

$$\begin{aligned}\frac{1}{12}:\frac{4}{9} &= \frac{1}{12}*\frac{9}{4}\\ &= \frac{9:3}{48:3}\\ &= \frac{3}{16}\end{aligned}$$

$$\begin{aligned}3+\frac{5}{12} &= \frac{3*12}{1*12}+\frac{5}{12}\\ &= \frac{36}{12}+\frac{5}{12}\\ &= \frac{41}{12}\end{aligned}$$

$$\begin{aligned}3*\frac{5}{12} &= \frac{3}{1}*\frac{5}{12}\\ &= \frac{15:3}{12:3}\\ &= \frac{5}{4}\end{aligned}$$

$$\begin{aligned}3:\frac{5}{12} &= \frac{3}{1}:\frac{5}{12}\\ &= \frac{3}{1}*\frac{12}{5}\\ &= \frac{36}{5}\end{aligned}$$

1.23

$$\begin{aligned}
2 * \left(\frac{3}{8} + \frac{1}{4}\right) &= 2 * \left(\frac{4}{4} * \frac{3}{8} + \frac{1}{4} * \frac{8}{8}\right) \\
&= 2 * \left(\frac{12}{32} + \frac{8}{32}\right) \\
&= 2 * \frac{20}{32} \\
&= \frac{2 * 20}{32} \\
&= \frac{40 : 2}{32 : 2} \\
&= \frac{20 : 2}{16 : 2} \\
&= \frac{10 : 2}{8 : 2} \\
&= \frac{5}{4}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{5}{6} - \frac{2}{9}\right) * \frac{3}{5} &= \left(\frac{5 * 3}{6 * 3} - \frac{2 * 2}{9 * 2}\right) * \frac{3}{5} \\
&= \left(\frac{15}{18} - \frac{4}{18}\right) * \frac{3}{5} \\
&= \frac{11}{18} * \frac{3}{5} \\
&= \frac{33 : 3}{90 : 3} \\
&= \frac{11}{30}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{5}{36} + \frac{1}{12}\right) : \frac{2}{9} &= \left(\frac{5}{36} + \frac{1 * 3}{12 * 3}\right) : \frac{2}{9} \\
&= \left(\frac{5}{36} + \frac{3}{36}\right) : \frac{2}{9} \\
&= \frac{8}{36} * \frac{9}{2} \\
&= \frac{72}{72} \\
&= 1
\end{aligned}$$

$$\begin{aligned}\left(\frac{7}{6}-\frac{2}{9}\right)*\left(\frac{1}{5}+\frac{1}{4}\right) &= \left(\frac{7*3}{6*3}-\frac{2*2}{9*2}\right)*\left(\frac{1*4}{5*4}+\frac{1*5}{4*5}\right)\\ &= \left(\frac{21}{18}-\frac{4}{18}\right)*\left(\frac{4}{20}+\frac{5}{20}\right)\\ &= \frac{17}{18}*\frac{9}{20}\\ &= \frac{17}{\cancel{18}_2}*\frac{\cancel{9}_1}{20}\\ &= \frac{17*1}{20*2}\\ &= \frac{17}{40}\end{aligned}$$

1.24

a)

$$\begin{aligned}\frac{\frac{2}{3}}{\frac{5}{6}} &= \frac{\frac{2}{3}*\frac{6}{1}}{\frac{5}{6}*\frac{6}{1}}\\ &= \frac{\frac{2}{\cancel{3}_1}*\frac{\cancel{6}_2}{1}}{\frac{5}{\cancel{6}_2}*\frac{\cancel{6}_2}{1}}\\ &= \frac{\frac{4}{1}}{\frac{10}{2}}\\ &= \frac{4}{5}\end{aligned}$$

b)

$$\begin{aligned}\frac{\frac{21}{36}}{\frac{14}{45}} &= \frac{21}{36}*\frac{45}{14}\\ &= \frac{\cancel{21}_3}{\cancel{36}_4}*\frac{\cancel{45}_5}{\cancel{14}_2}\\ &= \frac{3}{4}*\frac{5}{2}\\ &= \frac{15}{8}\end{aligned}$$

c)

$$\begin{aligned}
\frac{\frac{3}{2}+\frac{5}{8}}{\frac{1}{4}+\frac{25}{2}} &= \frac{\frac{3*8}{2*8}+\frac{5*2}{8*2}}{\frac{1*2}{4*2}+\frac{25*4}{2*4}} \\
&= \frac{\frac{24}{16}+\frac{10}{16}}{\frac{2}{8}+\frac{100}{8}} \\
&= \frac{\frac{34}{16}}{\frac{102}{8}} \\
&= \frac{34}{16} * \frac{8}{102} \\
&= \frac{\cancel{34}_{1}}{\cancel{16}_{2}} * \frac{\cancel{8}_{1}}{\cancel{102}_{3}} \\
&= \frac{1}{2} * \frac{1}{3} \\
&= \frac{1}{6}
\end{aligned}$$

d)

$$\begin{aligned}
\frac{3+\frac{4}{3}}{\frac{5}{12}+5} &= \frac{\frac{3*3}{1*3}+\frac{4*1}{3*1}}{\frac{5*1}{12*1}+\frac{5*12}{1*12}} \\
&= \frac{\frac{9}{3}+\frac{4}{3}}{\frac{5}{12}+\frac{60}{12}} \\
&= \frac{\frac{13}{3}}{\frac{65}{12}} \\
&= \frac{13}{3} * \frac{12}{65} \\
&= \frac{\cancel{13}_{1}}{\cancel{3}_{1}} * \frac{\cancel{12}_{4}}{\cancel{65}_{5}} \\
&= \frac{1}{1} * \frac{4}{5} \\
&= \frac{4}{5}
\end{aligned}$$

1.30

a)

$$\begin{aligned}
2x-5y+3x+7y+1 &= 2x+3x-5y+7y+1 \\
&= 5x+2y+1
\end{aligned}$$

b)

$$\begin{aligned}
a^2+2a+3+a^2-3a-1 &= a^2+a^2+2a-3a+3-1 \\
&= 2a^2-a+2
\end{aligned}$$

c)

$$\begin{aligned} 2x^2 + x + y^2 - 2x - 2y^2 &= 2x^2 + y^2 - 2y^2 + x - 2x \\ &= 2x^2 - y^2 - x \end{aligned}$$

d)

$$\begin{aligned} 2xy + xy^2 - x^2y - 2xy^2 - yx &= 2xy - yx + xy^2 - 2xy^2 - x^2y \\ &= xy + xy^2 - x^2y \end{aligned}$$

1.31

a)

$$\begin{aligned} (5x + y) + (2x - y) &= 5x + y + 2x - y \\ &= 7x + y - y \\ &= 7x \end{aligned}$$

b)

$$\begin{aligned} a + 2b - (-a + b) &= a + 2b + a - b \\ &= a + a + 2b - b \\ &= 2a + b \end{aligned}$$

c)

$$\begin{aligned} (x^2 + 2x + 1) - (x^2 - 2x + 1) &= x^2 + 2x + 1 - x^2 + 2x - 1 \\ &= x^2 - x^2 + 2x + 2x + 1 - 1 \\ &= 4x + 1 - 1 \\ &= 4x \end{aligned}$$

d)

$$\begin{aligned} 2a^2 - a - 3 + (-a^2 + a + 3) &= 2a^2 - a - 3 - a^2 + a + 3 \\ &= 2a^2 - a^2 - a + a - 3 + 3 \\ &= a^2 \end{aligned}$$

1.32

a)

$$\begin{aligned} 2(x + 4) &= 2 * x + 2 * 4 \\ &= 2x + 8 \end{aligned}$$

b)

$$\begin{aligned} -2(t-3) &= -2*t-(-2*3) \\ &= -2t+2*3 \\ &= -2t+6 \end{aligned}$$

c)

$$\begin{aligned} 3(2x+1)-2(3x+1) &= 3*2x+3*1-2*3x-2*1 \\ &= 6x+3-6x-2 \\ &= 6x-6x+3-2 \\ &= 3-2 \\ &= 1 \end{aligned}$$

d)

$$\begin{aligned} 5(x^2+3x+2)-5(x^2+1) &= 5*x^2+5*3x+5*2-5*x^2-5*1 \\ &= 5x^2+15x+10-5x^2-5 \\ &= 5x^2-5x^2+15x+10-5 \\ &= 15x+10-5 \\ &= 15x+5 \end{aligned}$$

1.33

a)

$$\begin{aligned} 2(2a-b)+3(-2a+3b) &= 2*2a-2*b+3*-2a+3*3b \\ &= 4a-2b-6a+9b \\ &= 4a-6a-2b+9b \\ &= -2a+7b \end{aligned}$$

b)

$$\begin{aligned} 2a(ab-b^2)-2b(a^2-ab) &= 2a*ab-2a*b^2-2b*a^2+2b*ab \\ &= 2ba^2-2ab^2-2ba^2+2ab^2 \\ &= 2ba^2-2ba^2+2ab^2-2ab^2 \\ &= 0 \end{aligned}$$

c)

$$\begin{aligned} (x+1)(2x-3) &= x+1*(2x-3) \\ &= 2x^2+1-3 \\ &= x+2x-3 \\ &= 3x-3 \end{aligned}$$

d)

$$\begin{aligned}5(x^2+3x+2)-5(x^2+1) &= 5*x^2+5*3x+5*2-5*x^2-5*1\\ &= 5x^2+15x+10-5x^2-5\\ &= 5x^2-5x^2+15x+10-5\\ &= 15x+10-5\\ &= 15x+5\end{aligned}$$

1.40

a)

$$\begin{aligned}\frac{a}{2}+\frac{a}{3}+\frac{a}{6} &= \frac{a}{2*3}+\frac{a}{3*2}+\frac{a}{6}\\ &= \frac{a}{6}+\frac{a}{6}+\frac{a}{6}\\ &= \frac{x}{y}\end{aligned}$$

b)

$$\begin{aligned}2a(ab-b^2)-2b(a^2-ab) &= 2ba^2-2ab^2-2ba^2+2ab^2\\ &= 2ba^2-2ba^2+2ab^2-2ab^2\\ &= 0\end{aligned}$$

c)

$$(x+1)(2x-3) =$$

d)

$$\begin{aligned}5(x^2+3x+2)-5(x^2+1) &= 5x^2+15x+10-5x^2-5\\ &= 15x+10-5\\ &= 15x+5\end{aligned}$$

1.50

a)

$$\begin{aligned}3^2 &= 3*3\\ &= 9\end{aligned}$$

b)

$$\begin{aligned}(-3)^2 &= (-3)*(-3)\\ &= 9\end{aligned}$$

c)

$$\begin{aligned} 3^3 &= 3 * 3 * 3 \\ &= 27 \end{aligned}$$

d)

$$\begin{aligned} (-3)^2 &= (-3) * (-3) \\ &= 9 \end{aligned}$$

1.51

a)

$$\begin{aligned} 3^2 * 3^3 &= 3 * 3 * 3 * 3 * 3 \\ &= 3^2+^3 \\ &= 3^5 \\ &= 243 \end{aligned}$$

b)

$$\begin{aligned} 2^4 * 2^6 &= 2^4+^6 \\ &= 2^10 \end{aligned}$$

c)

$$\begin{aligned} 5^3 * 5 &= 5^3+^1 \\ &= 5^4 \end{aligned}$$

d)

$$\begin{aligned} 10^2 * 10^3 * 10^5 &= 10^2 +^3 +^5 \\ &= 10^10 \end{aligned}$$

e)

$$\begin{aligned} (2 * 10^4) * (5 * 10^3) &= 2 * 5 * 10^4 * 10^3 \\ &= 10 * 10^4+^3 \\ &= 10 * 10^7 \\ &= 10^7+^1 \\ &= 10^8 \end{aligned}$$

1.52

a)

$$\begin{aligned}\frac{2^4}{2^3} &= 2^{4-3} \\ &= 2^1 \\ &= 2\end{aligned}$$

b)

$$\begin{aligned}\frac{10^5}{10^3} &= 10^{5-3} \\ &= 10^2 \\ &= 100\end{aligned}$$

c)

$$\begin{aligned}\frac{4^3 * 4^2}{4^4} &= \frac{4^{3+2}}{4^4} \\ &= \frac{4^5}{4^4} \\ &= 4^{5-4} \\ &= 4^1 \\ &= 4\end{aligned}$$

d)

$$\begin{aligned}\frac{3^8 * 3^6}{3^5 * 3^7} &= \frac{3^{8+6}}{3^{5+7}} \\ &= \frac{3^1 4}{3^1 2} \\ &= 3^{14-12} \\ &= 3^2 \\ &= 9\end{aligned}$$

e)

$$\begin{aligned}\frac{2*10^5*6*10^2}{4*10^4} &= \frac{2*6*10^{5+2}}{4*10^4}\\ &= \frac{12*10^7}{4*10^4}\\ &= \frac{12*10^{7-4}}{4}\\ &= \frac{12*10^3}{4}\\ &= \frac{12*1000}{4}\\ &= \frac{12000}{4}\\ &= 3000\end{aligned}$$

1.53

a)

$$5^0 = \frac{1}{5}$$

b)

$$(-2)^0 = \frac{1}{-2}$$

c)

$$5^{-1} = \frac{1}{5^1}$$

d)

$$2^{-4} = \frac{1}{2^4}$$

e)

$$10^{-2} = \frac{1}{10^2}$$

f)

$$10^0 = 1$$

g)

$$10^{-4} = \frac{1}{10^4}$$

1.54

a)

$$\begin{aligned} 2^3 - 2^{-4} &= 2^3 - \frac{1}{2^4} \\ &= \frac{1 * 2^3}{2^4} \\ &= \frac{1 * 8}{16} \\ &= \frac{8}{16} \\ &= 2 \end{aligned}$$

b)

$$\begin{aligned} 3^{-4} * 3^5 &= 3^{-4+5} \\ &= 3^1 \\ &= 3 \end{aligned}$$

c)

$$\begin{aligned} \frac{3^{-2}}{3^{-3}} &= \frac{3^3}{3^2} \\ &= 3^{3-2} \\ &= 3^1 \\ &= 3 \end{aligned}$$

d)

$$\begin{aligned} \frac{2^{-3} * 2^5}{2^3 * 2^{-1}} &= \frac{2^{-3+5}}{2^{3-1}} \\ &= \frac{2^2}{2^2} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$$

e)

$$\begin{aligned} \frac{a^4 * a^{-3}}{a^{-2} * a} &= \frac{a^{4-3}}{a^{-2+1}} \\ &= \frac{a^1}{a^{-1}} \\ &= a * a^1 \\ &= a^2 \end{aligned}$$

1.54

a)

$$\begin{aligned} 2^3 - 2^{-4} &= 2^3 - \frac{1}{2^4} \\ &= \frac{1 * 2^3}{2^4} \\ &= \frac{1 * 8}{16} \\ &= \frac{8}{16} \\ &= 2 \end{aligned}$$

b)

$$\begin{aligned} 3^{-4} * 3^5 &= 3^{-4+5} \\ &= 3^1 \\ &= 3 \end{aligned}$$

c)

$$\begin{aligned} \frac{3^{-2}}{3^{-3}} &= \frac{1}{3^2} * \frac{1}{3^3} \\ &= \frac{1}{3^{2+3}} \\ &= \frac{1}{3^5} \\ &= \frac{1}{243} \\ &= 243 \end{aligned}$$

d)

$$\begin{aligned} \frac{2^{-3} * 2^5}{2^3 * 2^{-1}} &= \frac{2^{-3+5}}{2^{3-1}} \\ &= \frac{2^2}{2^2} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$$

$e)$

$$\begin{aligned}\frac{a^4 * a^{-3}}{a^{-2} * a} &= \frac{a^{4-3}}{a^{-2+1}} \\ &= \frac{a^1}{a^{-1}} \\ &= a * a^1 \\ &= a^2\end{aligned}$$