Sinus forkurs matte oppgaver

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1.10

a)

$$-5 \in \mathbb{Z}$$

b)

$$-5 \notin \mathbb{N}$$

c)

$$\frac{2}{3} \notin \mathbb{Z}$$

d)

$$\frac{2}{3} \in \mathbb{Q}$$

e)

$$\sqrt{5} \in \mathbb{R}$$

f)

$$\sqrt{5} \notin \mathbb{Q}$$

1.11

$$2 \in \{1, 2, 3, 4\}$$

$$3 \notin \{0, 1, 2, 4\}$$

$$1, 5 \notin \{1, 2, 3, 4\}$$

$$-1 \notin \{-2, 1, 0, 1\}$$

1.12

$$\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

$$\{21, 23, 25, 27, 29, 31, 33, 35\}$$

$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

1.13

$$\{1, 2, 3, 4\} \backslash \{4\} = \{1, 2, 3\}$$

$$\{1, 2, 3, 4\} \backslash \{2, 4\} = \{1, 3\}$$

$$\{1, 2, 3, 4\} \backslash \{1, 5\} = \{2, 3, 4\}$$

$$\mathbb{Z} \backslash \mathbb{N} = \{..., -4, -3, -2, -1, 0\}$$

1.14

a)

$$\begin{aligned} 4 * 2^2 &= 4 * 4 \\ &= 16 \end{aligned}$$

b)

$$\begin{aligned} 4 * (-2)^2 &= 4 * 4 \\ &= 16 \end{aligned}$$

c)

$$\begin{aligned} 5 - 3^2 &= 5 - 9 \\ &= -4 \end{aligned}$$

d)

$$\begin{aligned} (5 - 3)^2 &= 2^2 \\ &= 4 \end{aligned}$$

e)

$$\begin{aligned} -2^2 + 3^2 - 2 * (-2) &= -4 + 9 + 4 \\ &= 9 \end{aligned}$$

f)

$$\begin{aligned} -(-2)^2 + (-3)^2 - 2^2 &= -(4) + 9 - 4 \\ &= 1 \end{aligned}$$

g)

$$\begin{aligned} (-3)^2 + 5 * (-3) + 6 &= 9 - 15 + 6 \\ &= -5 + 6 \\ &= 0 \end{aligned}$$

Oppgave 1.15

a)

$$\begin{aligned} 2(7-5) + 2 &= 2 * 2 + 2 \\ &= 4 + 2 = 6 \end{aligned}$$

b)

$$\begin{aligned} -3(4-12) + 2 * 3^2 &= -3 * -8 + 2 * 9 \\ &= 24 + 18 = 42 \end{aligned}$$

c)

$$\begin{aligned} -(8-4) - (3)^2 &= -4 - 9 \\ &= -13 \end{aligned}$$

d)

$$\begin{aligned} -2^4 + 3(17 - 3^2) + (3 * 4^2 - 2 * 5^2) &= -2^4 + 3 * 8 + (3 * 4^2 - 2 * 5^2) \\ &= -2^4 + 3(17 - 3^2) - 2 \\ &= -16 + 24 - 2 \\ &= 8 - 2 \\ &= 6 \end{aligned}$$

Oppgave 1.16

a)

$$\begin{aligned} 2(2 * 2 - 2)^2 &= 2(4-2)^2 \\ &= 2 * 4 \\ &= 8 \end{aligned}$$

b)

$$-2^6 + (-2)^6 = -64 + 64 = 0$$

c)

$$\begin{aligned} 4(3-2)^3 - 3(2-3)^3 &= 4 * 1 - 3 * (-1) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

d)

$$\begin{aligned} 4(2^2-3)^5 - 3(2^3-3^2)^5 &= 4 * 1^5 - 3 * (-1)^5 \\ &= 4 * 1 - 3 * -1 \\ &= 4 - (-3) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

Oppgave 1.17

a)

$$\begin{aligned} 2(2 * 2 - 2)^2 &= 2(4-2)^2 \\ &= 2 * 4 \\ &= 8 \end{aligned}$$

b)

$$-2^6 + (-2)^6 = -64 + 64 = 0$$

c)

$$\begin{aligned} 4(3-2)^3 - 3(2-3)^3 &= 4 * 1 - 3 * (-1) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

d)

$$\begin{aligned} 4(2^2-3)^5 - 3(2^3-3^2)^5 &= 4 * 1^5 - 3 * (-1)^5 \\ &= 4 * 1 - 3 * -1 \\ &= 4 - (-3) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

Oppgave 1.20

a)

$$\begin{aligned}\frac{4}{6} &= \frac{4:2}{6:2} \\ &= \frac{2}{3}\end{aligned}$$

b)

$$\begin{aligned}\frac{9}{15} &= \frac{9:3}{15:3} \\ &= \frac{3}{5}\end{aligned}$$

c)

$$\begin{aligned}\frac{18}{21} &= \frac{18:3}{21:3} \\ &= \frac{6}{7}\end{aligned}$$

d)

$$\begin{aligned}\frac{42}{54} &= \frac{42:6}{54:6} \\ &= \frac{7}{9}\end{aligned}$$

Oppgave 1.21

a)

$$\begin{aligned}\frac{72}{120} &= \frac{72:8}{120:8} \\ &= \frac{9}{15} \\ &= \frac{9:3}{15:3} \\ &= \frac{3}{5}\end{aligned}$$

b)

$$\begin{aligned}\frac{126}{294} &= \frac{126:7}{294:7}\\ &= \frac{18:2}{42:2}\\ &= \frac{9:3}{21:3}\\ &= \frac{3}{7}\end{aligned}$$

c)

$$\begin{aligned}\frac{132}{198} &= \frac{132:2}{198:2}\\ &= \frac{66:3}{99:3}\\ &= \frac{22:11}{33:11}\\ &= \frac{2}{3}\end{aligned}$$

d)

$$\begin{aligned}\frac{153}{51} &= \frac{153:3}{51:3}\\ &= \frac{51}{17}\end{aligned}$$

exmaples

$$\begin{aligned}3 * \frac{2}{3} &= \frac{3}{1} * \frac{3}{3} - \frac{2}{3}\\ &= \frac{7}{3} = \frac{3+3+1}{3}\\ &= \frac{3}{3} + \frac{3}{3} + \frac{1}{3}\\ &= 1 + 1 + \frac{1}{3} = 2\frac{1}{3}\end{aligned}$$

1.22

$$\begin{aligned}\frac{1}{12}+\frac{4}{9} &= \frac{9}{9}*\frac{1}{12}+\frac{4}{9}*\frac{12}{12}\\ &= \frac{9}{108}+\frac{48}{108}\\ &= \frac{57:3}{108:3}\\ &= \frac{19}{36}\end{aligned}$$

$$\begin{aligned}\frac{1}{12}*\frac{4}{9} &= \frac{4:2}{108:2}\\ &= \frac{2:2}{54:2}\\ &= \frac{1}{27}\end{aligned}$$

$$\begin{aligned}\frac{1}{12}:\frac{4}{9} &= \frac{1}{12}*\frac{9}{4}\\ &= \frac{9:3}{48:3}\\ &= \frac{3}{16}\end{aligned}$$

$$\begin{aligned}3+\frac{5}{12} &= \frac{3*12}{1*12}+\frac{5}{12}\\ &= \frac{36}{12}+\frac{5}{12}\\ &= \frac{41}{12}\end{aligned}$$

$$\begin{aligned}3*\frac{5}{12} &= \frac{3}{1}*\frac{5}{12}\\ &= \frac{15:3}{12:3}\\ &= \frac{5}{4}\end{aligned}$$

$$\begin{aligned}3:\frac{5}{12} &= \frac{3}{1}:\frac{5}{12}\\ &= \frac{3}{1}*\frac{12}{5}\\ &= \frac{36}{5}\end{aligned}$$

1.23

$$\begin{aligned}
2 * \left(\frac{3}{8} + \frac{1}{4}\right) &= 2 * \left(\frac{4}{4} * \frac{3}{8} + \frac{1}{4} * \frac{8}{8}\right) \\
&= 2 * \left(\frac{12}{32} + \frac{8}{32}\right) \\
&= 2 * \frac{20}{32} \\
&= \frac{2 * 20}{32} \\
&= \frac{40 : 2}{32 : 2} \\
&= \frac{20 : 2}{16 : 2} \\
&= \frac{10 : 2}{8 : 2} \\
&= \frac{5}{4}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{5}{6} - \frac{2}{9}\right) * \frac{3}{5} &= \left(\frac{5 * 3}{6 * 3} - \frac{2 * 2}{9 * 2}\right) * \frac{3}{5} \\
&= \left(\frac{15}{18} - \frac{4}{18}\right) * \frac{3}{5} \\
&= \frac{11}{18} * \frac{3}{5} \\
&= \frac{33 : 3}{90 : 3} \\
&= \frac{11}{30}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{5}{36} + \frac{1}{12}\right) : \frac{2}{9} &= \left(\frac{5}{36} + \frac{1 * 3}{12 * 3}\right) : \frac{2}{9} \\
&= \left(\frac{5}{36} + \frac{3}{36}\right) : \frac{2}{9} \\
&= \frac{8}{36} * \frac{9}{2} \\
&= \frac{72}{72} \\
&= 1
\end{aligned}$$

$$\begin{aligned}\left(\frac{7}{6}-\frac{2}{9}\right)*\left(\frac{1}{5}+\frac{1}{4}\right) &= \left(\frac{7*3}{6*3}-\frac{2*2}{9*2}\right)*\left(\frac{1*4}{5*4}+\frac{1*5}{4*5}\right)\\ &= \left(\frac{21}{18}-\frac{4}{18}\right)*\left(\frac{4}{20}+\frac{5}{20}\right)\\ &= \frac{17}{18}*\frac{9}{20}\\ &= \frac{17}{\cancel{18}_2}*\frac{\cancel{9}_1}{20}\\ &= \frac{17*1}{20*2}\\ &= \frac{17}{40}\end{aligned}$$

1.24

a)

$$\begin{aligned}\frac{\frac{2}{3}}{\frac{5}{6}} &= \frac{\frac{2}{3}*\frac{6}{1}}{\frac{5}{6}*\frac{6}{1}}\\ &= \frac{\frac{2}{\cancel{3}_1}*\frac{\cancel{6}_2}{1}}{\frac{5}{\cancel{6}_2}*\frac{\cancel{6}_2}{1}}\\ &= \frac{\frac{4}{1}}{\frac{10}{2}}\\ &= \frac{4}{5}\end{aligned}$$

b)

$$\begin{aligned}\frac{\frac{21}{36}}{\frac{14}{45}} &= \frac{21}{36}*\frac{45}{14}\\ &= \frac{\cancel{21}_3}{\cancel{36}_4}*\frac{\cancel{45}_5}{\cancel{14}_2}\\ &= \frac{3}{4}*\frac{5}{2}\\ &= \frac{15}{8}\end{aligned}$$

c)

$$\begin{aligned}
\frac{\frac{3}{2}+\frac{5}{8}}{\frac{1}{4}+\frac{25}{2}} &= \frac{\frac{3*8}{2*8}+\frac{5*2}{8*2}}{\frac{1*2}{4*2}+\frac{25*4}{2*4}} \\
&= \frac{\frac{24}{16}+\frac{10}{16}}{\frac{2}{8}+\frac{100}{8}} \\
&= \frac{\frac{34}{16}}{\frac{102}{8}} \\
&= \frac{34}{16} * \frac{8}{102} \\
&= \frac{\cancel{34}_{1}}{\cancel{16}_{2}} * \frac{\cancel{8}_{1}}{\cancel{102}_{3}} \\
&= \frac{1}{2} * \frac{1}{3} \\
&= \frac{1}{6}
\end{aligned}$$

d)

$$\begin{aligned}
\frac{3+\frac{4}{3}}{\frac{5}{12}+5} &= \frac{\frac{3*3}{1*3}+\frac{4*1}{3*1}}{\frac{5*1}{12*1}+\frac{5*12}{1*12}} \\
&= \frac{\frac{9}{3}+\frac{4}{3}}{\frac{5}{12}+\frac{60}{12}} \\
&= \frac{\frac{13}{3}}{\frac{65}{12}} \\
&= \frac{13}{3} * \frac{12}{65} \\
&= \frac{\cancel{13}_{1}}{\cancel{3}_{1}} * \frac{\cancel{12}_{4}}{\cancel{65}_{5}} \\
&= \frac{1}{1} * \frac{4}{5} \\
&= \frac{4}{5}
\end{aligned}$$

1.30

a)

$$\begin{aligned}
2x-5y+3x+7y+1 &= 2x+3x-5y+7y+1 \\
&= 5x+2y+1
\end{aligned}$$

b)

$$\begin{aligned}
a^2+2a+3+a^2-3a-1 &= a^2+a^2+2a-3a+3-1 \\
&= 2a^2-a+2
\end{aligned}$$

c)

$$\begin{aligned} 2x^2 + x + y^2 - 2x - 2y^2 &= 2x^2 + y^2 - 2y^2 + x - 2x \\ &= 2x^2 - y^2 - x \end{aligned}$$

d)

$$\begin{aligned} 2xy + xy^2 - x^2y - 2xy^2 - yx &= 2xy - yx + xy^2 - 2xy^2 - x^2y \\ &= xy + xy^2 - x^2y \end{aligned}$$

1.31

a)

$$\begin{aligned} (5x + y) + (2x - y) &= 5x + y + 2x - y \\ &= 7x + y - y \\ &= 7x \end{aligned}$$

b)

$$\begin{aligned} a + 2b - (-a + b) &= a + 2b + a - b \\ &= a + a + 2b - b \\ &= 2a + b \end{aligned}$$

c)

$$\begin{aligned} (x^2 + 2x + 1) - (x^2 - 2x + 1) &= x^2 + 2x + 1 - x^2 + 2x - 1 \\ &= x^2 - x^2 + 2x + 2x + 1 - 1 \\ &= 4x + 1 - 1 \\ &= 4x \end{aligned}$$

d)

$$\begin{aligned} 2a^2 - a - 3 + (-a^2 + a + 3) &= 2a^2 - a - 3 - a^2 + a + 3 \\ &= 2a^2 - a^2 - a + a - 3 + 3 \\ &= a^2 \end{aligned}$$

1.32

a)

$$\begin{aligned} 2(x + 4) &= 2 * x + 2 * 4 \\ &= 2x + 8 \end{aligned}$$

b)

$$\begin{aligned} -2(t-3) &= -2*t-(-2*3) \\ &= -2t+2*3 \\ &= -2t+6 \end{aligned}$$

c)

$$\begin{aligned} 3(2x+1)-2(3x+1) &= 3*2x+3*1-2*3x-2*1 \\ &= 6x+3-6x-2 \\ &= 6x-6x+3-2 \\ &= 3-2 \\ &= 1 \end{aligned}$$

d)

$$\begin{aligned} 5(x^2+3x+2)-5(x^2+1) &= 5*x^2+5*3x+5*2-5*x^2-5*1 \\ &= 5x^2+15x+10-5x^2-5 \\ &= 5x^2-5x^2+15x+10-5 \\ &= 15x+10-5 \\ &= 15x+5 \end{aligned}$$

1.33

a)

$$\begin{aligned} 2(2a-b)+3(-2a+3b) &= 2*2a-2*b+3*-2a+3*3b \\ &= 4a-2b-6a+9b \\ &= 4a-6a-2b+9b \\ &= -2a+7b \end{aligned}$$

b)

$$\begin{aligned} 2a(ab-b^2)-2b(a^2-ab) &= 2a*ab-2a*b^2-2b*a^2+2b*ab \\ &= 2ba^2-2ab^2-2ba^2+2ab^2 \\ &= 2ba^2-2ba^2+2ab^2-2ab^2 \\ &= 0 \end{aligned}$$

c)

$$\begin{aligned} (x+1)(2x-3) &= x*2x+x*-3+1*2x-3 \\ &= 2x^2-x3+2x-3 \\ &= 2x^2-x-3 \end{aligned}$$

d)

$$\begin{aligned}5(x^2+3x+2)-5(x^2+1) &= 5*x^2+5*3x+5*2-5*x^2-5*1\\ &= 5x^2+15x+10-5x^2-5\\ &= 5x^2-5x^2+15x+10-5\\ &= 15x+10-5\\ &= 15x+5\end{aligned}$$

1.34

a)

$$\begin{aligned}(2x-1)(x+3)+(x-1)(x-4) &= 2x*x+2x*3-1*x-1*3+x*x+x*-4-1*x-1*-4\\ &= 3x^2-3+4\\ &= 3x^2+1\end{aligned}$$

b)

$$\begin{aligned}2(x-1)(2x+3) &= 2(x*2x+x*3-1*2x-1*3)\\ &= 2(2x^2+3x-2x-3)\\ &= 2(2x^2+x-3)\\ &= 2*2x^2+2*x+2*(-3)\\ &= 4x^2+2x-6\end{aligned}$$

c)

$$\begin{aligned}(x+3)(4x-1)-(2x+1)(2x-3) &= x*4x+x*(-1)+3*4x+3*(-1)-2x*2x-2x*(-3)-1\\ &= 4x^2-x+12x-3-4x^2+6x-2x+3\\ &= 4x^2-4x^2-x+12x+6x-2x-3+3\\ &= 15x\end{aligned}$$

d)

$$\begin{aligned}\frac{3}{4}(t+3)(8t-4) &= (t+3)(\frac{3}{4}*8t-\frac{3}{4}*4)\\ &= (t+3)(\frac{3*\cancel{8}_2t}{\cancel{4}_1}-\frac{3*\cancel{4}}{\cancel{4}})\\ &= (t+3)(3*2t-3)\\ &= (t+3)(6t-3)\\ &= (t*6t+t*(-3))+(3*6t+3*(-3))\\ &= 6t^2-3t+18t-9\\ &= 6t^2+15t-9\end{aligned}$$

1.40

a)

$$\begin{aligned}\frac{a}{2}+\frac{a}{3}+\frac{a}{6} &= \frac{a*3}{2*3}+\frac{a*2}{3*2}+\frac{a}{6}\\ &= \frac{3a}{6}+\frac{2a}{6}+\frac{a}{6}\\ &= \frac{\not{6}a}{\not{6}}\\ &= a\end{aligned}$$

b)

$$\begin{aligned}\frac{1}{2a}+\frac{1}{3a}+\frac{1}{6a} &= \frac{1*3}{2*3a}+\frac{1*2}{3*2a}+\frac{1}{6a}\\ &= \frac{3}{6a}+\frac{2}{6a}+\frac{1}{6a}\\ &= \frac{\not{6}}{\not{6}a}\\ &= \frac{1}{a}\end{aligned}$$

c)

$$\begin{aligned}\frac{2}{x}+\frac{3}{2x}-\frac{4}{3x} &= \frac{2*6}{6*x}+\frac{3*3}{3*2x}-\frac{4*2}{2*3x}\\ &= \frac{12}{6x}+\frac{9}{6a}-\frac{8}{6x}\\ &= \frac{13}{6x}\end{aligned}$$

1.41

a)

$$\begin{aligned}\frac{2a}{3}*\frac{6}{a} &= \frac{2\not{a}}{\not{3}_1}*\frac{\not{6}_2}{\not{a}}\\ &= \frac{4}{1}\\ &= 4\end{aligned}$$

b)

$$\begin{aligned}\frac{2x^2}{3y}*\frac{5y^2}{4x} &= \frac{2x}{3}*\frac{5y}{4}\\ &= \frac{10xy:2}{12:2}\\ &= \frac{5xy}{6}\end{aligned}$$

c)

$$\begin{aligned}\frac{8a}{5} : \frac{4a}{15} &= \frac{\cancel{8}^{\cancel{4}}\cancel{a}}{\cancel{5}_{1}} * \frac{\cancel{15}^{3}}{\cancel{4}_{\cancel{2}}\cancel{a}} \\ &= \frac{\cancel{4}_{2}}{1} * \frac{3}{\cancel{2}_{1}} \\ &= \frac{6}{1} \\ &= 6\end{aligned}$$

d)

$$\begin{aligned}\frac{6a}{5} : 2a &= \frac{\cancel{6}^{3}\cancel{a}}{5} * \frac{1}{\cancel{2}_{1}\cancel{a}} \\ &= \frac{3}{5}\end{aligned}$$

1.42

a)

$$\begin{aligned}\frac{2}{3} * \frac{5}{a} + \frac{1}{2} * \frac{7}{3a} &= \frac{10}{3a} + \frac{7}{6a} \\ &= \frac{10 * 2}{3a * 2} + \frac{7}{6a} \\ &= \frac{20}{6a} + \frac{7}{6a} \\ &= \frac{27 : 3}{6a : 3} \\ &= \frac{9}{2a}\end{aligned}$$

b)

$$\begin{aligned}\frac{2}{x} * \left(\frac{5x}{3} - \frac{7x}{6}\right) &= \frac{2}{x} * \left(\frac{5x * 2}{3 * 2} - \frac{7x}{6}\right) \\ &= \frac{2}{x} * \left(\frac{10x}{6} - \frac{7x}{6}\right) \\ &= \frac{\cancel{2}}{x} * \frac{3x}{\cancel{6}_{3}} \\ &= \frac{3x}{3x} \\ &= 1\end{aligned}$$

c)

$$\begin{aligned}\left(\frac{x^2}{3}+\frac{5x}{6}\right):\frac{x}{12} &= \left(\frac{x^2*2}{3*2}+\frac{5x}{6}\right):\frac{x}{12}\\ &= \left(\frac{2x^2}{6}+\frac{5x}{6}\right):\frac{x}{12}\\ &= \left(\frac{2x^2+5x}{6}\right):\frac{x}{12}\\ &= \left(\frac{2x^2+5x}{6}\right)*\frac{12}{x}\\ &= \left(\frac{2x^{\not 1}+5\not{x}}{\not{6}}\right)*\frac{\not{12}^{2}}{\not{x}}\\ &= (2x+5)*2\\ &= 4x+10\end{aligned}$$

1.43

a)

$$\begin{aligned}\frac{2x+3}{4}-\frac{x+1}{4} &= \frac{(2x+3)-(x-1)}{4}\\ &= \frac{2x-x+3-1}{4}\\ &= \frac{x+2}{4}\end{aligned}$$

b)

$$\begin{aligned}\frac{a+2}{2}-\frac{2a-1}{6} &= \frac{3\,(a+2)}{2*3}-\frac{(2a-1)}{6}\\ &= \frac{3a+6}{6}-\frac{2a-1}{6}\\ &= \frac{3a-2a+6+1}{6}\\ &= \frac{a+7}{6}\end{aligned}$$

c)

$$\begin{aligned}\frac{x+2}{2x}-\frac{2x-1}{3x} &= \frac{3(x+2)}{2x*3}-\frac{2(2x-1)}{3x*2}\\ &= \frac{3x+6-4x+2}{6x}\\ &= \frac{-x+8}{6x}\end{aligned}$$

d)

$$\begin{aligned}\frac{2}{a}+\frac{a-2}{2a}-\frac{a+3}{3a} &= \frac{2*6}{a*6}+\frac{3(a-2)}{2a*3}-\frac{2(a+3)}{3a*2}\\ &= \frac{12}{6a}+\frac{3a-6}{6a}-\frac{2a-6}{6a}\\ &= \frac{12-6-6+3a-2a}{6a}\\ &= \frac{\cancel{a}}{6\cancel{a}}\\ &= \frac{1}{6}\end{aligned}$$

1.44

a)

$$\begin{aligned}\frac{\frac{2x}{5}+\frac{1}{2}}{\frac{x}{2}-\frac{1}{10}} &= \frac{\frac{2x*2}{5*2}+\frac{1*5}{2*5}}{\frac{x*5}{2*5}-\frac{1}{10}}\\ &= \frac{\frac{4x}{10}+\frac{5}{10}}{\frac{5x}{10}-\frac{1}{10}}\\ &= \frac{4x+5}{10}*\frac{10}{5x-1}\\ &= \frac{4x+5}{\cancel{10}}*\frac{\cancel{10}}{5x-1}\\ &= \frac{4x+5}{5x-1}\end{aligned}$$

b)

$$\begin{aligned}\frac{\frac{1}{x}+\frac{1}{2}}{1+\frac{2}{x}} &= \frac{\frac{1*2}{x*2}+\frac{1*x}{2*x}}{\frac{1*x}{x}+\frac{2}{x}}\\ &= \frac{\frac{\cancel{2x}}{\cancel{2x}}}{\frac{\cancel{x}+2}{\cancel{x}}}\\ &= \frac{1}{2}\end{aligned}$$

c)

$$\begin{aligned}\frac{\frac{1}{a}-\frac{2}{b}}{\frac{2}{a}-\frac{1}{b}} &= \frac{\frac{1*b}{a*b}-\frac{2*a}{b*a}}{\frac{2*b}{a*b}-\frac{1*a}{b*a}} \\ &= \frac{\frac{b}{ab}-\frac{2a}{ab}}{\frac{2b}{ab}-\frac{a}{ab}} \\ &= \frac{\frac{b-2a}{ab}}{\frac{2b-a}{ab}} \\ &= \frac{b-2a}{\cancel{ab}} * \frac{\cancel{ab}}{2b-a} \\ &= \frac{b-2a}{2b-a}\end{aligned}$$

d)

$$\begin{aligned}\frac{\frac{1}{x}+\frac{1}{6}}{\frac{1}{2x}-\frac{1}{3x}} &= \frac{\frac{1*6}{x*6}+\frac{1*x}{6*x}}{\frac{1*3}{2x*3}-\frac{1*2}{3x*2}} \\ &= \frac{\frac{6}{6x}+\frac{x}{6x}}{\frac{3}{6x}-\frac{2}{6x}} \\ &= \frac{6+x}{\cancel{6x}} * \frac{\cancel{6x}}{3-2} \\ &= \frac{6+x}{3-2} \\ &= \frac{6+x}{1} \\ &= 6+x\end{aligned}$$

1.50

a)

$$\begin{aligned}3^2 &= 3*3 \\ &= 9\end{aligned}$$

b)

$$\begin{aligned}(-3)^2 &= (-3)*(-3) \\ &= 9\end{aligned}$$

c)

$$\begin{aligned}3^3 &= 3*3*3 \\ &= 27\end{aligned}$$

d)

$$\begin{aligned}(-3)^3 &= (-3) * (-3) * (-3) \\ &= -27\end{aligned}$$

1.51

a)

$$\begin{aligned}3^2 * 3^3 &= 3 * 3 * 3 * 3 * 3 \\ &= 3^2+^3 \\ &= 3^5\end{aligned}$$

b)

$$\begin{aligned}2^4 * 2^6 &= 2^4+^6 \\ &= 2^10\end{aligned}$$

c)

$$\begin{aligned}5^3 * 5 &= 5^3+^1 \\ &= 5^4\end{aligned}$$

d)

$$\begin{aligned}10^2 * 10^3 * 10^5 &= 10^2 +^3 +^5 \\ &= 10^10\end{aligned}$$

e)

$$\begin{aligned}(2 * 10^4) * (5 * 10^3) &= 2 * 5 * 10^4 * 10^3 \\ &= 10 * 10^4+^3 \\ &= 10 * 10^7 \\ &= 10^7+^1 \\ &= 10^8\end{aligned}$$

1.52

a)

$$\begin{aligned}\frac{2^4}{2^3} &= 2^{4-3} \\ &= 2^1 \\ &= 2\end{aligned}$$

b)

$$\begin{aligned}\frac{10^5}{10^3} &= 10^{5-3}\\ &= 10^2\\ &= 100\end{aligned}$$

c)

$$\begin{aligned}\frac{4^3 * 4^2}{4^4} &= \frac{4^{3+2}}{4^4}\\ &= \frac{4^5}{4^4}\\ &= 4^{5-4}\\ &= 4^1\\ &= 4\end{aligned}$$

d)

$$\begin{aligned}\frac{3^8 * 3^6}{3^5 * 3^7} &= \frac{3^{8+6}}{3^{5+7}}\\ &= \frac{3^1 4}{3^1 2}\\ &= 3^{14-12}\\ &= 3^2\\ &= 9\end{aligned}$$

e)

$$\begin{aligned}\frac{2 * 10^5 * 6 * 10^2}{4 * 10^4} &= \frac{2 * 6 * 10^{5+2}}{4 * 10^4}\\ &= \frac{12 * 10^7}{4 * 10^4}\\ &= \frac{12 * 10^{7-4}}{4}\\ &= \frac{12 * 10^3}{4}\\ &= \frac{12 * 1000}{4}\\ &= \frac{12000}{4}\\ &= 3000\end{aligned}$$

1.53

a)

$$\begin{aligned} 5^0 &= \frac{5}{1} \\ &= 1 \end{aligned}$$

b)

$$\begin{aligned} (-2)^0 &= \frac{-2}{1} \\ &= 1 \end{aligned}$$

c)

$$5^{-1} = \frac{1}{5}$$

d)

$$\begin{aligned} 2^{-4} &= \frac{1}{2^4} \\ &= \frac{1}{16} \end{aligned}$$

e)

$$\begin{aligned} 10^{-2} &= \frac{1}{10^2} \\ &= \frac{1}{100} \end{aligned}$$

f)

$$10^0 = 1$$

g)

$$\begin{aligned} 10^{-4} &= \frac{1}{10^4} \\ &= \frac{1}{10\,000} \end{aligned}$$

1.54

a)

$$\begin{aligned} 2^3 - 2^{-4} &= 2^3 - \frac{1}{2^4} \\ &= \frac{1 * 2^{\cancel{3}}}{\cancel{2^4}_2} \\ &= \frac{1}{2} \end{aligned}$$

b)

$$\begin{aligned} 3^{-4} * 3^5 &= 3^{-4+5} \\ &= 3^1 \\ &= 3 \end{aligned}$$

c)

$$\begin{aligned} \frac{3^{-2}}{3^{-3}} &= \frac{3^3}{3^2} \\ &= 3^{3-2} \\ &= 3^1 \\ &= 3 \end{aligned}$$

d)

$$\begin{aligned} \frac{2^{-3} * 2^5}{2^3 * 2^{-1}} &= \frac{2^{-3+5}}{2^{3-1}} \\ &= \frac{2^2}{2^2} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$$

e)

$$\begin{aligned} \frac{a^4 * a^{-3}}{a^{-2} * a} &= \frac{a^{4-3}}{a^{-2+1}} \\ &= \frac{a^1}{a^{-1}} \\ &= a * a^1 \\ &= a^2 \end{aligned}$$

1.60

a)

$$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$$

b)

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

c)

$$\left(\frac{1}{10}\right)^3 = \frac{1^3}{10^3} = \frac{1}{1000}$$

d)

$$\left(-\frac{2}{3}\right)^4 = -\frac{2^4}{3^4} = \frac{-2 * -2 * -2 * -2}{3 * 3 * 3 * 3} = \frac{16}{81}$$

e)

$$\frac{a^4 * a^{-3}}{a^{-2} * a} = \frac{a^{4-3}}{a^{-2+1}} = \frac{a^1}{a^{-1}} = a * a^1 = a^2$$

1.61

a)

$$\begin{aligned}\left(\frac{2}{3}\right)^3 * 3^3 &= \frac{2^3 * 3^3}{3^3} \\ &= \frac{8 * 27}{27} \\ &= \frac{2 * 2 * 2 * 3 * 3 * 3}{3 * 3 * 3} \\ &= \frac{2 * 2 * 2 * \cancel{3} * \cancel{3} * \cancel{3}}{\cancel{3} * \cancel{3} * \cancel{3}} \\ &= \frac{8}{1} \\ &= 8\end{aligned}$$

b)

$$\begin{aligned}\frac{2^5}{5^2} * \left(\frac{5}{2}\right)^3 &= \frac{2^5 * 5^3}{5^2 * 2^3} \\ &= \frac{2 * 2 * 2 * 2 * 2 * 5 * 5 * 5}{5 * 5 * 2 * 2 * 2} \\ &= \frac{2 * 2 * \cancel{2} * \cancel{2} * \cancel{2} * 5 * \cancel{5} * \cancel{5}}{\cancel{5} * \cancel{5} * \cancel{2} * \cancel{2} * \cancel{2}} \\ &= \frac{4 * 5}{1} \\ &= \frac{20}{1} \\ &= 20\end{aligned}$$

c)

$$\begin{aligned}\left(\frac{x}{2}\right)^2 &= \frac{x^2}{2^2} \\ &= \frac{x^2}{4}\end{aligned}$$

d)

$$\begin{aligned}3^5 * \left(\frac{x}{3}\right)^4 &= \frac{3^5 * x^4}{3^4} \\ &= \frac{3 * \cancel{3} * \cancel{3} * \cancel{3} * \cancel{3} * x * x * x * x}{\cancel{3} * \cancel{3} * \cancel{3} * \cancel{3}} \\ &= 3x^4\end{aligned}$$

1.62

$a)$

$$\begin{aligned}\left(5 * 10^3\right)^3 &= 5^3 * 10^{3*3} \\ &= 5^3 * 10^9 \\ &= 125 * 10^9 \\ &= 125\,000\,000\,000\end{aligned}$$

$b)$

$$\begin{aligned}\left(2 * 10^2\right)^{-1} &= 2^{-1} * 10^{2*(-1)} \\ &= 2^{-1} * 10^{-2} \\ &= \frac{1}{2 * 10^2} \\ &= \frac{1}{200} \\ &= 0.005\end{aligned}$$

$c)$

$$\begin{aligned}\left(3 * 10^{-3}\right)^2 * \left(3 * 10^{-2}\right)^{-1} &= 3^2 * 10^{-3*2} * 3^{-1} * 10^{-2*(-1)} \\ &= 3^2 * 10^{-6} * 3^{-1} * 10^2 \\ &= 3^{2-1} * 10^{-6+2} \\ &= 3 * 10^{-4} \\ &= \frac{3}{10^4} \\ &= \frac{3}{10\,000} \\ &= 0.0003\end{aligned}$$

$d)$

$$\begin{aligned}\frac{5 * 10^{-2} * 9 * 10^4}{3 * 10^3} &= \frac{5 * 9 * 10^{-2+4}}{3 * 10^3} \\ &= \frac{5 * 9 * 10^2}{3 * 10^3} \\ &= \frac{5 * \cancel{9}_3 * \cancel{100}}{\cancel{3} * \cancel{1000}_{10}} \\ &= \frac{5 * 3}{10} \\ &= \frac{15}{10} \\ &= 1.5\end{aligned}$$

1.63

a)

$$\begin{aligned} x^7 * \left(x^{-2}\right)^3 &= x^7 * x^{-6} \\ &= x^{7-6} \\ &= x \end{aligned}$$

b)

$$\begin{aligned} \left(2x^{-2}\right)^{-1} * 2x^{-3} &= 2^{-1}x^{(-2)*(-1)} * 2x^{-3} \\ &= 2^{-1}x^2 * 2x^{-3} \\ &= 2^{-1}x^{2-3} \\ &= 2^{-1+1}x^{-1} \\ &= x^{-1} \\ &= \frac{1}{x} \end{aligned}$$

c)

$$\begin{aligned} \frac{\left(2a^2\right)^{-2} * \left(2a^{-3}\right)^2}{\left(2^2a^{-1}\right)^3 * (2a)^{-4}} &= \frac{2^{-2}a^{2*(-2)} * 2^2a^{-3*2}}{2^{2*3}a^{-1*3} * 2^{-4}a^{-4}} \\ &= \frac{2^{-2+2} * a^{-4-6}}{2^{6-4} * a^{-3-4}} \\ &= \frac{\cancel{a^{-10}}}{2^2 * \cancel{a^{-7}}} \\ &= \frac{a^{-3}}{4} \\ &= \frac{1}{a^3} * \frac{1}{4} \\ &= \frac{1}{4a^3} \end{aligned}$$

d)

$$\frac{\left(x^2y^{-2}\right)^{-1}\left(x^2\right)^2y^3}{\left(xy^2\right)^{-3}} = \frac{x^{2(-1)}y^{-2(-1)}x^{2*2}y^3}{x^{-3}y^{2(-3)}}$$
$$= \frac{x^{-2}y^2x^4y^3}{x^{-3}y^{-6}}$$
$$= \frac{x^{-2+4}y^{2+3}}{x^{-3}y^{-6}}$$
$$= \frac{x^2y^5}{x^{-3}y^{-6}}$$
$$= x^{2-(-3)}y^{5-(-6)}$$
$$= x^5y^{11}$$

1.64

a) **1**

$$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}}$$
$$= \frac{1}{2^2} * \frac{3^2}{1}$$
$$= \frac{9}{4}$$
$$= \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2}$$
$$= \frac{9}{4}$$
$$= \frac{3}{2}$$

2

$$\left(\frac{3}{5}\right)^{-3} = \frac{3^{-3}}{5^{-3}}$$
$$= \frac{1}{3^3} * \frac{5^3}{1}$$
$$= \frac{125}{27}$$

$$\left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3}$$
$$= \frac{125}{27}$$

3

$$\left(\frac{6}{7}\right)^3 = \frac{6^{-3}}{7^{-3}}$$
$$= \frac{1}{6^3} * \frac{7^3}{1}$$
$$= \frac{343}{216}$$

$$\left(\frac{7}{6}\right)^{-3} = \frac{7^3}{6^3}$$
$$= \frac{343}{216}$$

b)

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{a^n}$$

c)

$$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}}$$
$$= \frac{1}{a^n} * \frac{b^n}{1}$$
$$= \frac{b^n}{a^n}$$

1.70

a)

b)

$$\sqrt{\frac{49}{64}} = \frac{\sqrt{49}}{\sqrt{64}}$$
$$= \frac{7}{8}$$

c)

$$\sqrt{\frac{1}{121}} = \frac{\sqrt{1}}{\sqrt{121}}$$
$$= \frac{1}{11}$$

d)

$$\begin{aligned}\sqrt{\frac{18}{98}} &= \frac{\sqrt{9} * \sqrt{2}}{\sqrt{49} * \sqrt{2}} \\ &= \frac{3}{7}\end{aligned}$$

1.80

a)

$$\begin{aligned}\sqrt{\frac{16}{25}} &= \frac{\sqrt{16}}{\sqrt{25}} \\ &= \frac{4}{5}\end{aligned}$$

b)

$$\begin{aligned}\sqrt{\frac{49}{64}} &= \frac{\sqrt{49}}{\sqrt{64}} \\ &= \frac{7}{8}\end{aligned}$$

c)

$$\begin{aligned}\sqrt{\frac{1}{121}} &= \frac{\sqrt{1}}{\sqrt{121}} \\ &= \frac{1}{11}\end{aligned}$$

d)

$$\begin{aligned}\sqrt{\frac{18}{98}} &= \frac{\sqrt{9} * \sqrt{2}}{\sqrt{49} * \sqrt{2}} \\ &= \frac{3}{7}\end{aligned}$$

1.81

a)

$$\begin{aligned}\sqrt{18} &= 3\sqrt{2} \\ \sqrt{18} &= \sqrt{9 * 2}\end{aligned}$$

b)

$$\begin{aligned}\sqrt{12} &= 2\sqrt{3} \\ &= \sqrt{4*3}\end{aligned}$$

c)

$$\begin{aligned}\sqrt{48} &= 4\sqrt{3} \\ &= \sqrt{16*3}\end{aligned}$$

1.82

a)

$$\begin{aligned}\sqrt{27} &= \sqrt{9*3} \\ &= 3\sqrt{3}\end{aligned}$$

b)

$$\begin{aligned}\sqrt{75} &= \sqrt{5*15} \\ &= \sqrt{5}*\sqrt{5}*\sqrt{3} \\ &= \sqrt{25*3} \\ &= 5\sqrt{3}\end{aligned}$$

c)

$$\begin{aligned}\sqrt{162} &= \sqrt{81*2} \\ &= 9\sqrt{2}\end{aligned}$$

1.83

a)

$$\sqrt[3]{27} = 3$$

b)

$$\sqrt[3]{1000} = 10$$

c)

$$\sqrt[3]{-64} = -4$$

d)

$$\begin{aligned}
\sqrt[3]{-8000} &= \sqrt{-80 * -10 * -10} \\
&= \sqrt{-20 * 4 * 5 * 2 * 5 * 2} \\
&= \sqrt{-5 * 4 * 4 * 5 * 2 * 5 * 2} \\
&= \sqrt{-5 * 5 * 5 * 2 * 2 * 2 * 2 * 2 * 2} \\
&= \sqrt{-20 * -20 * -20} && = -20
\end{aligned}$$

1.84

a)

$$\sqrt[4]{13} = 3,6$$

b)

$$\sqrt[5]{20,5} = 4,5$$

c)

$$\sqrt[3]{15} = 1,96$$

d)

$$\sqrt[3]{-52} = -2,68$$

e)

$$\sqrt[3]{-27000} = -12,81$$

f)

$$\sqrt[4]{-12} = 1,364$$

1.83

a)

$$\sqrt[3]{10^3} = 10$$

b)

$$\sqrt[3]{10{-}6} = 10^{-9}$$

c)

d)

$$\begin{aligned}
\sqrt[3]{-8000} &= \sqrt{-80 * -10 * -10} \\
&= \sqrt{-20 * 4 * 5 * 2 * 5 * 2} \\
&= \sqrt{-5 * 4 * 4 * 5 * 2 * 5 * 2} \\
&= \sqrt{-5 * 5 * 5 * 2 * 2 * 2 * 2 * 2 * 2} \\
&= \sqrt{-20 * -20 * -20} && = -20
\end{aligned}$$

2.10

a)

$$\begin{aligned}
2x + 1 &= 5 \\
2x &= 5 - 1 \\
2x &= 4 \\
\frac{2x}{2} &= \frac{4}{2} \\
x &= 2
\end{aligned}$$

b)

$$\begin{aligned}
3x - 1 &= x + 2 \\
3x - x &= 2 + 1 \\
2x &= 3 \\
\frac{2x}{2} &= \frac{3}{2} \\
x &= 1,5
\end{aligned}$$

c)

$$\begin{aligned}
-2x + 2 &= 2x - 2 \\
-2x - 2x &= -2 - 2 \\
-4x &= -4 \\
\frac{-4x}{-4} &= \frac{-4}{-4} \\
x &= 1
\end{aligned}$$

d)

$$\begin{aligned} 2x + 2 &= -3x + 7 \\ 2x + 3x &= 7 - 2 \\ 5x &= 5 \\ \frac{5x}{5} &= \frac{5}{5} \\ x &= 1 \end{aligned}$$

2.11

a)

$$\begin{aligned} 12x - 13 &= 9x - 7 \\ 12x - 9x &= -7 + 13 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

b)

$$\begin{aligned} -7x + 11 &= 2x - 3 \\ -7x - 2x &= -3 - 11 \\ -9x &= -14 \\ \frac{-9x}{-9} &= \frac{-14}{-9} \\ x &= \frac{14}{9} \end{aligned}$$

c)

$$\begin{aligned} 0,02x + 0,7 &= -0,03x + 0,2 \\ 0,02x + 0,03x &= 0,2 - 0,7 \\ 0,05x &= -0,5 \\ \frac{0,05x}{0,05} &= \frac{-0,5}{-0,05} \\ x &= 10 \end{aligned}$$

2.12

a)

$$\begin{aligned} 12x - 13 &= 9x - 7 \\ 12x - 9x &= -7 + 13 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

b)

$$\begin{aligned}
-7x + 11 &= 2x - 3 \\
-7x - 2x &= -3 - 11 \\
-9x &= -14 \\
\frac{-9x}{-9} &= \frac{-14}{-9} \\
x &= \frac{14}{9}
\end{aligned}$$

c)

$$\begin{aligned}
0,02x + 0,7 &= -0,03x + 0,2 \\
0,02x + 0,03x &= 0,2 - 0,7 \\
0,05x &= -0,5 \\
\frac{0,05x}{0,05} &= \frac{-0,5}{-0,05} \\
x &= 10
\end{aligned}$$