Models for the detection of Diabetes

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1 Bolie's diabetes model

Jachym dopise teoreticky uvod

$$V\frac{\mathrm{d}H}{\mathrm{d}t} = -F_1(H) + F_2(G) + x \tag{1}$$

$$V\frac{\mathrm{d}G}{\mathrm{d}t} = -F_3(H,G) - F_4(H,G) + y \tag{2}$$

For changes around equilibrium:

vi. bod 1 dopsat assumptions

$$V\frac{\mathrm{d}h}{\mathrm{d}t} = -F_1(H_0+h) + F_2(G_0+g) + x \tag{3}$$

$$V\frac{\mathrm{d}g}{\mathrm{d}t} = -F_3(H_0+h, G_0+g) - F_4(H_0+h, G_0+g) + y \tag{4}$$

Linearization:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\underbrace{\frac{1}{V}\frac{\partial F_1(H_0)}{\partial H}}_{\alpha}h + \underbrace{\frac{1}{V}\frac{\partial F_2(G_0)}{\partial G}}_{\beta}g + O(2) + \dots \tag{5}$$

$$\frac{\mathrm{d}g}{\mathrm{d}t} = -\frac{1}{V}\frac{\partial F_3(H_0,G_0)}{\partial H}h - \frac{1}{V}\frac{\partial F_3(H_0,G_0)}{\partial G}g - \frac{1}{V}\frac{\partial F_4(H_0,G_0)}{\partial H}h - \frac{1}{V}\frac{\partial F_4(H_0,G_0)}{\partial G}g + \mathcal{O}(2) + \dots \tag{6}$$

$$= -\underbrace{\left(\frac{1}{V}\frac{\partial F_3(H_0,G_0)}{\partial H} + \frac{\partial F_4(H_0,G_0)}{\partial H}\right)}_{\gamma}h - \underbrace{\left(\frac{1}{V}\frac{\partial F_3(H_0,G_0)}{\partial G} + \frac{\partial F_4(H_0,G_0)}{\partial G}\right)}_{\delta}g + \mathcal{O}(2) + \dots \tag{7}$$

Linearized:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\alpha h + \beta g$$

$$\frac{\mathrm{d}g}{\mathrm{d}t} = -\gamma h - \delta g$$

diskuze znamenek (bod 2), tabulka s popisem velicin

Symbol	Meaning	Dimension
V	volume	L
x	rate of insulin injection	units h^{-1}
y	rate of glucose injection	$\mathrm{g\,h}^{-1}$
H	insulin concentration	units L^{-1}
H_0	insulin concentration equilibrium	units L^{-1}
h	insulin concentration changes	units L^{-1}
G	glucose concentration	$\mathrm{g\,L}^{-1}$
G_0	glucose concentration equilibrium	$\mathrm{g\,L}^{-1}$
g	glucose concentration changes	$\mathrm{g\,L}^{-1}$
$F_1(H)$	rate of insulin destruction	units h^{-1}
$F_2(G)$	rate of insulin production	units h^{-1}
$F_3(H,G)$	rate of liver accumulation of glucose	$\mathrm{g\,h}^{-1}$
$F_4(H,G)$	rate of tissue utilization of glucose	$\mathrm{g\,h}^{-1}$

Table 1: Diabetes model parameters.

2 Linearized model solution

bod 3, redukce na 2nd order, char. polynom, reseni; stabilita, kdy je stabilni periodicky/aperiodicky

$$\ddot{g} + (\alpha + \delta)\dot{g} + (\beta\gamma + \delta\alpha)g = S(t) \tag{8}$$

$$\lambda^2 + (\alpha + \delta)\lambda + (\beta\gamma + \delta\alpha) = 0 \tag{9}$$

$$\lambda_{1,2} = \frac{1}{2}\left(-(\alpha + \delta) \pm \sqrt{(\alpha - \delta)^2 - 4\beta\gamma}\right) \tag{10}$$

Since both $(\alpha + \delta)$ and $(\beta\gamma + \delta\alpha)$ are positive, the solutions are always stable, going to zero with $t \to \infty$.

3 Bolie's diabetes test

predpoklada kriticky tlumene reseni

bod 4, povidani o testu

ploty reseni g,h, diskuze viz. 4

nabizi se vlozit reseny priklad, napr. 4 na str. 108

4 Ackermann's diabetes test

predpoklada kmitave reseni

opsat par rovnic ze strany 105/106

vyresit ten stejny priklad jako vyse