

Models for the detection of Diabetes

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1 Bolie's diabetes model

Jachym dopise teoreticky uvod

$$V \frac{dH}{dt} = -F_1(H) + F_2(G) + x \quad (1)$$

$$V \frac{dG}{dt} = -F_3(H, G) - F_4(H, G) + y \quad (2)$$

For changes around equilibrium:

vi. bod 1 dopsat assumptions

$$V \frac{dh}{dt} = -F_1(H_0 + h) + F_2(G_0 + g) + x \quad (3)$$

$$V \frac{dg}{dt} = -F_3(H_0 + h, G_0 + g) - F_4(H_0 + h, G_0 + g) + y \quad (4)$$

Linearization:

$$\frac{dh}{dt} = - \underbrace{\frac{1}{V} \frac{\partial F_1(H_0)}{\partial H}}_{\alpha} h + \underbrace{\frac{1}{V} \frac{\partial F_2(G_0)}{\partial G}}_{\beta} g + \mathcal{O}(2) + \dots \quad (5)$$

$$\frac{dg}{dt} = - \frac{1}{V} \frac{\partial F_3(H_0, G_0)}{\partial H} h - \frac{1}{V} \frac{\partial F_3(H_0, G_0)}{\partial G} g - \frac{1}{V} \frac{\partial F_4(H_0, G_0)}{\partial H} h - \frac{1}{V} \frac{\partial F_4(H_0, G_0)}{\partial G} g + \mathcal{O}(2) + \dots \quad (6)$$

$$= - \underbrace{\left(\frac{1}{V} \frac{\partial F_3(H_0, G_0)}{\partial H} + \frac{\partial F_4(H_0, G_0)}{\partial H} \right)}_{\gamma} h - \underbrace{\left(\frac{1}{V} \frac{\partial F_3(H_0, G_0)}{\partial G} + \frac{\partial F_4(H_0, G_0)}{\partial G} \right)}_{\delta} g + \mathcal{O}(2) + \dots \quad (7)$$

Linearized:

$$\begin{aligned} \frac{dh}{dt} &= -\alpha h + \beta g \\ \frac{dg}{dt} &= -\gamma h - \delta g \end{aligned}$$

diskuze znamenek (bod 2), tabulka s popisem velicin

Symbol	Meaning	Dimension
V	volume	L
x	rate of insulin injection	units h ⁻¹
y	rate of glucose injection	g h ⁻¹
H	insulin concentration	units L ⁻¹
H_0	insulin concentration equilibrium	units L ⁻¹
h	insulin concentration changes	units L ⁻¹
G	glucose concentration	g L ⁻¹
G_0	glucose concentration equilibrium	g L ⁻¹
g	glucose concentration changes	g L ⁻¹
$F_1(H)$	rate of insulin destruction	units h ⁻¹
$F_2(G)$	rate of insulin production	units h ⁻¹
$F_3(H, G)$	rate of liver accumulation of glucose	g h ⁻¹
$F_4(H, G)$	rate of tissue utilization of glucose	g h ⁻¹

Table 1: Diabetes model parameters.

2 Linearized model solution

bod 3, redukce na 2nd order, char. polynom, reseni; stabilita, kdy je stabilni periodicky/apperiodicky

$$\ddot{g} + (a + d)\dot{g} + (bc + da)g = S(t) \quad (8)$$

$$\lambda^2 + (a + d)\lambda + (bc + da) = 0 \quad (9)$$

$$\lambda_{1,2} = \frac{1}{2} \left(-(a + d) \pm \sqrt{(a + d)^2 - 4(bc + da)} \right) \quad (10)$$

Since both $(a + d)$ and $(bc + da)$ are positive, the solutions are always stable, going to zero with $t \rightarrow \infty$.

3 Bolie's diabetes test

predpoklada kriticky tlumene reseni

bod 4, povidani o testu

ploty reseni g,h, diskuze viz. 4

nabizi se vlozit reseny priklad, napr. 4 na str. 108

4 Ackermann's diabetes test

predpoklada kmitave reseni

opsat par rovnic ze strany 105/106

vyresit ten stejný priklad jako vyše