Vehicle Dynamics

Lateral dynamics: Steady-State Cornering

Outline

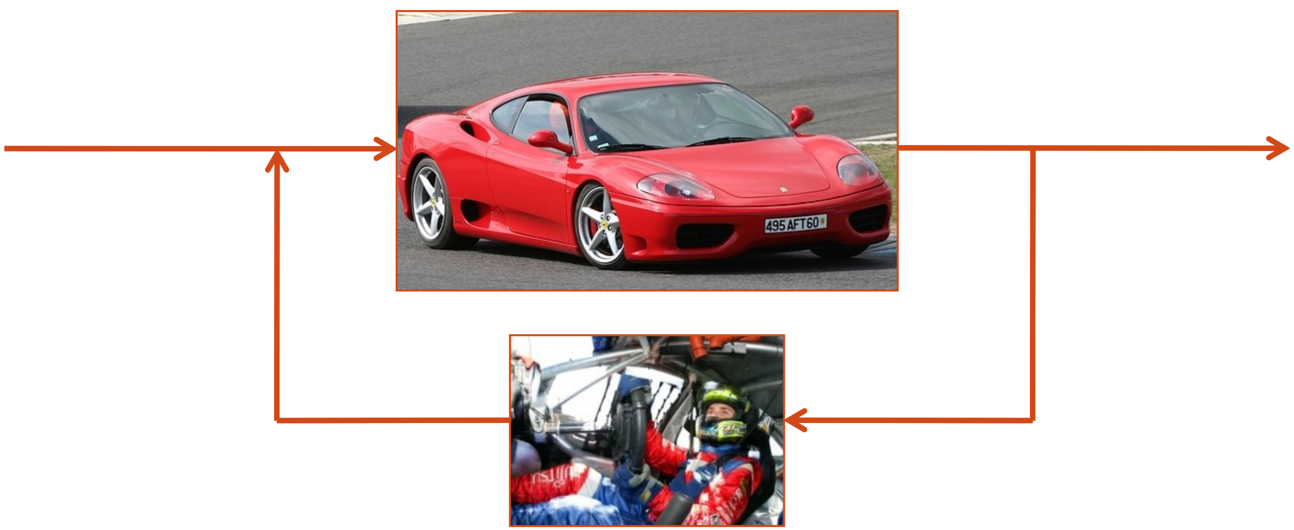
- Handling performance in general
- Cornering at low speed
- Cornering at high speed
- Standard manoeuvres for understeer gradient measurement
    - Constant Radius, Constant Speed and Constant Steering Angle methods
- Load transfer during cornering

Handling

Handling performance in a vehicle is linked to:

- Responsiveness of the vehicle to driver's directional input;
- Ease of control during cornering

Handling = overall performance of driver-vehicle system → "closed-loop"



Open-loop manoeuvre for handling assessment

Open-loop manoeuvre: predefined input-history and measurement of vehicle response

- Steering angle
- Throttle/braking pedal

?

Objective performance indicator: **understeer gradient**

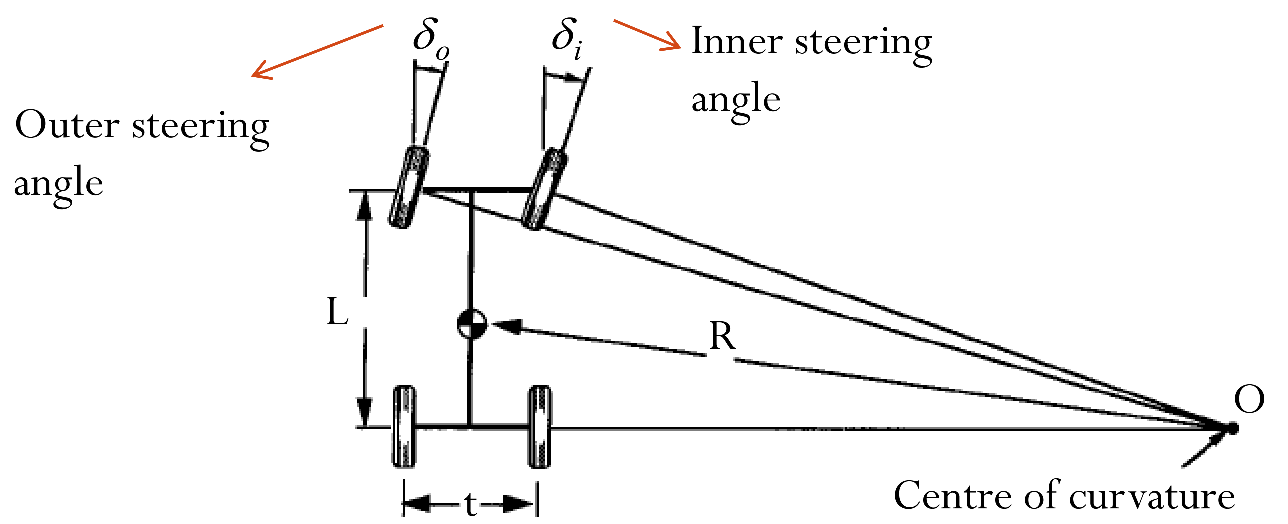
NOTE:
The understeer gradient is measured during **steady-state** manoeuvres: (quasi-)constant longitudinal speed, lateral acceleration, roll-angle, turn radius)

Low-speed cornering

Assumption: vehicle speed sufficiently low so that:

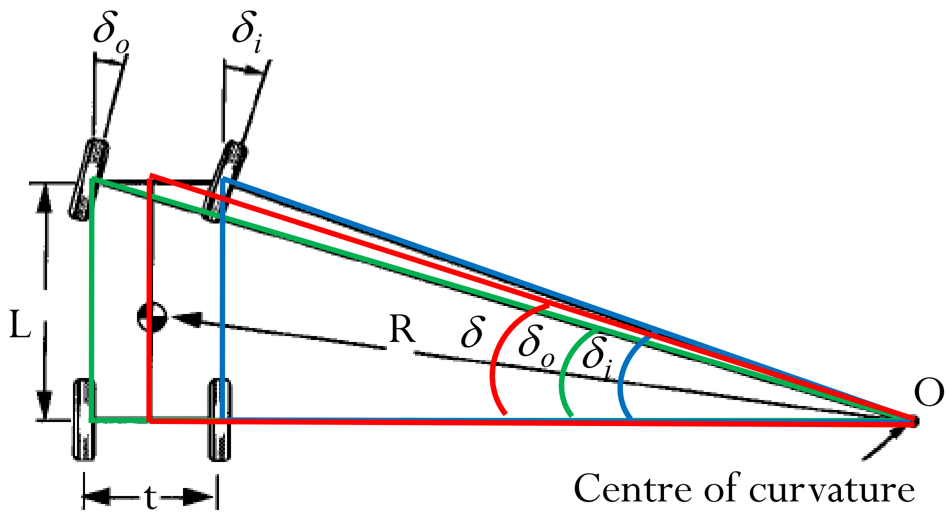
$$a_y = \frac{V^2}{R} \cong 0 \quad \Rightarrow$$

Lateral forces at tire-road contact patches are negligible



- O is located along the normal line to the longitudinal axis (rear axle direction of motion)
- For a kinematically correct steering, normal lines to front wheels must pass through O

Ackerman steering angle



$$\begin{cases} \tan(\delta_o) \cong \dfrac{L}{R+t/2} \\ \\ \tan(\delta_i) \cong \dfrac{L}{R-t/2} \end{cases}$$

Assumption:
small angles

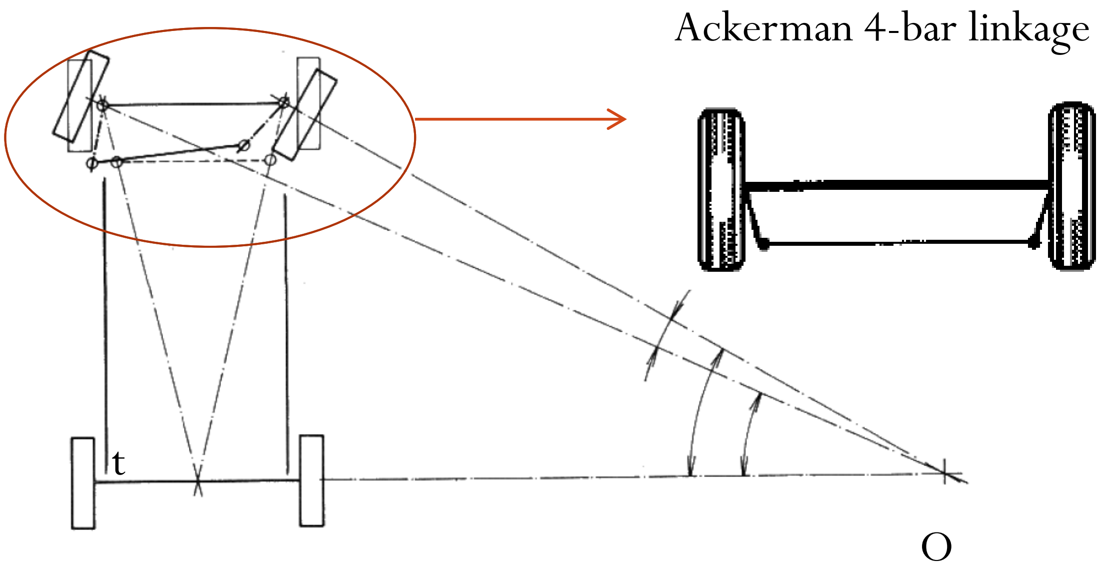
$$\delta_o \cong \frac{L}{R+t/2}$$

$$\delta_i \cong \frac{L}{R-t/2}$$

By def: Ackerman angle = average steering angle at the front wheels

$$\boxed{\delta_{Ack} \cong \frac{L}{R}}$$

Ackerman geometry



$$\delta_o \cong \frac{L}{R + t/2}$$

$$\delta_i \cong \frac{L}{R - t/2}$$

Notes:

- Ackerman geometry fulfilled → resistant torque at the steering wheel increases with the steering angle
- Small deviations have poor impact on cornering behaviour
- Huge deviations cause wear to front tires

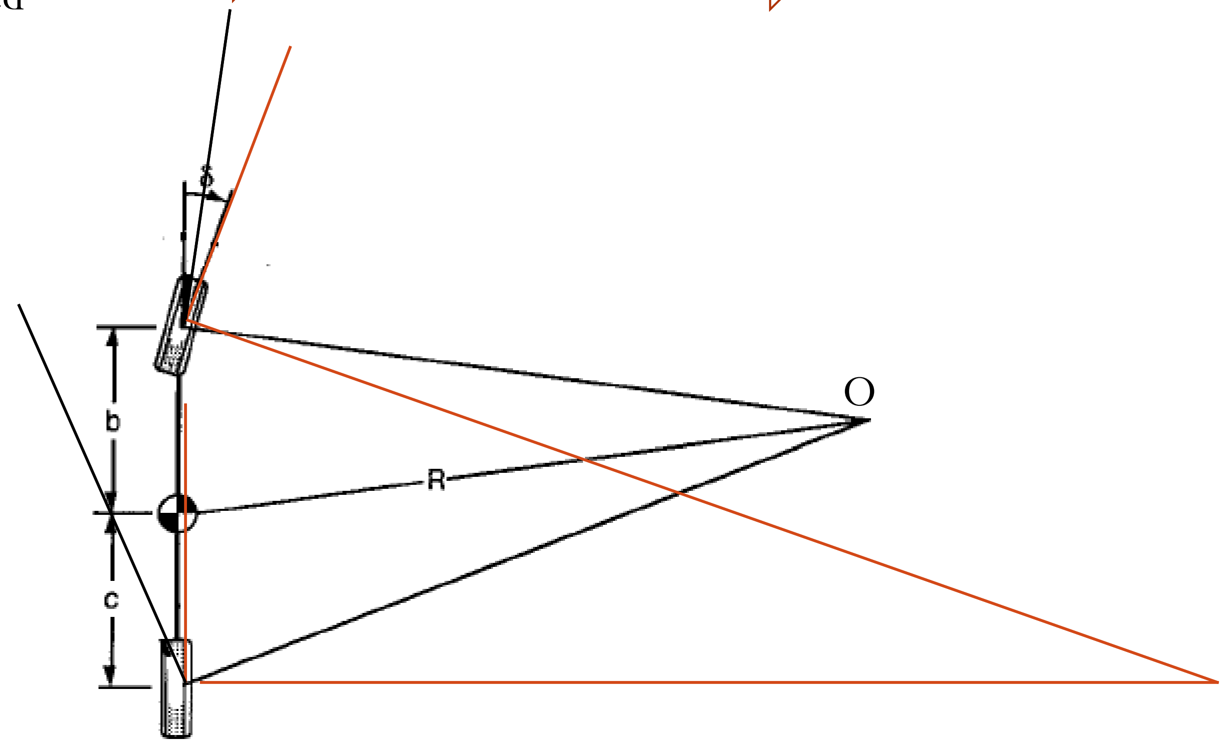
High-speed cornering: bicycle model

$$\delta_o \cong \frac{L}{R + t/2}$$

$$\delta_i \cong \frac{L}{R - t/2}$$

Cornering at high speed $\Rightarrow$ R >> L, t $\Rightarrow$ $\delta_o \cong \delta_i$

Bicycle model (single track)



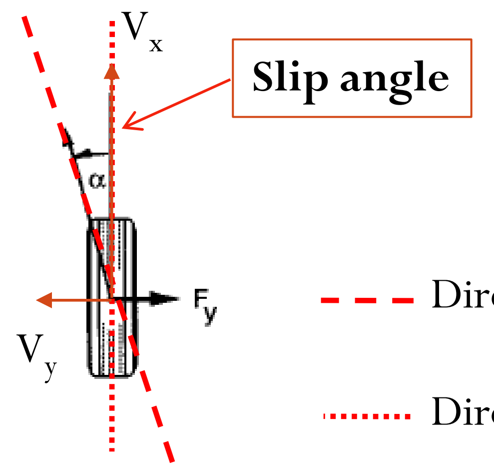
Question: what looks wrong in the picture?

High-speed cornering: lateral (or cornering) forces at tires

Lateral acceleration (and force) non negligible → centrifugal force must be balanced by lateral (cornering) forces at the tires

⇩

Tires undergo <u>lateral slip</u> while rotating around their axis



Centrifugal (lateral) force

Direction of travel

≠

Direction of heading

Centripetal acceleration V^2/R

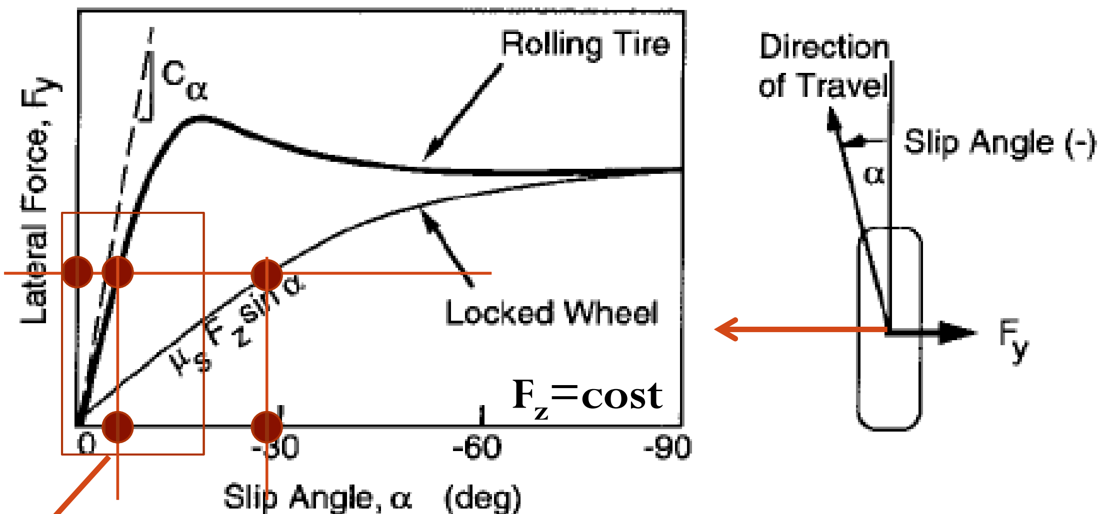
Centre of curvature

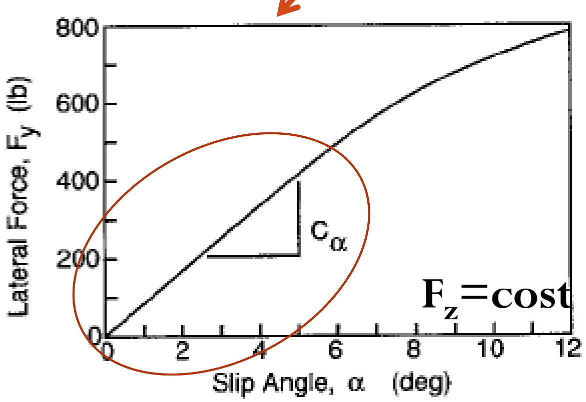
Note: - the cornering force is perpendicular to the direction of heading

- the centre of curvature is located along the normal to the direction of travel

High-speed cornering: lateral (or cornering) forces at tires

Lateral force as a function of the slip angle for a given value of vertical load





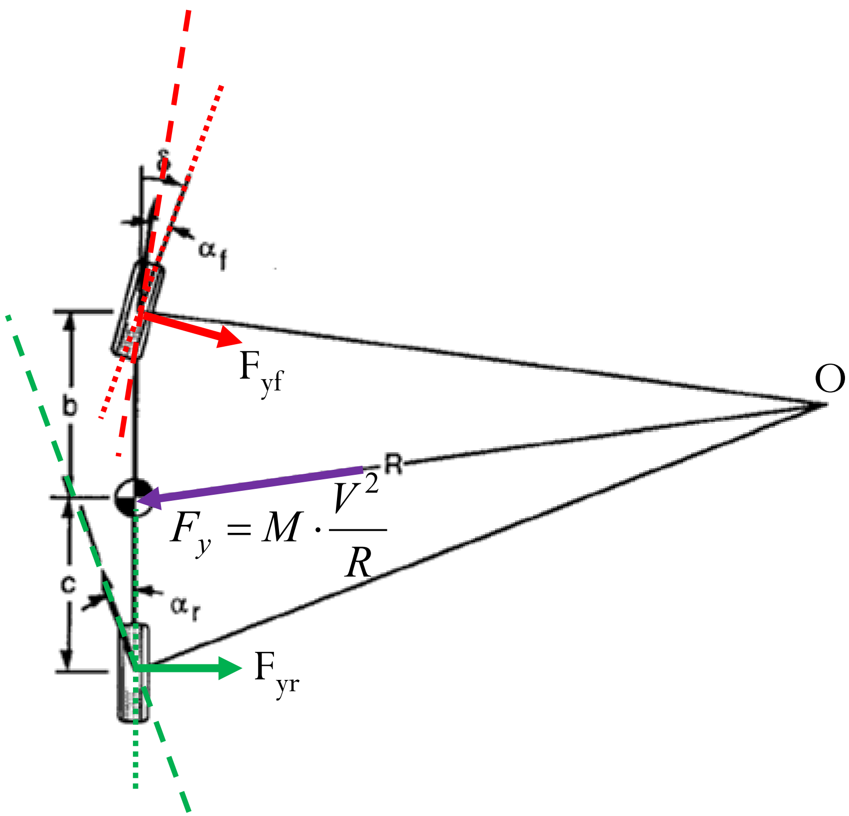
For small values of slip angle F_y is linearly increasing with α:

$$F_y = C_\alpha \cdot \alpha$$

C_α ⇨ **Cornering Stiffness**

High-speed cornering: slip angle & lateral forces at tires

A vehicle driven along a turn of radius R at a longitudinal speed V will undergo a centripetal (lateral) acceleration equal to V^2/R.



Travel direction at front wheels

Heading direction at front wheels

α_f → slip angle at front wheels

$F_{yf}=C_{\alpha f}\alpha_f$ → lateral (cornering) force at front wheels

Travel direction at rear wheels

Heading direction at rear wheels

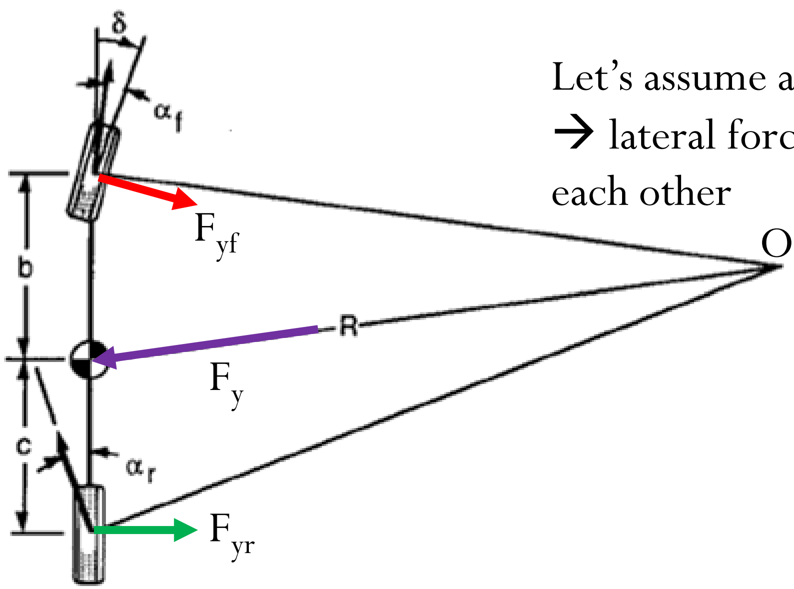
α_r → slip angle at rear wheels

$F_{yr}=C_{\alpha r}\alpha_r$ → lateral (cornering) force at rear wheels

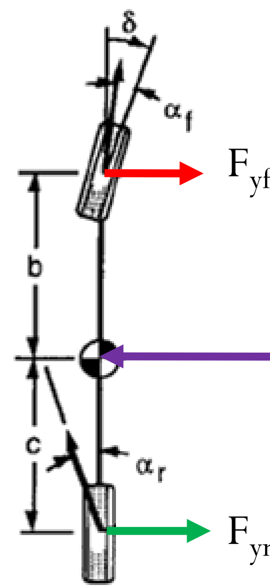
High-speed cornering: free body diagram

Note: - the cornering force is perpendicular to the direction of heading

- the centre of curvature is located along the normal to the direction of travel

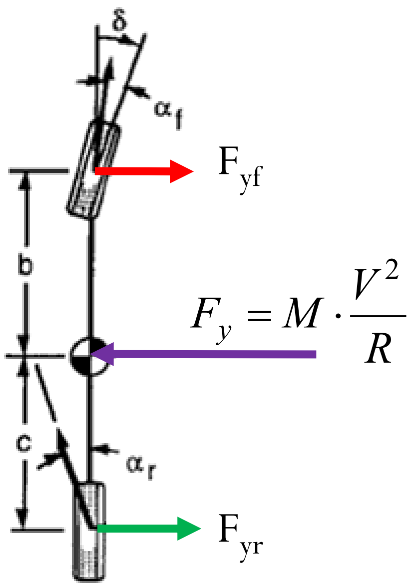


Let's assume angles are small → lateral forces are parallel to each other



$$F_y = M \cdot \frac{V^2}{R}$$

High-speed cornering: Dynamic equilibrium



Translation in the lateral direction: $$F_{yf} + F_{yr} = M \cdot \frac{V^2}{R} \quad (1)$$

Rotation around the CG: $$F_{yf} \cdot b - F_{yr} \cdot c = 0 \quad (2)$$

$$F_{yf} = \frac{c}{b} F_{yr} \quad (3)$$

Replacing eq.(3) in eq. (1) gives:

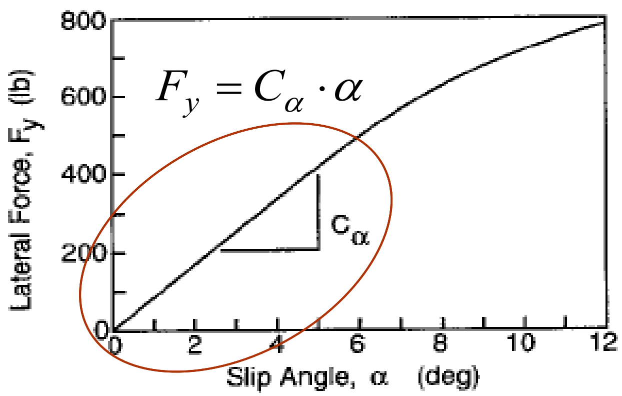
$$F_{yf} + F_{yr} = F_{yr}(c/b + 1) = F_{yr}\, L/b = M \cdot \frac{V^2}{R}$$

$$\Rightarrow \left\{ \begin{array}{l} F_{yr} = \frac{Mb}{L} \cdot \frac{V^2}{R} = \frac{W_{rs}}{g} \cdot \frac{V^2}{R} \\ \\ F_{yf} = \frac{Mc}{L} \cdot \frac{V^2}{R} = \frac{W_{fs}}{g} \cdot \frac{V^2}{R} \end{array} \right.$$

- W_{fs} and W_{rs} are the static loads on the front and rear axis respectively

High-speed cornering: slip angles

- From lateral forces we can compute lateral slip angles at front and rear wheels
- Let's assume to be in the F_y-α linear region:



$$\alpha_f = \frac{F_{yf}}{C_{\alpha f}} = \frac{W_{fs}}{C_{\alpha f} \cdot g} \cdot \frac{V^2}{R}$$

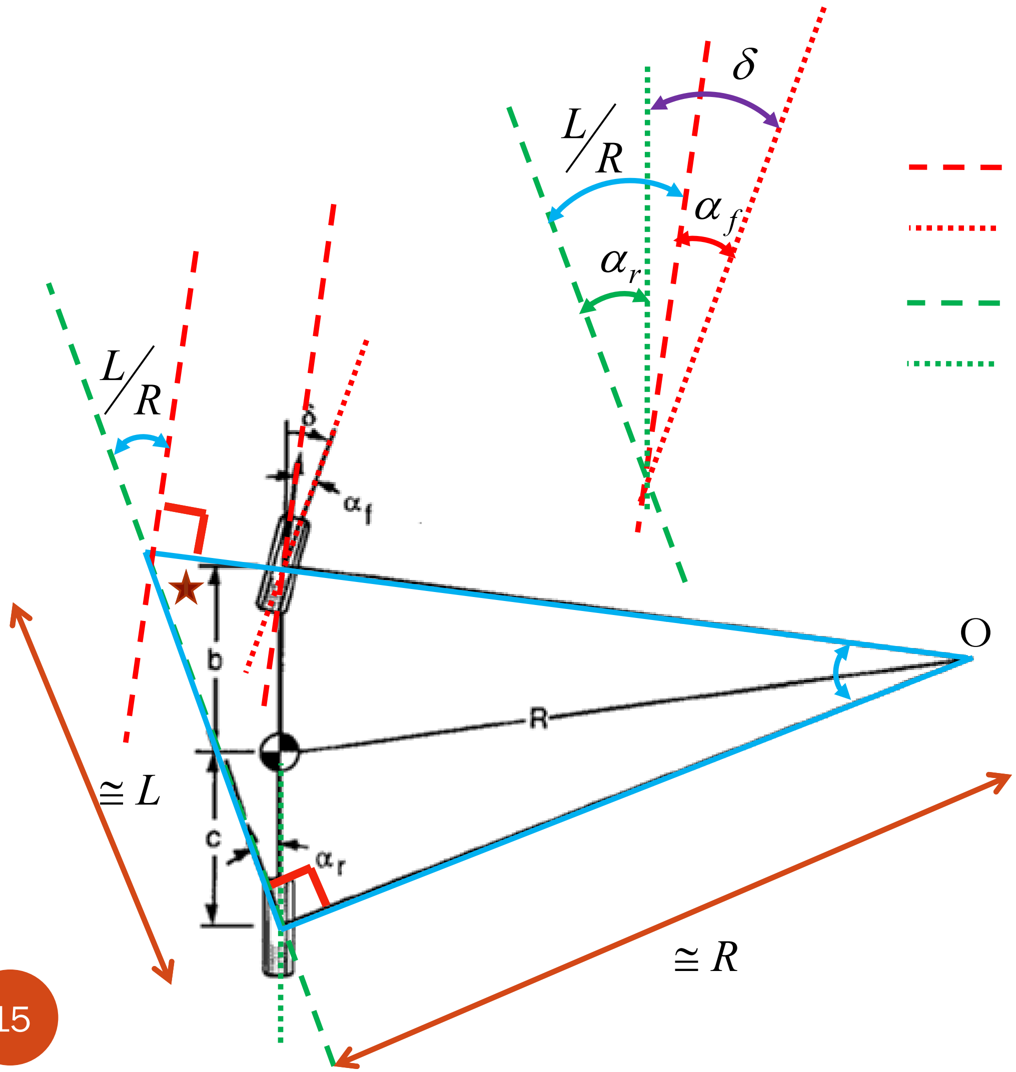
$$\alpha_r = \frac{F_{yr}}{C_{\alpha r}} = \frac{W_{rs}}{C_{\alpha r} \cdot g} \cdot \frac{V^2}{R}$$

Note that on each axle:

- if the static vertical load increases → the slip angle increases as well
- if the cornering stiffness increases → the slip angle decreases
- if the lateral acceleration increases → the slip angle increases as well

High-speed cornering: steering angle

δ is the steering angle at the front wheels (angle between the heading and the travel direction



- - - Travel direction at front wheels
······ Heading direction at front wheels
- - - Travel direction at rear wheels
······ Heading direction at rear wheels

$$\delta = 57.3 \cdot \frac{L}{R} + \alpha_f - \alpha_r$$

High-speed cornering: Understeer Gradient

We know that the slip angles are:

$$\alpha_f = \frac{F_{yf}}{C_{\alpha f}} = \frac{W_f}{C_{\alpha f} \cdot g} \cdot \frac{V^2}{R}$$

$$\alpha_r = \frac{F_{yr}}{C_{\alpha r}} = \frac{W_r}{C_{\alpha r} \cdot g} \cdot \frac{V^2}{R}$$

$$\delta = 57.3 \cdot \frac{L}{R} + \alpha_f - \alpha_r$$

Lateral acceleration a_y

$$\delta = 57.3\frac{L}{R} + \boxed{\frac{1}{g}\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)}\boxed{\frac{V^2}{R}}$$

$$\boxed{K = \frac{1}{g}\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right) = \frac{\alpha_f - \alpha_r}{a_y}}$$

Understeer Gradient [deg/g]

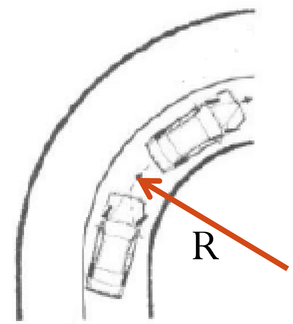
$$\boxed{\delta = 57.3\frac{L}{R} + K \cdot a_y}$$

Steering angle [deg]

High-speed cornering: Understeer Gradient

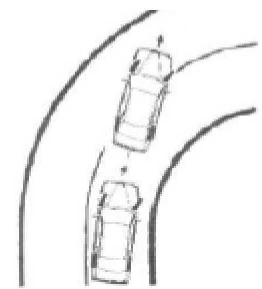
$$\delta = 57.3\frac{L}{R} + Ka_y = 57.3\frac{L}{R} + \alpha_f - \alpha_r$$

1. **Neutral steering $K=0$ → $\alpha_f = \alpha_r$:** the required steering angle is equal to the Ackerman angle (independent from the speed)



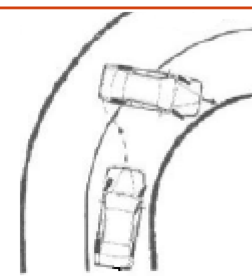
$K=0$

2. **Understeering $K>0$ → $\alpha_f > \alpha_r$:** the required steering angle increases with the speed



$K>0$

3. **Oversteering $K<0$ → $\alpha_f < \alpha_r$:** the required steering angle decreases when the speed increases

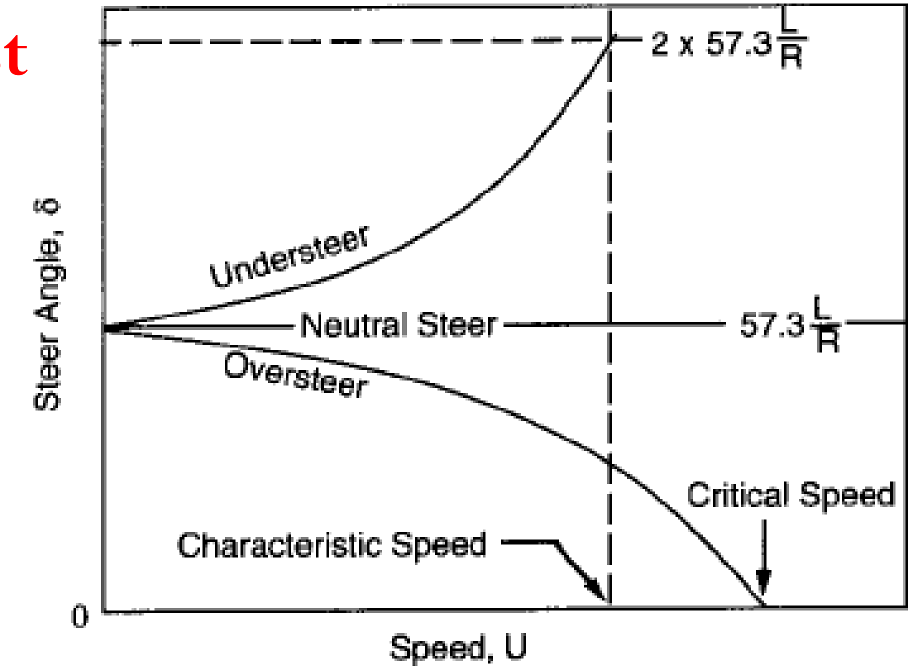


$K<0$

High-speed cornering: Understeer Gradient

R = cost

$$\delta = 57.3\frac{L}{R} + Ka_y$$

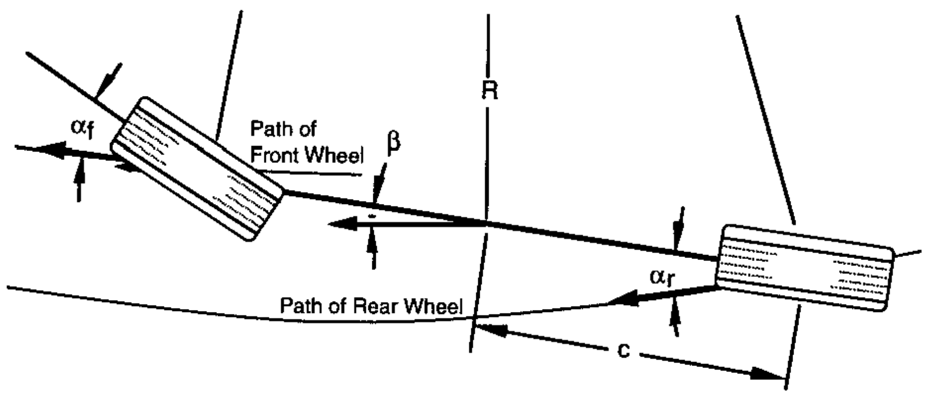


• **Characteristic speed**: speed at which the steer angle is two times as big as the Ackerman angle in an understeering vehicle $V_{char} = \sqrt{57.3\ L\ g/K}$

• **Critical speed**: speed at which the steer angle is equal to zero in an understeering vehicle (unstable condition) $V_{crit} = \sqrt{-\ 57.3\ L\ g/K}$

Sideslip angle

Angle between the longitudinal axis and local direction of travel (velocity vector) at CG



Sideslip angle as a function of speed

$$\beta = 57.3\, c/R - \alpha_r$$

$$= 57.3\, c/R - W_r V^2/(C_{\alpha r}\, g\, R)$$

Speed at which the sideslip angle vanishes

$$V_{\beta=0} = \sqrt{57.3\, g\, c\, C_{\alpha r}/W_r}$$

Note: independent of R

a_y

Test methods for the estimate of K

ISO 4138 standard

$$\delta = 57.3\frac{L}{R} + K \cdot a_y$$

Test method	Constant	Varied	Measured
Constant radius	Radius	Speed	Steering-wheel angle
Constant steering-wheel angle	Steering-wheel angle	Speed	Radius
Constant speed with discrete turn radii	Speed	Radius	Steering-wheel angle
Constant speed with discrete steering-wheel angles	Speed	Steering-wheel angle	Radius

Vehicle equipped with a proper set of sensors

NOTE:

The understeer gradient is measured during **steady-state** manoeuvres: (quasi-)constant longitudinal speed, lateral acceleration, roll-angle, turn radius)

Constant Radius Method

Method	Costant	Varying	Measured
Constant radius	Radius of curvature	Vehicle speed	Steering angle

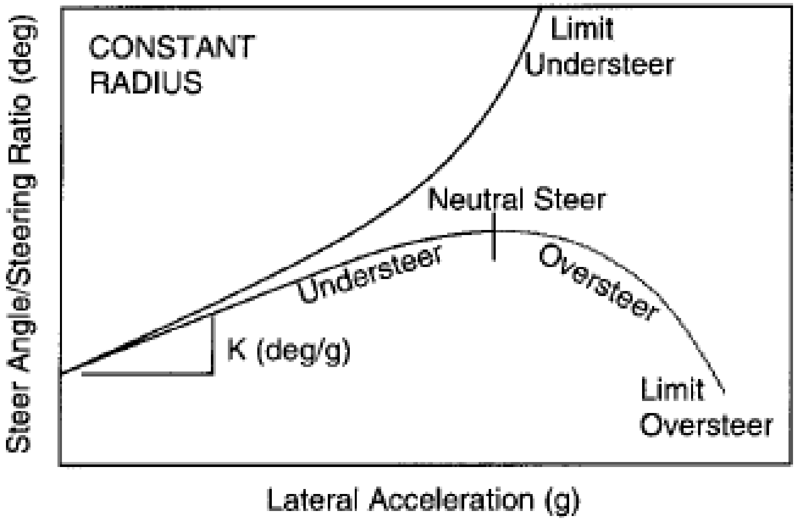
1) Vehicle driven along the turn at low speed→ Ackerman angle
2) Further tests are done while increasing the speed slowly (maximum jerk equal to 0.1 $m/s^2/s$)
3) From V and R the lateral acceleration is derived (in g): $a_y = V^2 / gR$
4) At each lap the steer angle **$\boldsymbol{\delta}$** is measured
5) The curve **$\boldsymbol{\delta} - \mathbf{a}_y$** is defined by points, from which

K is derived by differentiation

$$\frac{\partial \delta}{\partial a_y} = \frac{\partial}{\partial a_y}(57.3\frac{L}{R}) + \frac{\partial}{\partial a_y}(K \cdot a_y)$$

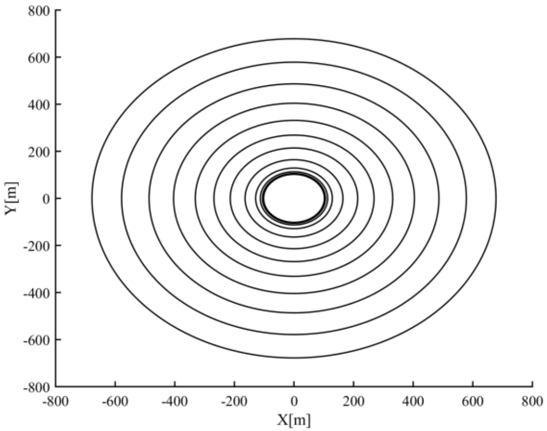
$$\Downarrow$$

$$K = \frac{\partial \delta}{\partial a_y}$$



Constant Steering Angle Method

Method	Costant	Varying	Measured
Constant steering angle	Steering wheel angle	Speed	Radius of curvature



1. δ is set to a fixed value and a circular trajectory is followed at low speed

3. The speed is increased with discrete values and limited jerk (0,1 $m/s^2/s$)

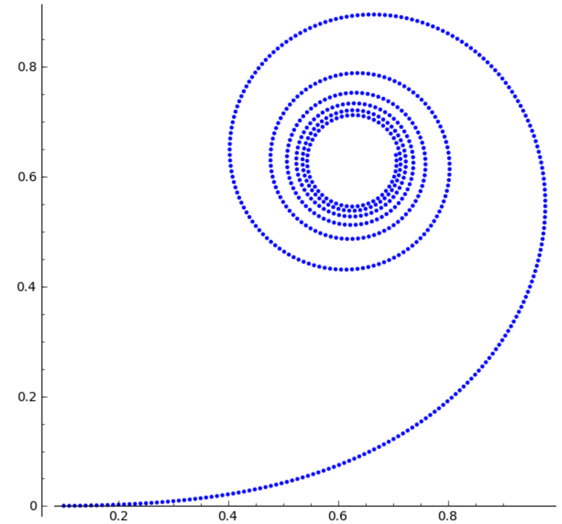
4. From measured data K is calculated as

$$K(a_y(s)) = \frac{\left(\delta - \frac{180}{\pi}\frac{L}{R(s)}\right)}{a_y(s)}$$

Constant Speed Method

Metodo	Parametro Costante	Parametro Variabile	Parametri Misurati
Constant Speed	Velocità longitudinale	Angolo di sterzo al volante	• Angolo di sterzo al volante • Accelerazione Laterale

1) Si aumenta progressivamente l'angolo di sterzo al volante;
2) Il raggio R della curva percorsa viene determinato dai valori di V e a_y : $R = V^2 / a_y$
3) Per ogni prova viene misurato l'angolo di sterzo al volante e tramite lo **steering ratio** si risale all'angolo di sterzo alle ruote anteriori
4) Si plotta l'angolo di sterzo alle ruote in funzione dell'accelerazione laterale
5) Il gradiente di sottosterzo è dato dalla differenza tra la pendenza della curva δ - a_y e la pendenza della curva di Ackerman



Constant Speed Method

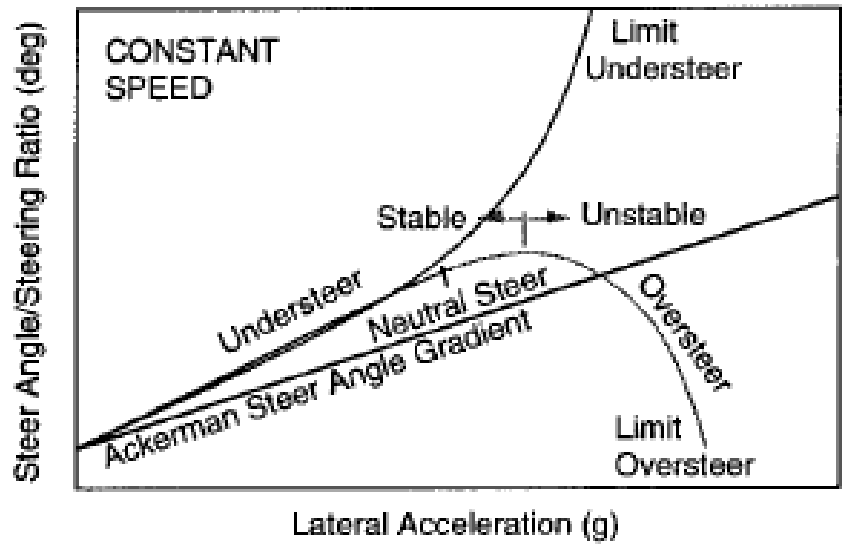
$$R = V^2 / a_y \quad \Rightarrow \quad \delta = 57.3\frac{L}{R} + K \cdot a_y = 57.3\frac{La_y}{V^2} + K \cdot a_y$$

L'understeering gradient si ottiene ancora una volta derivando rispetto ad a_y l'espressione:

$$\frac{\partial \delta}{\partial a_y} = \frac{\partial}{\partial a_y}\left(57.3\frac{La_y}{V^2}\right) + \frac{\partial}{\partial a_y}(K \cdot a_y)$$

$$\Downarrow$$

$$K = \frac{\partial \delta}{\partial a_y} - 57.3\frac{L}{V^2}$$



Il gradiente di sottosterzo è la differenza tra la pendenza della curva δ- a_y e la pendenza della retta rappresentante l'angolo di Ackerman in funzione dell'accelerazione laterale.

La V è costante, al variare del raggio della curva varia anche l'accelerazione laterale. Ad ogni valore dell'accelerazione laterale corrisponde un raggio R e quindi un certo valore dell'angolo di Ackerman;

ISO 3888 – Obstacle avoindance

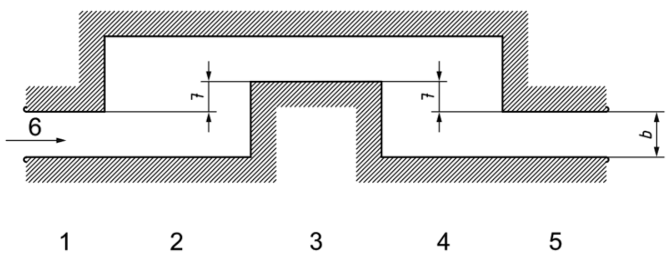
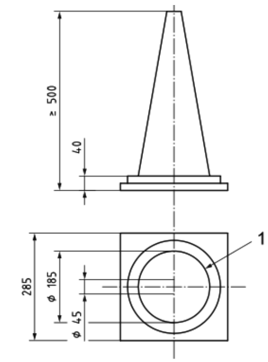


Figure 4: track for Severe Lane Change test

Section	Length [m]	Lane Offset [m]	Width b [m]
1	12	-	1.1 x vehicle width + 0.25
2	13.5	-	-
3	11	1	vehicle width +1
4	12.5	-	-
5	12	-	1.3 x vehicle width + 0.25 (min ≥ 3)

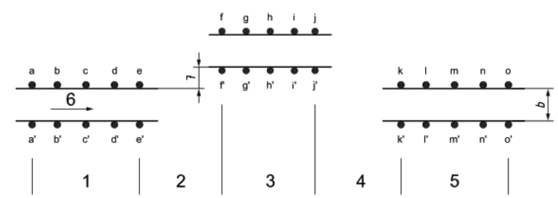
Table 2: Obstacle avoidance track dimension



Key

1 base circle of cone

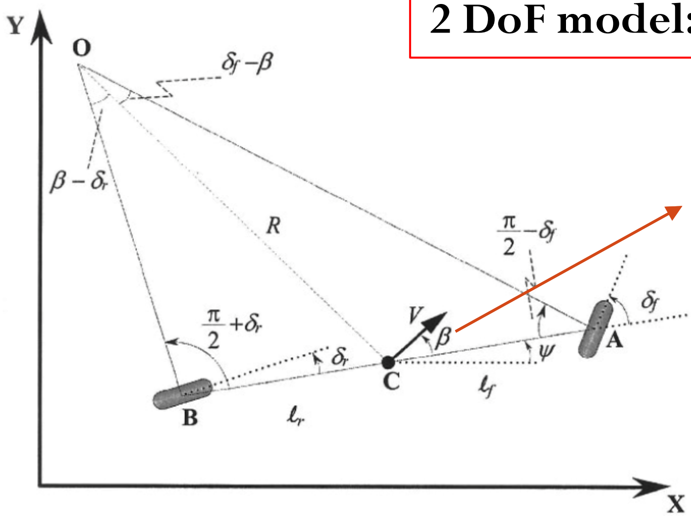
Figure 2 — Cone used for obstacle avoidance track delimitation



- Enter first section with the highest gear enabling at least 2000 rpm engine speed
- After two meters throttle is released
- Test is successful if no cone is hit

Kinematic bicycle model

2 DoF model: V, β



Sideslip angle=Slip angle at CG

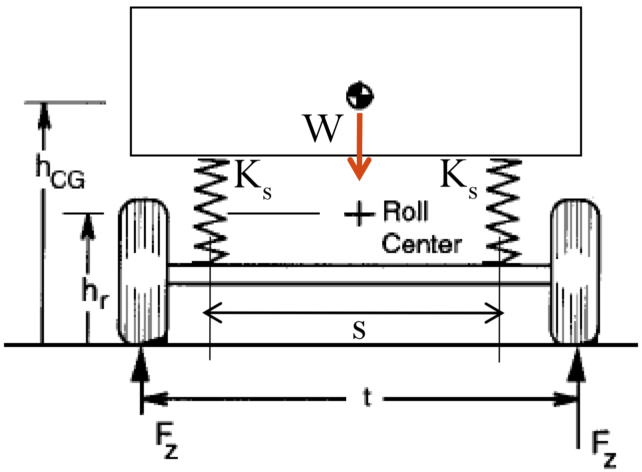
$$\begin{cases} \dot{\psi} = \dfrac{V}{R} \\ \{\tan(\delta_f) - \tan(\delta_r)\}\cos(\beta) = \dfrac{\ell_f + \ell_r}{R} \end{cases}$$

$$\boxed{\begin{aligned} \dot{\psi} &= \frac{V\cos(\beta)}{\ell_f + \ell_r}\left(\tan(\delta_f) - \tan(\delta_r)\right) \\ \dot{X} &= V\cos(\psi + \beta) \\ \dot{Y} &= V\sin(\psi + \beta) \end{aligned}}$$

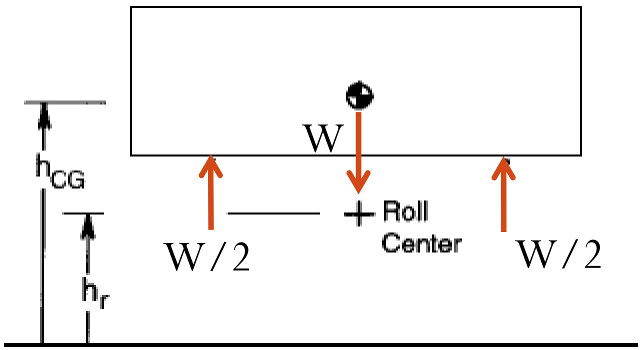
Rajamani

Load transfer during cornering: low speed

If the later acceleration is negligible, the vertical load W is distributed equally on the two wheels of an axle.

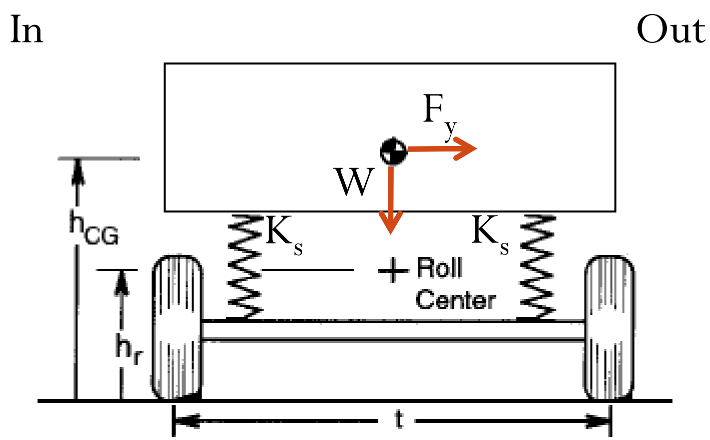


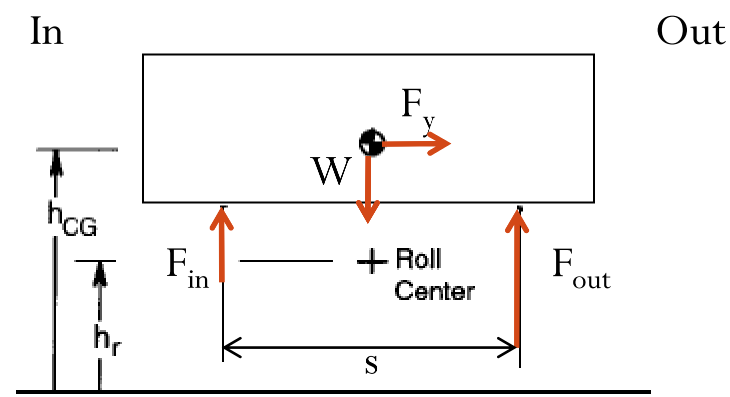
Both springs of the suspension undergo a compression equal to $(W/2)/K_s$ in comperison to the unloaded condition



Load transfer during cornering: high speed

The sprung mass is subject to a lateral force equal to $M\ a_y$





Equilibrium in the vertical direction: $F_{in} + F_{out} = W$

Equilibrum to rotation around roll center: $$F_y(h_{CG} - h_r) = \underbrace{F_{out}\frac{s}{2} - F_{in}\frac{s}{2}}_{M_\Phi}$$

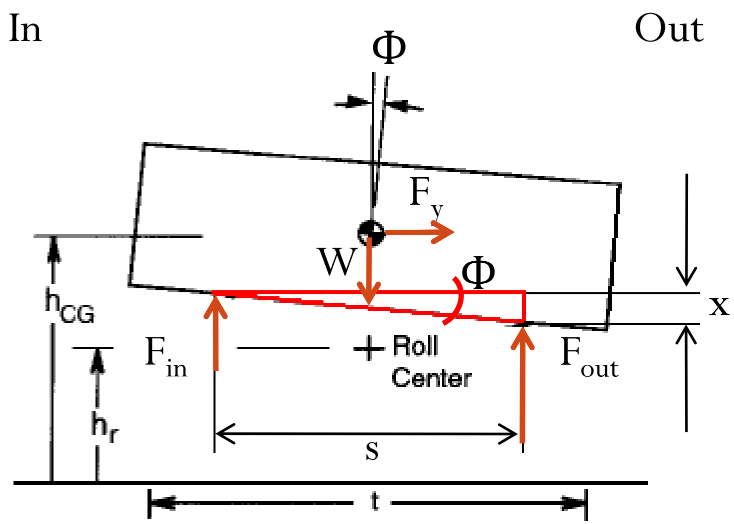
M_Φ Roll torque

$$\boxed{\begin{aligned} F_{in} &= \frac{W}{2} - \frac{F_y(h_{CG} - h_r)}{s} \\ F_{out} &= \frac{W}{2} + \frac{F_y(h_{CG} - h_r)}{s} \end{aligned}}$$

$$\frac{\Delta F_z}{2} = \frac{F_y(h_{CG} - h_r)}{s}$$

Load transfer during cornering: roll stiffness

The sprung mass undergoes a rotation ϕ (roll angle):



For small angles :

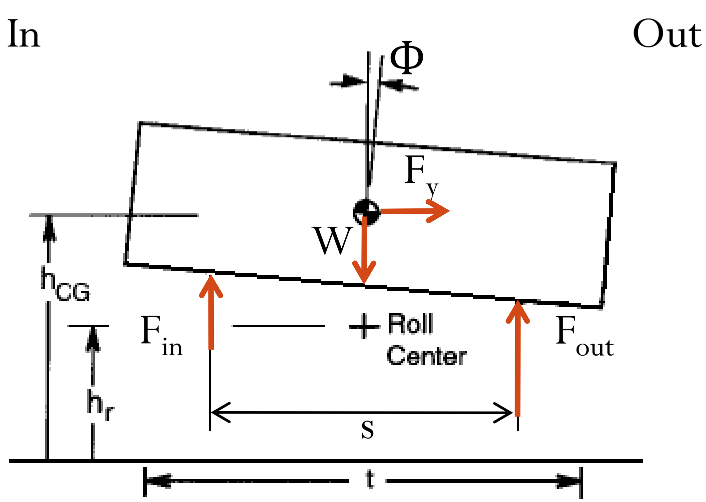
$$\phi \cong \frac{x}{s} \quad \Rightarrow \quad x = \phi \cdot s$$

The torque generated by the suspension springs around the roll center (roll torque) is equal to:

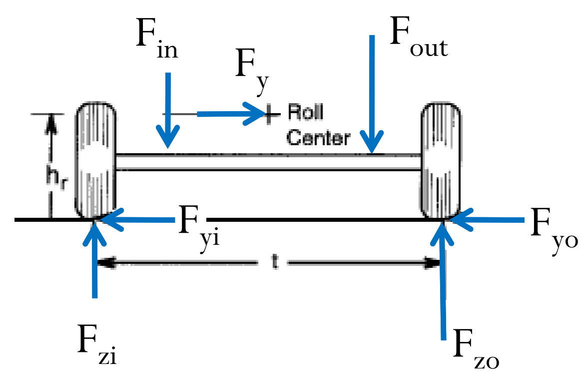
$$M_\phi = \frac{s}{2}(F_{out} - F_{in}) = \frac{s}{2} \cdot K_s x = \frac{1}{2} s^2 K_s \phi = K_\phi \cdot \phi$$

K_ϕ is the roll stiffness of the suspension: $\boxed{K_\phi = \frac{1}{2} s^2 K_s}$

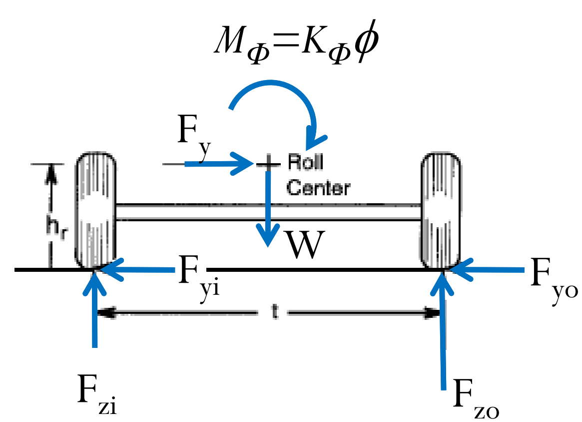
Load transfer during cornering



Forces and torques are transferred from the sprung mass to the axle



- $F_{in} + F_{out} = W$
- The roll torque generated by the springs is M_Φ



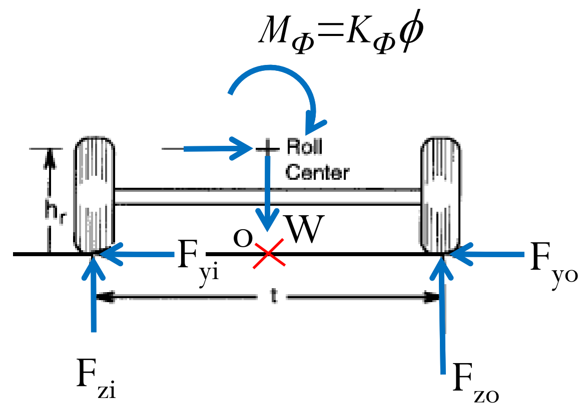
Load transfer during cornering

Rotational equilibrum around o:

$$F_{zo}\frac{t}{2} - F_{zi}\frac{t}{2} = F_y h_r + K_\phi \phi$$

$$\Downarrow$$

$$F_{zo} - F_{zi} = \underbrace{\boxed{\frac{2F_y h_r}{t}}}_{(1)} + \underbrace{\boxed{\frac{2K_\phi \phi}{t}}}_{(2)} = 2\Delta F_z$$



(1) Load transfer due to the lateral force (zero if $h_c = 0$)

(2) Load transfer due to vehicle roll

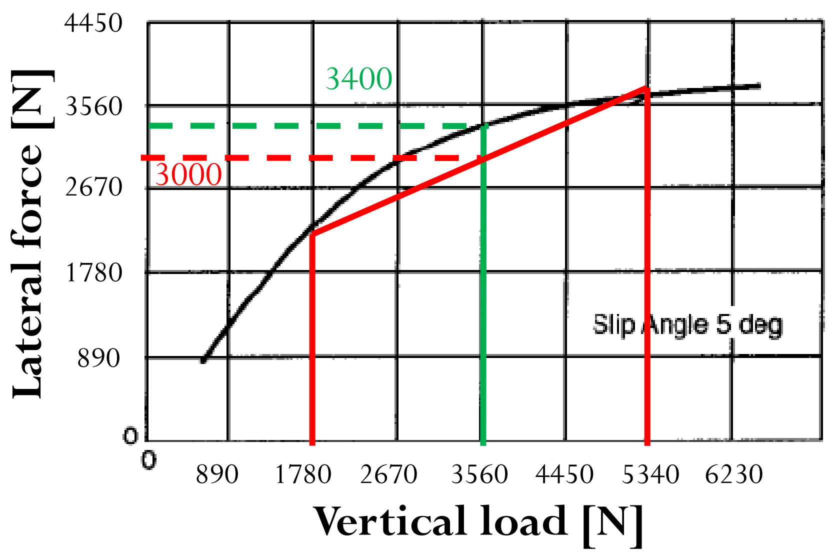
Note: anti-roll bars affect the roll stiffness and hence the understeer gradient, such that:

$K_{\phi f} \uparrow \; \Rightarrow \; K \uparrow$ understeering

$K_{\phi r} \uparrow \; \Rightarrow \; K \downarrow$ oversteering

Lateral force vs vertical load: effect of load transfer

Non linear relationship between lateral force and vertical load for a given value of slip angle



Load transfer during cornering → Lower value of lateral force → Impact on understeer gradient