Lateral Dynamics

Steady-state cornering

Handling

"Handling" is a loosley used term meant to imply the responsiveness of a vehicle to driver inputs or the ease of control. This makes handling an overall measure of the vehicle-driver combination. The driver and vehicle is a "closed-loop" system - the driver observes the vehicles motion, and corrects his input to achieve desired motion. To characterize only the vehicle, "open-loop" behavior is used. this refers to vehicle response to specific steering inputs.

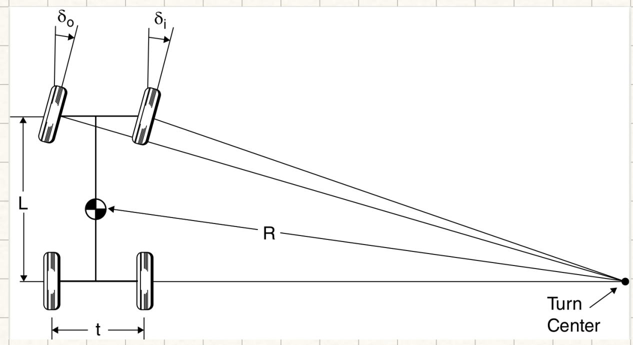
Most used measurement is the understeer gradient; a measure of performance under steady-state conditions.

Low-Speed turning

At low speeds, the lateral forces on the tires are negligible.

If the rear wheels have no slip angle, the center of turn must lie on the projection of the rear axle. Likewise, the perpendicular line from each front wheel should pass through the same point, the center of the turn. If they do not pass through the same point, the wheels will "fight" each other in the turn, with each wheel experiencing some sideslip.

For proper geometry in the turn (assuming small angles), the steer angles are given by:

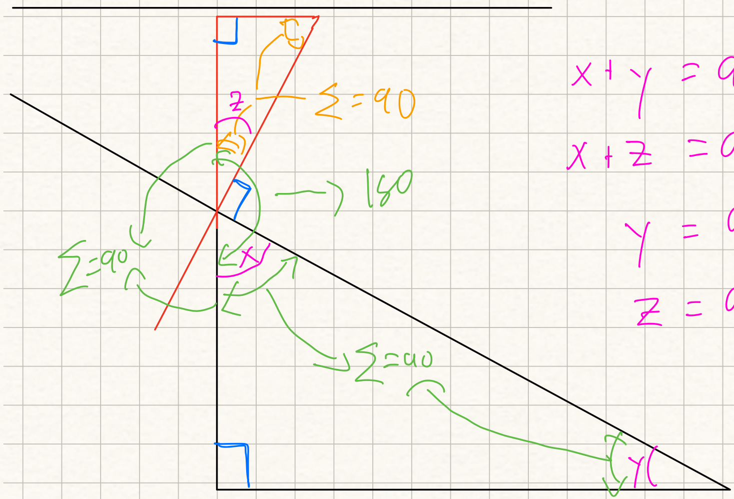


$$\delta_o = \tan^{-1}\left(\frac{L}{R+\frac{t}{2}}\right) \cong \frac{L}{R+\frac{t}{2}}$$

$$\delta_i = \tan^{-1}\left(\frac{L}{R-\frac{t}{2}}\right) \cong \frac{L}{R-\frac{t}{2}}$$

The average angle of the front wheels (again assuming small angles) is defined as the Ackerman Angle:

$$\boxed{\delta = \frac{L}{R}}$$



$$x + y = 90$$

$$x + z = 90$$

$$\left.\begin{aligned} y &= 90 - x \\ z &= 90 - x \end{aligned}\right\} \; y = z$$

When Ackerman geometry fulfilled, the resistant torque at the steering wheel increases with the steering angle

High-speed cornering

Lateral acceleration will be present. To counteract this, tires must develop lateral forces and slip angles will be present at each wheel.

Tire cornering forces

The angle between its direction of heading and its direction of travel is called the slip angle, α.

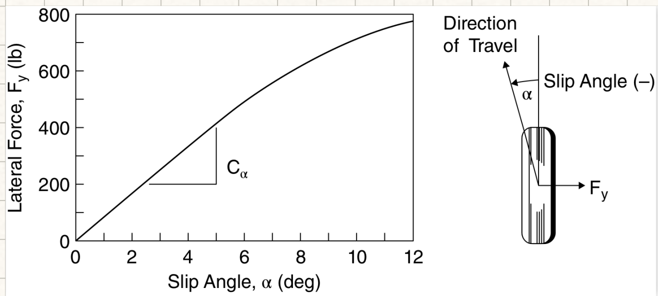
Lateral force, F_y, is called cornering force when camber angle zero.

At a given tire load, the cornering force grows with slip angle.

At low angles ($<5^\circ$), the relationship is linear, hence, allowing the cornering force to be described as:

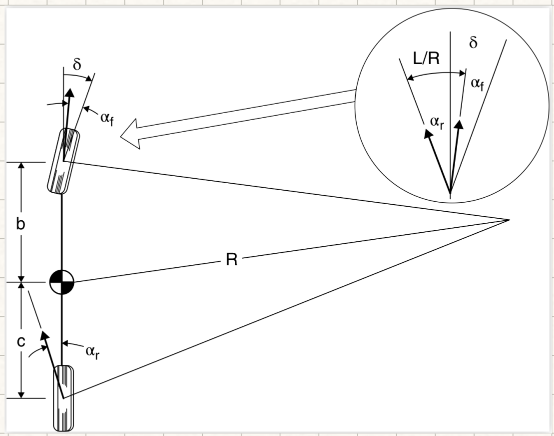
$$F_y = C_\alpha \alpha$$

The proportionality constant, C_α, is called the cornering stiffness. Defined as the slope of the curve for F_y vs. α at $\alpha = 0$.



Cornering equations

At high speeds the radius of turn is much larger than the wheelbase of the vehicle. Small angles can be assumed, and the difference between road wheel steer angles on the outside and inside front wheels can become negligible. For convenience, the two front wheels may be represented by a single wheel at a steer angle, δ, with a cornering force equivalent to both tires. The same assumption is made for the rear wheels.



For a vehicle traveling forward with a speed V, the sum of the forces in the lateral direction from the tires must equal the mass times the centripetal acceleration

$$\sum F_y = F_{yf} + F_{yr} = \frac{MV^2}{R}$$

where

F_{yf}: lateral force at front axle
F_{yr}: lateral force at rear axle
M: mass of vehicle
V: forward velocity
R: radius of turn

For the vehicle to be in a moment equilibrium about the C.G., the sum of the moments from the front and rear lateral forces must be zero:

$$F_{yf} \cdot b - F_{yr} \cdot c = 0$$

$$\Rightarrow \quad F_{yf} = F_{yr} \frac{c}{b} \quad \Leftrightarrow \quad F_{yr} = \frac{b}{c} F_{yf}$$

This gives us:

$$\frac{MV^2}{R} = F_{yf} + F_{yr} = F_{yr}\frac{c}{b} + F_{yr} = F_{yr}\left(\frac{c}{b}+1\right) = F_{yr}\frac{c+b}{b} = F_{yr}\frac{L}{b}$$

$$\Rightarrow \quad F_{yr} = \frac{MV^2}{R}\frac{b}{L} = \frac{Mb}{L}\frac{V^2}{R}$$

$$W_{fs} = W\frac{c}{L}, \quad W_{rs} = W\frac{b}{L}, \quad W = Mg \Rightarrow M = \frac{W}{g}$$

$$F_{yr} = \frac{Mb}{L}\frac{V^2}{R} = \frac{W}{g}\frac{b}{L}\frac{V^2}{R} = \frac{W_{rs}}{g}\frac{V^2}{R}$$

Same can be done for F_{yf}:

$$\frac{MV^2}{R} = F_{yf} + \frac{b}{c}F_{yf} = F_{yf}\left(1+\frac{b}{c}\right) = F_{yf}\frac{c+b}{c} = F_{yf}\frac{L}{c}$$

$$\Rightarrow \quad F_{yf} = \frac{Mc}{L}\frac{V^2}{R} = \frac{W}{g}\frac{c}{L}\frac{V^2}{R} = \frac{W_{fs}}{g}\frac{V^2}{R}$$

Back to the slip angle:

$$F_y = C_\alpha \alpha$$

$$F_{yf} = C_{\alpha f}\alpha_f, \quad F_{yr} = C_{\alpha r}\alpha_r$$

$$\Rightarrow \quad \alpha_f = \frac{F_{yf}}{C_{\alpha f}}, \quad \alpha_r = \frac{F_{yr}}{C_{\alpha r}}$$

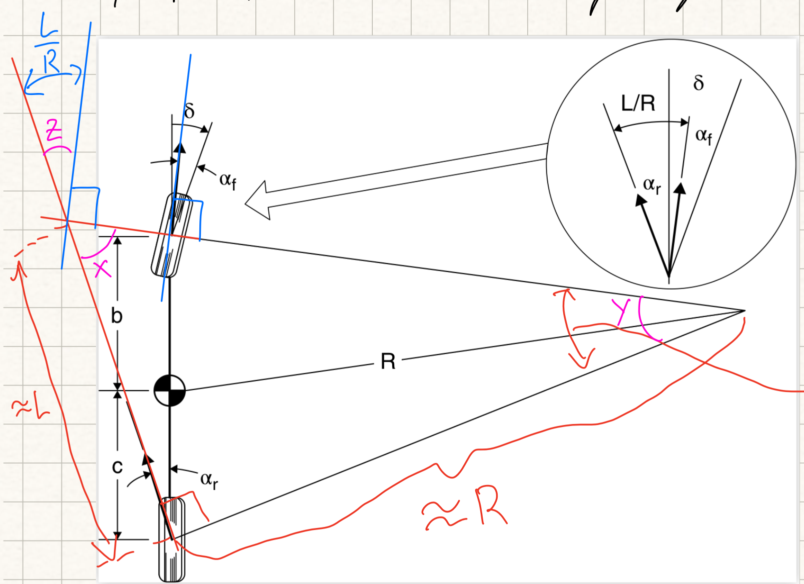
Subsituting with what we found earlier:

$$\boxed{\begin{aligned}\alpha_f &= \frac{1}{C_{\alpha f}}\frac{W_{fs}}{g}\frac{V^2}{R} = \frac{W_{fs}V^2}{gC_{\alpha f}R} \\ \alpha_r &= \frac{1}{C_{\alpha r}}\frac{W_{rs}}{g}\frac{V^2}{R} = \frac{W_{rs}V^2}{gC_{\alpha r}R}\end{aligned}}$$

Note that on each axle:

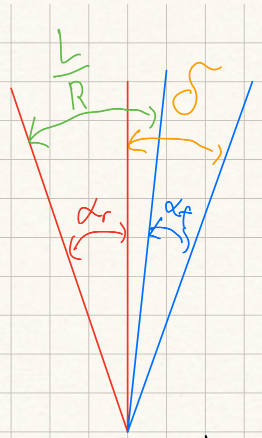
- Static vertical load increases $\longrightarrow$ slip angle increases
- Cornering stiffness increases $\longrightarrow$ slip angle decreases
- lateral acceleration increases $\left(\frac{V^2}{R}\right) \longrightarrow$ slip angle increases

Now, to find the steering angle:



$\rightarrow \frac{L}{R}$ (assuming small angles)

$$\left.\begin{aligned} x + y = 90^\circ &\Rightarrow y = 90^\circ - x \\ x + z = 90^\circ &\Rightarrow z = 90^\circ - x \end{aligned}\right\} \Rightarrow y = z$$



$$\boxed{\delta = 57.3\frac{L}{R} + \alpha_f - \alpha_r}$$

$$\delta = 57.3\frac{L}{R} + \alpha_f - \alpha_r$$

$$F_{yf} = C_{\alpha f}\,\alpha_f \Rightarrow \alpha_f = \frac{F_{yf}}{C_{\alpha f}} = \frac{W_{fs}}{g C_{\alpha f}}\frac{V^2}{R}$$

$$F_{yr} = C_{\alpha r}\,\alpha_r \Rightarrow \alpha_r = \frac{F_{yr}}{C_{\alpha r}} = \frac{W_{rs}}{g C_{\alpha r}}\frac{V^2}{R}$$

$$\delta = 57.3\frac{L}{R} + \frac{W_{fs}V^2}{gC_{\alpha f}R} - \frac{W_{rs}}{gC_{\alpha r}}\frac{V^2}{R}$$

$$= 57.3\frac{L}{R} + \underbrace{\frac{1}{g}\left(\frac{W_{fs}}{C_{\alpha f}} - \frac{W_{rs}}{C_{\alpha r}}\right)}_{K\text{ - understeer gradient}}\underbrace{\frac{V^2}{R}}_{\text{lateral acceleration } a_y}$$

$$\boxed{\delta = 57.3\frac{L}{R} + K a_y}$$

Understeer gradient

The equation above describes how the steer angle of the vehicle must be changed with the radius of the turn, R, or the lateral acceleration,

$\frac{V^2}{R}$. The term $\left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right]$ determines the magnitude and direction of the steering inputs required. It consists of two terms, each of which is the ratio of the load on the axle (front or rear) to the cornering stiffness of the tires on the axle. This is called the "Understeer gradien", and will be denoted by K, with units $[\text{deg}/g]$. We have three possibilities:

1) **Neutral steer:** $K = 0 \rightarrow \alpha_f = \alpha_r$

On a constant-radius turn, no change in steer angle is required as the speed is varied. The steer angle required to make the turn will be equivalent to the Ackerman angle, $57.3\frac{L}{R}$. Physically, this means that there is a balance on the vehicle such that the force of the lateral acceleration at C.G. causes an identical increase in slip angle at both the front and the rear wheels.

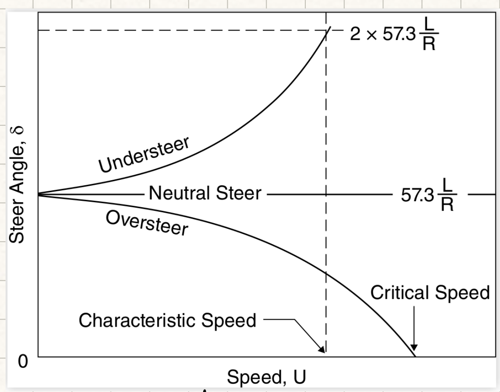
2) **Understeer:** $K > 0 \rightarrow \alpha_f > \alpha_r$

On a constant-radius turn, the steer angle will have to increase with speed in proportion to K times the lateral acceleration in g's. Therefore the steer angle increases linearly with the lateral acceleration and with the square of the speed. The lateral acceleration at C.G. causes the front wheels to slip sideways to a greater extent than at the rear wheels. To develop the lateral force at the front wheels necessary to maintain the radius of turn, the front wheels must be steered to a greater angle.

3) **Oversteer:** $K < 0 \rightarrow \alpha_f < \alpha_r$

On a constant-radius turn, the steer angle will have to decrease as the speed (and lateral acceleration) is increased. In this case, the lateral acceleration at C.G. causes the slip angle on the rear wheels to increase more than at the front. The outward drift at the rear turns the front wheels inwards, diminishing the radius of the turn. The increase in lateral acceleration that follows causes the rear to drift out even further and the process continues unless the steer angle is reduced to maintain the turn radius.

With a neutral-steer vehicle, the steer angle to follow the curve at any speed is the Ackerman angle. With understeer, the angle increases with the square of the speed, reaching twice the initial angle at the characteristic speed. In oversteer, the steer angle decreases with the square of the speed and becomes zero at the critical speed value.



Characteristic speed

The speed at which the steer angle required make any turn is twice the Ackerman angle:

$$\delta = 2 \cdot \left(57.3 \frac{L}{R}\right)$$

$$2\left(57.3\frac{L}{R}\right) = 57.3\frac{L}{R} + K a_y$$

$$\Rightarrow \quad K a_y = 57.3\frac{L}{R}$$

Since a_y is a function of speed squared, the characteristic speed is:

$$V_{char} = \sqrt{57.3\frac{Lg}{K}}$$

Critical speed

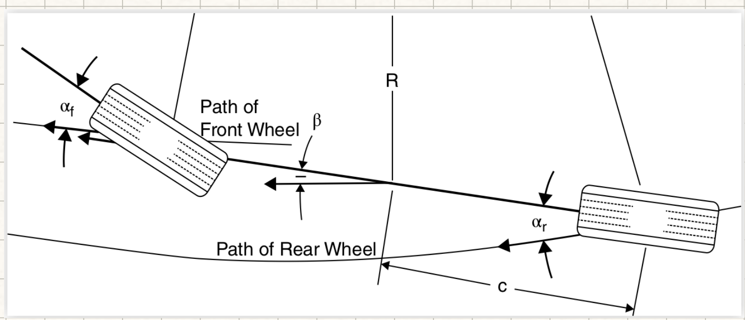
In oversteer, a critical speed will exist above which the vehicle is unstable.

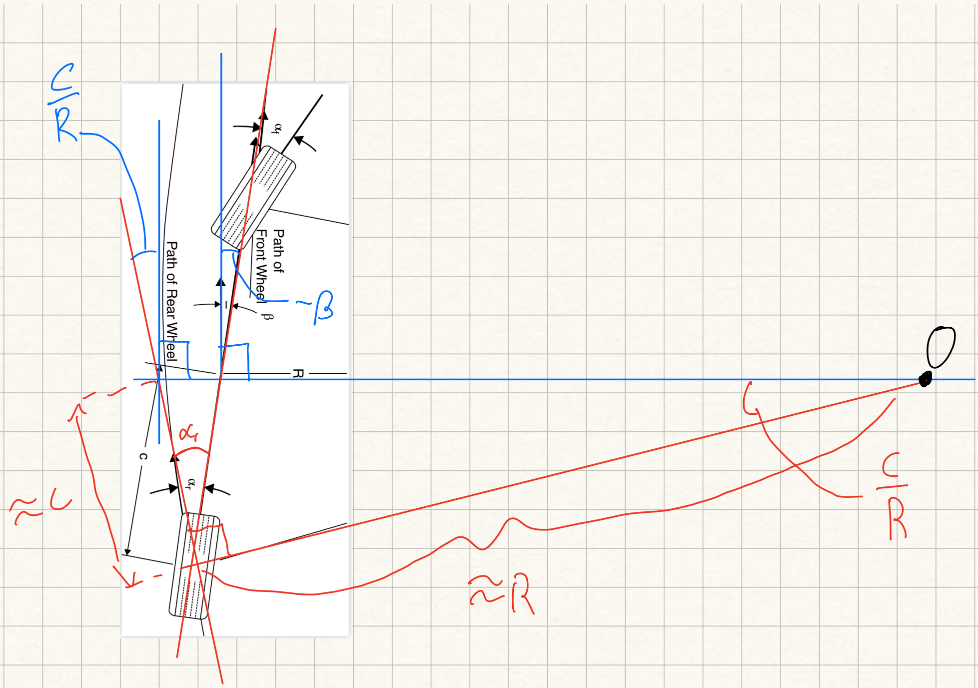
$$V_{crit} = \sqrt{-57.3\frac{Lg}{K}}$$

K is negative so the expression under the root is positive. Note that the critical speed is dependent on the wheelbase; for a given level of oversteer, long-wheelbased vehicles have a higher critical speed than short-wheelbased vehicles.

Sideslip angle

At any point on the vehicle a side slip angle may be defined as the angle between the longitudinal axis and the local direction of travel. In general, the sideslip angle will be different at any point on a car during cornering.





$$57.3\frac{c}{R}-\beta=\alpha_r \Rightarrow$$

$$\boxed{\begin{aligned}\beta &= 57.3\frac{c}{R}-\alpha_r\\ &= 57.3\frac{c}{R}-\frac{W_r\,V^2}{g C_{\alpha r} R}\end{aligned}}$$

The speed at which the side slip angle becomes zero is:

$$V_{\beta=0}=\sqrt{57.3\,\frac{c\,g\,C_{\alpha r}}{W_r}}$$

which is independent of the turn radius R.

Test methods for the estimate of K

Constant radius method

1) Vehicle driven along the turn at low speed $\longrightarrow$ Ackerman angle
2) Further tests are done while increasing the speed slowly (maximum jerk

is $0.1\ m/s^2/s$)

3) From V and R the lateral acceleration is derived (in g)

$$a_y = \frac{V^2}{gR}$$

4) At each lap the steer angle δ is measured

5) The curve $\delta - a_y$ is defined by points, from which K is derived by differentiation

$$\frac{\partial \delta}{\partial a_y} = \frac{\partial}{\partial a_y}\left(57.3 \frac{L}{R}\right) + \frac{\partial}{\partial a_y}\left(K a_y\right)$$

$$\Rightarrow \quad K = \frac{\partial \delta}{\partial a_y}$$

Constant steering angle method

1) δ is set to a fixed value and a circular trajectory is followed at low speed

2) the speed is increased with discrete values and limited jerk ($0.1\ m/s^2/s$)

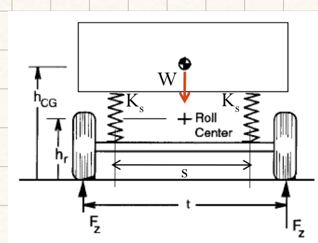
3) From measured data K is calculated as

$$K(a_y(s)) = \frac{\delta - \frac{180}{\pi}\frac{L}{R(s)}}{a_y(s)}$$

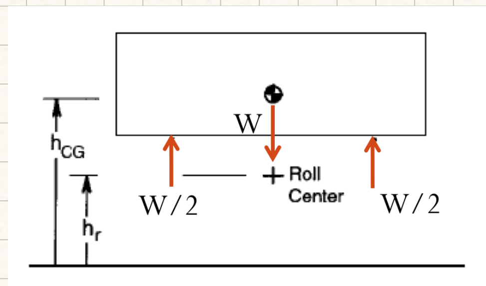
Load transfer during cornering

Low speed

If the lateral acceleration is negligible, the vertical load W is distributed equally on

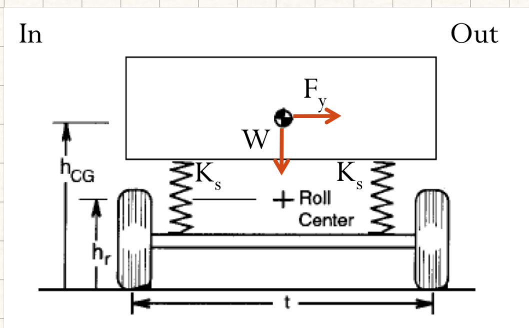


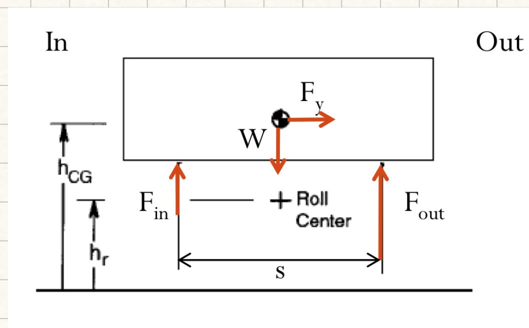
the two wheels of an axle
Both springs of the suspension undergo a compression equal to $(W/2)/K_s$ in comparison to the unloaded condition.



High speed

The sprung mass is subject to a lateral force equal to Ma_y





Equilibrium in the vertical direction: $F_{in} + F_{out} = W$
Equilibrium to rotation around roll center:

$$F_y(h_{CG} - h_r) = \underbrace{F_{out}\frac{s}{2} - F_{in}\frac{s}{2}}_{M_\phi \text{ roll torque}}$$

$F_{out} = W - F_{in}$, $F_{in} = W - F_{out}$

$$F_{in} = \frac{2}{s}\left[F_{out}\frac{s}{2} - F_y(h_{CG} - h_r)\right] = W - F_{in} - \frac{2F_y(h_{CG} - h_r)}{s}$$

$$\Rightarrow \quad F_{in} = \frac{W}{2} - \frac{F_y(h_{CG} - h_r)}{s}$$

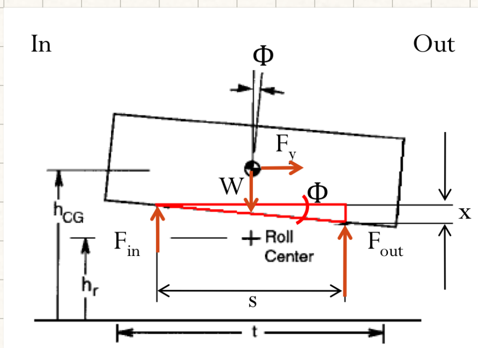
$$F_{out} = \frac{2}{s}\left[F_y(h_{CG}-h_r) + F_{in}\frac{s}{2}\right] = W - F_{out} + \frac{2}{s}F_y(h_{CG}-h_r)$$

$$\Rightarrow \quad F_{out} = \frac{W}{2} + \frac{F_y(h_{CG}-h_r)}{s}$$

$$\frac{\Delta F_z}{2} = \frac{F_y(h_{CG}-h_r)}{s}$$

Roll stiffness

The sprung mass undergoes a rotation ϕ (roll angle):



For small angles:

$$\phi \cong \frac{x}{s} \quad \Rightarrow \quad x = \phi s$$

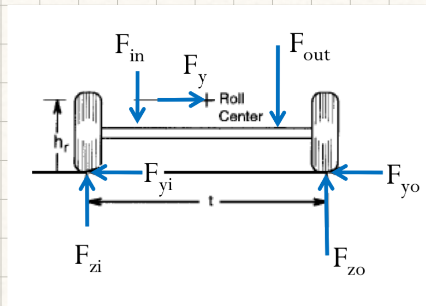
The torque generated by the suspension springs around the roll center (roll torque) is equal to:

$$M_\phi = \frac{s}{2}(F_{out}-F_{in}) = \frac{s}{2}k_s x = \underbrace{\left(\frac{1}{2}s^2 k_s\right)}_{K_\phi}\phi = K_\phi \phi$$

K_ϕ is the roll stiffness of the suspension:

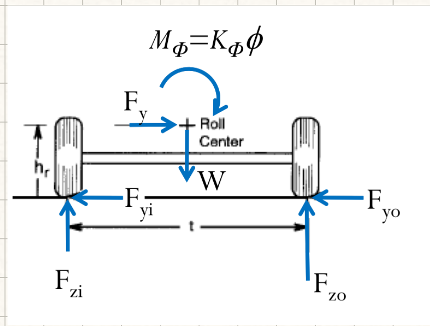
$$K_\phi = \frac{1}{2}s^2 k_s$$

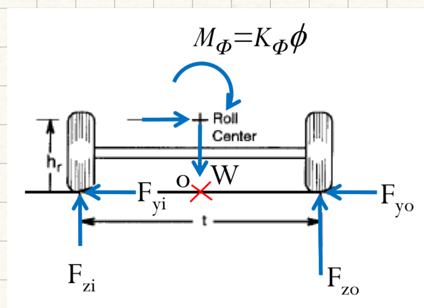
Forces and torques are transferred from the spring mass to the axle



$$F_{in} + F_{out} = W$$

The roll torque generated by the springs is M_ϕ





Rotational equilibrium around o:

$$F_{zo}\frac{t}{2} - F_{zi}\frac{t}{2} = F_y h_r + K_\phi \phi$$

$$\Rightarrow F_{zo} - F_{zi} = \underbrace{\frac{2F_y h_r}{t}}_{1} + \underbrace{\frac{2K_\phi \phi}{t}}_{2} = 2\Delta F_z$$

1: Load transfer due to the lateral force

2: Load transfer due to vehicle roll

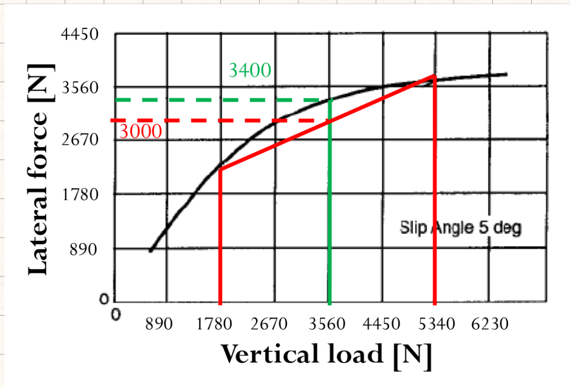
Anti-roll bars affect the roll stiffness and hence the understeer gradient, such that:

$K_{\phi f} \uparrow \longrightarrow K \uparrow$ understeering

$K_{\phi r} \uparrow \longrightarrow K \downarrow$ oversteering

Lateral force vs. vertical load: effect of load transfer

Non-linear relationship between lateral force and vertical load for a given value of slip angle;



Load transfer during cornering

↓

Lower value of lateral force

↓

Impact on understeer gradient