Formulas

1) Fundamentals

Static load on each axle:

$$W_{fs} = W \frac{c}{L}$$

$$W_{rs} = W \frac{b}{L}$$

Assuming not static:

$$W_f = \frac{1}{L}\left[W\cos(\theta)\cdot c - W\sin(\theta)\cdot h - D_A h_a - \frac{W}{g} a_x h\right]$$

$$W_r = \frac{1}{L}\left[W\cos(\theta)\cdot b + W\sin(\theta)\cdot h + D_A h_a + \frac{W}{g} a_x h\right]$$

2) Longitudinal Dynamics

Upper limit:

$$a_x = \frac{F_x}{M} = \frac{gP}{VW}$$

$1 \text{ kW} = 0{,}746 \text{ HP}$

Power-limited acceleration

Engine: $T_c = T_E - I_E \alpha_E$

Transmission: $T_D = (T_c - I_T \alpha_E) n_T$

Differential: $T_A = (T_D - I_D \alpha_D) n_F = F_x r + I_w \alpha_w$

Kinematic relationships:

$$a_x = \alpha_w r \qquad \alpha_D = \alpha_w n_F \qquad \alpha_E = \alpha_D n_T = \alpha_w n_F n_T$$

Rotational equilibrium of powertrain:

$$F_x = \frac{T_E n_{TF} \eta_{TF}}{r} - \left[(I_E + I_T) n_{TF}^2 + I_D n_F^2 + I_w \right] \frac{a_x}{r^2}$$

Translational equilibrium of vehicle:

$$M a_x = F_x - R_x - D_A - R_{hx} - W \sin\theta$$

Simplified power-limited acceleration:

$$(M + M_r) a_x = \frac{T_E n_{TF} \eta_{TF}}{r} - R_x - D_A - R_{hx} - W \sin\theta$$

Mass factor:

$$\frac{M + M_r}{M} = 1 + 0.04 + 0.0025 n_{TF}^2$$

Traction-Limited Acceleration

$$F_x = \mu W \quad , \; \mu \text{ friction coefficient}$$

Transverse weight shift due to drive torque:

Axle in equilibrium:

$$\sum T = \left(\frac{W_r}{2} + W_y \right) \frac{t}{2} - \left(\frac{W_r}{2} - W_y \right) \frac{t}{2} + T_s - T_D = 0$$

$$\Rightarrow W_y = \frac{T_D - T_s}{t} \quad , \; T_D \text{ torque from driveshaft on axle}$$

T_s torque due to suspension roll stiffness

$$T_D = F_x \frac{r}{n_F}$$

Roll torque:

$$T_{sf} = K_{\phi f}\, \phi \qquad , \; K_{\phi f}: \text{front susp. roll stiffness}$$

$$T_{sr} = K_{\phi r}\, \phi \qquad , \; K_{\phi r}: \text{rear susp. roll stiffness}$$

$$K_\phi = K_{\phi f} + K_{\phi r}$$

$$\phi = \frac{T_D}{K_\phi} = \frac{T_D}{K_{\phi f} + K_{\phi r}}$$

$$T_{sr} = K_{\phi r} \frac{T_D}{K_{\phi f} + K_{\phi r}}$$

$$W_y = \frac{F_x r}{n_F t}\left[1 - \frac{K_{\phi r}}{K_{\phi r} + K_{\phi f}}\right] = \frac{F_x r}{n_F t} \frac{K_{\phi f}}{K_\phi}$$

Net load on rear axle during acceleration:

$$W_r = W\left(\frac{b}{L} + \frac{a_x}{g}\frac{h}{L}\right)$$

Neglecting rolling resistance and drag

$$W_r = W\left(\frac{b}{L} + \frac{F_x}{Mg}\frac{h}{L}\right)$$

Maximum tractive force:

Solid rear axle with non-locking diff.:

$$F_{x_{max}} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu + \frac{2\mu r}{n_F t}\frac{K_{\phi f}}{K_\phi}}$$

Solid rear axle with locking diff. / independent rear susp.:

$$F_{x_{max}} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu}$$

Rolling Resistance

$$R_x = f_r W$$, f_r rolling resistance coefficient

Aerodynamic drag

$$D_A = \frac{1}{2} \rho C_D A V^2$$

Road grade

$$R_G = W \sin\theta$$

Braking Force

$$F_b = \frac{T_b - I_w \alpha_w}{r}$$

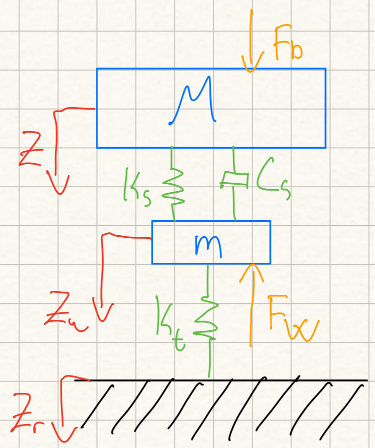
Simplified: $F_b = \frac{T_b}{r}$

Braking coefficient:

$$\mu = \frac{F_x}{F_z}$$

Slip: $$Slip = \frac{V - \omega r}{V}$$

4) Ride



Ride rate: effective stiffness

$$RR = \frac{K_s K_t}{K_s + K_t}$$

Undamped frequency:

$$\omega_n = \sqrt{\frac{RR}{M}}$$

Damped frequency:

$$\omega_d = \omega_n \sqrt{1 - \xi_s^2}$$

ξ_s damping ratio: 0,2 - 0,4

$$M\ddot{z} + C_s \dot{z} + K_s z = C_s \dot{z}_u + K_s z_u + F_b$$

$$m\ddot{z}_u + C_s\dot{z}_u + (k_s + k_t)\,z_u = C_s\dot{z} + k_s z + k_t z_r + F_w$$

Analytical frequencies:

If $k_t \gg k_s \rightarrow \quad \omega_1 = \sqrt{\frac{k_s}{M}} \quad , \quad \omega_2 = \sqrt{\frac{k_t}{m}}$

Skyhook Control

$$C_s = \begin{cases} C_{max} \, , & \dot{z}_s(\dot{z}_s - \dot{z}_u) \geq 0 \\ C_{min} \, , & \dot{z}_s(\dot{z}_s - \dot{z}_u) \leq 0 \end{cases}$$

5) Lateral Dynamics

Low speed cornering

Steer angles (assuming small angles):

$$\delta_o = \frac{L}{R + \frac{t}{2}}$$

$$\delta_i = \frac{L}{R - \frac{t}{2}}$$

Average angle, Ackerman angle:

$$\delta = 57{,}3\,\frac{L}{R}$$

High speed cornering

Cornering force:

$F_y = \alpha C_\alpha \quad , \quad C_\alpha$ cornering stiffness

Sum of lateral forces:

$$\sum F_y = F_{yf} + F_{yr} = \frac{MV^2}{R}$$

To have moment equilibrium:

$$F_{yf} \cdot b - F_{yr} \cdot c = 0 \quad \Rightarrow \quad F_{yf} = F_{yr}\frac{c}{b} \iff F_{yr} = F_{yf}\frac{b}{c}$$

This gives us:

$$F_{yr} = \frac{Mb}{L}\frac{V^2}{R} = \frac{W_r}{g}\frac{V^2}{R}$$

$$F_{yf} = \frac{Mc}{L}\frac{V^2}{R} = \frac{W_f}{g}\frac{V^2}{R}$$

Need to find slip angle:

$$F_y = C_\alpha \alpha \quad \Rightarrow \quad \alpha = \frac{F_y}{C_\alpha}$$

$$\alpha_f = \frac{W_f V^2}{C_{\alpha f} g R}$$

$$\alpha_r = \frac{W_r V^2}{C_{\alpha r} g R}$$

Steering angle:

$$\delta = 57.3\frac{L}{R} + \alpha_f - \alpha_r$$

$$= 57.3\frac{L}{R} + \underbrace{\frac{1}{g}\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)}_{K}\underbrace{\frac{V^2}{R}}_{a_y}$$

$$\delta = 57.3\frac{L}{R} + K a_y$$

Characteristic speed:

$$V_{char} = \sqrt{57.3\frac{Lg}{K}}$$

Critical speed:

$$V_{crit} = \sqrt{-57.3\frac{Lg}{K}}$$

Sideslip angle:

$$\beta = 57.3\frac{c}{R} - \alpha_r$$

$$= 57.3\frac{c}{R} - \frac{W_r V^2}{C_{\alpha r} g R}$$

$$V_{\beta=0} = \sqrt{57.3\frac{c g C_{\alpha r}}{W_r}}$$

Load transfer during cornering

Equilibrium in vertical direction:

$$F_{in} + F_{out} = W$$

Equilibrium to rotation around roll center

$$F_y(h_{GS}-h_r) = \underbrace{F_{out}\frac{s}{2} - F_{in}\frac{s}{2}}_{M_\phi \text{ roll torque}}$$

$$F_{in} = \frac{W}{2} - \frac{F_y(h_{GS}-h_r)}{s}$$

$$F_{out} = \frac{W}{2} + \frac{F_y(h_{GS}-h_r)}{s}$$

$$\frac{\Delta F_z}{2} = \frac{F_y(h_{CG}-h_r)}{s}$$

Roll angle sprung mass (for small angles):

$$\phi = \frac{x}{s} \Rightarrow x = \phi s$$

Torque generated by suspension springs around roll center

$$M_\phi = \frac{s}{2}(F_{out} - F_{in}) = \frac{s}{2} k_s x = \frac{1}{2} s^2 k_s \phi = k_\phi \phi$$

Rotational equilibrium around ground point:

$$F_{zo}\frac{t}{2} - F_{zi}\frac{t}{2} = F_y h_r + k_\phi \phi$$

$$\Rightarrow F_{zo} - F_{zi} = \underbrace{\frac{2 F_y h_r}{t}}_{1} + \underbrace{\frac{2 k_\phi \phi}{t}}_{2} = 2 \Delta F_z$$

1: Load transfer lateral force

2: Load transfer vehicle roll

6) Tires

Characteristics:

P 225 / 60 R 16 93 Z

1 2 3 4 5 6 7

1: Application (Passenger, Truck, ...)

2: Section widt [mm]

3: Aspect ratio $\frac{\text{height}}{\text{width}}$

4: Construction (Radial, Bias)

5: Rim diameter [in] 1 in = 2,54 cm

6: Load index

7: Speed rating