Lecture 4 – Optimality conditions for unconstrained optimization

The maximum of multiple convex functions are convex.
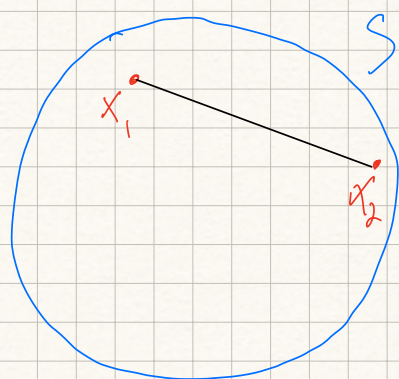
Definition of a Convex Set

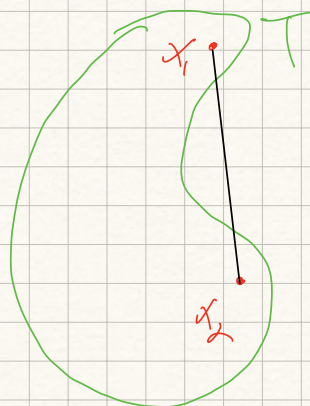A set $S \subseteq \mathbb{R}^n$ is convex if, for any two points $x_1, x_2 \in S$ and any $\lambda \in [0, 1]$:
$$\lambda x_1 + (1-\lambda) x_2 \in S$$

Geometric Interpretation:
A set is convex if the line segment connecting any two points in the set lies entirely within the set.

Convex

Not convex

Convex function:

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2)$$

The graph of a convex function always lies below the line segment connecting any two points on the graph