Exercises

From chat GPT

Driving uphill on road with 5° incline at steady 90 km/h $= 25\ m/s$

$m = 1500\ kg$

Frontal area $A = 2{,}4\ m^2$

Drag coefficient $C_d = 0{,}32$

Tire radius $r = 0{,}33\ m$

Air density $\rho = 1{,}225\ kg/m^3$

Engine torque at current operating point $T_E = 150\ Nm$

Drivetrain efficiency $\eta = 90\%$, $n_F = 4{,}2$

$$(M+M_r)\,a_x = \frac{T_E\, n_{TF}\, \eta}{r} - R_x - D_A - \underbrace{R_{hx}}_{=0} - W\sin\theta$$

$a_x = 0$

$R_x = f_r W = 0{,}015\, g \cdot 1500\ kg = 0{,}015 \cdot 9{,}81\ m/s^2 \cdot 1500\ kg$

$= 220{,}725\ N$

$D_A = \frac{1}{2}\rho C_d A V^2 = \frac{1}{2} \cdot 1{,}225\ kg/m^3 \cdot 0{,}32 \cdot 2{,}4\ m^2 \cdot (25\ m/s)^2$

$= \frac{1}{2} \cdot 1{,}225 \cdot 0{,}32 \cdot 2{,}4\ kg/m \cdot 25^2\ m^2/s^2$

$= \frac{1}{2} \cdot 1{,}225 \cdot 0{,}32 \cdot 2{,}4 \cdot 25^2\ kg \cdot m/s^2 = 294\ N$

$W\sin\theta = M \cdot g \cdot \sin(5) = 1500\ kg \cdot 9{,}81\ m/s^2 \cdot 0{,}087$

$$= \underline{1280{,}205\ N}$$

$$(M+M_r)\,a_x = \frac{T_E\, n_{TF}\, \eta}{r} - R_x - D_A - W \sin\theta$$

$$\frac{T_E\, n_{TF}\, \eta}{r} = R_x + D_A + W \sin\theta$$

$$n_{TF} = \frac{r}{T_E\, \eta}\left[R_x + D_A + W\sin\theta\right]$$

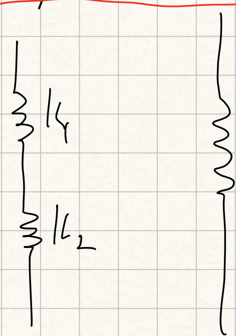
$$= \frac{0{,}33\,m}{150\,Nm \cdot 0{,}9}\left[220{,}725\,N + 294\,N + 1280{,}205\,N\right]$$

$$= \frac{0{,}33}{135\,N} \cdot 1794{,}93\,N$$

$$\approx \underline{4{,}39}$$

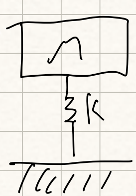
$$n_{TF} = n_T \cdot n_F \Rightarrow n_T = \frac{n_{TF}}{n_F} = \frac{4{,}39}{4{,}2} \approx 1{,}04 \Rightarrow \text{4th gear}$$

<u>Springs in series</u>



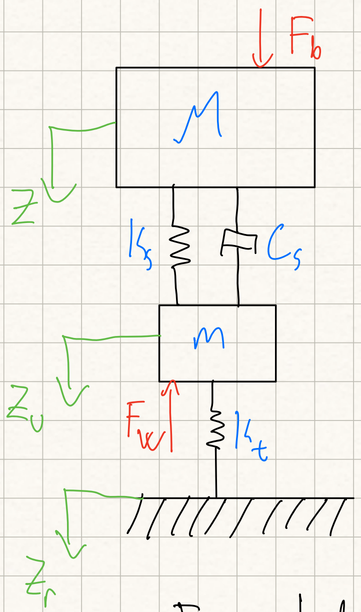
$$k_{eq} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \frac{1}{\frac{k_2}{k_1 k_2} + \frac{k_1}{k_2 k_1}}$$

$$= \frac{1}{\frac{k_1 + k_2}{k_1 k_2}} = \frac{k_1 k_2}{k_1 + k_2}$$



natural frequency: $\omega_n = \sqrt{\frac{k}{m}}$

Quarter-car model



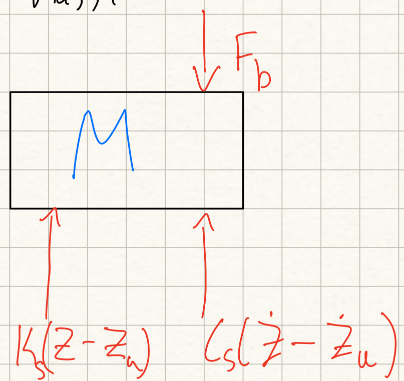
Consider free-body diagram for each, then we get:

$$M\ddot{z} + C_s\dot{z} + k_s z = C_s\dot{z}_u + k_s z_u + F_b$$

$$m\ddot{z}_u + C_s\dot{z}_u + (k_s+k_t)z_u = C_s\dot{z} + k_s z + k_t z_r + F_w$$

Free body diagram of each:

Spring mass:



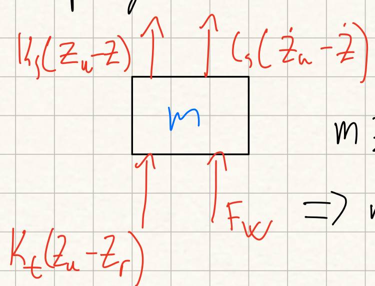
Which gives us:

$$M\ddot{z} = -C_s(\dot{z}-\dot{z}_u) - k_s(z-z_u) + F_b$$

$$M\ddot{z} = -C_s\dot{z} + C_s\dot{z}_u - k_s z + k_s z_u + F_b$$

$$\Rightarrow M\ddot{z} + C_s\dot{z} + k_s z = C_s\dot{z}_u + k_s z_u + F_b$$

Unspring mass:



$$m\ddot{z}_u = -k_s(z_u-z) - C_s(\dot{z}_u-\dot{z}) - k_t(z_u-z_r) + F_w$$

$$\Rightarrow m\ddot{z}_u + C_s\dot{z}_u + (k_s+k_t)z_u = C_s\dot{z} + K_s z + k_t z_r + F_w$$

Want to find sprung mass motion in response to road displacement inputs, forces at the axle, and forces applied directly to the sprung mass

Amesim frequency response function

$m_s = 500\ \text{kg}\ ,\quad m_u = 50\ \text{kg}$

$k_s = 30\,000\ \text{N/m}\ ,\quad k_t = 100\,000\ \text{N/m}$

$c_s = 1000\ \text{N/m/s}$

$$\omega_1 = \sqrt{\frac{k_s}{m_s}} = \sqrt{\frac{30\,000\ \text{N/m}}{500\ \text{kg}}} = \sqrt{60\ 1/s^2} \approx 7{,}75\ \text{rad/s}$$

$$N = \text{kg}\,\text{m/s}^2 \qquad \frac{\text{N/m}}{\text{kg}} = \frac{\frac{\text{kg}\,\text{m/s}^2}{\text{m}}}{\text{kg}} = \frac{\text{kg/s}^2}{\text{kg}} = 1/s^2$$

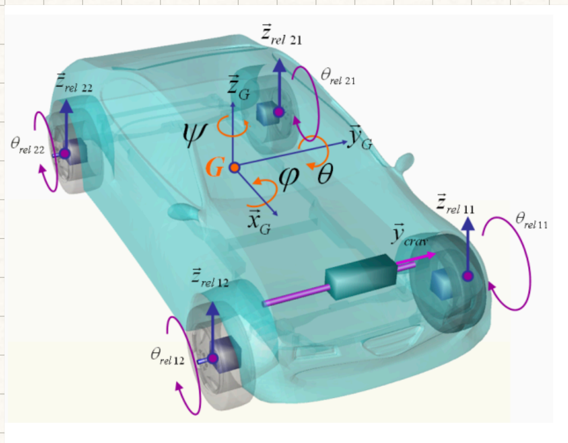
$$f_1 = \frac{1}{2\pi}\, 7{,}75\ \text{rad/s} \approx 1{,}23\ \text{Hz}$$

$$\omega_2 = \sqrt{\frac{k_t}{m_u}} = \sqrt{\frac{100\,000\ \text{N/m}}{50\ \text{kg}}} = \sqrt{2000\ 1/s^2} \approx 44{,}72\ \text{rad/s}$$

$$f_2 = \frac{1}{2\pi}\, 44{,}72\ \text{rad/s} \approx 7{,}12\ \text{Hz}$$

$$\frac{\ddot{Z}M}{F_b} = \left[\frac{\text{m/s}^2 \cdot \text{kg}}{N} = \frac{\text{kg}\,\text{m/s}^2}{\text{kg}\,\text{m/s}^2} = 1\right] \Rightarrow \text{dimensionless}$$

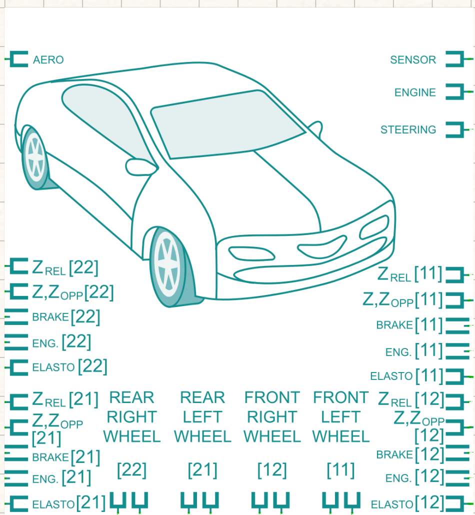
Amesim full 15 DOF model



Where the DOFs come from:

Car body: 6 DOF
Steering rack: 1 DOF
4 spindle body: 4×1 DoF
4 wheel body: 4×1 DoF

} 15 DOF



Z_{REL}: suspension (input), outputs vertical relative velocity + displacement of the suspension

Z, Z_{OPP}: For advanced suspension modeling Inputs are current wheel lift, steering rack displacement and opposite wheel lift. Outputs are velocity and displacement of wheel, steering rack and opposite wheel, for the front axles. For the rear axles, only velocity and displacement of wheel and opposite wheel.

BRAKE: Input: brake torque applied on axle, output is relative rotary velocity

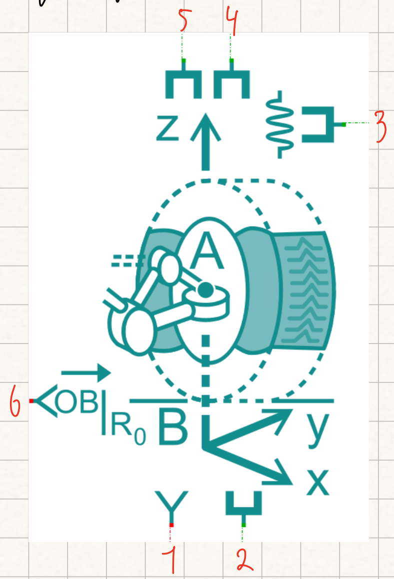
ENG.: Input is engine torque

ELASTO: Can model the deformation of the axle under forces and moments due to its inner compliance.

Wheels:

Left port: 3D mechanical port related to wheel

Right port: 3D mechanical port related to spindle



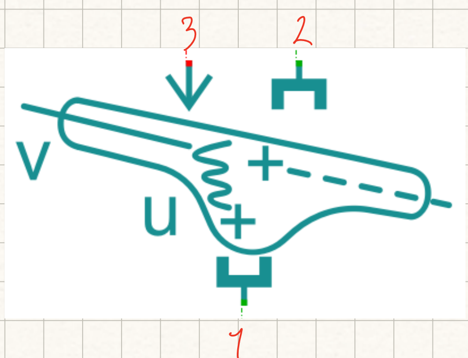
Tire kinematics model:

Port 3: tire stiffness

Port 6: Signal port for dynamic localization of point B. used by aerodynamic models,

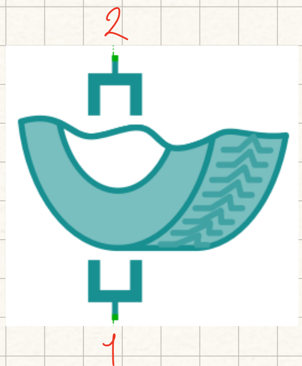
Port 2: outputs camber angle and self-rotating angle

This model is used to compute all kinematic elements of center of tire contact (point B). These elements are used by belt model to determine characteristic input values of tire model.



Tire belt model:

Allows computation of characteristic inputs of tire model



Tire model:

Generates the contact force at tire/road interface.

<u>Upper limit</u> longitudinal acceleration:

$$F_x = M a_x \Rightarrow a_x = \frac{F_x}{M}$$

Power is force times speed: $P = FV \Rightarrow F = \frac{P}{V}$

Weight is mass times gravity acc: $W = Mg \Rightarrow M = \frac{W}{g}$

$$a_x = \frac{F_x}{M} = \frac{P}{V}\frac{g}{W}$$

For BMW:

$P = 137\ HP$, $M = 1{,}5$ tons, $V = 100\ km/h$

$P = 137\ HP = 137 \cdot 0{,}746\ kW = 102{,}202\ kW = 102\ 202\ W$

$= 102\ 202\ N\frac{m}{s}$

$$\boxed{1\ kW = 0{,}746\ HP}$$

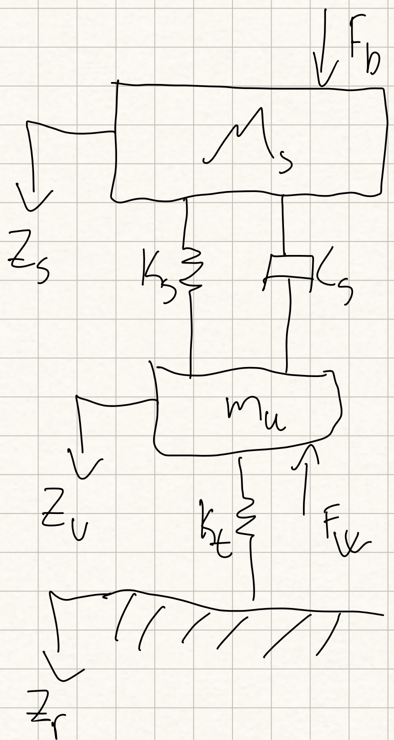
$M = 1{,}5$ tonns $= 1500\ kg \Rightarrow W = Mg = 1500\ kg \cdot 9{,}81\ m/s^2 = 14\ 715\ N$

$V = 100\ km/h = \frac{100}{3{,}6}\ m/s \approx 27{,}78\ m/s$

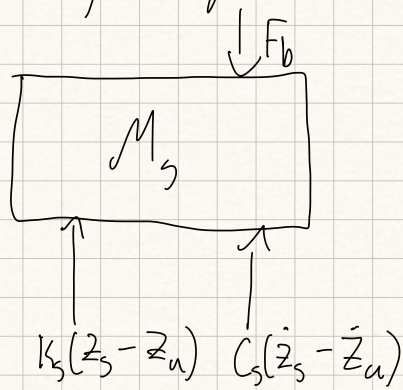
$$a_x = \frac{F_x}{M} = \frac{P/V}{W/g} = \frac{P}{V}\frac{g}{W} = \frac{g}{V} \cdot \frac{P}{W}$$

$$= \frac{9{,}81\,m/s^2}{27{,}78\,m/s} \cdot \frac{102\,202\,\cancel{N}\,\frac{m}{s}}{14\,715\,\cancel{N}} \approx 0{,}353\,\tfrac{1}{s} \cdot 6{,}945\,m/s \approx 2{,}45\,m/s^2$$

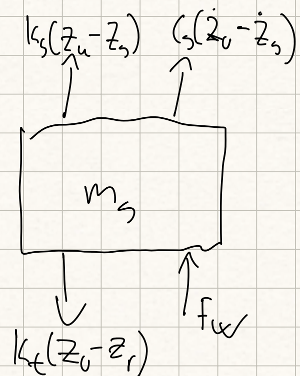
Quarter car di nuovo



Free body diagrams



$$M_s \ddot{z}_s + c_s \dot{z}_s + k_s z_s = c_s \dot{z}_u + k_s z_u + F_b$$



$$m_s \ddot{z}_u + c_s \dot{z}_u + (k_s + k_t) z_u = c_s \dot{z}_s + k_s z_s + k_t z_r + F_w$$