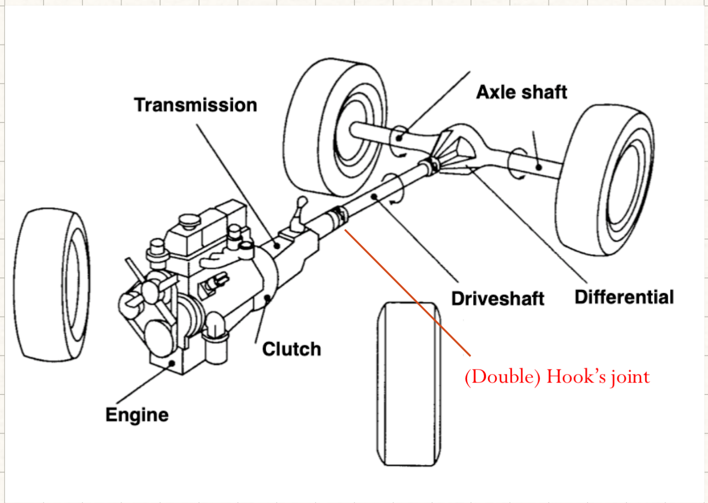
Longitudinal Dynamics

Acceleration Performance

Powertrain

- Engine: provides power for traction
- Clutch: allows to dis-engage engine shaft from driveshaft
- Transmission: allows to match engine speed with vehicle speed through gearsets
- Driveshaft: power transfer
- Differential: turns power flow by 90° and allows speed differentiation for the traction wheels
- Axle shaft: transmits power to the wheel



Internal Combustion Engine

Spark-ignition 4-stroke engine

- Piston + connecting rod + crank: From reciprocating motion of the piston (caused by expanding forces of gases) to rotational motion of the crankshaft

- Fly wheel:
  rotating disk for energy storing
- Inlet and outlet valves:
  Fresh air/fuel mixture in
  Exhaust gases out
- Camshaft:
  Synchronized valves activation
- Spark plug:
  electric spark-induced ignition of air/fuel mixture in the combustion chamber $\longrightarrow$ gas expansion
  Different ignition in diesel engines

Power/Torque relationship

$$P = T \cdot \Omega \qquad \left[W = \frac{Nm}{s} \right]$$

Power (kW) = 0.746 HP

Limited torque and speed range $\longrightarrow$ need for transmission
$\longrightarrow$ need for clutch

Upper limit longitudinal acceleration

$$a_x = \frac{F_x}{M} = \frac{g}{V} \cdot \underbrace{\frac{P}{W}}_{\text{specific power}}$$

Clutch

Function: engine-to-transmission connection enabling
- Disengagement of a gear when the vehicle is stationary and the engine is running
- Smooth transmission of the engine power to the traction wheels at vehicle start
- Gradual loading of the engine at each gear shifting
- To permit the engaging of the gears when the vehicle is in motion

Transmission

Why we need it?
1) To keep the engine running even when the vehicle is not moving (with clutch engaged)
2) To be able to drive the car backwards by shifting to reverse gear
3) To carry high loads OR climb steep slopes as well as achieve high speed on straight roads

3 → achieved by changing the transmission ratio between input and output shaft

Typical gear ratios in 5-speed gearboxes:

1st gear: 3-4:1

2nd gear: 2-2.5:1

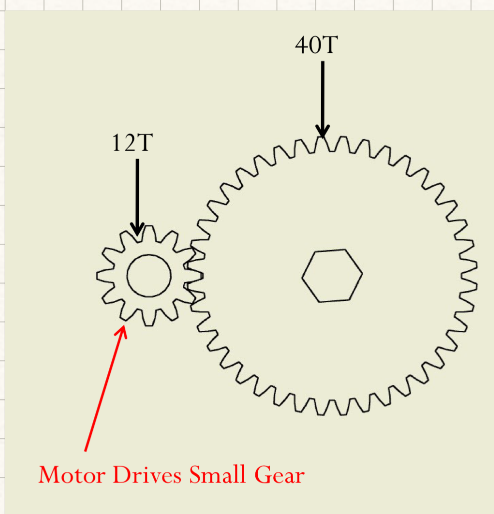
3rd gear: 1.4-1.6:1

4th gear: 1-1.2:1

5th gear: 0.7-0.8:1

Reverse: -3.1:1

Gear (transmission) ratio



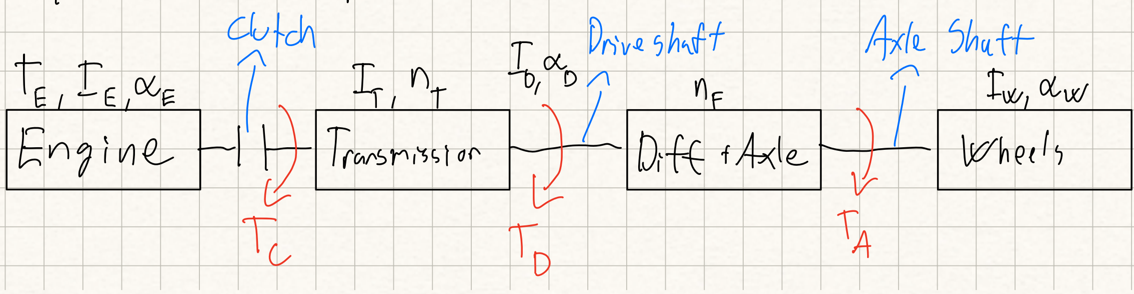
Reduction: $\frac{40}{12} = 3.33$

The gearbox reduces speed

The gearbox increases torque

Power-Limited Acceleration

Dynamics of powertrain:



Engine: $T_c = T_E - I_E \alpha_E$

Transmission: $T_D = (T_c - I_T \alpha_E) n_T$

Diff + axle: $T_A = F_x r + I_w \alpha_w = (T_D - I_D \alpha_D) n_F$

Kinematic relationships:

$a_x = \alpha_w r$ $\quad \alpha_D = n_F \alpha_w$ $\quad \alpha_E = n_T \alpha_D = n_T n_F \alpha_w$

Want to find tractive forces at ground:

$$F_x = \frac{1}{r}\left[T_A - I_w \alpha_w\right]$$

$$= \frac{1}{r}\left[(T_D - I_D \alpha_D) n_F - I_w \alpha_w\right]$$

$$= \frac{1}{r}\left[([T_c - I_T \alpha_E] n_T - I_D \alpha_D) n_F - I_w \alpha_w\right]$$

$$= \frac{1}{r}\left[([T_E - I_E \alpha_E - I_T \alpha_E] n_T - I_D \alpha_D) n_F - I_w \alpha_w\right]$$

$$= \frac{1}{r}\left[([T_E - I_E n_T n_F \alpha_w - I_T \alpha_E] n_T - I_D n_F \alpha_w) n_F - I_w \alpha_w\right]$$

$$= \frac{1}{r}\left[\left(\left[T_E - I_E n_T n_F \frac{a_x}{r} - I_T n_T n_F \frac{a_x}{r}\right] n_T - I_D n_F \frac{a_x}{r}\right) n_F - I_w \frac{a_x}{r}\right]$$

$$= \frac{1}{r}\left[\left(T_E n_T - I_E n_T^2 n_F \frac{a_x}{r} - I_T n_T^2 n_F \frac{a_x}{r} - I_D n_F \frac{a_x}{r}\right) n_F - I_w \frac{a_x}{r}\right]$$

$$= \frac{1}{r}\left[T_E n_T n_F - I_E n_T^2 n_F^2 \frac{a_x}{r} - I_T n_T^2 n_F^2 \frac{a_x}{r} - I_D n_F^2 \frac{a_x}{r} - I_w \frac{a_x}{r}\right]$$

$[n_T n_F = n_{TF}]$

$$= \frac{1}{r}\left[T_E n_{TF} - \left(I_E n_{TF}^2 + I_T n_{TF}^2 + I_D n_F^2 + I_w\right)\frac{a_x}{r}\right]$$

$$= \frac{1}{r}\left[T_E n_{TF} - \left([I_E + I_T] n_{TF}^2 + I_D n_F^2 + I_W\right)\frac{a_x}{r}\right]$$

$$F_x = \frac{T_E n_{TF}}{r} - \left[(I_E + I_T) n_{TF}^2 + I_D n_F^2 + I_W\right]\frac{a_x}{r^2}$$

have to add an efficiency coefficient to account for mechanical and viscous losses

$$\boxed{F_x = \frac{T_E n_{TF} \eta_{TF}}{r} - \left[(I_E + I_T) n_{TF}^2 + I_D n_F^2 + I_W\right]\frac{a_x}{r^2}}$$

where η_{TF} is the combined efficiency of transmission and final drive, typically ~80%-90%

This equation is the rotational equilibrium of the powertrain, we also have the translational equilibrium of the vehicle:

$$M a_x = F_x - R_x - D_A - R_{hx} - W \sin\theta$$

R_x = rolling resistance

D_A = aerodynamic drag

R_{hx} = hitch (towing) forces

F_x includes engine torque and rotational inertia terms. As a convenience, the rotational inertias from previous equation where we found F_x are often lumped in with the mass of the vehicle to obtain a simplified equation:

$$\boxed{(M+M_r)\,a_x = \frac{T_E\, n_{TF}\, \eta_{TF}}{r} - R_x - D_A - R_{hx} - W \sin\theta}$$

where

M_r = equivalent mass of the rotating components

The combination of the two masses $M+M_r$ is called the "effective mass", and the ratio $\frac{(M+M_r)}{M}$ is the "mass factor", often represented as

$$\frac{(M+M_r)}{M} = 1 + 0.04 + 0.0025\, n_{TF}^2$$

Traction-Limited Acceleration

Presuming adequate power from the engine, acceleration may be limited by the coefficient of friction between tire and road. In that case, F_x is limited by:

$$F_x = \mu W$$

where

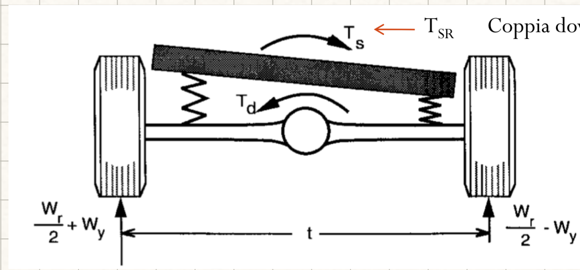
μ: peak coefficient of friction

W: weight on drive wheels

Weight on drive wheel depends on the static plus the dynamic load due to acceleration, and on any transverse shift of load due to drive torque.

Transverse weight shift due to drive torque

Transverse weight shift occurs on all solid drive axels, whether on the front or rear.



The drive shaft into the differential imposes a torque T_D on the axle. As will be seen, the chassis may roll, compressing and extending springs on opposite sides of the vehicle such that a torque due to suspension roll stiffness, T_{sr} is produced. Any difference in these two must be absorbed as a difference in weight on the two wheels. Non-locking axle: torque delivered to both wheels limited by the traction limit on the most lightly loaded wheel.

Axle in equilibrium:

$$\sum T_O = \left(\frac{W_r}{2} + W_y - \frac{W_r}{2} + W_y\right)\frac{t}{2} + T_s - T_D = 0$$

$$\Rightarrow \quad W_y = \frac{T_D - T_s}{t}$$

Can relate T_D to drive force:

$$T_D = F_x \frac{r}{n_F}$$

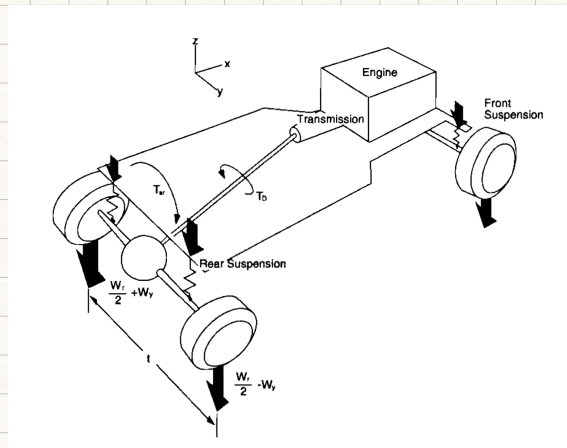
where

F_x: total drive force from the two rear wheels

r: tire radius

n_F: final drive ratio

Need to determine roll torque produced by suspension, requires analysis of whole vehicle, because reaction of drive torque on chassis attempts to roll the chassis on both front and rear suspensions.



Drive torque reaction at engine/transmission is transferred to the frame and distributed between front and rear suspension.

Roll torque produced by suspension is proportional to roll angle of the chassis. Then:

$$T_{sf} = K_{\phi f}\phi$$

$$T_{sr} = K_{\phi r}\phi$$

$$K_\phi = K_{\phi f} + K_{\phi r}$$

where

T_{sf}: roll torque front susp.

T_{sr}: roll torque rear susp.

$K_{\phi f}$: front susp. roll stiffness

$K_{\phi r}$: rear susp. roll stiffness

K_ϕ: Total roll stiffness

T_{sr} can be related to roll angle, roll angle can be related to drive torque. Roll angle is drive torque divided by total roll stiffness:

$$\phi = \frac{T_D}{K_\phi} = \frac{T_D}{K_{\phi f} + K_{\phi r}}$$

There fore

$$T_{sr} = K_{\phi r} \frac{T_D}{K_{\phi f} + K_{\phi r}}$$

Which gives us:

$$W_y = \frac{F_x r}{n_F t}\left[1 - \frac{K_{\phi r}}{K_{\phi r} + K_{\phi f}}\right] = \frac{F_x r}{n_F t} \frac{K_{\phi f}}{K_\phi}$$

This gives magnitude of the lateral load transfer as a function of tractive force and vehicle parameters (final drive ratio n_F, tread of the axle t, tire radius r, susp. roll stiffness K_ϕ)

Net load on rear axle during acceleration will be its static plus dynamic component.

$$W_r = W\left(\frac{b}{L} + \frac{a_x}{g} \frac{h}{L}\right)$$

Neglecting rolling resistance and aerodynamic drag force, acceleration is simply tractive force divided by vehicle mass

$$W_r = W\left(\frac{b}{L} + \frac{F_x}{Mg} \frac{h}{L}\right)$$

Traction limits

Solving for F_x gives final expression for maximum tractive force that can be developed by a solid rear axle with a non-locking differential.

$$F_{x_{max}} = \frac{\mu \frac{W b}{L}}{1 - \frac{h}{L}\mu + \frac{2 \mu r}{n_F t} \frac{K_{\phi f}}{K_\phi}}$$

For a solid rear axle with locking differential, additional tractive force can be obtained from the other wheel up to its traction limits such that the last term in the denominator drops out. Also the case in independent rear suspension because the drive line torque reaction is picked up by the chassis-mounted differential. In both cases:

$$F_{x_{max}} = \frac{\mu \frac{W b}{L}}{1 - \frac{h}{L}\mu}$$

Braking Manoeuvre

Secondary resistant forces

1) Rolling resistance:

$$R_x = R_{xF} + R_{xR} = f_r(W_F + W_R) = f_r W$$

f_r is the rolling resistance coefficient. Total force is independent of the distribution of loads on the axles (static or dynamic). Rolling resistance forces are nominally equivalent to about 0.01 g.

2) Aerodynamic drag

$$D_A = \frac{1}{2} \rho C_D A V^2$$

drag from air resistance depends on the dynamic pressure, and is therefore proportional to the square of the speed. At low speed it is negligible. At normal highway speeds, may contribute around 0.03g

3) Powertrain

- Frictions in bearings and gears
- Engine break

4) Road grade

Grade is defined as rise over run (vertical over horizontal distance). The additional force on the vehicle arising from grade, R_g, is:

$$R_g = W \sin\theta$$

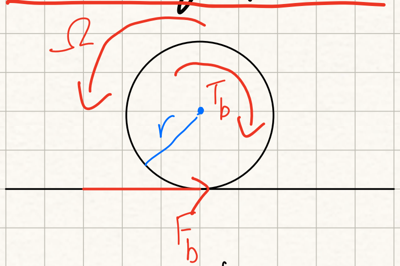
For small angles typical of most grades:

$$\theta(\text{rad}) \cong \text{grade} = \text{rise}/\text{run}$$

$$R_g = W\sin\theta \cong W\theta$$

Thus a grade of 4% will be equivalent to a deceleration of $\pm 0.04\,g$.

Braking force



The torque produced by the brakes acts to generate a braking force at the ground and to decelerate the wheels and driveline components.

NSL: $\sum F = ma$, $\sum T = I\alpha$

$$T_b - F_b r = I_w \alpha_w \quad \Rightarrow \quad F_b = \frac{T_b - I_w \alpha_w}{r}$$

Where

- r: rolling radius of tires
- I_w: rotational inertia of wheels (and drive components)
- α_w: rotational deceleration of wheels

Except during a wheel lockup process, α_w is related to the deceleration of the vehicle through the radius of the wheel $(\alpha_w = \frac{a_x}{r})$, and I_w may be lumped in with the vehicle mass for convenience in calculations. In that case we get:

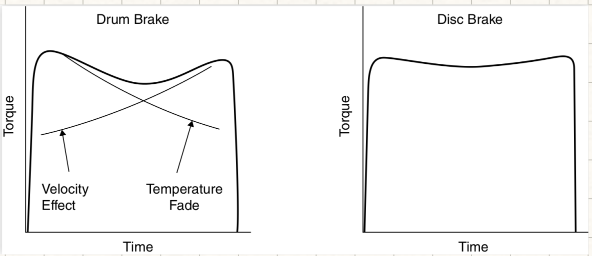
$$F_b = \frac{T_b}{r}$$

The torque produced by the brakes, T_b, is a function of the actuation effort, P_a, but to levels that vary with speed and the energy absorbed (through the temperatures generated). This relationship can be represented as:

$$T_b = f(P_a, \text{Velocity}, \text{Temperature})$$

Disc brake vs. drum brake

On drum brakes, the torque will often exhibit a "sag" in the middle part of the stop. May be due to the combination of temperature fade



and velocity effects (torque increases as velocity decreases). Disc brakes normally shows less torque variation in the course of a stop.

Tire-road friction

The brake force can only increase to the limit of the frictional coupling between the tire and road.

Surface adhesion arises from the intermolecular bonds between the rubber and the aggregate in the road surface.
The bulk hysteresis mechanism represents energy loss in the rubber as it deforms when sliding over the aggregate in the road.
Both depend on small amount of slip ocurring at the tire-road interface. Additional slip is observed as a result of the deformation of the rubber elements of the tire thread as they deform to develop and sustain the braking force.
Because of these mechanisms, brake force and slip coexist.
Brake force can be expressed as a coefficient $\mu = \frac{F_x}{F_z}$

Slip of the tire is defined by the ratio of slip velocity in the contact patch (forward velocity minus tire circumferential speed) to forward velocity:

$$\text{Slip} = \frac{V - \omega r}{V}$$

where

V: Vehicle forward velocity

ω: Tire rotational speed (rad/sec)

The brake coefficient deriving from adhesive and hysteretic friction increases with slip up to about 10 to 20% in magnitude depending on conditions. The peak coefficient is a key property, μ_p. It establishes the maximum braking force that can be obtained from the particular tire-road friction pair.

Exercise

$$D_x = -a_x = \frac{F_{xT}}{m} = cost$$

stopping time:

$$V = V_0 + at \qquad [m/s = m/s + m/s^2 \cdot s]$$

Since stopping, $V = 0\ m/s$

$$\Rightarrow V_0 = -at \quad , \quad -a = D_x$$

$$\Rightarrow t = \frac{V_0}{D_x}$$

stopping distance:

$$d = \frac{V_0 + V_f}{2} t \qquad \text{(average velocity)} \cdot \text{time} \quad [m = m/s \cdot s], \quad V_f = 0$$

$$V_f = V_0 + at \Rightarrow t = \frac{V_0}{D_x}$$

$$d = \frac{V_0}{2} \cdot \frac{V_0}{D_x} = \frac{V_0^2}{2D_x}$$