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Group 2

Covid-19 predictions

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1. Introduction

To be able to understand and predict the future spread of the covid-19 virus the science community needs to create well-functioning prediction models to be prepared for rises and drops in certain regions or cities. These models, that are based on data of how the virus behaves and how it has affected the regions past weeks, can aid in predicting the future of the pandemic and how to best respond to it.

Because of the enormous effect the virus has had on the world, large amount of data has been collected. Using this data, we have tried and tested 12 different models to figure out what model is the best at successfully predicting the future spread of the covid-19 virus. Some of the biggest challenges when making these models are that the virus is evolving constantly, and that the spread is heavily influenced by a large number of factors, such as individual behavior, population density etc. Despite these challenges, creating models can be a useful tool if they are created and used in a thoughtful manner. This essay will comment on these models and analyze their strengths and weaknesses, while also comparing them to each other.

2. Our own models

We made three models on our own to predict the covid cases that we will evaluate and compare to our other given models (see appendix for details about the other models). Our goal with these models is to be able to predict the cases of covid for a given week.

2.1 Our model 1

Model 10 is $Y_t = Y_{t-1} + (last\ change) * (0.8^h)$

Table 2.1: Our model 1 (model 10)

VARIABLE	MEANING
$\boldsymbol{Y_t}$	Number of cases per 100 000 inhabitants (adult population) and week.

When looking at the historical data of how the spreading of the covid virus seemed to behave we noticed that when the virus started to spread it did so very fast and seemed to do so exponentially. However, when the cases per 100 000 inhabitants started going down it did so much slower, and slower relative to its own number the faster it went towards 0. Having this in mind, and plotting the time series data we hypothesized that the amount of cases had reached a peak and was going to go down, and this model aims to correctly follow such a downfall of covid cases by using the naive 3 model but slightly modifying it by multiplying the last change with 0.8 raised to h(the prediction horizon). This way we succeed in slowing down the momentum of the naive 3 model by scaling down the predicted drop in covid cases. One reason for this approach is that we had a hard time beating the naive 3 model, but that by modifying it we would be able to get the best model available.

2.2 Our model 2

model 11 is $Y_t = \phi_0 + \alpha_1 X_{1,t-1} + \alpha_2 X_{2,t-1} + \alpha_3 X_{3,t-1} + \alpha_4 X_{4,t-1} + e_t$

Table 2.2: Our model 2 (model 11)

VARIABLE	MEANING
$\boldsymbol{Y_t}$	Number of cases per 100 000 inhabitants (adult population) and week.
$\boldsymbol{X_1}$	Positivity of the kommun for that week. (proportion of tests that were positive out of those performed that week)
$\boldsymbol{X_2}$	Number of tests per 100 000 inhabitants (adult population) and week.
$\boldsymbol{X_3}$	Number of ambulance calls assessed as suspected covid-19 by ambulance personnel, per 100 000 inhabitants (adult population) and week.
$\boldsymbol{X_4}$	Daily average of patients per week, regardless of where the patient lives, i.e. they can live in other regions. (Approx. HOSP + ICU)

For second model (model 11) we choose to do a timeseries regression with variables that were associated with number of suspected cases of covid (see table 2.2). The reason why this is a good indicator for covid cases the coming week is because it is very associated with the number of cases. After testing a bunch of different variables that were associated with amount of cases, we found that this was the best model with that reasoning.

2.3 Our model 3

model 12 is $Y_t = \phi_0 + \alpha_1 X_{1,t-1} + \alpha_2 X_{2,t-1} + e_t$

Table 2.3: Our model 3 (model 12)

VARIABLE	MEANING
$\boldsymbol{Y_t}$	Number of cases per 100 000 inhabitants (adult population) and week.
$\boldsymbol{X_1}$	Positivity of the kommun for that week. (proportion of tests that were positive out of those performed that week)
$\boldsymbol{X_2}$	Number of tests per 100 000 inhabitants (adult population) and week.

For our third model (see table 2.3) we took all the significant variables of our second model (see table 2.2) which turned out to be X_1 and X_2. We thought that these variables were more reliable to draw conclusions from since they are significant hence the forecast results would be more reliable.

3. Predictions one week ahead

Table 3.1 Error measures for predictions with h=1

		ME	MAE	MSE	MPE	MAPE
MODEL 1	Naive 1	-222.59	222.59	56 394.53	-49.02%	49.02%
MODEL 2	Naive 2	65.2	65.2	6889.22	17.89%	17.89%
MODEL 3	Naive 3	31.03	50.91	3175.17	9.36%	13.23%
MODEL 4	ARIMA(1.1.0)	51.7	54,88	5072.56	14.52%	15.14%
MODEL 5	ARIMA(0.1.1)	51.69	62.39	5625.1	14.65%	16.78%
MODEL 6	ARIMA(1.1.1)	58.3	58.3	5182.36	15.82%	15.82%
MODEL 7	TS Regress"$X_{1,t-1}$"	110.73	116.84	23921.91	31.29%	32.48%
MODEL 8	TS Regress "$X_{2,t-1}$"	-98.72	98.72	13576.72	-20.56%	20.56%
MODEL 9	TS Regress "$X_{3,t-1}$"	106.78	108.43	17504.46	-21.9%	22.4%
MODEL 10	Own model 1	37.87	52.23	3688.48	11.06	13.86%
MODEL 11	Own model 2	35.43	74.43	8271.42	12.27%	19.85%
MODEL 12	Own model 3	65.75	84.53	11799.2	19.63%	23.28%

In table 3.1 are the results from task 1. Here we can see our key figures for all 12 models. All models have calculated week 15, 16 and 17 using the data from week 14, 15 and 16. Thus, the prediction horizon has been one week. For each model we have calculated the mean error, the mean absolute error, the mean squared error, the mean percentage error and the mean absolute percentage error (see appendix 6.1 model 1 to see how they are calculated). All these error measurements give important information on how the model have performed when predicting data for the weeks we have information about. Looking at these results we can see that the best performing model in terms of ME was the naive 3 model, followed by our model 1 then model 2. For MAE the best model was the naive 3 model, followed by our model 1 and model 2. For MSE the best performing model was, again the naive 3 model, which makes sense since this is just the ME squared. The same goes for the MPE, the order will be the same. For MAPE the best three performing models were naive 3, our model1 and the ARIMA(1.1.0). We can conclude that our model1 and the naive 3 models outclassed the others in terms of prediction accuracy. We would however argue that the mean error, and all error measurements based on the mean error can be misleading since the negative and positive values cancel each other out. This means that the predictions could be way off, but if it is equally bad in both directions (positive and negative) the mean error will be good. Therefore, we think this measurement should be valued higher than the mean error, unless there is some value in just being on average right, regardless of the variance.

4. Predictions 3 weeks ahead using data from week 14

Table 4.1 Error measures for predictions with h=1, h=2, h=3

		ME	MAE	MSE	MPE	MAPE
MODEL 1	Naive 1	-229.25	222.25	58,603.11	-50.83%	50.83%
MODEL 2	Naive 2	89.43	89.43	14045.28	24.9%	24.9%
MODEL 3	Naive 3	6.8	40.97	2160.68	3.76%	10.71%
MODEL 4	ARIMA(1.1.0)	68.06	71.24	10,183.4	19.67%	20.29%
MODEL 5	ARIMA(0.1.1)	61.53	72.46	9833.49	18.27%	20.39%
MODEL 6	ARIMA(1.1.1)	93.83	93.83	15,031.93	25.99%	25.99%
MODEL 7	TS Regress "$X_{1,t-1}$"	96.78	102.88	19,317.06	27.64%	28.83%
MODEL 8	TS Regress "$X_{2,t-1}$"	-111.96	111.96	17,604.58	-23.44%	23.44%
MODEL 9	TS Regress "$X_{3,t-1}$"	-77.93	94.73	11,599.49	-15.02%	20.12%
MODEL 10	Own model 1	24.28	48.53	4286.21	8.63%	13.43%
MODEL 11	Own model 2	18.68	64.56	6372.09	8.08%	17.14%
MODEL 12	Own model 3	89.01	107.78	19898.54	26.23%	29.89%

In table 4.1 we have error measurements for all models when using data from week 14 to predict values for week 15, 16 and 17. Hence we predict 1, 2 and 3 weeks into the future (see appendix 6.2). Since we have the actual data for these weeks, we are able to measure how good each prediction is. We can clearly see that model 3 (Naive 3) outclasses all the other models in terms of all error measurements, meaning that it gives very accurate predictions. The second-best performing model is our model 1 and model 2, since we earlier stated that absolute measurements in better, we can state that our model 1 has the better predictions. Our model 2 is performing much better when only using data from week 14 and forecasting 3 weeks into the future. Model 3 and our model 1 was the best models both when forecasting 1 week into the future and forecasting 3 weeks into the future meaning that we think these models performs the best when forecasting week 18, 19 and 20 that we do not have data on.

5. Predictions 3 weeks ahead using data from week 17

Table 5.1

Using data up to and including week 17 Predict h= 1, i.e. predict week 18	
Data: w17	**Pred**
Naive 1	195.60
Naive 2	330.01
Naive 3	195.60
Arima(1.1.0)	269.60
Arima(0.1.1)	284.81
Arima(1.1.1)	269.92
Sole regressor $X_{1,t-1}$	490.02
Sole regressor $X_{2,t-1}$	209.61
Sole regressor $X_{3,t-1}$	315.97
Own model 1	222.48
Own model 2	359.23
Own model 3	409.43

Table 5.2

Using data up to and including week 17 - Predict h= 2;i.e. predict week 19	
Data: w17	**Pred**
Naive 1	195.60
Naive 2	330.01
Naive 3	61.191
Arima(1.1.0)	242.45
Arima(0.1.1)	284.81
Arima(1.1.1)	234.27
Sole regressor $X_{1,t-2}$	515.89
Sole regressor $X_{2,t-2}$	215.92
Sole regressor $X_{3,t-2}$	314.13
Own model 1	153.67
Own model 2	339.14
Own model 3	473.55

Table 5.3

Using data up to and including week 17 - Predict h= 3;i.e. predict week 20	
Data: w17	**Pred**
Naive 1	195.60
Naive 2	330.01
Naive 3	-207.63
Arima(1.1.0)	230.25
Arima(0.1.1)	284.81
Arima(1.1.1)	213.13
Sole regressor $X_{1,t-3}$	533.62
Sole regressor $X_{2,t-3}$	223.99
Sole regressor $X_{3,t-3}$	308.91
Own model 1	63.376
Own model 2	322.96
Own model 3	534.33

Table 5.4

Our model 1	
Week	Prediction
Week 18	222.48
Week 19	153.67
Week 20	63.376

In table 5.1, table 5.2 and table 5.3 we see predictions for all different models 3 weeks into the future using data from week 17. We do not have any data for week 18, 19 and 20 which means that we cannot evaluate the predictions in the same way as we did for week 15, 16 and 17. Considering the results from table 5.3 it would be fair to guess that model 3 will perform best when predicting these weeks. Although, model 3 predict -207.63 covid cases for week 20 which is not possible. We get more reasonable values for our model 1 meaning that we think that it will perform better when predicting week 18, 19 and 20. Therefore we think that the values in table 5.4 will be the best prediction for those weeks.

6. Appendix

6.1 Predictions 1 week ahead

Model 1

$Y_t = sample\ mean\ of\ Y$

Naive 1	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	206.9332	514.10791	-307.17	307.17	94 353.41	(-307.17 /514.11) * 100 = -59.75%	59.75%
Week 16	w15	214.0768	464.42300	-250.35	250.35	62 675.12	(-250.35 / 464.42) * 100 = -53.91%	53.91%
Week 17	w16	219.7665	330.01230	-110.25	110.25	12 155.06	(-110.25 /330.01) * 100 = -33.41%	33.41%
			ME = (-307.17 + -250.35 + -110.25) / 3 = -222.59					
			MAE = (307.17 + 250.35 + 110.25) / 3 = 222.59					
			MSE = (94,353.41 + 62,675.12 + 12,155.06) / 3 = 56,394.53					
			MPE = (-59.75% + -53.91% + -33.41%) / 3 = -49.02%					
			MAPE = (59.75% + 53.91% + 33.41%) / 3 = 49.02%					

Model 2

$Y_t = last\ level\ value\ of\ Y_t$

Naive 2	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	525.6139	514.1079	11.5	11.5	132.5	2.24%	2.24%
Week 16	w15	514.1079	464.42300	49.69	49.69	2469.1	10.7%	10.7%
Week 17	w16	464.423	330.01230	134.41	134.41	18066.05	40.73%	40.73%
			ME = 65.2					
			MAE = 65.2					
			MSE = 6889.22					
			MPE = 17.89%					
			MAPE = 17.89%					

Model 3

$Y_t = Last\ Y + h * (last\ change)$, h being the prediction horizon

Naive 3	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	484.297	514.10791	-29.81	29.81	888.64	-5.8%	5.8%
Week 16	w15	502.6019	464.42300	38.18	38.18	1457.71	8.22%	8.22%
Week 17	w16	414.7381	330.01230	84.73	84.73	7179.17	25.67%	25.67%
			ME = 31.03					
			MAE = 50.91					
			MSE = 3175.17					
			MPE = 9.36%					
			MAPE = 13.23%					

Model 4

Arima(1.1.0)

Arima(1.1.0)	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	509.3502	514.10791	-4.76	4.76	22.66	-0.93%	0.93%
Week 16	w15	509.5980	464.42300	45.18	45.18	2041.23	9.73%	9.73%
Week 17	w16	444.6976	330.01230	114.69	114.69	13153.8	34.75%	34.75%
			ME = 51.7					
			MAE = 54,88					
			MSE = 5072.56					
			MPE = 14.52%					
			MAPE = 15.14%					

Model 5

Arima(0.1.1)

Arima(0.1.1)	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	497.7067	514.10791	-16.41	16.41	269.29	-3.19%	3.19%
Week 16	w15	519.3314	464.42300	54.91	54.91	3015.11	11.82%	11.82%
Week 17	w16	446.5913	330.01230	116.58	116.58	13590.9	35.33%	35.33%
			ME = 51.69					
			MAE = 62.39					
			MSE = 5625.1					
			MPE = 14.65%					
			MAPE = 16.78%					

Model 6

Arima(1.1.1)

Arima(1.1.1)	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	528.3972	514.10791	14.29	14.29	204.2	2.78%	2.78%
Week 16	w15	509.7619	464.42300	45.34	45.34	2055.72	9.76%	9.76%
Week 17	w16	445.2795	330.01230	115.27	115.27	13287.17	34.93%	34.93%
			ME =58.3					
			MAE = 58.3					
			MSE = 5182.36					
			MPE = 15.82%					
			MAPE = 15.82%					

Model 7

model 7 is $Y_t = \phi_0 + \alpha_1 X_{1,t-1} + e_t$

TS Regress "$\boldsymbol{X_{1,t-1}}$"	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	504.947	514.10791	-9.16	9.16	83.91	-1.78%	1.78%
Week 16	w15	553.1948	464.42300	88.77	88.77	7880.11	19.11%	19.11%
Week 17	w16	582.5986	330.01230	252.59	252.59	63,801.71	76.54%	76.54%
			ME = 110.73					
			MAE = 116.84					
			MSE = 23921.91					
			MPE = 31.29%					
			MAPE = 32.48%					

Model 8

model 8 is $Y_t = \phi_0 + \alpha_1 X_{2,t-1} + e_t$

TS Regress "$\boldsymbol{X_{2,t-1}}$"	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	351.6315	514.10791	-162.48	162.48	26399.75	-31.6%	31.6%
Week 16	w15	345.6384	464.42300	-118.78	118.78	14108.69	-25.58%	25.58%
Week 17	w16	315.12	330.01230	-14.89	14.89	221.71	-4.51%	4.51%
			ME =-98.72					
			MAE = 98.72					
			MSE = 13576.72					
			MPE = -20.56%					
			MAPE = 20.56%					

Model 9

model 9 is $Y_t = \phi_0 + \alpha_1 X_{3,t-1} + e_t$

TS Regress "$\boldsymbol{X_{3,t-1}}$"	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	366.9278	514.10791	-147.19	147.19	21664.9	-28.63%	28.63%
Week 16	w15	288.7991	464.42300	-175.62	175.62	30842.38	-37.81%	37.81%
Week 17	w16	332.4757	330.01230	2.47	2.47	6.1	0.75%	0.75%
			ME =-106.78					
			MAE = 108.43					
			MSE = 17504.46					
			MPE = -21.9%					
			MAPE = 22.4%					

Model 10

Model 10 is $Y_t = Y_{t-1} + h * (last\ change) * (0.8^h)$

Own model 1	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	492.5604	514.10791	-21.55	21.55	464.4	-4.2%	4.2%
Week 16	w15	504.9031	464.42300	40.48	40.48	1638.63	8.72%	8.72%
Week 17	w16	424.6751	330.01230	94.67	94.67	8962.41	28.67%	28.67%
			ME = 37.87					
			MAE = 52.23					
			MSE = 3688.48					
			MPE = 11.06%					
			MAPE = 13.86%					

Model 11

model 11 is $Y_t = \ \phi_0 + \alpha_1 X_{1,t-1} + \alpha_2 X_{2,t-1} + \alpha_3 X_{3,t-1} + \alpha_4 X_{4,t-1} + \ e_t$

Own model 2	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	455.6097	514.10791	-58.5	58.5	3422.25	-11.38%	11.38%
Week 16	w15	484.3233	464.42300	19.9	19.9	396.01	4.28%	4.28%
Week 17	w16	474.9172	330.01230	144.9	144.9	20996.01	43.9%	43.9%
			ME = 35.43					
			MAE = 74.43					
			MSE = 8271.42					
			MPE = 12.27%					
			MAPE = 19.85%					

Model 12

model 12 is $Y_t = \ \phi_0 + \alpha_1 X_{1,t-1} + \alpha_2 X_{2,t-1} + \ e_t$

Own model 3	h = 1 Window expanding 3 times, i.e. doing three h = 1 predictions							
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14	485.9241	514.10791	-28.18	28.18	794.11	-5.48%	5.48%
Week 16	w15	509.3209	464.42300	44.9	44.9	2016.01	9.67%	9.67%
Week 17	w16	510.5363	330.01230	180.52	180.52	32587.47	54.7%	54.7%
			ME = 65.75					
			MAE = 84.53					
			MSE = 11799.2					
			MPE =19.63%					
			MAPE = 23.28%					

6.2 Predictions 3 weeks ahead using data from week 14

Model 1

$Y_t = sample\ mean\ of\ Y$

Naive 1								
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	206.9332	514.10791	-307.18	307.18	94,359.55	-59.75%	59.75%
Week 16	w14 h = 2	206.9332	464.42300	-257.49	257.49	66,301.1	-55.44%	55.44%
Week 17	w14 h = 3	206.9332	330.01230	-123.08	123.08	15,148.69	-37.3%	37.3%
			ME = -229.25					
			MAE = 229.25					
			MSE = 58,603.11					
			MPE = -50.83%					
			MAPE = 50.83%					

Model 2

$Y_t = last\ level\ value\ of\ Y_t$

Naive 2								
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	525.6139	514.10791	11.5	11.5	132.25	2.24%	2.24%
Week 16	w14 h = 2	525.6139	464.42300	61.19	61.19	3744.22	13.18%	13.18%
Week 17	w14 h = 3	525.6139	330.01230	195.6	195.6	38259.36	59.27%	59.27%
			ME =89.43					
			MAE = 89.43					
			MSE = 14045.28					
			MPE = 24.9%					
			MAPE = 24.9%					

Model 3

$Y_t = \ Last\ Y\ +\ h * (last\ change),$ h being the prediction horizon

Naive 3								
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	484.297	514.10791	-29.81	29.81	888.64	-5.8%	5.8%
Week 16	w14 h = 2	442.9801	464.42300	-21.44	21.44	459.67	-4.62%	4.62%
Week 17	w14 h = 3	401.6632	330.01230	71.65	71.65	5133.72	21.71%	21.71%
			ME =6.8					
			MAE = 40.97					
			MSE = 2160.68					
			MPE = 3.76%					
			MAPE = 10.71%					

Model 4

Arima(1.1.0)

Arima(1.1.0)								
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	509.3502	514.10791	-4.76	4.76	22.66	-0.93%	0.93%
Week 16	w14 h = 2	502.9482	464.42300	38.53	38.53	1484.56	8.3%	8.3%
Week 17	w14 h = 3	500.4282	330.01230	170.42	170.42	29,042.98	51.64%	51.64%
			ME =68.06					
			MAE = 71.24					
			MSE = 10,183.4					
			MPE = 19.67%					
			MAPE = 20.29%					

Model 5

Arima(0.1.1)

Arima(0.1.1)								
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	497.7067	514.10791	-16.4	16.4	268.96	-3.19%	3.19%
Week 16	w14 h = 2	497.7067	464.42300	33.29	33.29	1108.22	7.17%	7.17%
Week 17	w14 h = 3	497.7067	330.01230	167.7	167.7	28,123.29	50.82%	50.82%
			ME =61.53					
			MAE = 72.46					
			MSE = 9833.49					
			MPE = 18.27%					
			MAPE = 20.39%					

Model 6

Arima(1.1.1)

Arima(1.1.1)								
Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	528.3972	514.10791	14.29	14.29	204.2	2.78%	2.78%
Week 16	w14 h = 2	530.2181	464.42300	65.8	65.8	4329.64	14.17%	14.17%
Week 17	w14 h = 3	531.4095	330.01230	201.4	201.4	40,561.96	61.03%	61.03%
			ME =93.83					
			MAE = 93.83					
			MSE = 15,031.93					
			MPE = 25.99%					
			MAPE = 25.99%					

Model 7

For h = 1, model 7 is $Y_t = \phi_0 + \alpha_1 X_{1,t-1} + e_t$
For h = 2, model 7 is $Y_t = \phi_0 + \alpha_1 X_{1,t-2} + e_t$
For h = 3, model 7 is $Y_t = \phi_0 + \alpha_1 X_{1,t-3} + e_t$

Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	504.947	514.10791	-9.16	9.16	83.91	-1.78%	1.78%
Week 16	w14 h = 2	533.4814	464.42300	69.06	69.06	4769.28	14.87%	14.87%
Week 17	w14 h = 3	560.4424	330.01230	230.43	230.43	53,097.98	69.83%	69.83%
			ME =96.78					
			MAE = 102.88					
			MSE = 19,317.06					
			MPE = 27.64%					
			MAPE = 28.83%					

Model 8

For h = 1, model 8 is $Y_t = \phi_0 + \alpha_1 X_{2,t-1} + e_t$
For h = 2, model 8 is $Y_t = \phi_0 + \alpha_1 X_{2,t-2} + e_t$
For h = 3, model 8 is $Y_t = \phi_0 + \alpha_1 X_{2,t-3} + e_t$

Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	337.4252	514.10791	-176.68	176.68	31,215.82	-34.37%	34.37%
Week 16	w14 h = 2	324.3778	464.42300	-140.04	140.04	19,611.2	-30.15%	30.15%
Week 17	w14 h = 3	310.8633	330.01230	-19.15	19.15	366.72	-5.8%	5.8%
			ME =-111.96					
			MAE = 111.96					
			MSE = 17,604.58					
			MPE = -23.44%					
			MAPE = 23.44%					

Model 9

For h = 1, model 9 is $Y_t = \phi_0 + \alpha_1 X_{3,t-1} + e_t$
For h = 2, model 9 is $Y_t = \phi_0 + \alpha_1 X_{3,t-2} + e_t$
For h = 3, model 9 is $Y_t = \phi_0 + \alpha_1 X_{3,t-3} + e_t$

Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	366.9278	514.10791	-147.18	147.18	21,661.95	-28.63%	28.63%
Week 16	w14 h = 2	352.6078	464.42300	-111.81	111.81	12,501.48	-24.08%	24.08%
Week 17	w14 h = 3	355.2129	330.01230	25.2	25.2	635.04	7.64%	7.64%
			ME =-77.93					
			MAE = 94.73					
			MSE = 11,599.49					
			MPE = -15.02%					
			MAPE = 20.12%					

Our model 1

For h = 1, model 10 is $Y_t = Y_{t-1} + 1 * (last\ change) * (0.8^1)$
For h = 2, model 10 is $Y_t = Y_{t-1} + 2 * (last\ change) * (0.8^2)$
For h = 3, model 10 is $Y_t = Y_{t-1} + 3 * (last\ change) * (0.8^3)$

Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	484.297	514.10791	-29.81	29.81	888.64	-5.8%	5.8%
Week 16	w14 h = 2	457.8542	464.42300	-6.57	6.57	43.16	-1.41%	1.41%
Week 17	w14 h = 3	439.2229	330.01230	109.21	109.21	11,926.82	33.09%	33.09%
Model 10			ME =24.28					
			MAE = 48.53					
			MSE = 4286.21					
			MPE = 8.63%					
			MAPE =13.43%					

Our model 2

For h = 1, model 11 is $Y_t = \phi_0 + \alpha_1 X_{1,t-1} + \alpha_2 X_{2,t-1} + \alpha_3 X_{3,t-1} + \alpha_4 X_{4,t-1} + e_t$
For h = 2, model 11 is $Y_t = \phi_0 + \alpha_1 X_{1,t-2} + \alpha_2 X_{2,t-2} + \alpha_3 X_{3,t-2} + \alpha_4 X_{4,t-2} + e_t$
For h = 3, model 11 is $Y_t = \phi_0 + \alpha_1 X_{1,t-3} + \alpha_2 X_{2,t-3} + \alpha_3 X_{3,t-3} + \alpha_4 X_{4,t-3} + e_t$

Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	455.6097	514.10791	-58.5	58.5	3422.25	-11.38%	11.38%
Week 16	w14 h = 2	454.1035	464.42300	-10.32	10.32	106.5	-2.22%	2.22%
Week 17	w14 h = 3	454.8628	330.01230	124.85	124.85	15,587.52	37.83%	37.83%
Model 11	Using data	Pred.	ME =18.68					
	w14 h=1		MAE = 64.56					
	w14 h=2		MSE = 6372.09					
	w14 h=3		MPE = 8.08%					
			MAPE = 17.14%					

Our model 3

For h = 1, model 12 is $Y_t = \phi_0 + \alpha_1 X_{1,t-1} + \alpha_2 X_{2,t-1} + e_t$
For h = 2, model 12 is $Y_t = \phi_0 + \alpha_1 X_{1,t-2} + \alpha_2 X_{2,t-2} + e_t$
For h = 3, model 12 is $Y_t = \phi_0 + \alpha_1 X_{1,t-3} + \alpha_2 X_{2,t-3} + e_t$

Predict	Using Data	Pred.	True	Error	|Error|	Error^2	%Error	|%Error|
Week 15	w14 h = 1	485.9541	514.10791	-28.16	28.16	792.99	-5.48%	5.48%
Week 16	w14 h = 2	524.443	464.42300	60.02	60.02	3602.4	12.92%	12.92%
Week 17	w14 h = 3	565.1168	330.01230	235.16	235.16	55,300.23	71.26%	71.26%
Model 12			ME = 89.01					
			MAE = 107.78					
			MSE = 19898.54					
			MPE = 26.23%					
			MAPE = 29.89%					

6.3 Predictions 3 weeks ahead using data from week 17

Model 1

Data: w17	**H**	**Week**	**Pred**
Naive 1	1	18	222.2164
	2	19	222.2164
	3	20	222.2164

Model 2

Data: w17	**H**	**Week**	**Pred**
Naive 2	1	18	330.0123
	2	19	330.0123
	3	20	330.0123

Model 3

Data: w17	**H**	**Week**	**Pred**
Naive 3	1	18	195.6016
	2	19	61.19089
	3	20	-207.6305

Model 4

Data: w17	**H**	**Week**	**Pred**
Arima(1.1.0)	1	18	269.6028
	2	19	242.4523
	3	20	230.2498

Model 5

Data: w17	**H**	**Week**	**Pred**
Arima(0.1.1)	1	18	284.8098
	2	19	284.8098
	3	20	284.8098

Model 6

Data: w17	**H**	**Week**	**Pred**
Arima(1.1.1)	1	18	269.9151
	2	19	234.2696
	3	20	213.1272

Model 7

Data: w17	H	Week	Pred
Sole regressor $X_{1,t-1}$	1	18	490.0185
Sole regressor $X_{1,t-2}$	2	19	515.8878
Sole regressor $X_{1,t-3}$	3	20	533.6221

Model 8

Data: w17	H	Week	Pred
Sole regressor $X_{2,t-1}$	1	18	209.6067
Sole regressor $X_{2,t-2}$	2	19	215.9158
Sole regressor $X_{2,t-3}$	3	20	223.9938

Model 9

Data: w17	H	Week	Pred
Sole regressor $X_{3,t-1}$	1	18	315.9684
Sole regressor $X_{3,t-2}$	2	19	314.1329
Sole regressor $X_{3,t-3}$	3	20	308.9057

Model 10

Data: w17	H	Week	Pred
Own model1	1	18	222.4837
	2	19	153.6655
	3	20	63.37587

Model 11

Data: w17	H	Week	Pred
Own model2	1	18	359.2287
	2	19	339.1388
	3	20	322.9623

Model 12

Data: w17	H	Week	Pred
Own model3	1	18	409.426
	2	19	473.548
	3	20	534.3361