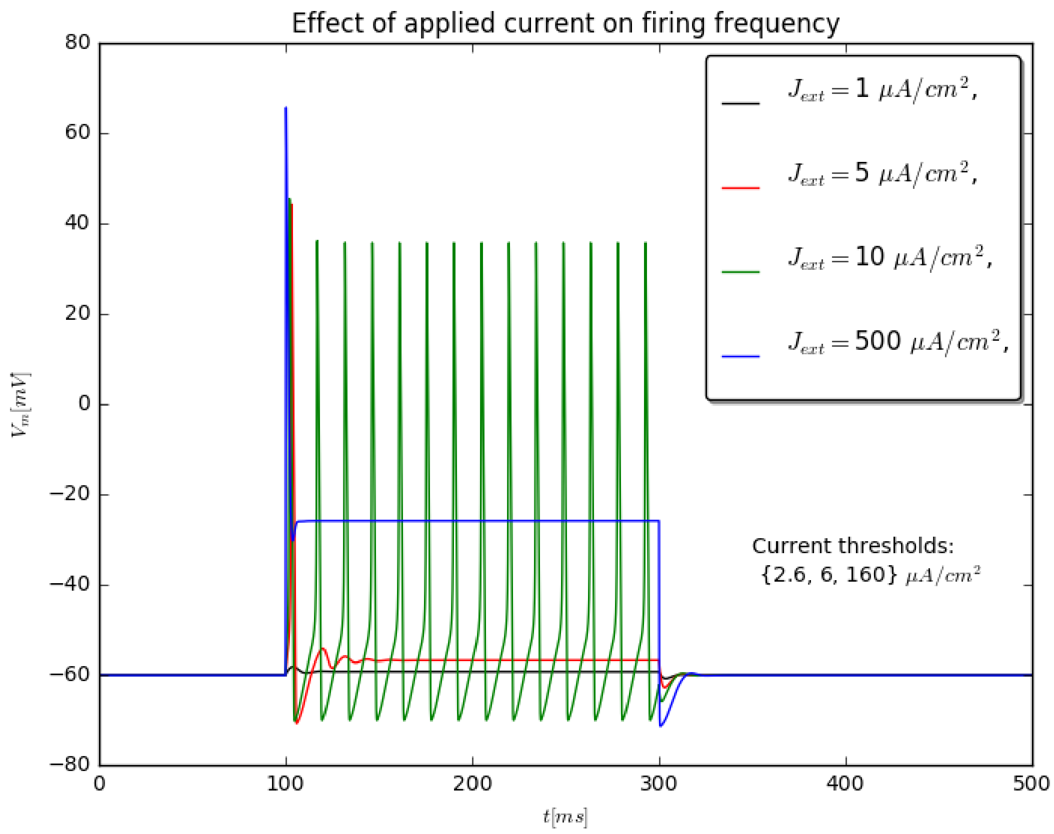
Remarks and Notes:

Part 1

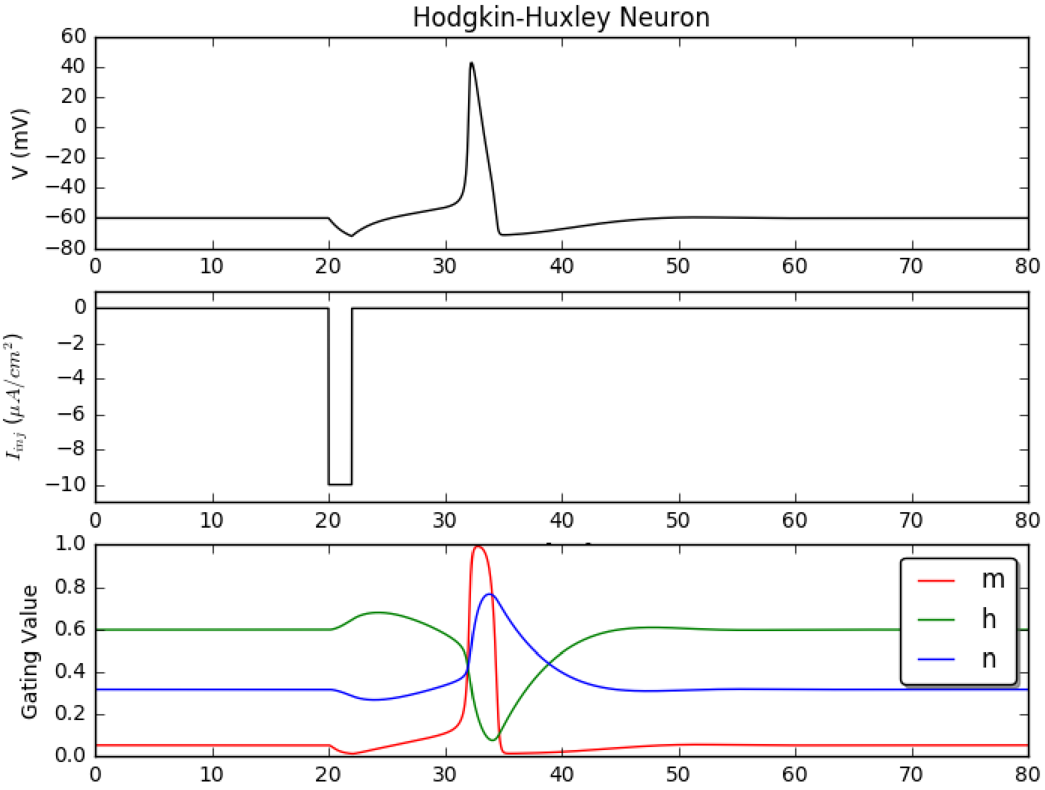
- max, min found frequency is 153, 63 for J_ext 110, 7.50 respectively
    - Otherwise not considered as firing, voltage doesn't rise above selected threshold (-10mV) and frequency drops to zero
        - Note that the current thresholds as well as minimal firing rate depend on the choice of threshold above which we decide to count the occurring voltage change as "firing" (here –10mV)
    - Why?
        - 1) it is too low current to raise voltage above threshold value
        - 2) Na+ voltage gated channels open so frequently that K+ channels don't manage to catch up. This leads to net efflux of charge from the cell which is not compensated and to permanent depolarization. This also attenuates the sodium concentration gradient and after some time, when Na channels are open, there is only weak influx of charge into the cell, which is not sufficient to excite action potential (only minor peaks can be observed).



Part 2

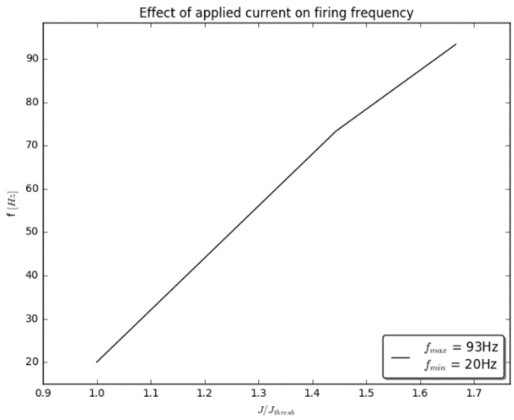
- For combination of t_pulse = 2 ms and J_ext = -10 uA/cm2 –> first firing observed
- Why?
    - Negative injected current leads to hyperpolarization (dv/dt is negative). Once the current is shut off, there is a net influx of charge (sodium) into the cell, so as to reestablish resting potential. Moreover, potassium conductance is effectively reduced, which limits is efflux.

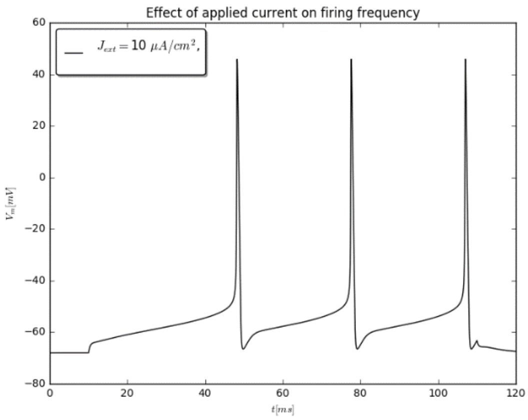
- This however overshoots and gets into positive feedback loop

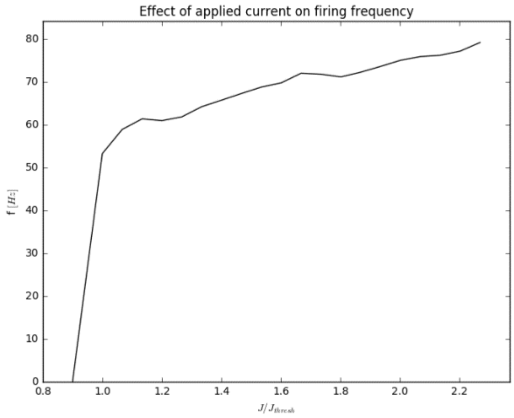


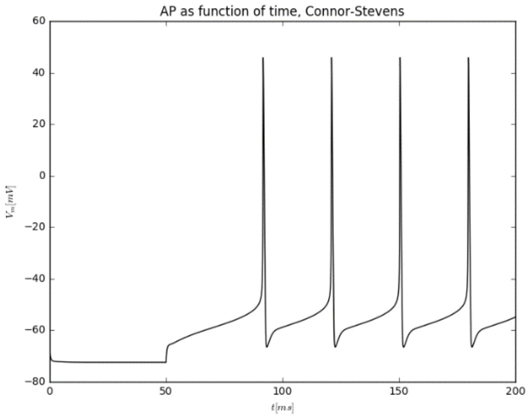
Part 3

- SubplotA –
  - starting point as well as xAxis values depend on the choice of 'firing' threshold, here -20mV was chosen for voltage which corresponds to (~ 8 uA/cm2 of injected current)
- SubplotC
  - Again depends on selection of voltage threshold
  - The xaxis can be slightly displaced by choosing a bit different value of current (here ~ 6 uA/cm2)
- SubplotD
  - The current needs to be stepped to positive (here -10 uA/cm2 for 50ms and then +10 for rest) value to cause firing after hyperpolarization. This is also what is described in the book.
- Subplot B
  - The rebound at the end is just corresponding to turn off in current



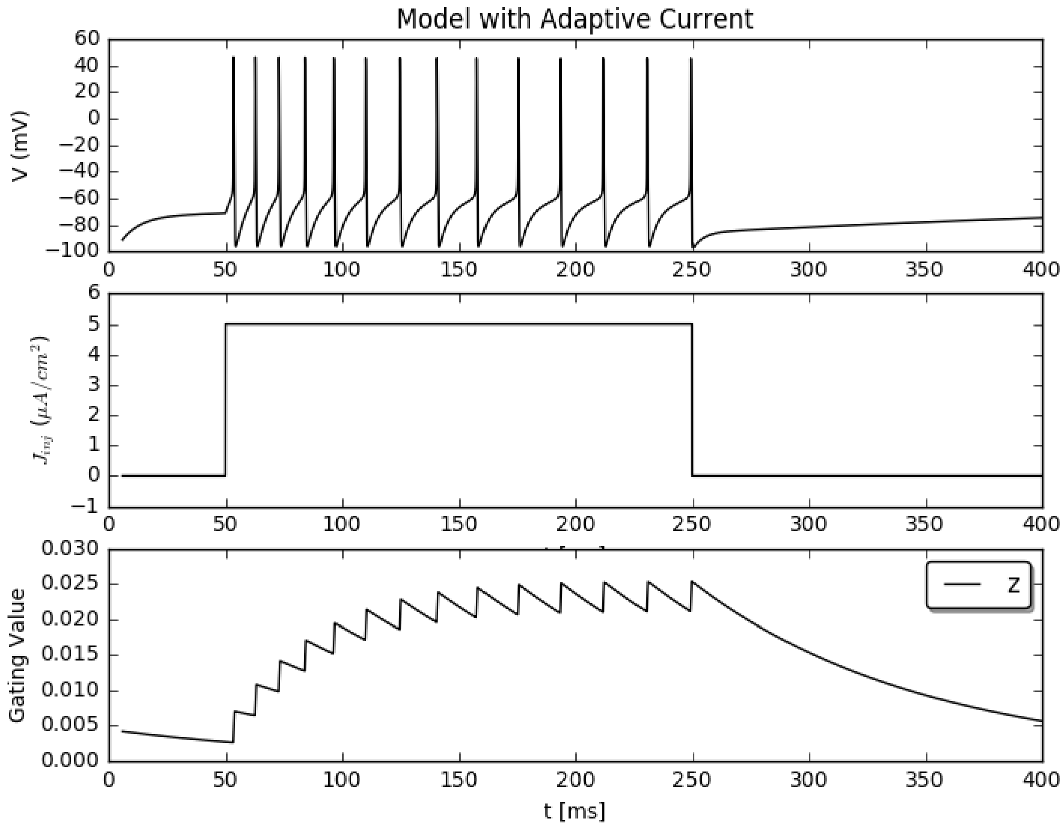






Part 4

- Explanation for the shape of z-curve:
  - M-channels have low time constant, thus each action potential increases the amount of open M-channels by more or less fixed amount and the corresponding potassium current subtracts from the applied stimulus, thus causing a decrease in the firing frequency.



Part 5

1) Propagation velocity vs. Temperature

Propagation velocity v_{AP} for temperature $T = 6.3$℃ is $0.109\ cm/ms$. I found this one difficult to explain, how it could be that at a physiological temperature the propagation velocity drops to zero??

2) Propagation velocity vs. medium resistivity

The velocity is known to be a product of distance travelled and inverse time. Qualitatively we may express it as

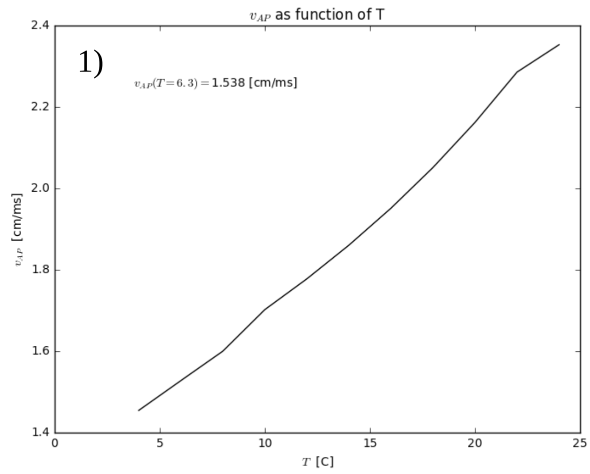
$$\vartheta \propto \frac{\lambda}{\tau},$$

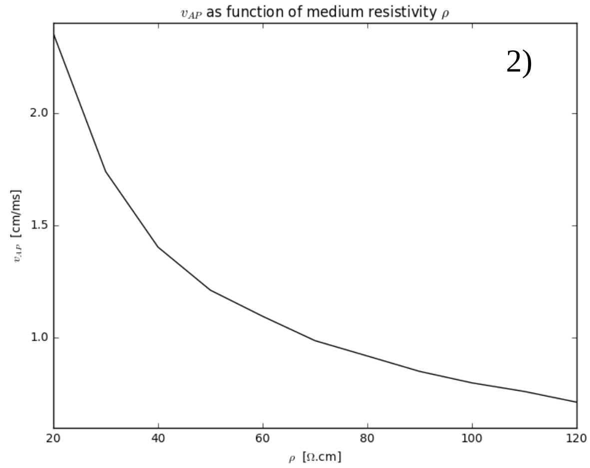
Where λ is a characteristic length and τ is the characteristic time of the system. The rate change in the system is described via exponential dependencies and we have:

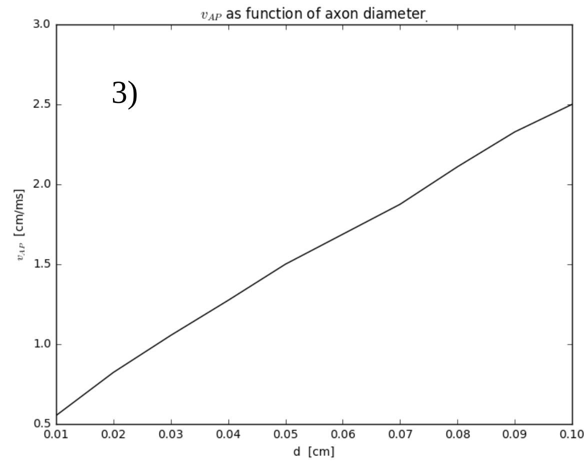
$$\lambda = \sqrt{\frac{d \cdot R_m}{4R_i}}, \qquad \tau = R_m \cdot C_m,$$

where R_m is membrane resistance (i.e. inverse permeability, in our case expressed by Conductance terms), $R_i = \frac{\rho \Delta z}{\pi r^2}$ is the resistance of axonal medium (axoplasm) and $C_m$is the membrane capacitance. For velocity of propagation of action potential, we may then write

$$\vartheta \propto \frac{\lambda}{\tau} = \frac{1}{C_m}\sqrt{\frac{d}{4R_m R_i}}.$$







Comparing this equation with obtained plot, we indeed observe that the velocity decreases as $1/\rho$.

3) Propagation velocity vs. Axon Diameter

Following the same line of thought as in previous case, we can see that the velocity increases roughly as $\sqrt{d}$. In simplified terms we now can see that to increase propagation velocity 2 (4) fold, we would need to have an axon of 4-times (16-times) the initial diameter. This is indeed impractical and that's also the rationale for having myelinated fibers, where the propagation velocity is much higher. Note, at this point, that if the axon was myelinated then we would observe more linear increase of propagation velocity with increasing diameter.