Automata in Software Verification and Testing

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March 5, 2024

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  - Shape analysis (majority of the work).
  - Automated software testing (described later).

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  - Goal: Derive reachable shapes of dynamic data structures.
  - Can be used to find bugs such as invalid memory dereferences, invalid frees, memory leaks.
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  - Have to to deal with infinite state spaces.
  - Application: Proving correctness of critical software systems (e.g., operating system kernel).

Automata Theory in Shape Analysis

- Data structures can be viewed as graphs.
- Graphs can be accepted by automata.
- An automaton can represent a set of graphs.
- Goal of analysis: Derive an automaton for each program location representing all possible shapes of data structures at the location,
  - So called shape invariant.
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Advantages

- Existing efficient algorithms for handling finite automata.
- Generality to represent various kinds of data structures.

Counterexample Validation and Abstraction Refinement for Shape Analysis based on Forest Automata

Holík, L., Hruška, M., Lengál, O., Rogalewicz, A., Vojnar, T.: Counterexample Validation and Interpolation-Based Refinement for Forest Automata. In *Proc. of VMCAI'17*, LNCS 10145, Springer, 2017.

Forest Automata Encoding of Heap

- A Forest Automaton (FA)* is a tuple of tree automata (TA).

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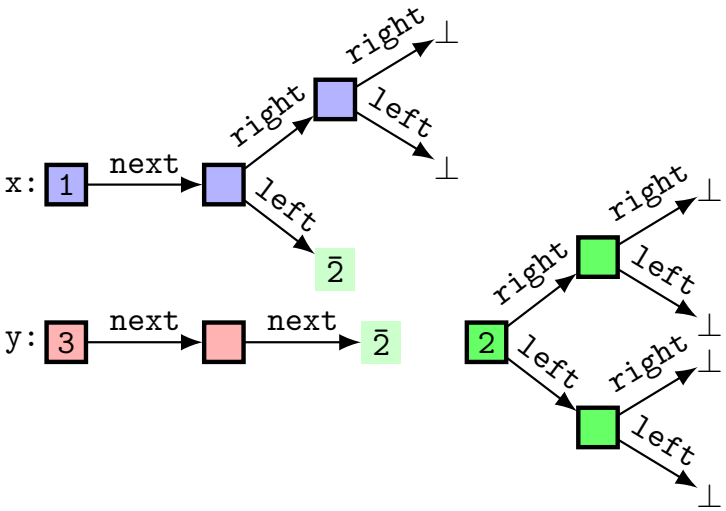
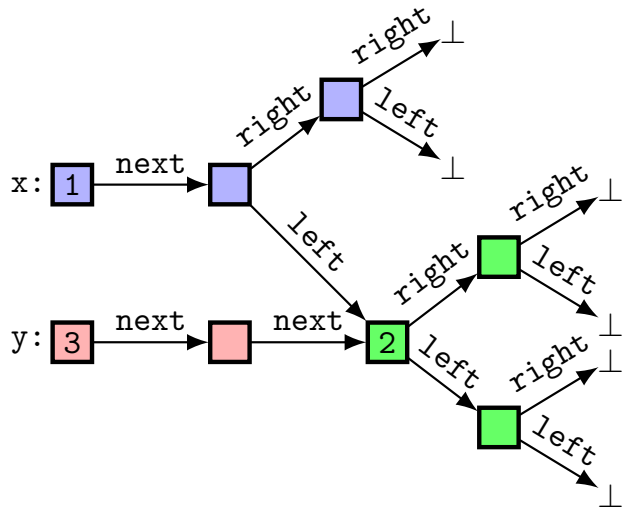
- Encoded heap graphs obtained by connecting leaves with the referenced roots.





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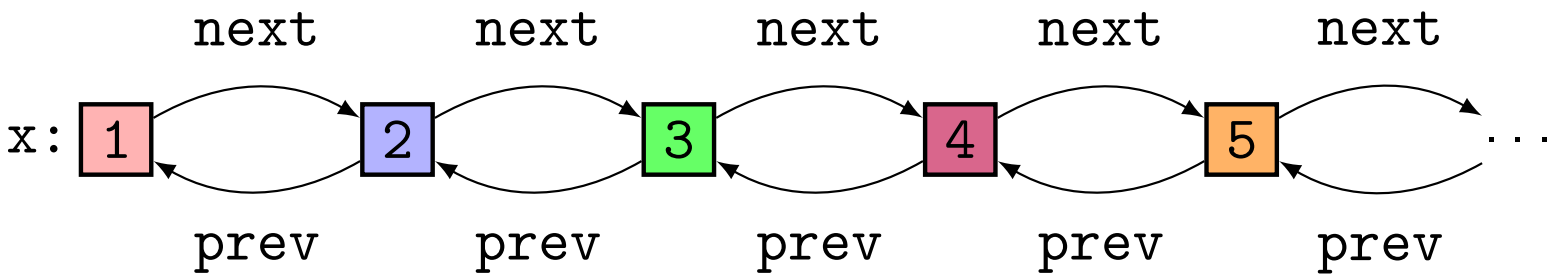
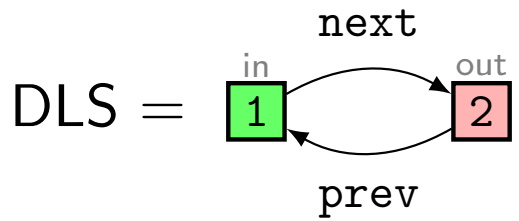
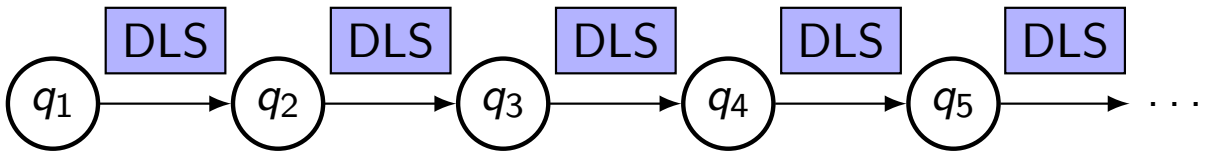
Boxes

- Boxes are used to extend the expressive power of FA.
- A box is an FA that can be used as a symbol of another FA.
- A box represents repeating subgraphs of a heap.
- FA having boxes in the alphabet are called hierarchical.







An Overview of Verification Method

- Based on Abstract Regular Tree Model Checking (ARTMC) [†] and Counterexample-guided Abstraction Refinement (CEGAR). [‡]
  - Sets of heap configurations are represented by automata.
  - Employs abstraction over automata to overapproximate the set of reachable configurations (allowing termination on ∞ state spaces).

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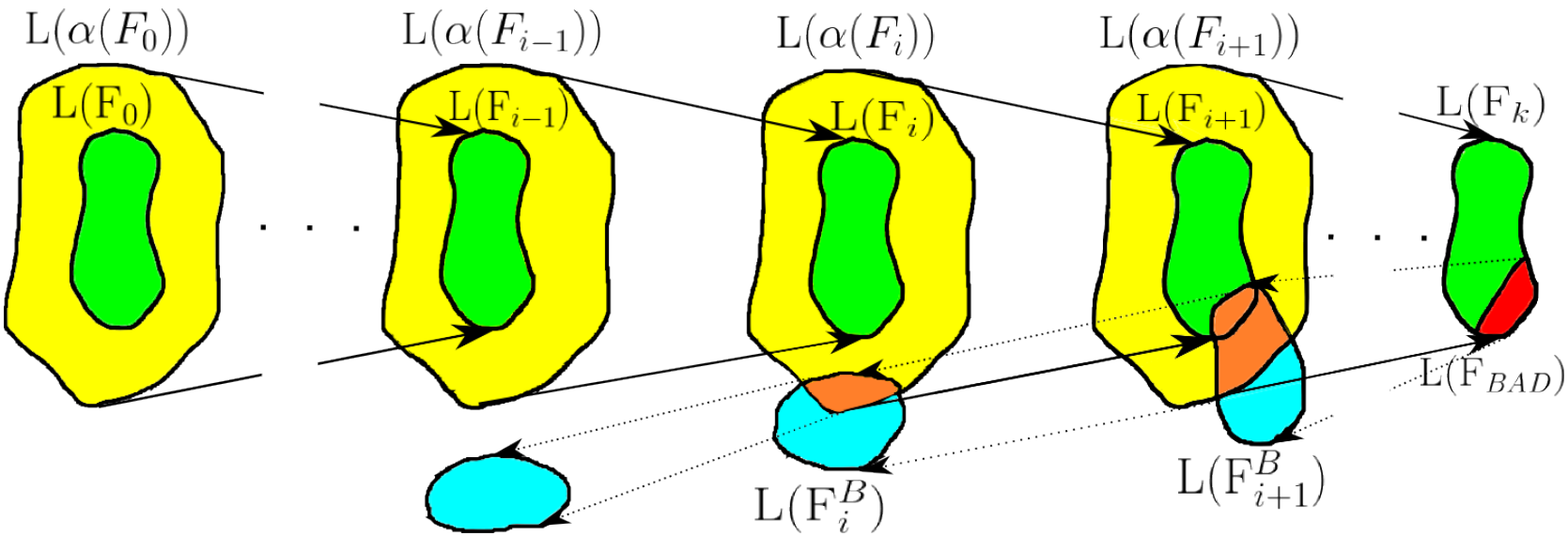

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Abstraction

- Overapproximates reachable configurations and accelerates analysis
- Collapses states in the same equivalence class of a relation $\sim$
- Height Abstraction
  - Equivalence $\sim_H$ is defined as

$$q_1 \sim_H q_2 \stackrel{DEF}{\equiv} L^n(q_1) = L^n(q_2)$$

    where $L^n(q)$ is the language of prefixes of trees accepted from $L(q)$ with height up to n
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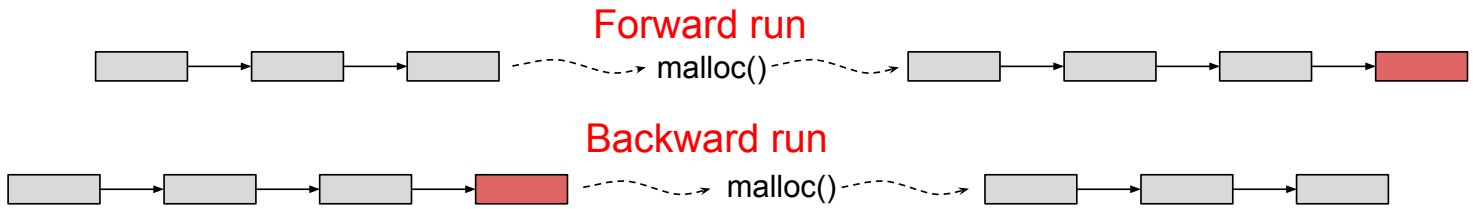
- Predicate Language Abstraction
  - Given a set of predicate languages $P = \{p_1, \ldots, p_n\}$, equivalence $\sim_P$ is defined as

$$q_1 \sim_P q_2 \stackrel{DEF}{\equiv} \forall p \in P : L(q_1) \cap p \neq \emptyset \Leftrightarrow L(q_2) \cap p \neq \emptyset$$

  - Refinement: Interpolating languages from counterexamples
  - Informed refinement

Backward Run and Abstraction for Forest Automata

- Counterexample validation via backward run.
- Ingredients for backward run:
  - Reversion of abstract transformations.



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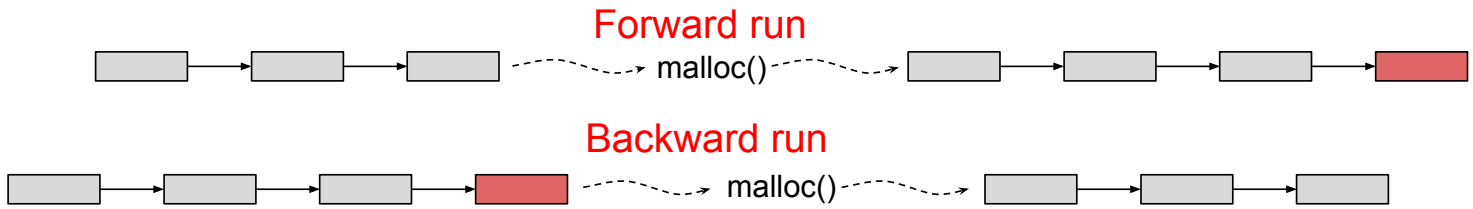

  - Reversion of abstraction $\rightarrow$ intersection of FA.
    - Consider FA $F_1 = (TA_1^1, \ldots, TA_n^1)$ and $F_2 = (TA_1^2, \ldots, TA_n^2)$, intersection is done component-wise using TA intersection, i.e., $F_1 \cap F_2 = (TA_1^1 \cap TA_1^2, \ldots, TA_n^1 \cap TA_n^2)$.
  - Precise reversion of folding and unfolding of boxes $\Rightarrow$ compatible form.

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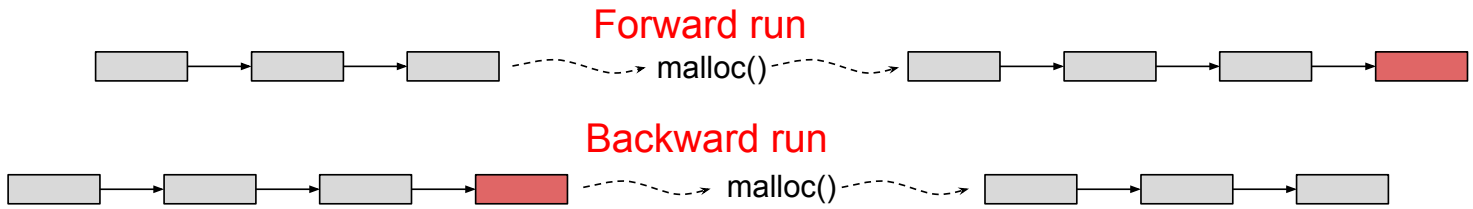

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  - Precise reversion of folding and unfolding of boxes $\Rightarrow$ compatible form.
- Ingredients for predicate language abstraction:
  - Predicate languages represented by TA.
  - New predicate TAs obtained by intersecting FAs from FW and BW run.

Experimental Evaluation

- Shape analysis based on Forest automata implemented in the Forester tool

Program	Status	LoC	Time [s]	Refnm	Preds	Program	Status	LoC	Time [s]	Refnm	Preds
SLL (delete)	safe	33	0.02	0	0	DLL (rev)	safe	39	0.70	0	0
SLL (bubblesort)	safe	42	0.02	0	0	CDLL	safe	32	0.02	0	0
SLL (insertsort)	safe	36	0.04	0	0	DLL (insertsort)	safe	42	0.56	0	0
SLLOfCSLL	safe	47	0.02	0	0	DLLOfCDLL	safe	54	1.76	0	0
SLL01	safe	70	1.20	1	1	**DLL01**	safe	73	0.65	2	2
CircularSLL	safe	49	3.57	3	3	**CircularDLL**	safe	52	37.22	18	24
OptPtrSLL	safe	59	1.90	3	3	**OptPtrDLL**	safe	62	1.87	5	5
QueueSLL	safe	71	11.32	10	10	**QueueDLL**	safe	74	44.68	14	14
GBSLL	safe	64	0.84	3	3	**GBDLL**	safe	71	1.89	4	4
GBSLLSent	safe	68	0.85	3	3	**GBDLLSent**	safe	75	2.19	4	4
RGSLL	safe	72	14.41	22	38	**RGDLL**	safe	76	78.76	26	26
WBSLL	safe	62	0.84	5	5	**WBDLL**	safe	71	1.37	7	7
SortedSLL	safe	76	227.12	15	15	**SortedDLL**	safe	82	36.67	11	11
EndSLL	safe	45	0.07	2	2	**EndDLL**	safe	49	0.10	3	3
TreeRB	error	130	0.08	0	0	**TreeWB**	error	125	0.05	0	0
TreeCnstr	safe	52	0.31	0	0	**TreeCnstr**	error	52	0.03	0	0
TreeOfCSLL	safe	109	0.57	0	0	**TreeOfCSLL**	error	109	0.56	1	3
TreeStack	safe	58	0.20	0	0	**TreeStack**	error	58	0.01	0	0
TreeDSW	safe	72	1.87	0	0	**TreeDSW**	error	72	0.02	0	0
TreeRootPtr	safe	62	1.43	0	0	**TreeRootPtr**	error	62	0.17	2	6
SkipList	safe	84	3.36	0	0	**SkipList**	error	84	0.08	1	1

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- 2017
  - Generating correctness witness (forest automaton representing shape invariants of program).
  - Counterexample analysis and abstraction refinement for hierarchical forest automata.
- Forester has never won any medal, but was able to verify difficult data structures (skip-lists, or various trees) which were not solved by any other tool.

Towards Efficient Shape Analysis with Tree Automata

Holík, L., Hruška, M. NETYS'21.

Towards Efficient Shape Analysis with Tree Automata (TA)

- Forest Automata have their limits:
  - Can't represent data structures such as grid (unbounded number of cut-points).
  - Are not closed under union (which complicates their manipulation during verification procedure, e.g., a fixpoint computation).

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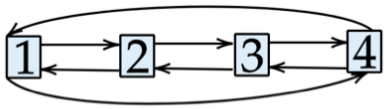
- Forest Automata have their limits:
  - Can't represent data structures such as grid (unbounded number of cut-points).
  - Are not closed under union (which complicates their manipulation during verification procedure, e.g., a fixpoint computation).
- We proposed a new automata capable of representing graphs with bounded tree-width.
- We sketched efficient algorithm for entailment for these automata.
- The new automata are based on tree automata but we work with single TA instead of tuple of TA (like in FA).

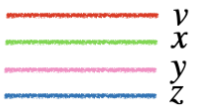
Tree Decomposition with Variables

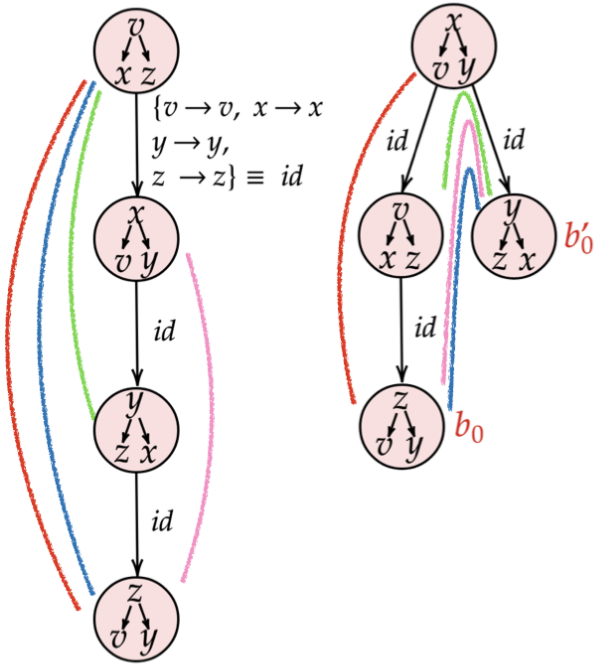
- How to represent a graph with bounded tree-width by a tree automaton? By tree decompositions with variables.

Graph of circular DLL



Tree decompositions:





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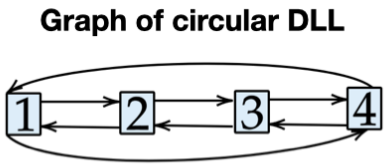

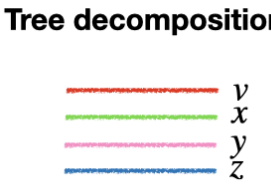

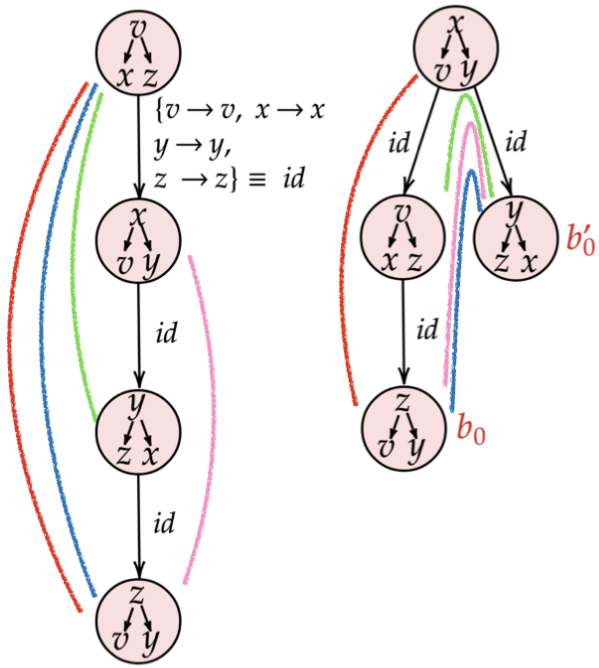

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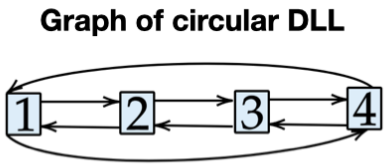

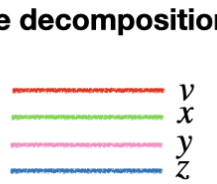

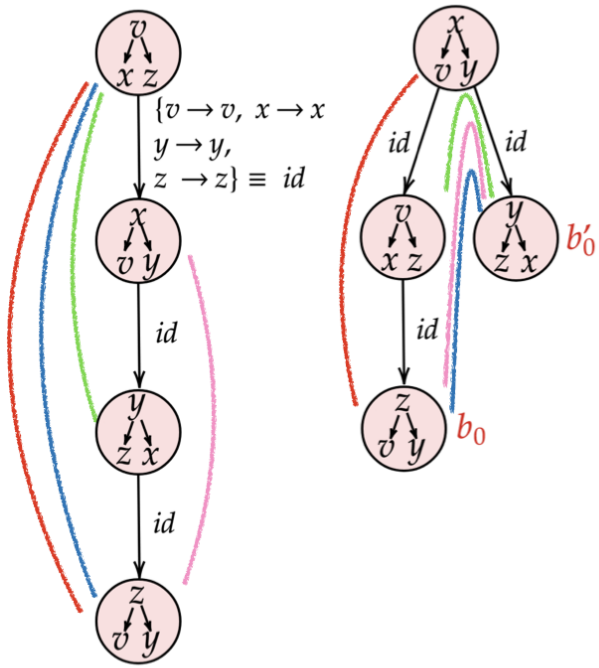

- Two operations needed to transform one decomposition to another:
  - Reconnection - moving the node b_0 under the node b_0'.
  - Rotation - changing orientation of the edges between two nodes.

Towards Entailment

- Phase is a set of all operations that could be performed at once (they don't interfere with each other).

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- The implementation of a tree automata phase at most doubles the number of variables and leads to an automaton that is of a polynomial size assuming a fixed tree-width of the original automaton.
- Since automata inclusion is EXPTIME-complete, we have entailment which is singly exponential (assuming a fixed maximum tree width).

Shape Analysis based on SMT Solving in 2LS Framework

Malík, V., Hruška, M., Schrammel P., Vojnar T. FMCAD'18.

Shape Analysis based on SMT Solving in 2LS Framework

- Shape analysis using SMT solving to compute the points-to relation between pointers and (abstract) memory addresses.
- Domain designed for representation of linked-lists.
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- Therefore more straightforward than the automata-based approaches, but lacks generality of automata based approaches.
- Implemented within the 2LS framework for program analysis.
  - Combination with other domains in the framework, e.g., the numerical domain.

Shape Analysis based on SMT Solving in 2LS Framework

- Verification procedure takes:
  - A first order formula over combination of SMT theories that represents the program in SSA form (i.e., transition relation).
  - A set of invariants based on predefined templates (proposed for various domains).
  - The property of interest (e.g., memory safety properties).

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  - The property of interest (e.g., memory safety properties).
- And verifies that there is no reachable invariants violate the properties of interest.

Shape Analysis based on SMT Solving in 2LS Framework

- Technically, unsatisfiability of this formula:
  - $\exists \vec{x}, \vec{x}'.\ \neg(\mathit{Init}(\vec{x}) \implies \mathcal{T}(\vec{x}, \vec{d})) \vee \neg(\mathcal{T}(\vec{x}, \vec{d}) \wedge \mathit{Trans}(\vec{x}, \vec{x}') \implies \mathcal{T}(\vec{x}', \vec{d}))$
  - where $\mathit{Init}(\vec{x})$ is an initial state of variables, $\mathcal{T}$ is a set of invariants based on templates, *Trans* formula representing program

Shape Analysis based on SMT Solving in 2LS Framework

- Technically, unsatisfiability of this formula:
  - $\exists \vec{x}, \vec{x}'.\ \neg(\mathit{Init}(\vec{x}) \implies \mathcal{T}(\vec{x}, \vec{d})) \vee \neg(\mathcal{T}(\vec{x}, \vec{d}) \wedge \mathit{Trans}(\vec{x}, \vec{x}') \implies \mathcal{T}(\vec{x}', \vec{d}))$
  - where $\mathit{Init}(\vec{x})$ is an initial state of variables, $\mathcal{T}$ is a set of invariants based on templates, *Trans* formula representing program
- The template for shape analysis is: $\mathcal{T}^S \equiv \bigwedge_{p_i^{lb} \in Ptr^{lb}} = \mathcal{T}^S_{p_i^{lb}}(d_{p_i^{lb}})$,
  - where $\mathcal{T}^S_{p_i^{lb}}(d_{p_i^{lb}}) \equiv (\bigvee_{a \in d_{p_i^{lb}}} p_i^{lb} = a)$.
  - Basically, it describes the points-to relation between a pointer (p_i^{lb}) and addresses a that the pointer may point to.

Experimental Evaluation

Table: Comparison of 2LS with other tools on examples needing reasoning about unbounded data structures and their stored data.

	2LS	**CPA-Seq**	**PredatorHP**	**Forester**	**Symbiotic**	**UAutomizer**
Calendar	2.88	timeout	false	unknown	timeout	timeout
Cart	23.70	timeout	false	unknown	timeout	timeout
Hash Function	3.65	8.51	unknown	unknown	unknown	timeout
MinMax	5.14	timeout	false	unknown	timeout	timeout
Packet Filter	431.00	timeout	timeout	unknown	unknown	timeout
Process Queue	6.62	7.68	timeout	unknown	timeout	timeout
Quick Sort	18.20	3.50	timeout	unknown	unknown	5.75
Running Example	1.24	timeout	timeout	unknown	timeout	unknown
SM1	0.53	timeout	0.31	false	timeout	timeout
SM2	0.55	5.41	false	false	timeout	14.50

Automated Software Testing (with Automata)

Generating Scenarios for Digital Twins of Distributed Manufacturing Execution Systems.

Fiedor, T., Hruška, M., Smrčka, A. EUROCAST'22.

Introduction

- Manufacturing Execution System (MES)
  - Software managing production in factory.
  - This includes communication with information system, machines, etc.
  - Difficult to test:
    - Distributed nature of manufacturing, different communication protocols, different formats of structured data.

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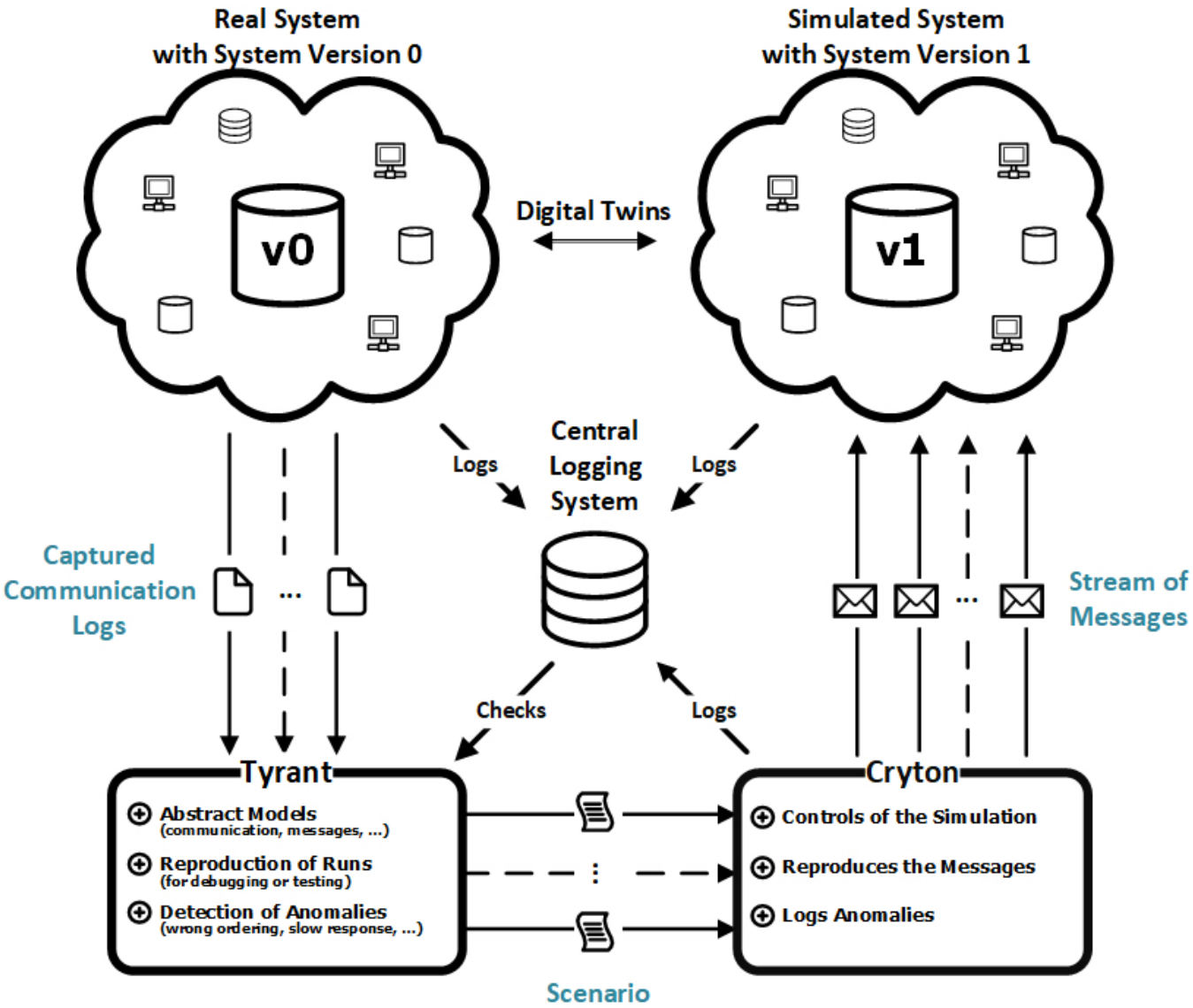
  - Not everything needs to be simulated, some software may be used in digital twin natively.
- Project solved with the Unis company (MES Pharis) and Masaryk University (Cryton for orchestrating digital twin)

Our Solution



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  - New strings in overapproximated langauge $\Rightarrow$ new series of events $\Rightarrow$ new scenario $\Rightarrow$ new test case.
- Implemented in the Tyrant tool.
  - Generated valid scenarios for orchestration of digital twin with deployed Pharis.
  - More engineering work needed to be deployed in production.

Conclusion

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- Work presented in the thesis:
  - Counterexample analysis and abstraction refinement for Forest automata.
  - Competing in SV-COMP, edition 2015-2018.
  - Chapter on forest automata in book on software verification with refined presentation of the approach.
  - Automata over graphs with bounded tree-width.
  - Shape analysis based on SMT solving.
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  - Automated testing of distributed manufacturing execution systems.
- Work not presented (or only mildly mentioned) in the thesis:
  - Connection of Predator and Symbiotic participating in SV-COMP'20.
  - The design and implementation of efficient automata library called MATA (TACAS'24).

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- Fiedor T., Hruška M., Smrčka A., *Software for measurement and evaluation of performance parameters.* 2021.
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The Definition of Graph Tree Width

- A tree decomposition of a graph $G = (V_G, E_G)$ is a tree $T = (V_T, E_T)$, such that:
  - $V_T \subseteq 2^{V_G}$
  - $\forall v_G \in V_G \, \exists v_T \in V_T : v_G \in v_T$
  - $\forall (u, v) \in E_G \, \exists v_T \in V_T : u, v \in v_T$
  - $\forall v_G \in V_G$: The set $\{v_t \in V_T \mid v_G \in v_T\}$ induces connected subtree
- A width of the given decomposition is the maximal cardinality of the nodes of the decomposition minus one.
- A tree width of the given graph is a minimal width of all possible tree decompositions of the given graph.