

Assignment 3 (ML for TS) - MVA

Martin Jolif martinjolif@gmail.com
Enzo Abdallah-Hervé enzo.abdallahherve@gmail.com

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1 Introduction

Objective. The goal is to implement (i) a signal processing pipeline with a change-point detection method and (ii) wavelets for graph signals.

Warning and advice.

- Use code from the tutorials as well as from other sources. Do not code yourself well-known procedures (e.g. cross validation or k-means), use an existing implementation.
- The associated notebook contains some hints and several helper functions.
- Be concise. Answers are not expected to be longer than a few sentences (omitting calculations).

Instructions.

- Fill in your names and emails at the top of the document.
- Hand in one report per pair of students.
- Rename your report and notebook as follows:
FirstnameLastname1_FirstnameLastname1.pdf and
FirstnameLastname2_FirstnameLastname2.ipynb.
For instance, LaurentOudre_CharlesTruong.pdf.
- Upload your report (PDF file) and notebook (IPYNB file) using the link given in the email.

2 Dual-tone multi-frequency signaling (DTMF)

Dual-tone multi-frequency signaling is a procedure to encode symbols using an audio signal. The possible symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, #, A, B, C, and D. A symbol is represented by a sum of cosine waves: for $t = 0, 1, \dots, T - 1$,

$$y_t = \cos(2\pi f_1 t / f_s) + \cos(2\pi f_2 t / f_s)$$

where each combination of (f_1, f_2) represents a symbols. The first frequency has four different levels (low frequencies), and the second frequency has four other levels (high frequencies); there are 16 possible combinations. In the notebook, you can find an example symbol sequence encoded with sound and corrupted by noise (white noise and a distorted sound).

Question 1

Design a procedure that takes a sound signal as input and outputs the sequence of symbols. To that end, you can use the provided training set. The signals have a varying number of symbols with a varying duration. There is a brief silence between each symbol.

Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values). Hint: use the time-frequency representation of the signals, apply a change-point detection algorithm to find the starts and ends of the symbols and silences, and then classify each segment.

Answer 1

For an input signal we apply the following procedure:

We process the spectrogram of the signal and subdivide it into several small signals through a change point detection algorithm that detect the starts and ends of the symbols and silences (*split_signal* function).

Then, on each subdivision of the input signal, we evaluate f_1 and f_2 by computing the discrete Fourier Transform of the subdivision and get the two frequencies f_1 and f_2 where the magnitude of the DFT is maximum (*find_frequencies_from_signal* function).

Then, we choose the closest frequencies from f_1 and f_2 in the frequencies from the DTMF telephone keypad. Thanks to the tuple of frequencies obtained just before, we obtain a symbol (*symbol_from_frequencies* function)!

Question 2

What are the two symbolic sequences encoded in the test set?

Answer 2

- Sequence 1: Our procedure predicted 723C99 instead of 721C99
- Sequence 2: Our procedure predicted 1 # 2 # correctly

3 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E . A specific node is denoted by v and a specific edge, by e . The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ and u_1, u_2, \dots, u_n respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \dots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^n \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \quad (1)$$

where $\hat{f} = [\hat{f}_1, \dots, \hat{f}_n]$ is the Fourier transform of f and \hat{g}_m are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the \hat{g}_m ; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel \hat{g}_m is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \quad (2)$$

where $a := \lambda_n / (M + 1 - R)$,

$$\hat{g}^U(\lambda) := \frac{1}{2} \left[1 + \cos \left(2\pi \left(\frac{\lambda}{aR} + \frac{1}{2} \right) \right) \right] \mathbb{1}(-Ra \leq \lambda < 0) \quad (3)$$

and $R > 0$ is defined by the user.

Question 3

Plot the kernel functions \hat{g}_m for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 1. What is the influence of R ?

Answer 3

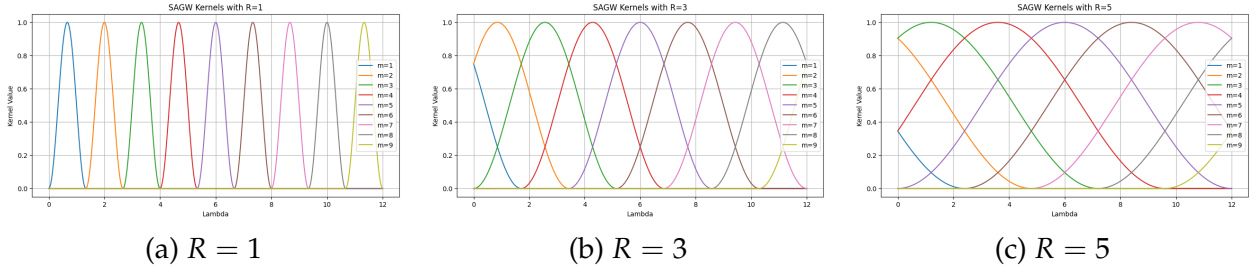


Figure 1: The SAGW kernels functions

The parameter R has an influence on the kernel functions \hat{g}_m , indeed, the bigger R is, the more the overlap between the functions \hat{g}_m is bigger.

We will study the Molene data set (the one we used in the last tutorial). The signal is the temperature.

Question 4

Construct the graph using the distance matrix and exponential smoothing (use the median heuristics for the bandwidth parameter).

- Remove all stations with missing values in the temperature.
- Choose the minimum threshold so that the network is connected and the average degree is at least 3.
- What is the time where the signal is the least smooth?
- What is the time where the signal is the smoothest?

Answer 4

The stations with missing values are: BATZ, BEG_MEIL, CAMARET, PLOUGONVELIN, RIEC SUR BELON, ST NAZAIRE-MONTOIR, PLOUAY-SA, VANNES-MEUCON, LANNAERO, PLOUDALMEZEAU, LANDIVISIAU, SIZUN, QUIMPER, OUessant-STIFF, LANVEOC, ARZAL, BREST-GUIPAVAS and BRIGNOGAN.

The threshold is equal to 0.83.

The signal is the least smooth at 2014-01-21 06:00:00.

The signal is the smoothest at 2014-01-24 19:00:00.

Question 5

(For the remainder, set $R = 3$ for all wavelet transforms.)

For each node v , the vector $[W_f(1, v), W_f(2, v), \dots, W_f(M, v)]$ can be used as a vector of features. We can for instance classify nodes into low / medium / high frequency:

- a node is considered low frequency if the scales $m \in \{1, 2, 3\}$ contain most of the energy,
- a node is considered medium frequency if the scales $m \in \{4, 5, 6\}$ contain most of the energy,
- a node is considered high frequency if the scales $m \in \{6, 7, 9\}$ contain most of the energy.

For both signals from the previous question (smoothest and least smooth) as well as the first available timestamp, apply this procedure and display on the map the result (one colour per class).

Answer 5

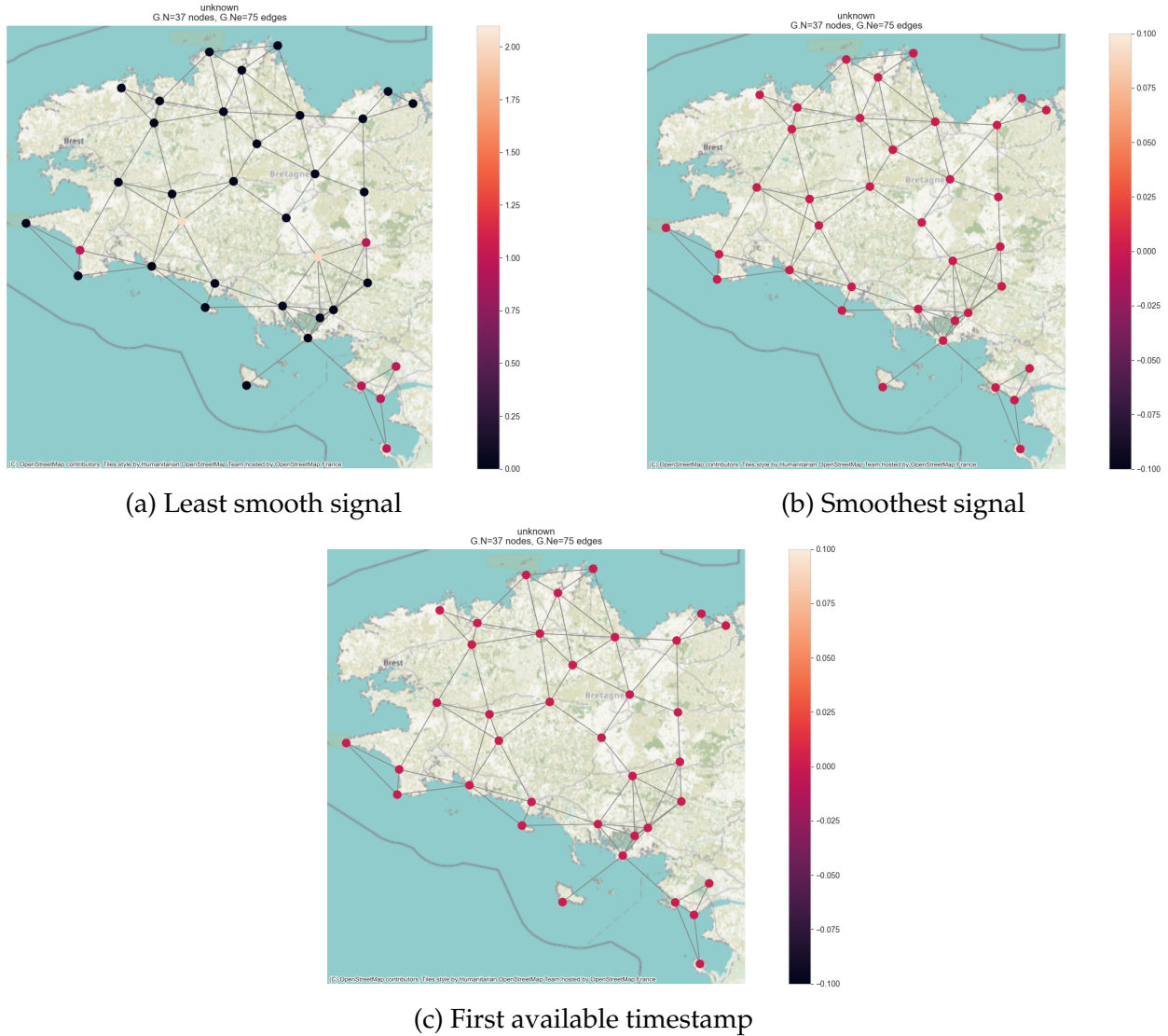


Figure 2: Classification of nodes into low / medium / high frequency

Question 6

Display the average temperature and for each timestamp, adapt the marker colour to the majority class present in the graph (see notebook for more details).

Answer 6

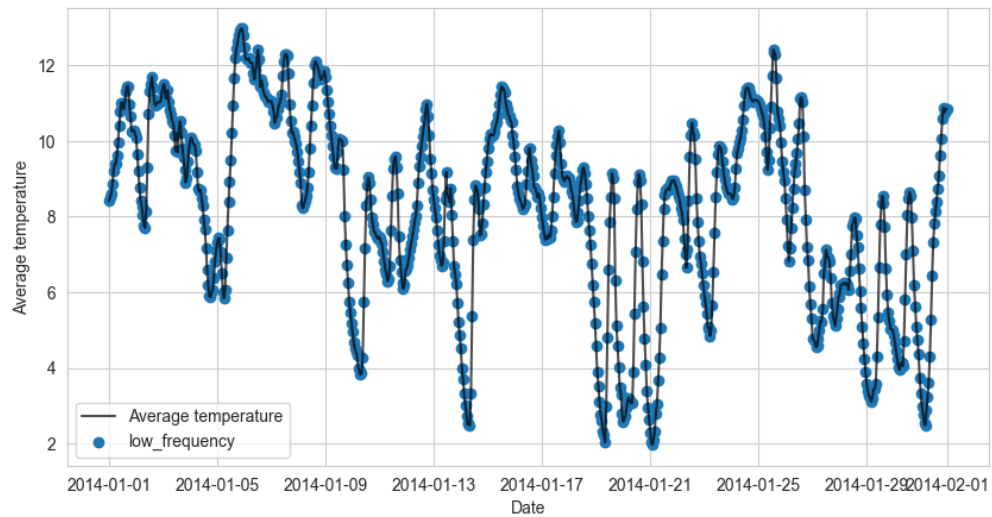


Figure 3: Average temperature. Markers' colours depend on the majority class.

Question 7

The previous graph G only uses spatial information. To take into account the temporal dynamic, we construct a larger graph H as follows: a node is now *a station at a particular time* and is connected to neighbouring stations (with respect to G) and to itself at the previous timestamp and the following timestamp. Notice that the new spatio-temporal graph H is the Cartesian product of the spatial graph G and the temporal graph G' (which is simply a line graph, without loop).

- Express the Laplacian of H using the Laplacian of G and G' (use Kronecker products).
- Express the eigenvalues and eigenvectors of the Laplacian of H using the eigenvalues and eigenvectors of the Laplacian of G and G' .
- Compute the wavelet transform of the temperature signal.
- Classify nodes into low/medium/high frequency and display the same figure as in the previous question.

Answer 7

- The Laplacian of H can be express as follows :

$$L_H = L_G \otimes L_{G'}$$

- Let $\lambda_1, \dots, \lambda_n$ be the eigenvalues of L_G and μ_1, \dots, μ_m the eigenvalues of $L_{G'}$, then the eigenvalues of L_H are $\lambda_i \mu_j$ for $i, j \in \{1, \dots, n\} \times \{1, \dots, m\}$. For the eigenvectors, if X_1, \dots, X_n and Y_1, \dots, Y_m are respectively the eigenvectors of L_G and $L_{G'}$ then the eigenvectors of L_H are $X_i \otimes Y_j$ for $i, j \in \{1, \dots, n\} \times \{1, \dots, m\}$.

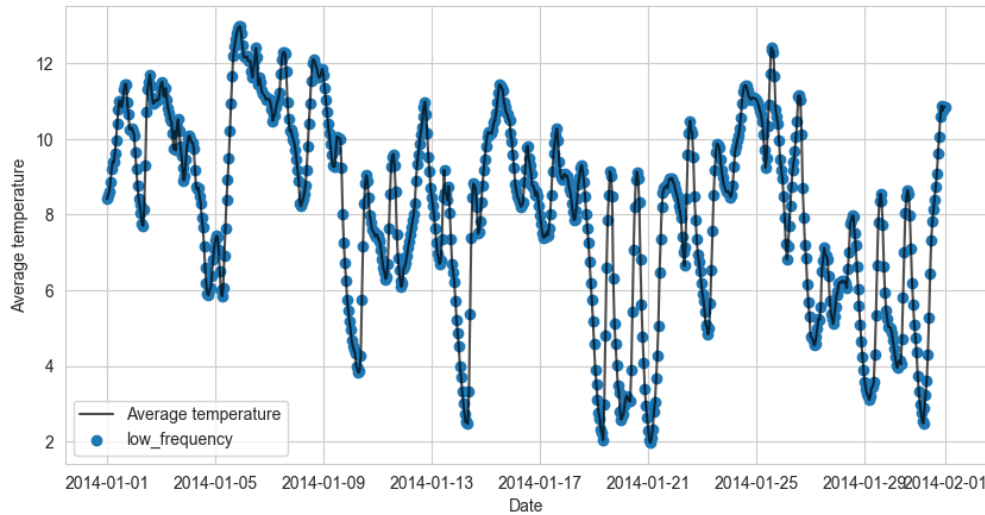


Figure 4: Average temperature. Markers' colours depend on the majority class.