

Probabilistic Inference on Noisy Time Series

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Abstract

Stuff

1. Introduction

We're trying to gather a bunch of algorithms to infer model parameters based on noisy time-series data.

1.1. Problem statement

We have some noisy time-series data and a forward model (simulation) that can be used to replicate it. We'd like to find out which parameter values are compatible with the experimental evidence.

- Observations $D = \{(t_1, z_1), \dots, (t_n, z_n)\}$ where t is time ($t_i > t_{i-1}$ for $i > 1$) and $z_i \in \mathbb{R}$ is a noisy measurement at time t_i .
- Forward model $f(t|\theta) \rightarrow \mathbb{R}$ with m parameters, bundled in θ .
- The parameters live in some bounded space $\theta \in \Theta \subset \mathbb{R}^m$
- *Only* normally distributed, time-independent noise with some unknown variance σ^2 , such that $z_i = f(t_i|\theta_{true}) + \mathcal{N}(0, \sigma^2)$.

1.1.1. Log likelihood

Some PINTS methods use the log-likelihood of the observations, given a proposed set of parameters θ . We define the likelihood in terms of the probability *density*

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of obtaining observations D for parameters θ :

$$l(\theta|D) \equiv f(D|\theta) \quad (1)$$

With purely random noise, the measurement error at any point in time is independent of the signal history, so that:

$$f(D|\theta) = \prod_{i=1}^n f(t_i, z_i|\theta) \quad (2)$$

With normally distributed noise, the probability density function for observing D is then:

$$f(x|\theta, \sigma) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i - f(t_i|\theta))^2}{2\sigma^2}\right) \quad (3)$$

And taking the logarithm, we find

$$\log l(\theta, \sigma|D) = -\frac{n}{2} \log(2\pi) - n \log(\sigma) - \frac{1}{2\sigma^2} \sum_{i=1}^n (z_i - f(t_i|\theta))^2 \quad (4)$$

so that the likelihood is maximised for

$$\min_{\theta, \sigma^2} \left[2n\sigma^2 \log(\sigma) + \sum_{i=1}^n (z_i - f(t_i|\theta))^2 \right] \quad (5)$$

We typically sneak σ into the parameter vector (and increment the number of parameters m by 1) to simply write $\log l(\theta|D)$.

1.1.2. Ionic currents

Models of ionic currents are based on ODEs, so that the forward model involves solving an initial-value problem $y(t|\theta) = \int_{t_0}^t \dot{y}(\tau|\theta) d\tau$. A few extra problems occur:

1. The ODE is forced/driven/paced by a function $q(t)$
2. The state of the ODE is not observed directly, but via a function $r(t, q(t), y(t|\theta), \theta)$.
3. The initial state $(t_0, y(t_0))$ is only approximately known.

The third problem is usually ignored, leaving

$$f(t|\theta) = r(t, q(t), \int \dot{y}(\tau, \theta) d\tau, \theta) \quad (6)$$

References