Probabilistic Inference on Noisy Time Series

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Abstract

Stuff

1. Introduction

We're trying to gather a bunch of algorithms to infer model parameters based on noisy time-series data.

1.1. Problem statement

We have some noisy time-series data and a forward model (simulation) that can be used to replicate it. We'd like to find out which parameter values are compatible with the experimental evidence.

- Observations $D = \{(t_1, z_1), ..., (t_n, z_n)\}$ where t is time ($t_i > t_{i-1}$ for $i > 1$) and $z_i \in \mathbb{R}$ is a noisy measurement at time t_i.
- Forward model $f(t|\theta) \rightarrow \mathbb{R}$ with m parameters, bundled in θ.
- The parameters live in some bounded space $\theta \in \Theta \subset \mathbb{R}^m$
- *Only* normally distributed, time-independent noise with some unknown variance σ^2, such that $z_i = f(t_i|\theta_{true}) + \mathcal{N}(0, \sigma^2)$.

1.1.1. Log likelihood

Some PINTS methods use the log-likelihood of the observations, given a proposed set of parameters θ. We define the likelihood in terms of the probability *density*

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of obtaining observations D for parameters θ:

$$l(\theta|D) \equiv f(D|\theta) \tag{1}$$

With purely random noise, the measurement error at any point in time is independent of the signal history, so that:

$$f(D|\theta) = \prod_{i=1}^{n} f(t_i, z_i|\theta) \tag{2}$$

With normally distributed noise, the probability density function for observing D is then:

$$f(x|\theta, \sigma) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i - f(t_i|\theta))^2}{2\sigma^2}\right) \tag{3}$$

And taking the logarithm, we find

$$\log l(\theta, \sigma|D) = -\frac{n}{2}\log(2\pi) - n\log(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(z_i - f(t_i|\theta))^2 \tag{4}$$

so that the likelihood is maximised for

$$\min_{\theta,\sigma^2}\left[n\log(\sigma) + \frac{1}{2\sigma^2}\sum_{i=1}^{n}(z_i - f(t_i|\theta))^2\right] \tag{5}$$

We typically sneak σ into the parameter vector (and increment the number of parameters m by 1) to simply write $\log l(\theta|D)$.

1.1.2. Cyclic voltammetry

It's fun.

1.1.3. Ionic currents

Models of ionic currents are based on ODEs, so that the forward model involves solving an initial-value problem $y(t|\theta) = \int_{t_0}^{t} \dot{y}(\tau|\theta)d\tau$. A few extra problems occur:

1. The ODE is forced/driven/paced by a function $q(t)$
2. The state of the ODE is not observed directly, but via a function $r(t, q(t), y(t|\theta), \theta)$.
3. The initial state $(t_0, y(t_0))$ is only approximately known.

The third problem is usually ignored, leaving

$$f(t|\theta) = r(t, q(t), \int \dot{y}(\tau, \theta)d\tau, \theta) \tag{6}$$

Finally, the noise is probably not normally distributed or independent, and the models often don't fit.

References