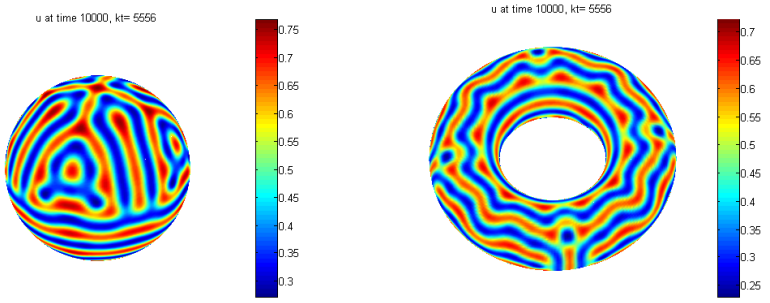
Surface Computing Project for CBL Summer School

Convergence study. As one refines the mesh and decreases time step, the numerical solution converges to the true solution. In other words, the numerical error, which is the "difference" between the numerical solution and the true solution, will decrease as mesh and time step sizes decrease. The goal of convergence study is to quantitatively measure the relationship between decrease of mesh/time-step size and the decrease of numerical error.

Please use the code ***example_heat_sphere.m*** to solve the diffusion on the unit sphere using mesh size dx equal to 0.2, 0.1, 0.05, 0.025 and measure the corresponding numerical errors. You can summarize the errors by a table or MATLAB figure (would be a plus). From the code given, what are the exact solution, final stopping time and time step? How do you measure the numerical errors? Can you comment on the relationship between mesh size dx and numerical error?

Pattern formation on a torus. We can model pattern formation on surfaces using the Gray–Scott reaction-diffusion equations:

$$u_t = \nu_u \Delta_{\mathcal{S}} u - uv^2 + F(1-u)$$
$$v_t = \nu_v \Delta_{\mathcal{S}} v + uv^2 - (F+k)v$$



Please modify the given code ***example_RD_sphere.m*** (which solves Gray–Scott equation on a sphere) to make it solve Gray–Scott on a torus. You (might only) need to change the closest point function ***cpf*** which computes the closest points and the parametrization function ***paramf*** which is for plotting. You can start with exactly the same initial conditions (with or without random perturbations) and parameters ν_u, ν_v, F, and k as in the ***example_RD_sphere.m*** code. Once you've got the code working on torus, you can change the ratio between ν_u and ν_v to see what new patterns you will get, and how the patterns change as time evolve.