Quiz

1. Classify the following PDEs as either elliptic, parabolic or hyperbolic
   - Laplace eq: $\Delta u = 0$
   - Poisson eq: $\Delta u = f(x, y, z)$
   - Heat or diffusion eq: $u_t = \Delta u$
   - Wave eq: $u_{tt} = \Delta u$
2. Use the Taylor expansion on the following equation and show that it is a good approximation to $u''(t)$ for small k: $\frac{u(t+k) - 2u(t) + u(t-k)}{k^2}$
3. Write down a method of lines discretisation of $u_t = -u_{xx} + u^3$ with boundary conditions $u(0, t) = u(1, t) = 0$. From this, write a time-step equation using (a) Forwards Euler and (b) Backwards Euler. How would you solve the equation produced by (b)?
4. Use big O notation to write down how the storage size (in memory) of a dense $N \times N$ matrix changes with respect to N. Consider the sparse L matrix introduced in the lectures: how does its size grow with N?

Quiz

5. Consider the solution of the heat equation $u_t = u_{xx}$ over time, given an initial condition $u(x, 0) = cos(\eta x)$. Discuss the time evolution of $u(x, t)$ for (a) large η, or (b) small η. If you were numerically solving this equation using a forward Euler method, how would the value of η determine (a) the spatial resolution h and (b) the time-step k?
6. Write down a method of lines discretisation of $u_t = u_{xx}$ with Neumann boundary conditions of $u_x(0, t) = u_x(1, t) = 0$
7. Use big O notation to write down how the size of L in d spatial dimensions varies with both N and d.
8. Write down the matrix A generated using the following MATLAB code:

```
v = [2 1 -1 -2 -5];
A = diag(v)
```