BIOENG 145 HW 3

Martin Kim (3034222416

February 16, 2021

1 MLE of Poisson distribution

Starting from the definition of MLE, we can take the logarithm of the whole expression because it is a monotonic and smooth function:

$$\begin{aligned}\hat{\lambda}_{\text{MLE}} &= \arg\max_{\lambda} \mathcal{L}(\lambda; x) = \arg\max_{\lambda} \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} = \arg\max_{\lambda} \ln \Big(\prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \Big) \\ &= \arg\max_{\lambda} \sum_{i=1}^{n} \ln \Big(\frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \Big) = \arg\max_{\lambda} \sum_{i=1}^{n} x_i \ln(\lambda) - \lambda - \ln(x_i!)\end{aligned}$$

Since our objective function is concave, we can find its maximum by taking the derivative of the expression and setting it equal to 0:

$$\frac{d}{d\lambda} \Big(\sum_{i=1}^{n} x_i \ln(\lambda) - \lambda - \ln(x_i!) \Big) = \sum_{i=1}^{n} \frac{x_i}{\lambda} - 1 = -n + \sum_{i=1}^{n} \frac{x_i}{\lambda}$$

$$\Rightarrow \frac{1}{\lambda} \sum_{i=1}^{n} x_i = n \ \Rightarrow \ \lambda = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\Rightarrow \hat{\lambda}_{\text{MLE}} = \frac{1}{n} \sum_{i=1}^{n} x_i \ \Rightarrow \ \text{QED}$$

2 Poisson counts

A. Using the result from the previous section, we see that:

$$\lambda_0 = \frac{1}{2n}\sum_{i=1}^{n} x_i + y_i, \quad \lambda_x = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \lambda_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

C.

$$\mathcal{L}_0(\lambda_0; x, y) = \prod_{i=1}^{n} \frac{\lambda_0^{x_i} e^{-\lambda_0}}{x_i!} \cdot \frac{\lambda_0^{y_i} e^{-\lambda_0}}{y_i!}, \quad \mathcal{L}_\alpha(\lambda_x, \lambda_y; x, y) = \prod_{i=1}^{n} \frac{\lambda_x^{x_i} e^{-\lambda_x}}{x_i!} \cdot \frac{\lambda_y^{y_i} e^{-\lambda_y}}{y_i!}$$

E. Our likelihood ratio is calculated to be 0.2959. The critical value for $p = 0.05$ with one degree of freedom 3.841. Thus, we cannot safely reject the null hypothesis.

`https://people.richland.edu/james/lecture/m170/tbl-chi.html`

Assignment 3 Code

Editing and saving the code:

- Go to **File > Download .ipynb** in order to work using the jupyter suite locally.
- Otherwise, work through Google Colab by: **File > Save a copy in Drive** to edit a personal copy.

Running the second cell (which reads files):

- In Google Colab, ensure that the Assignment 3 files are uploaded to the local runtime using the File icon on the left.
- Locally, ensure that the Assignment 3 files are placed in the same directory as the notebook.

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import poisson
```

In [2]:
```
two = pd.read_csv("q2_set_1.tsv", sep='\t', index_col=0)
```

2. Poisson Counts

2b. Compute Lambdas

In [3]:
```
two_x = two.iloc[0,0:5].to_numpy()
two_y = two.iloc[0,5:].to_numpy()
two_all = two.iloc[0,:].to_numpy()
```

In [4]:
```
lambda_0 = np.average(two_all)
lambda_x = np.average(two_x)
lambda_y = np.average(two_y)
print("lambda_0: " + str(lambda_0))
print("lambda_x: " + str(lambda_x))
print("lambda_y: " + str(lambda_y))
```

```
lambda_0: 5889.4
lambda_x: 5902.6
lambda_y: 5876.2
```

2d. Compute Likelihoods

Read the docs on [scipy.stats.poisson (https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.poisson.html)](https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.poisson.html).

To chain multiple products, you can use [np.prod (https://numpy.org/doc/stable/reference/generated/numpy.prod.html)](https://numpy.org/doc/stable/reference/generated/numpy.prod.html).

In [5]:

```
poisson_0 = poisson(lambda_0)
poisson_x = poisson(lambda_x)
poisson_y = poisson(lambda_y)
likelihood_0 = np.prod([poisson_0.pmf(val) for val in two_all])
likelihood_a = np.prod([poisson_x.pmf(x_i) for x_i in two_x] + [poisso
print("likelihood_0: " + str(likelihood_0))
print("likelihood_a: " + str(likelihood_a))
```

```
likelihood_0: 2.2655243807774852e-24
likelihood_a: 2.6267127365612815e-24
```

2e. Compute Likelihood Ratio

In [6]:

```
ratio = -2*np.log(likelihood_0/likelihood_a)
print("Likelihood ratio: " + str(ratio))
```

```
Likelihood ratio: 0.29585381503238717
```