Roll decay damping

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ABSTRACT

The roll damping of a ship at 0 knots is determined with time series from scale model roll decay tests and CFD simulations using a parameter identification technique (PIT).

INTRODUCTION

The present dataset contains roll decay tests from actual model tests, carried out at SSPA maritime-dynamics-laboratory (www.sspa.se) and also CFD simulations, using a method called FNPF. The roll damping is determined by identifying the roll damping parameters in a prescribed mathematical model for the decaying roll motion. The roll angle time series from a roll decay model test is shown in Fig.1. The analysis for this report can also be found at:

- GitHub: `https://github.com/martinlarsalbert/roll_decay_damping`
- Binder: `https://mybinder.org/v2/gh/martinlarsalbert/roll_decay_damping/HEAD?filepath=reports%2Freport%2F01.1.report.ipynb`

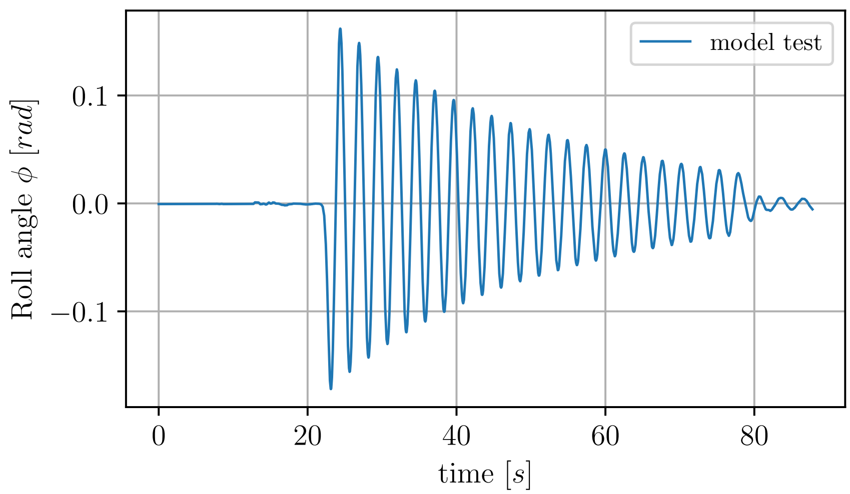

Figure 1: Roll decay time series

The oscillating motion can be described by a spring-mass-damper system as seen in Fig.2.

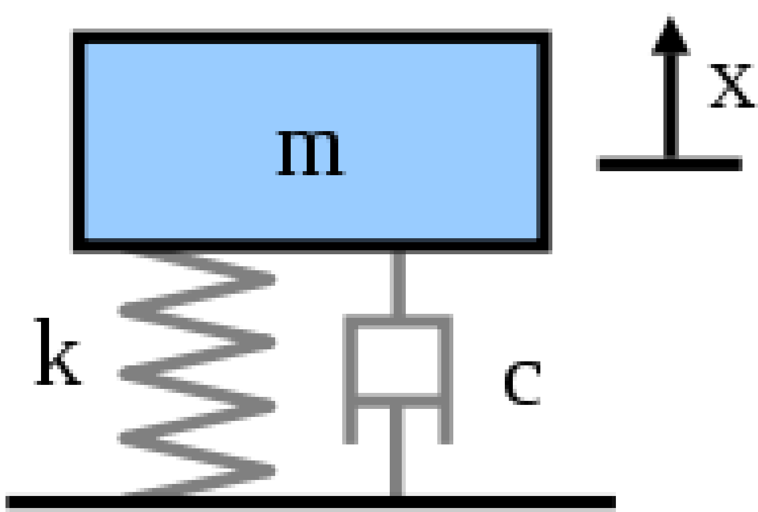

Figure 2: Spring-mass-damper system

This system can me described as the following ordinary differential equation:

$$A_{44}\ddot{\phi} + \mathrm{B}_{44}\left(\dot{\phi}\right) + \mathrm{C}_{44}\left(\phi\right) = 0 \tag{1}$$

Where $B_{44}(\dot{\phi})$ and $C_{44}(\phi)$ are the damping and stiffness models. A cubic model can be obtained by using cubic damping:

$$\mathrm{B}_{44}\left(\dot{\phi}\right) = B_1\dot{\phi} + B_2\left|\dot{\phi}\right|\dot{\phi} + B_3\dot{\phi}^3 \tag{2}$$

And cubic stiffness model:

$$\mathrm{C}_{44}\left(\phi\right) = C_1\phi + C_3\phi^3 + C_5\phi^5 \tag{3}$$

The total equation is then written:

$$A_{44}\ddot{\phi} + \left(B_1 + B_2\left|\dot{\phi}\right|\right)\dot{\phi} + \left(C_1 + C_3\phi^2 + C_5\phi^4\right)\phi = 0 \tag{4}$$

This equation does not have one unique solution however. If all parameters would be multiplied by a factor k these parameters would also yield as a solution to the equation. All parameters are therefore divided by the total inertia A_{44} (including added mass inertia), replacing the parameters with new normalized parameters such as: $B_{1A} = B_1/A_{44}$. The equation is now rewritten with these new parameters which have unique solutions:

$$\left(B_{1A} + B_{2A}\left|\dot{\phi}\right|\right)\dot{\phi} + \left(C_{1A} + C_{3A}\phi^2 + C_{5A}\phi^4\right)\phi + \ddot{\phi} = 0 \tag{5}$$

DATA

The data used in this study is described in Tab.1. There is one result with a pure FNPF simulation at 0 knots. For model test results, two tests are available at 0 knots and one test at 15.5 knots. There is also a result at 15.5 with a hybrid method, where semi empirical viscosity has been injected into the FNPF calculations.

Table 1: Data files

file	data file	Ship speed $[kts]$	Method
1	fnpf kvlcc2 rolldecay 0kn.csv	0.0	FNPF
2	model test 21337.csv	0.0	model test
3	model test 21338.csv	0.0	model test
4	model test 21340.csv	15.5	model test
5	fnpf kvlcc2 rolldecay 15-5kn ikeda dev.csv	15.5	hybrid

Fig. 3 shows the roll angle time series for all the tests. It can also be seen that test 1 and 5 also have time series for the roll angle velocity and acceleration from the conducted FNPF simulations. For the model test (2,3,4) velocities and accelerations were not measured during the tests.

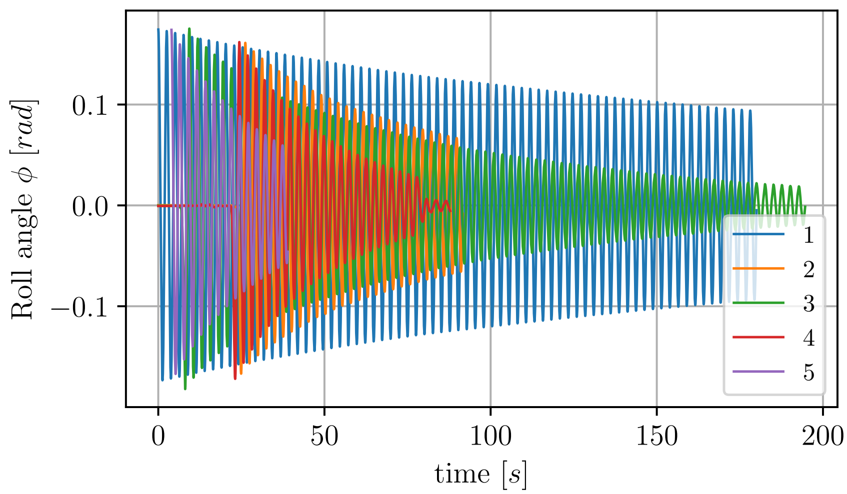

Figure 3: All tests

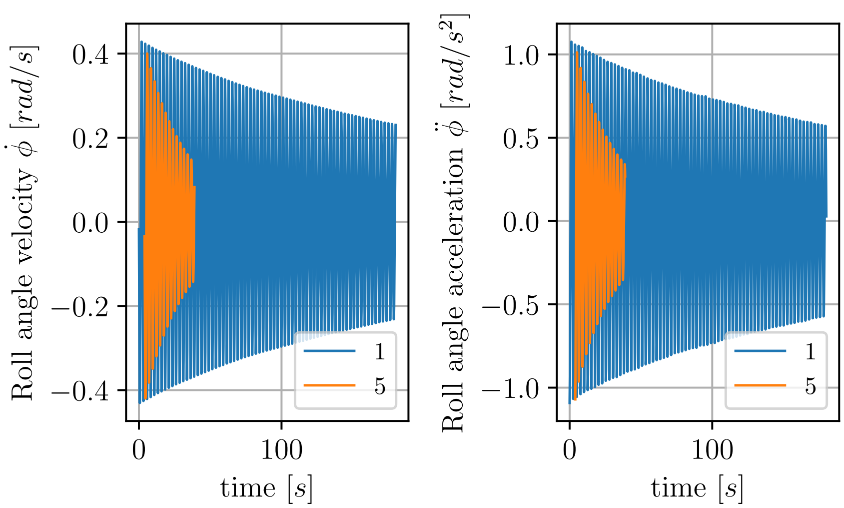

Figure 4: Tests with velocities and accelerations

ANALYSIS

FNPF

The FNPF results have the benifit of having all the three states: ϕ, $\dot{\phi}$ and $\ddot{\phi}$. This means that these time series can be inserted into the differential equation (Eq.5) and the parameters of the model can be estimated using linear regression with Ordinary Least Square method (OLS), solving the following regression:

$$y = X \cdot \beta + \epsilon \tag{6}$$

where:

- y is the dependent variable (also called "label").
- β is a vector with the regressed parameters.
- X is a matrix containing the independent variables (also called "features").

The roll decay equation can be expressed as a linear regression with

- y : the roll angle acceleration $\ddot{\phi}$
- β : contains all the parameters : B_1, B_2, C_1...
- X : contains all the time varying features such as: $|\dot{\phi}|\dot{\phi}$ etc.

The roll acceleration is put on the left hand side:

$$\begin{aligned} -\ddot{\phi} = B_{1A}\dot{\phi} + B_{2A}\left|\dot{\phi}\right|\dot{\phi} + B_{3A}\dot{\phi}^3 + C_{1A}\phi + C_{3A}\phi^3 \\ +C_{5A}\phi^5 \end{aligned} \tag{7}$$

The equation for the acceleration Eq.7 can now be rewritten as a linear regression (see in Eq.6) where β, X and y contain the following:

$$\beta = \begin{bmatrix} B_{3A} \\ C_{3A} \\ C_{5A} \\ B_{2A} \\ B_{1A} \\ C_{1A} \end{bmatrix} \tag{8}$$

$$X = \begin{bmatrix} \dot{\phi}^3 & \phi^3 & \phi^5 & \left|\dot{\phi}\right|\dot{\phi} & \dot{\phi} & \phi \end{bmatrix} \tag{9}$$

$$y = -\ddot{\phi} \tag{10}$$

The coefficients determined with Ordinary Least Square fit is shown in Tab.2. The mean value of these coefficients are presented togehter with 5% confidence level intervalls.

Table 2: Parameters estimation cubic model (5% confidence)

coeff	mean	P_{value}	$conf_{lower}$	$conf_{higher}$
B_{1A}	0.016	0.0	0.014	0.018
B_{2A}	-0.062	0.0	-0.076	-0.047
B_{3A}	0.098	0.0	0.072	0.124
C_{1A}	6.116	0.0	6.114	6.118
C_{3A}	-5.522	0.0	-5.807	-5.236
C_{5A}	254.093	0.0	244.923	263.264

Simulation

Fig.5 shows a simulation with the regressed parameters together with the original data from the FNPF simulations. The simulations are conducted by solving the intial value problem by Runge Kutta integration of Eq.7.

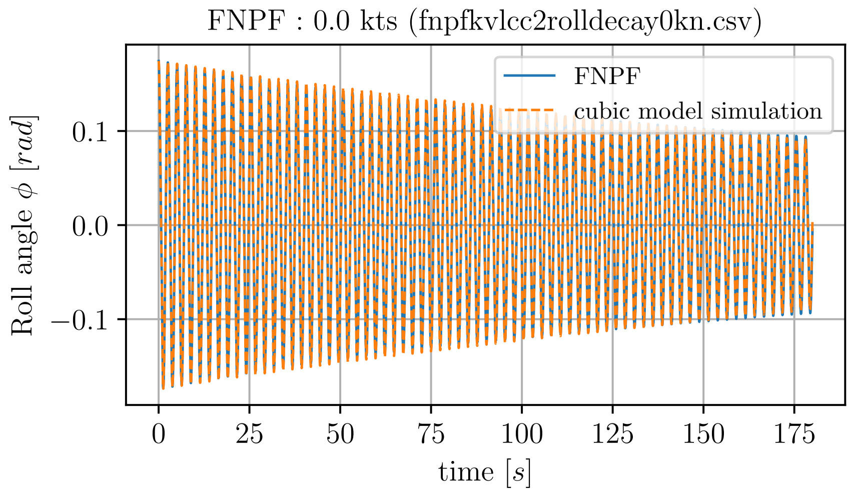

Figure 5: Simulation of roll decay test with parameters from the cubic model.

The P_{value} is shown for each parameter in Tab.2 testing the null hypothesis H_0 for each parameter. H_0 tests the probability that a certain parameter would be 0 (so that it can be removed). For this case the P_{value} is close to 0 for all parameters, which means that none of them should be removed. It can however be noted that the lower and upper limits of the 5% confidence intervalls differs quite a bit. In the following, a simpler linear model will therefore also be investigated, to see if there is a need to reduce the complexity of this model. For a linear model, the differential equation for the roll motions is reduced to:

$$-\ddot{\phi} = B_{1A}\dot{\phi} + C_{1A}\phi \tag{11}$$

The β and X is now expressed as:

$$\beta = \begin{bmatrix} B_{1A} \\ C_{1A} \end{bmatrix} \tag{12}$$

$$X = \begin{bmatrix} \dot{\phi} & \phi \end{bmatrix} \tag{13}$$

The parameter estimation for the linear model is shown in Tab.3. It can be noted that the upper and lower limits of the confidence intervalls are now much closer to each other. A simulation with this linear model is shown in Fig.6.

Table 3: Parameters estimation linear model (5% confidence)

coeff	mean	P_{value}	$conf_{lower}$	$conf_{higher}$
B_{1A}	0.007	0.0	0.007	0.007
C_{1A}	6.101	0.0	6.1	6.101

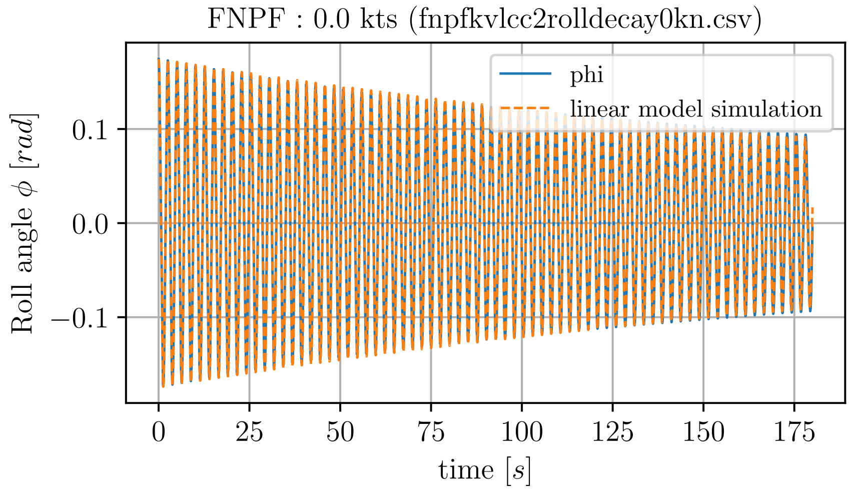

Figure 6: Simulation of roll decay test with parameters from the linear model.

The coefficient of determination R^2 is very similar between the cubic (0.997) and the linear model (0.989), when comparing simulated roll signal with the corresponding data from FNPF.

Model tests

Fig.7 shows the roll signal from one of the roll decay model tests. A sample from this signal as indicated by the “cut” in Fig.7 is used to regress the roll damping. The roll velocity and acceleration are however missing from this model test data. This means that the regression approach as used for the FNPF data cannot be used directly. Instead the velocity and acceleration are first estimated using numerical differentiation. The acceleration and velocity signals estimated in this way are very noisy as shown in Fig.8, where the roll angle measurement noise is included in the differentiation. The roll signal has been low pass filtered prior to the differentiation. The signal has been filtered using a linear digital filter twice: once forward and once backwards. This “filt-filt” approach ensures that the filter is not introducing a phase lag onto the signal, which is crucial for the present regression. A 5th order digital filter with 15 Hz cutoff frequency was used for the filtering. Results from a regression with this noisy data is shown in Fig.9, where the numerical acceleration and a 5% confidence intervall is also shown. Fig.10 shows the residual distribution. This distribution seems to be approximatelly normal distributed, which is confirmed when also looking at the normal probability plot in Fig.11.

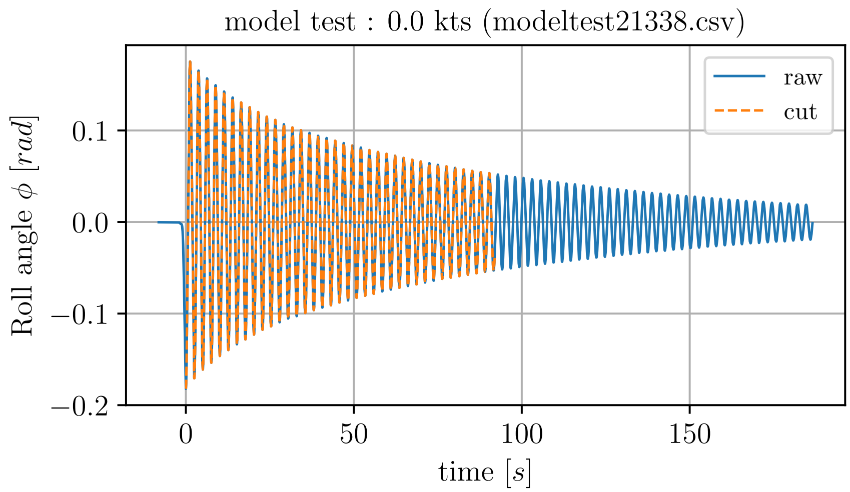

Figure 7: Roll decay model test

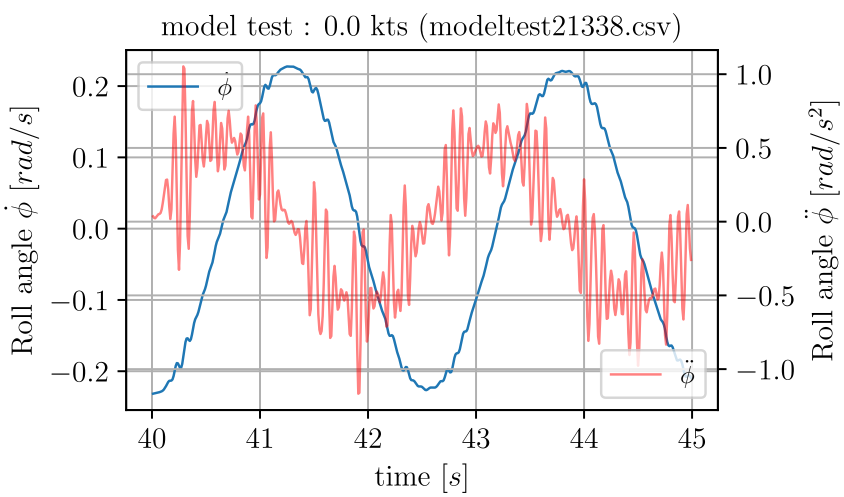

Figure 8: Numerical gradient of roll velocity and acceleration

Table 4: Parameters estimation cubic model

coeff	mean	P_{value}	$conf_{lower}$	$conf_{higher}$
B_{1A}	0.009	0.913	-0.149	0.166
B_{2A}	0.132	0.843	-1.177	1.441
B_{3A}	-0.133	0.918	-2.65	2.385
C_{1A}	6.046	0.0	5.887	6.205
C_{3A}	7.441	0.583	-19.155	34.037
C_{5A}	-88.159	0.847	-981.645	805.328

Table 5: Parameters estimation linear model

coeff	mean	P_{value}	$conf_{lower}$	$conf_{higher}$
B_{1A}	0.032	0.006	0.009	0.055
C_{1A}	6.115	0.0	6.059	6.172

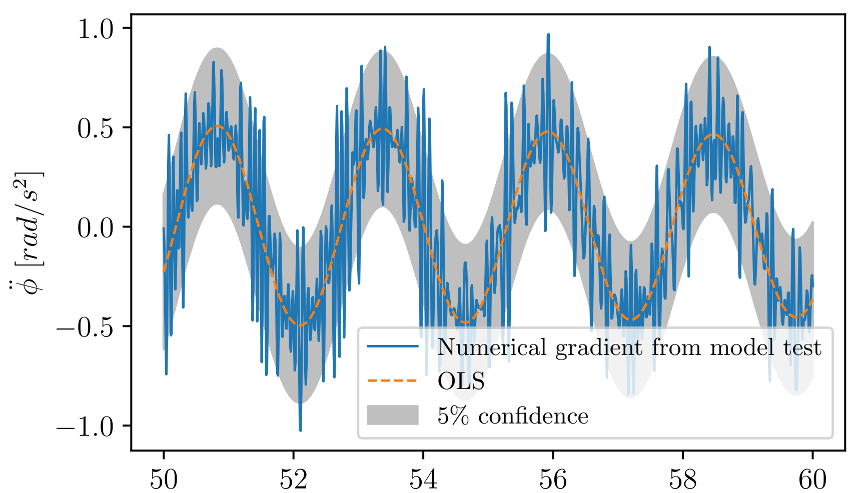

Figure 9: Roll acceleration from numerical gradient and OLS regression

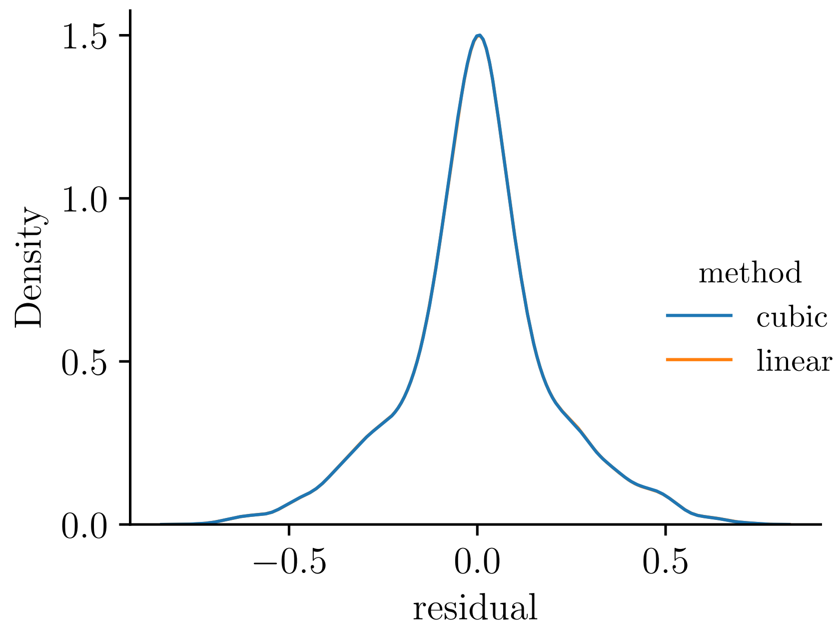

Figure 10: Roll acceleration residual of linear model

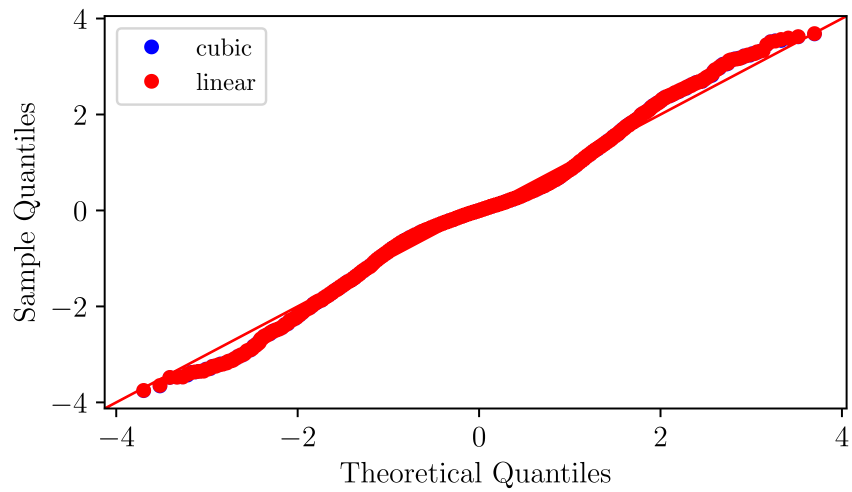

Figure 11: Residual Normal probability plot

The coefficient of determination R^2 is slightly higher for the cubic model (0.997) compared to the the linear model (0.982), when comparing simulated roll signal with the corresponding data from the model test.

Validation

The other model test at 0 knots (see Fig.12) is used for validation of the regression models.

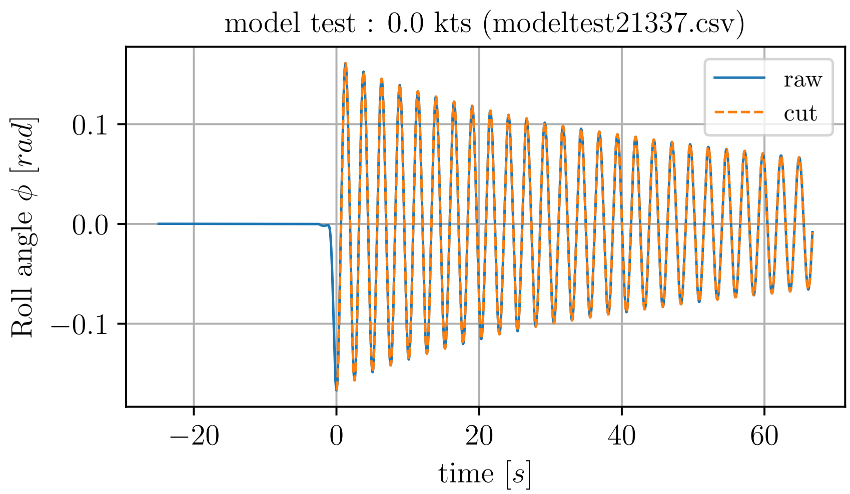

Figure 12: The other roll decay model test at 0 knots

Fig.13 shows a comparison with this model test and simulations with regressed parameters from the first model test.

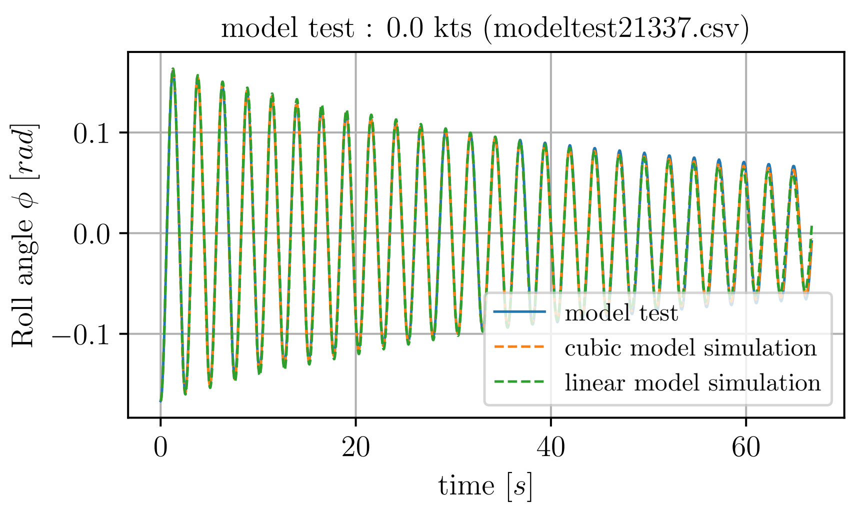

Figure 13: The other roll decay model test compared with corresponding simulations with linear and cubic models regressed from the first model test.

The coefficient of determination R^2 is similar for the cubic model (0.998) compared to the the linear model (0.992), when comparing simulated roll signal with the corresponding data from the model test.

CONCLUSIONS

The parameters in a cubic and a linear roll decay motion model have been estimated by fitting a linear regression with ordinary least square fit to roll decay tests time series at 0 knots ship speed. The regression was validated by comparing simulated roll signals (including the regressed parameters) with the original roll signals. The coefficient of determination R^2 was slightly higher for the cubic model (suggesting higher accuracy) compared to the linear model for the investigation of the FNPF data. For the model test data, a numerical differentiation was used to estimate the roll velocity and acceleration, as these signals were otherwise missing from the model test data. The numerical estimates for these signals were unfortunatelly very noisy. The residuals from the fitted models with these noisy signals was however found to be normal distributed. These residuals most likely origins from a normal distributed measurement noise. The noisy signals gave a large spread between the lower and upper limits of the regressed parameter confidence intervalls. But since the residuals were normal distributed, the mean values of these parameter estimations gave a high accuracy when evaluating with simulations. For the first model test, which was a very long test in time, the cubic model gave a bit better accuracy. Also using the regressed parameters to simulate a complete new dataset from the second model test (a test not seen by the regression) gave good accuracy for the two models. For a shorter time span and amplitude span the linear model gives good accuracy. For longer time and amplitude spans the cubic model seems to be a better alternative. The parameter confidence intervalls of the cubic model have larger spreadings, which will make direct comparison of parameter values, for instance between two different ships, or two different speeds more unreliable.