

IBM Research Data Science Course

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Agenda

Neural Networks

- Weights and Bias
- Activation Functions (Non-Linearity)
- Data Space Transformation
- Keras Code

High School Mathematics

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Self Aware Artificial Intelligence to destroy Humanity

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Looking at Data

Sepal Length	Sepal Width	Petal Length	Petal Width	Flower
4.9	3.0	1.4	0.2	Iris-Setosa
6.3	2.5	5	1.9	Iris-Virginica
5.7	2.8	4.5	1.3	Iris-Versicolor
6.3	3.3	4.7	1.6	Iris-Versicolor
4.7	3.2	1.3	0.2	Iris-Setosa
6.5	3	5.2	2	Iris-Virginica

Looking at Data

4-dimension Vector
Input

Sepal Length	Sepal Width	Petal Length	Petal Width	Flower
4.9	3.0	1.4	0.2	Iris-Setosa
6.3	2.5	5	1.9	Iris-Virginica
5.7	2.8	4.5	1.3	Iris-Versicolor
6.3	3.3	4.7	1.6	Iris-Versicolor
4.7	3.2	1.3	0.2	Iris-Setosa
6.5	3	5.2	2	Iris-Virginica

Looking at Data

4-dimension Vector Input

Sepal Length	Sepal Width	Petal Length	Petal Width
4.9	3.0	1.4	0.2
6.3	2.5	5	1.9
5.7	2.8	4.5	1.3
6.3	3.3	4.7	1.6
4.7	3.2	1.3	0.2
6.5	3	5.2	2

Flower

Iris-Setosa
Iris-Virginica
Iris-Versicolor
Iris-Versicolor
Iris-Setosa
Iris-Virginica

1
3
2
2
1
3

1-dimension Vector Output

Looking at Data

Sepal Length	Sepal Width	Petal Length	Petal Width
4.9	3.0	1.4	0.2
6.3	2.5	5	1.9
5.7	2.8	4.5	1.3
6.3	3.3	4.7	1.6
4.7	3.2	1.3	0.2
6.5	3	5.2	2

Flower
Iris-Setosa
Iris-Virginica
Iris-Versicolor
Iris-Versicolor
Iris-Setosa
Iris-Virginica

1
3
2
2
1
3

{ 1 , 2 , 3 }
1 0 0
0 0 1
0 1 0
0 1 0
1 0 0
0 0 1

3-dimension
Vector Output

Another way to vectorise labels (One-Hot)

How do we convert (transform) one vector to another?



Weights

$$\begin{array}{c} \text{(1 x 4)} \\ \boxed{\begin{array}{|c|c|c|c|} \hline 4.9 & 3.0 & 1.4 & 0.2 \\ \hline \end{array}} \\ \text{(0,1) (0,2) (0,3) (0,4)} \end{array} \times \begin{array}{c} \text{(4 x 3)} \\ \boxed{\begin{array}{|c|c|c|} \hline (0,0) & (0,1) & (0,2) \\ \hline (1,0) & (1,1) & (1,2) \\ \hline (2,0) & (2,1) & (2,2) \\ \hline (3,0) & (3,1) & (3,2) \\ \hline \end{array}} \end{array} = \boxed{\begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array}}$$

Weights

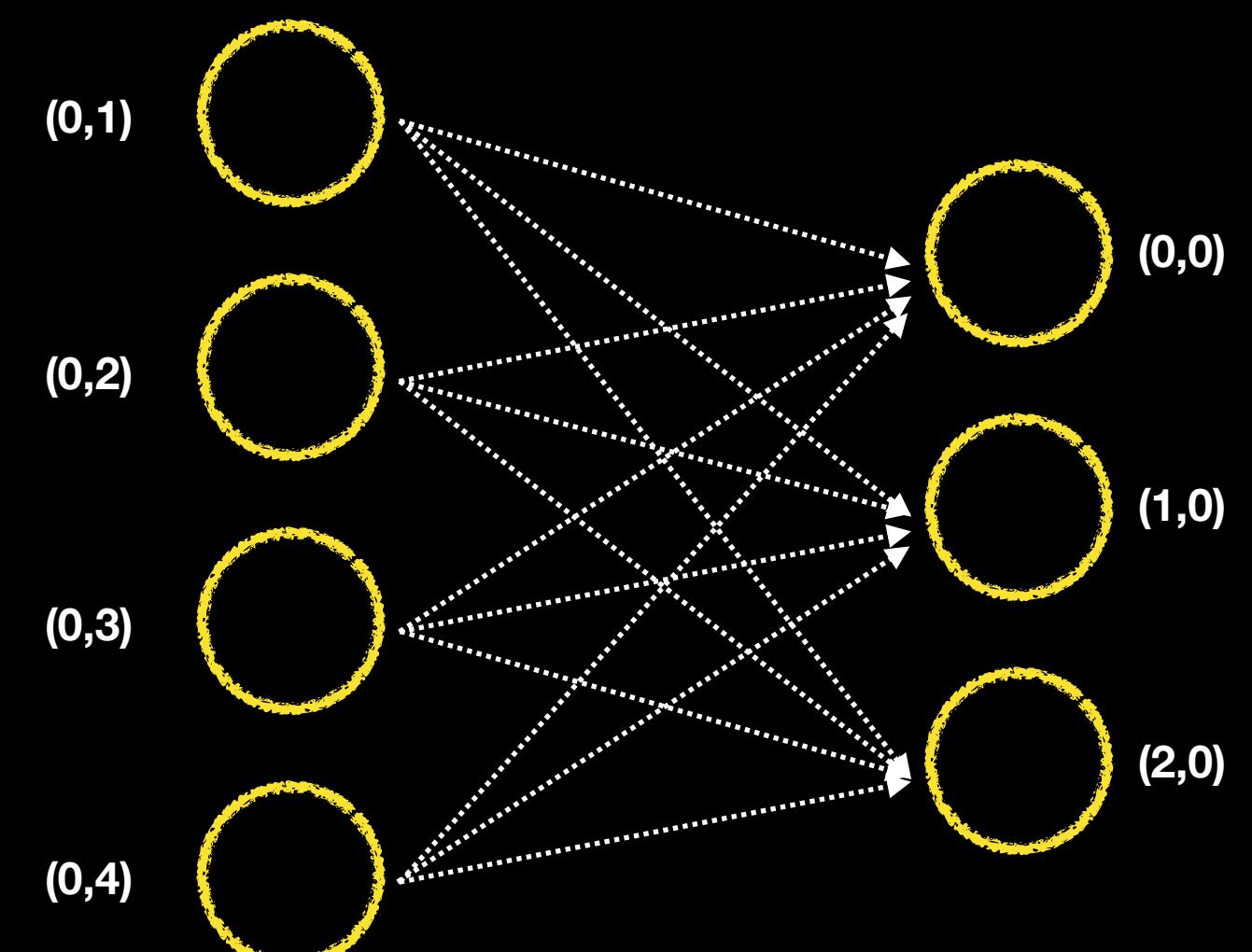
$$\begin{matrix} & & (1 \times 4) \\ \boxed{4.9 \quad 3.0 \quad 1.4 \quad 0.2} & & \\ (0,1) & (0,2) & (0,3) & (0,4) \end{matrix}$$

X

	(0,0)	(0,1)	(0,2)
	(1,0)	(1,1)	(1,2)
	(2,0)	(2,1)	(2,2)
	(3,0)	(3,1)	(3,2)

(4 x 3)

$$= \boxed{\begin{matrix} 1 & 0 & 0 \end{matrix}} \quad (1 \times 3)$$



Input

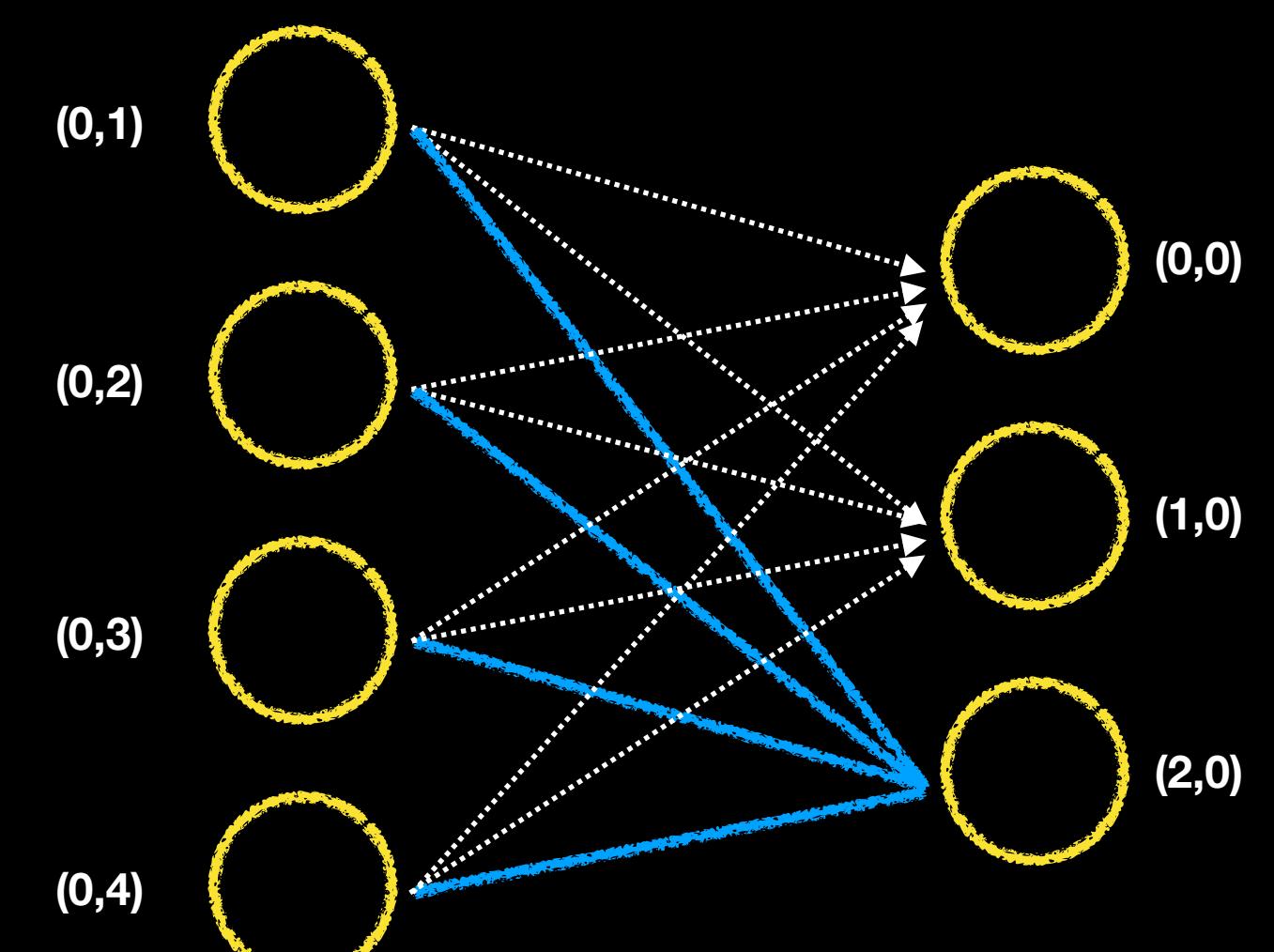
Output

Weights

(1 x 4)			
4 . 9	3 . 0	1 . 4	0 . 2
(0,1)	(0,2)	(0,3)	(0,4)

X

$$\begin{matrix} & & (4 \times 3) \\ & & \\ \begin{matrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \\ (2,0) & (2,1) & (2,2) \\ (3,0) & (3,1) & (3,2) \end{matrix} & = & \begin{matrix} (1 \times 3) \\ \boxed{1 \quad 0 \quad 0} \end{matrix} \end{matrix}$$



Input

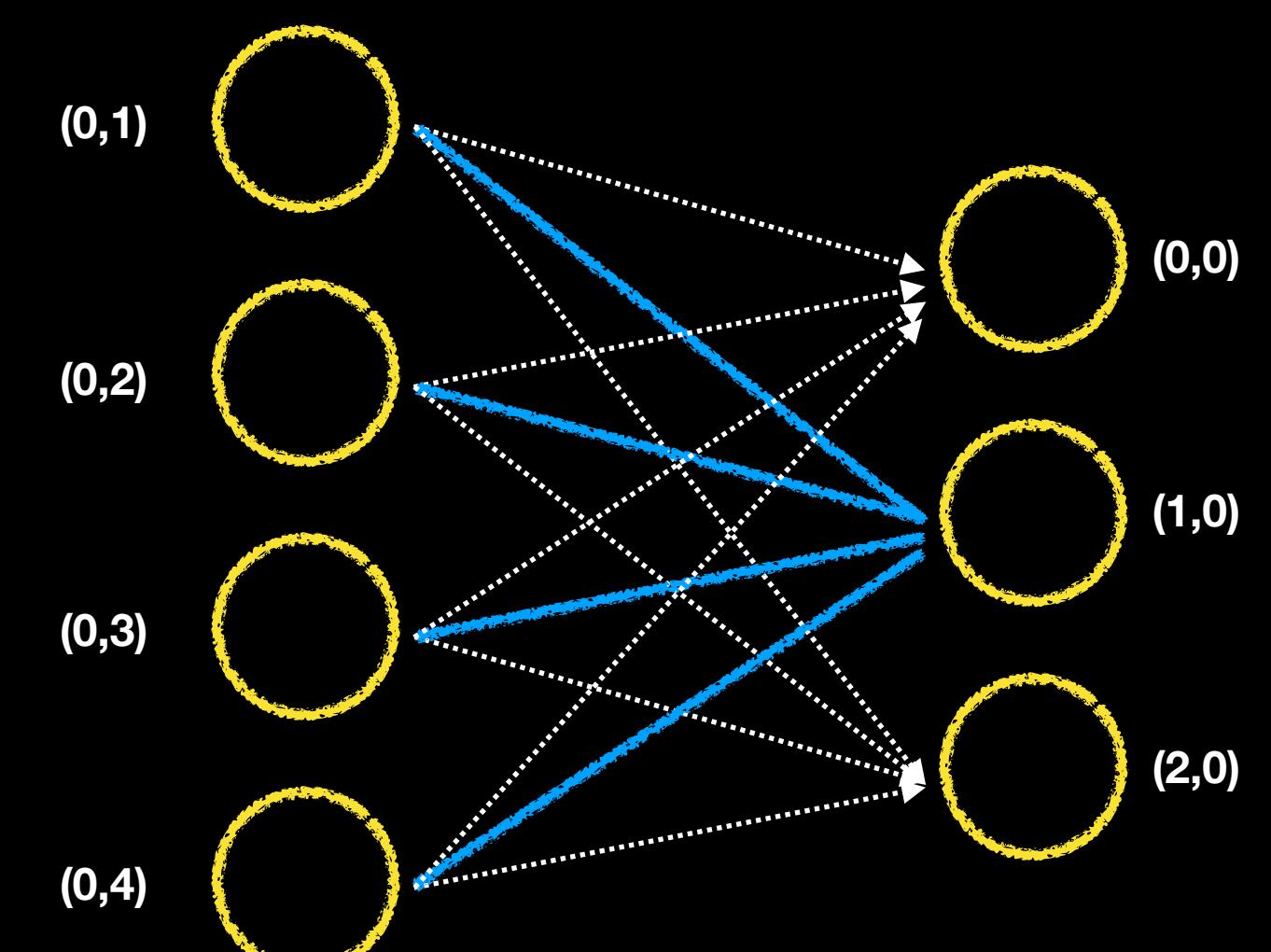
Output

Weights

(1 x 4)			
4 . 9	3 . 0	1 . 4	0 . 2
(0,1)	(0,2)	(0,3)	(0,4)

X

$$\begin{matrix} & & (4 \times 3) \\ & & \\ \begin{matrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \\ (2,0) & (2,1) & (2,2) \\ (3,0) & (3,1) & (3,2) \end{matrix} & = & \begin{matrix} (1 \times 3) \\ 1 & 0 & 0 \end{matrix} \end{matrix}$$



Input

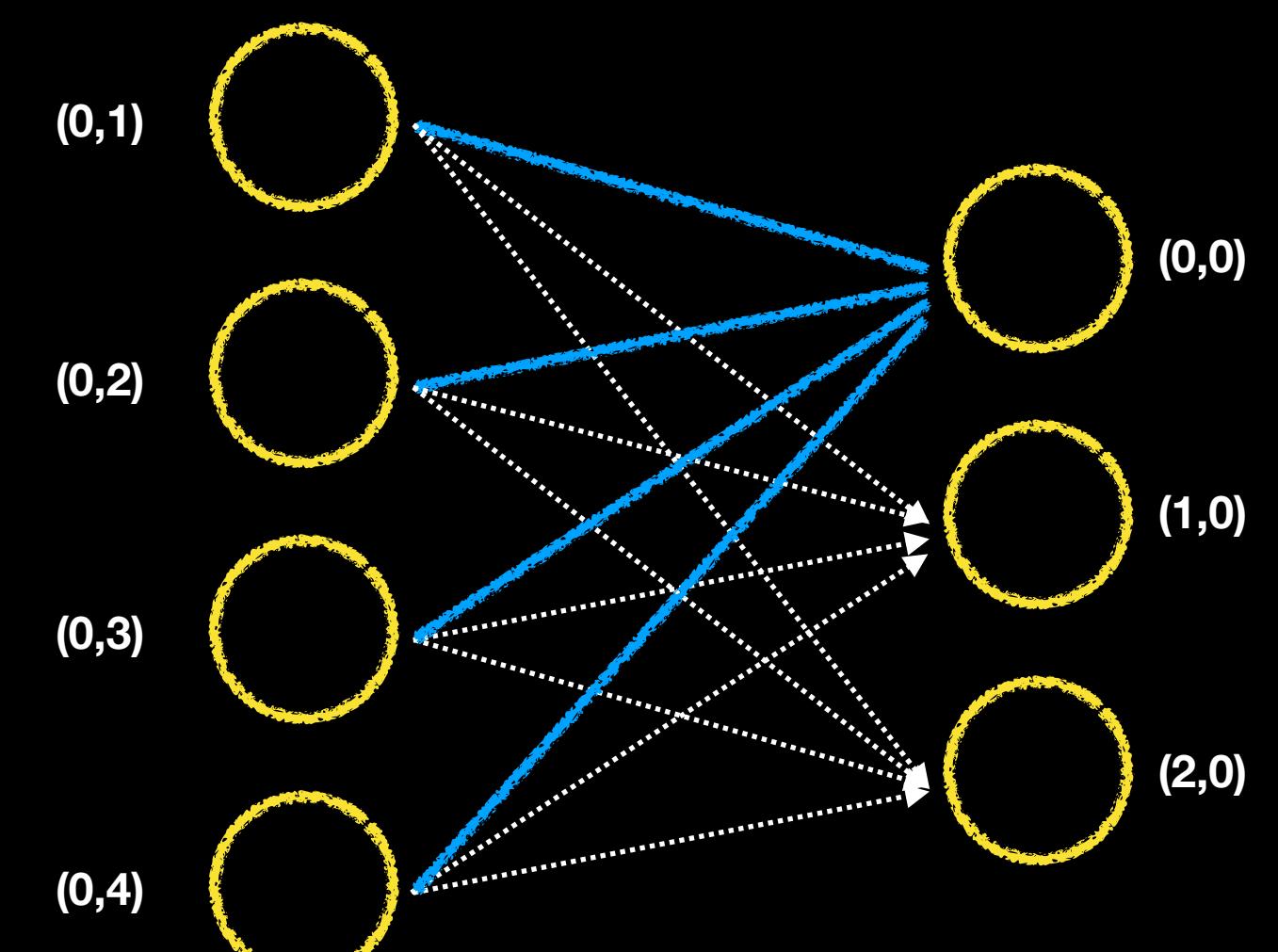
Output

Weights

(1 x 4)			
4 . 9	3 . 0	1 . 4	0 . 2
(0,1)	(0,2)	(0,3)	(0,4)

X

$$\begin{matrix} & & (4 \times 3) \\ \begin{array}{|c|c|c|} \hline (0,0) & (0,1) & (0,2) \\ \hline (1,0) & (1,1) & (1,2) \\ \hline (2,0) & (2,1) & (2,2) \\ \hline (3,0) & (3,1) & (3,2) \\ \hline \end{array} & = & \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array} \end{matrix}$$



Input

Output

Weights

(1 x 4)			
4 . 9	3 . 0	1 . 4	0 . 2
(0,1)	(0,2)	(0,3)	(0,4)

X

(0,0)	(0,1)	(0,2)
(1,0)	(1,1)	(1,2)
(2,0)	(2,1)	(2,2)
(3,0)	(3,1)	(3,2)

(4 x 3)

1	0	0

(1 x 3)

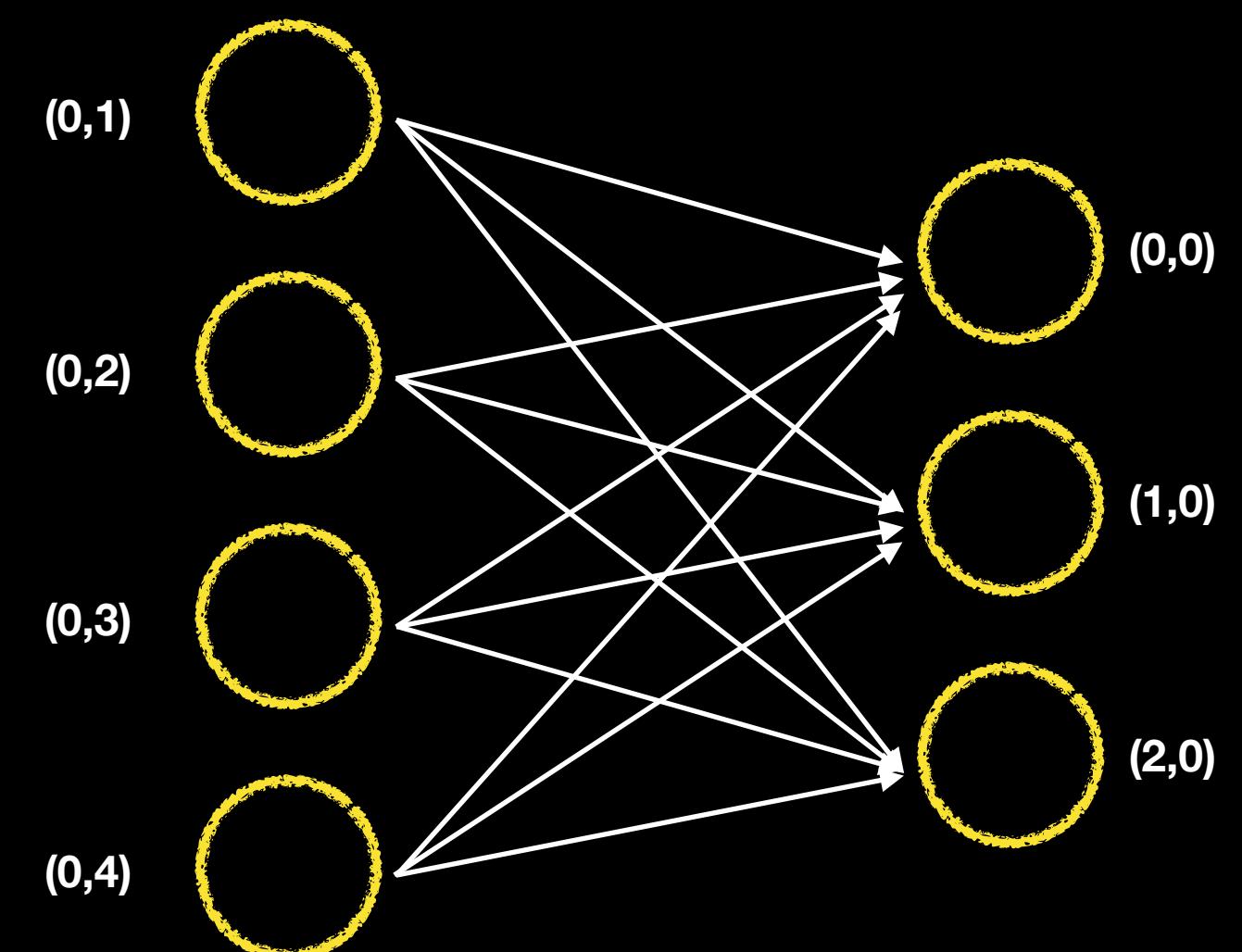
Input

.

Weights

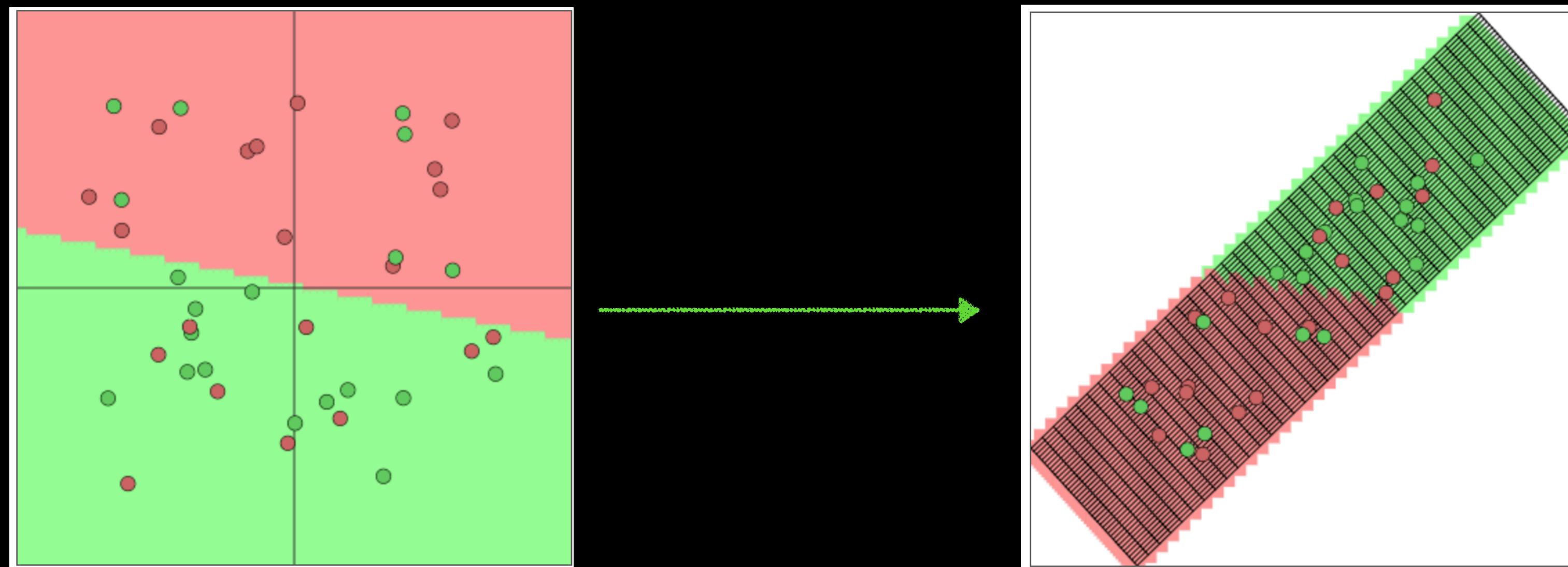
=

Output



Weights

**Another way to think about it : Which is the best transformation such that it is easy to draw a line between classes?
(Linearly Separable)**



Bias

$$\text{Input} \quad \boxed{\begin{matrix} 4.9 & 3.0 & 1.4 & 0.2 \end{matrix}} \quad (1 \times 4)$$

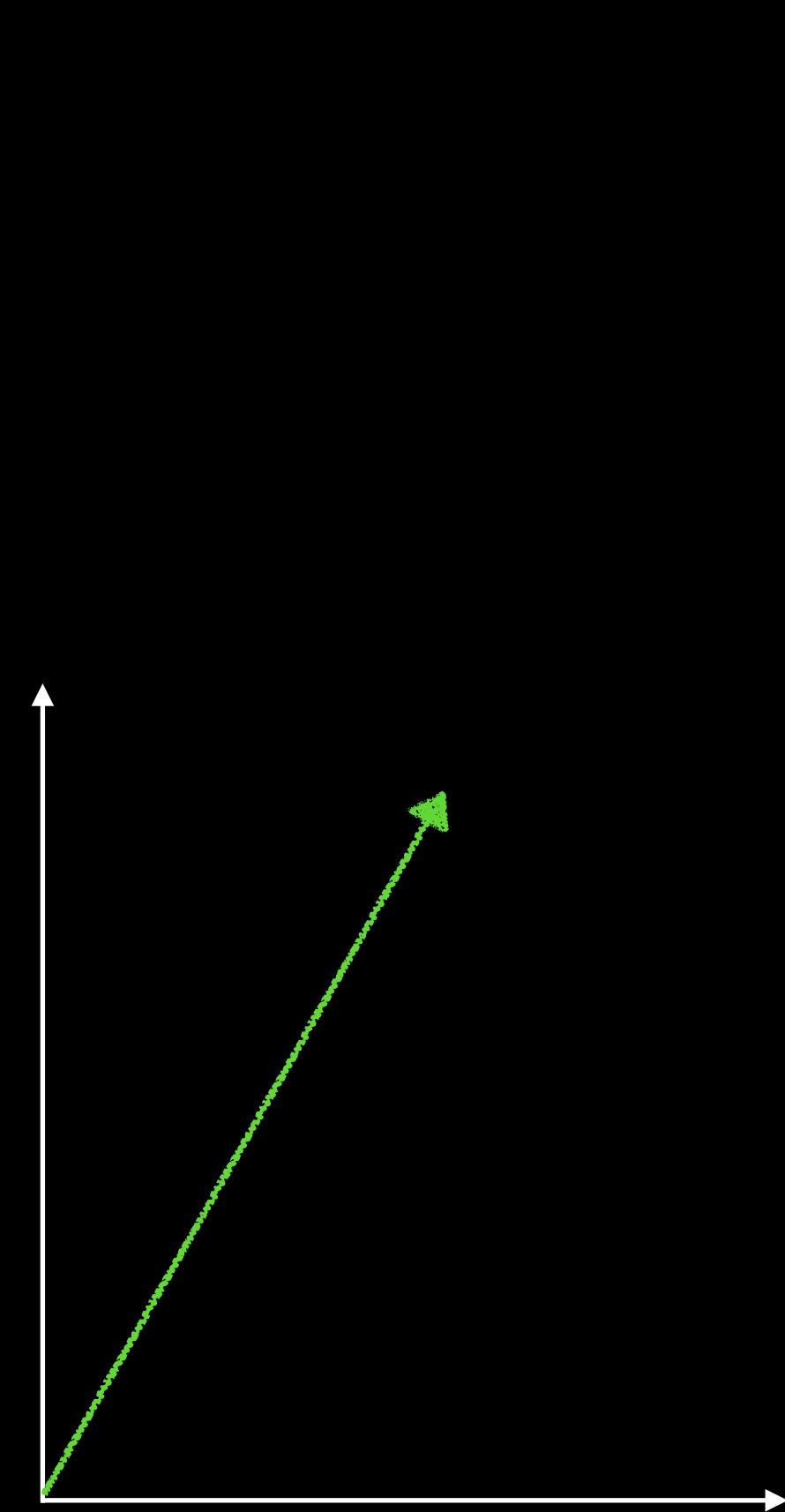
X

(0,0)	(0,1)	(0,2)
(1,0)	(1,1)	(1,2)
(2,0)	(2,1)	(2,2)
(3,0)	(3,1)	(3,2)

(4 x 3)

$$= \boxed{\begin{matrix} 1 & 0 & 0 \end{matrix}} \quad (1 \times 3)$$

Output



$$y = 2x$$

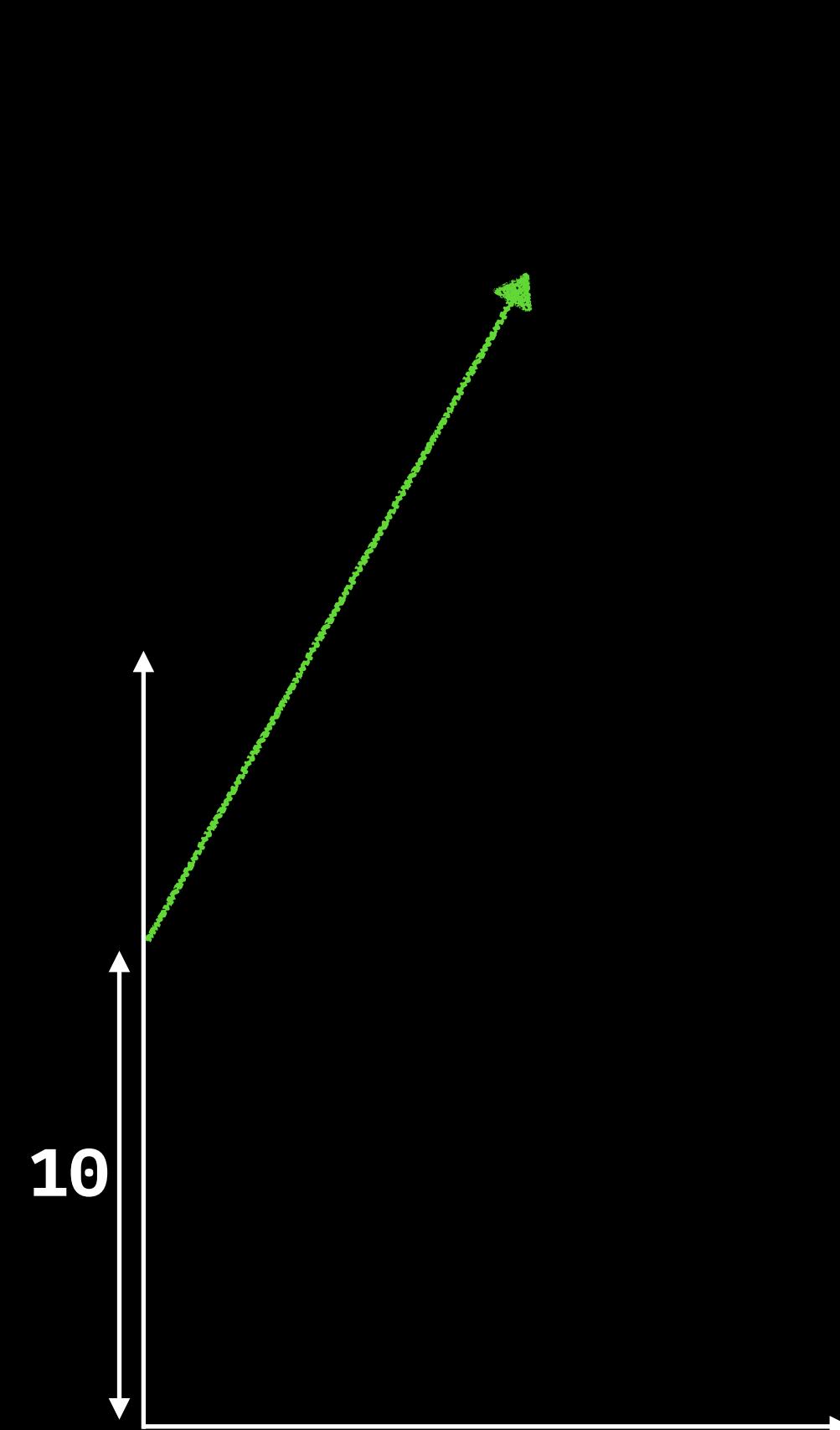
$$y = mx + c$$

Bias

$$\text{Input} \quad \boxed{\begin{matrix} 4.9 & 3.0 & 1.4 & 0.2 \\ (0,1) & (0,2) & (0,3) & (0,4) \end{matrix}} \quad \mathbf{x} \quad (1 \times 4)$$

$$\text{Weights} \quad \boxed{\begin{matrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \\ (2,0) & (2,1) & (2,2) \\ (3,0) & (3,1) & (3,2) \end{matrix}} \quad (4 \times 3)$$
$$= \boxed{\begin{matrix} 1 & 0 & 0 \end{matrix}} \quad (1 \times 3)$$

= Output



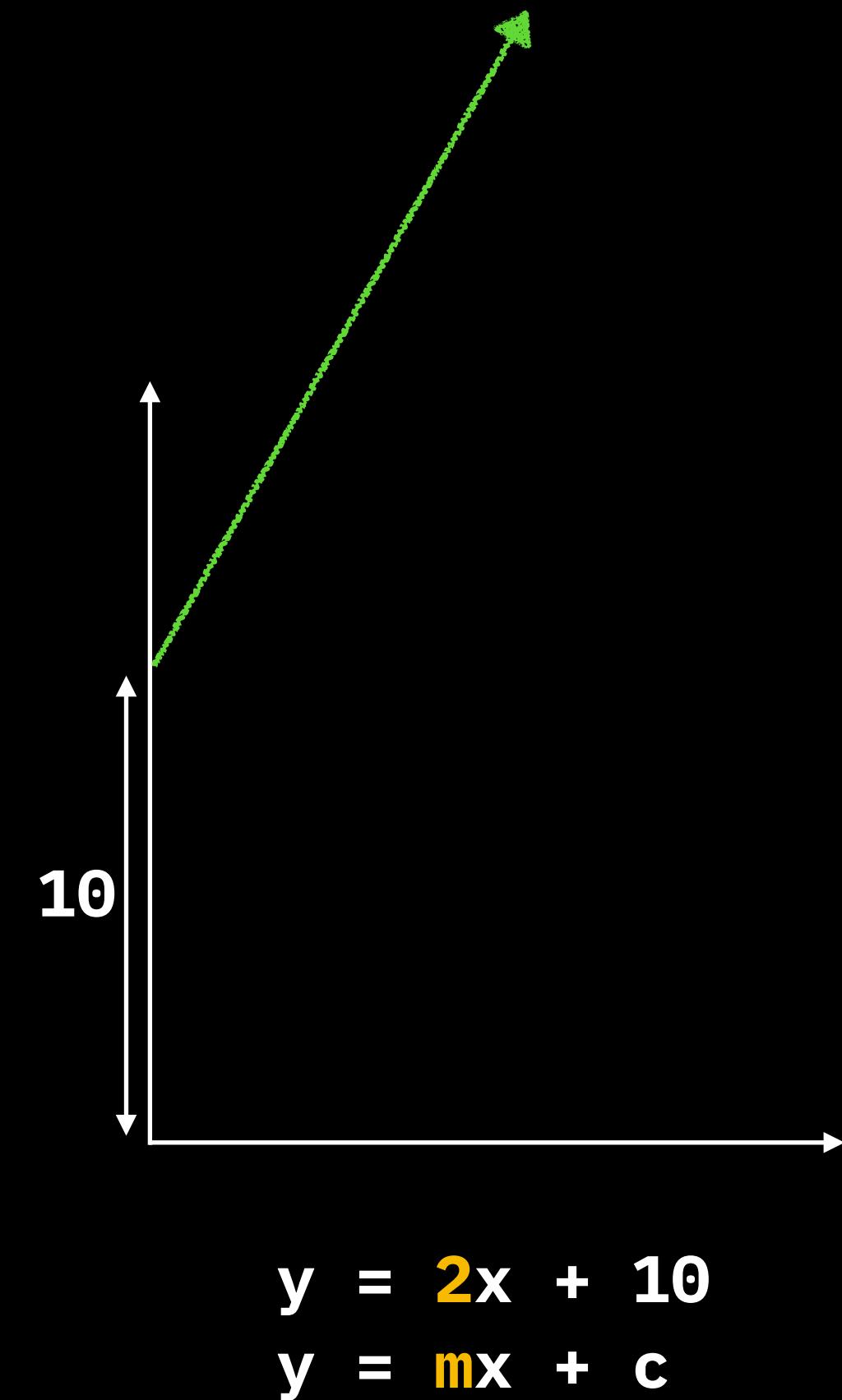
$$? \\ y = mx + c$$

Bias

$$\text{Input} \quad \boxed{\begin{matrix} 4.9 & 3.0 & 1.4 & 0.2 \\ (0,1) & (0,2) & (0,3) & (0,4) \end{matrix}} \quad \mathbf{x} \quad (1 \times 4)$$

$$\text{Weights} \quad \boxed{\begin{matrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \\ (2,0) & (2,1) & (2,2) \\ (3,0) & (3,1) & (3,2) \end{matrix}} \quad (4 \times 3)$$
$$= \boxed{\begin{matrix} 1 & 0 & 0 \end{matrix}} \quad (1 \times 3)$$

= Output

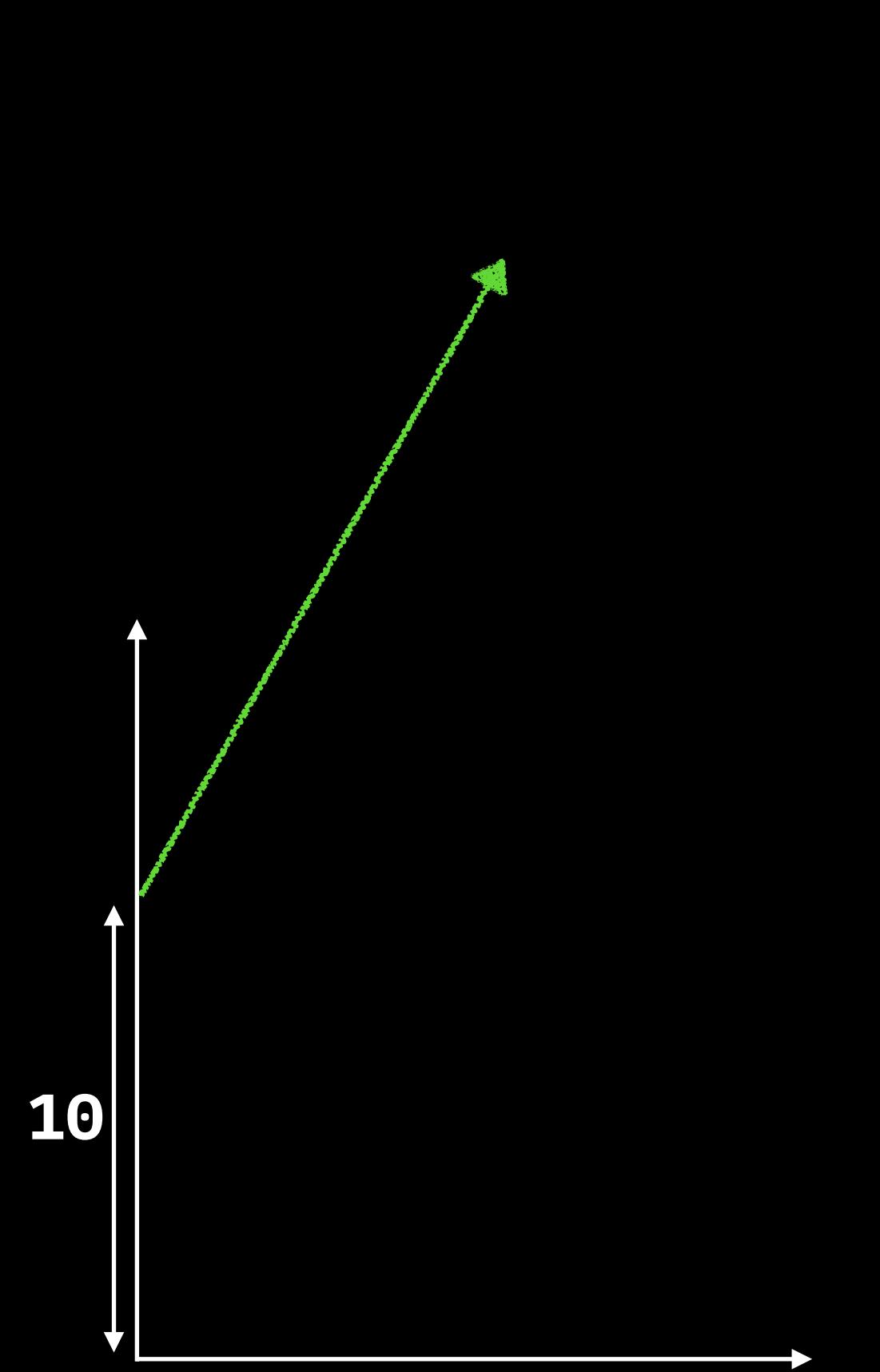


Bias

$$\text{Input} \quad \boxed{\begin{matrix} 4.9 & 3.0 & 1.4 & 0.2 \\ (0,1) & (0,2) & (0,3) & (0,4) \end{matrix}} \quad \mathbf{X} \quad (1 \times 4)$$

$$\text{Weights} \quad \boxed{\begin{matrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \\ (2,0) & (2,1) & (2,2) \\ (3,0) & (3,1) & (3,2) \end{matrix}} \quad (4 \times 3) = \boxed{\begin{matrix} 1 & 0 & 0 \end{matrix}} \quad (1 \times 3)$$

= Output



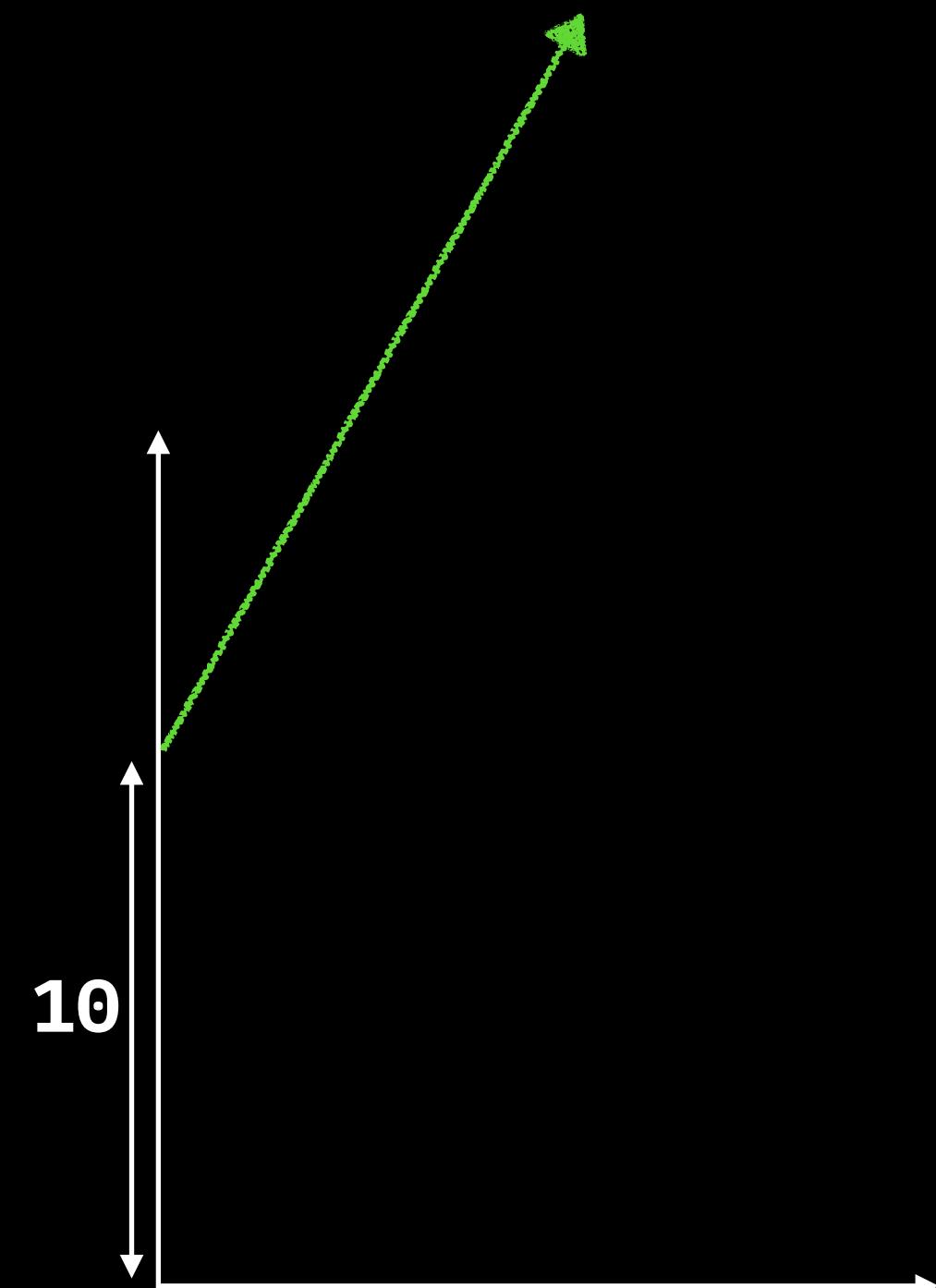
$$y = 2x + 10$$
$$y = mx + c \quad \text{Bias}$$

Bias

$$\begin{matrix} & \text{(1 x 4)} \\ \boxed{\begin{matrix} 4.9 & 3.0 & 1.4 & 0.2 \end{matrix}} & \times \end{matrix}$$

$$\begin{matrix} & \text{(4 x 3)} \\ \boxed{\begin{matrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \\ (2,0) & (2,1) & (2,2) \\ (3,0) & (3,1) & (3,2) \end{matrix}} & = \boxed{\begin{matrix} 1 & 0 & 0 \end{matrix}} \end{matrix}$$

Input * Weights + Bias = Activations



$$\begin{aligned} y &= 2x + 10 \\ y &= mx + c \end{aligned}$$

Training Process (Backpropagation)

Step 1 : Initially weights and bias are randomly assigned.

Step 2 : Vector -> Matrix Multiplication -> Some Output (Vector)

Step 3 : Compare with the actual value (Label)

Step 4 : See the difference and pass it back to all the layers behind it.

Step 5 : Update the weights and repeat from Step 2

Training Process (Finding the values of W,b)

Step 1 : Initialise the weights to something

(4 x 3)

(1 x 4)

4 . 9	3 . 0	1 . 4	0 . 2	
(0,1)	(0,2)	(0,3)	(0,4)	

x

1	1	1
1	1	1
1	1	1
1	1	1

Training Process (Finding the values of W,b)

Step 2 : Do the matrix multiplication with initialised weights.

(4 x 3)

(1 x 4)

4 . 9	3 . 0	1 . 4	0 . 2
(0,1)	(0,2)	(0,3)	(0,4)

x

1	1	1
1	1	1
1	1	1
1	1	1

2

9.5

(Prediction)



Training Process (Finding the values of W,b)

Step 3 : Compare with actual value (Label)

$$\begin{matrix} & \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \\ \boxed{4.9 \quad 3.0 \quad 1.4 \quad 0.2} \quad \mathbf{x} & = \quad \boxed{9.5 \quad 9.5 \quad 9.5} \quad (\text{Prediction}) \end{matrix}$$

↑
↓

Label :

1	0	0
---	---	---

Training Process (Finding the values of W,b)

Step 4 : See the difference and pass back the changes (gradients) to previous layers

$$\begin{matrix} & \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \\ \boxed{4.9 \quad 3.0 \quad 1.4 \quad 0.2} \quad \mathbf{x} & = \quad \boxed{9.5 \quad 9.5 \quad 9.5} \end{matrix}$$

Square Error : Square of difference.

$$E_{total} = \sum \frac{1}{2}(target - output)^2 = 84.25$$

Label :

1	0	0
---	---	---

Training Process (Finding the values of W,b)

Step 4 : See the difference and pass back the changes (gradients) to previous layers

$$\begin{matrix} & \boxed{4.9} & 3.0 & 1.4 & 0.2 \\ \text{x} & \end{matrix} \times \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} = \boxed{\begin{matrix} 9.5 & 9.5 & 9.5 \end{matrix}}$$

Error = 84.25

Label : 1 | 0 | 0

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2}(target_{o1} - out_{o1})^{2-1} * -1 + 0$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1})$$

Training Process (Finding the values of W,b)

Step 4 : See the difference and pass back the changes (gradients) to previous layers

$$\begin{matrix} & \begin{matrix} 1 & 1 & 1 \end{matrix} \\ \begin{matrix} 4.9 & 3.0 & 1.4 & 0.2 \end{matrix} & \times & \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} & = & \begin{matrix} 9.5 & 9.5 & 9.5 \end{matrix} \end{matrix}$$

Error = 84.25

Label :

1	0	0
---	---	---

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2}(target_{o1} - out_{o1})^{2-1} * -1 + 0$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1})$$



Gradients (Derivatives)

Training Process (Finding the values of W,b)

Step 5 : Update the Weights

$$\begin{matrix} 4.9 & 3.0 & 1.4 & 0.2 \end{matrix} \boxed{\mathbf{x}} \begin{matrix} 1-\delta & 1-\delta & 1-\delta \\ 1-\delta & 1-\delta & 1-\delta \\ 1-\delta & 1-\delta & 1-\delta \\ 1-\delta & 1-\delta & 1-\delta \end{matrix} = \begin{matrix} 9.5 & 9.5 & 9.5 \end{matrix}$$

Error = 84.25

Label : $\begin{matrix} 1 & 0 & 0 \end{matrix}$

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2}(target_{o1} - out_{o1})^{2-1} * -1 + 0$$

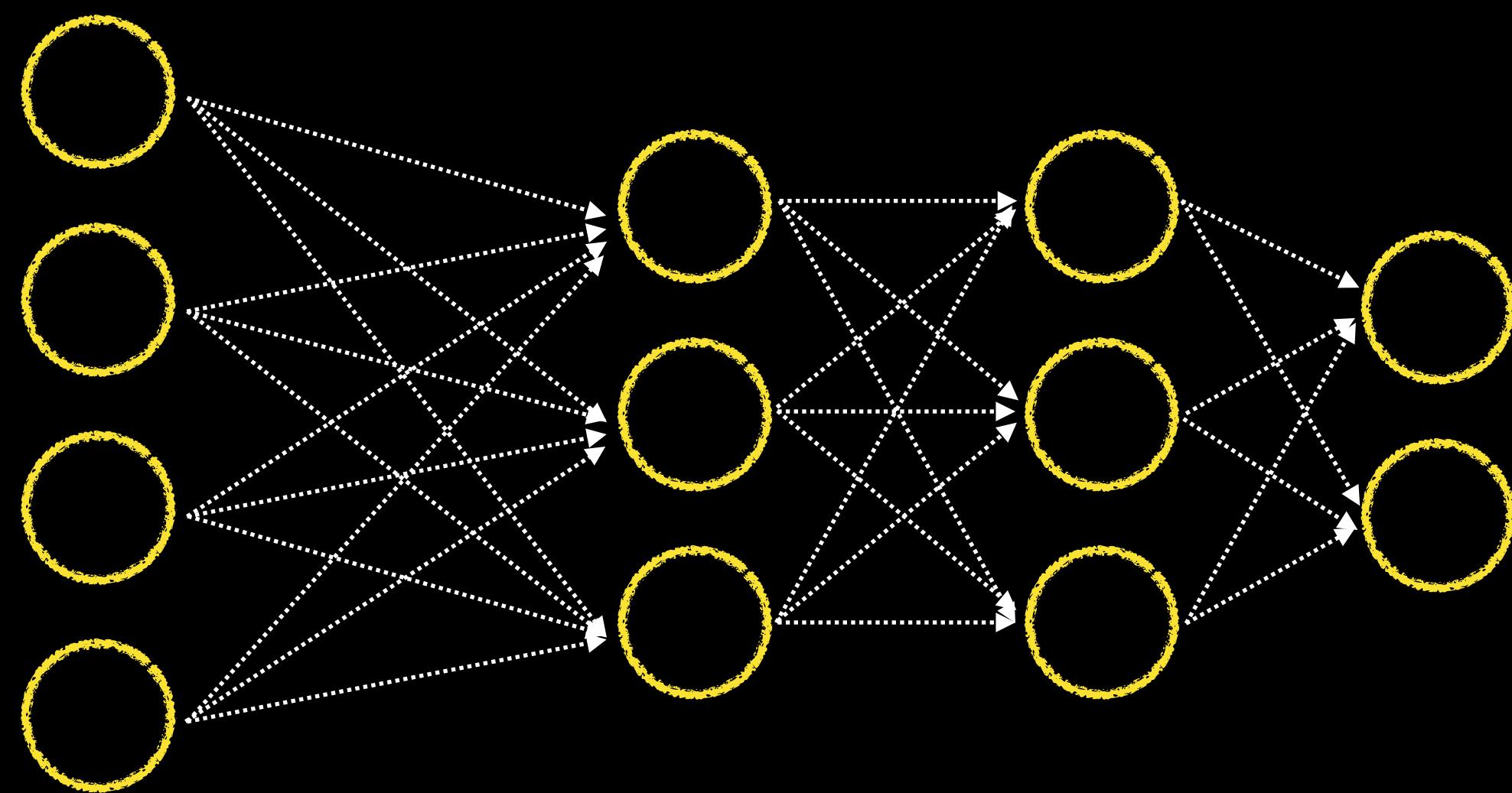
$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1})$$



Gradients (Derivatives)

Subtract from each weight a small fraction of its corresponding derivative (Delta Rule)

Fully Connected Layer



Activations

Input * Weights + Bias = Activations

Activations

Input * Weights + Bias = Activations-Pre-Activations

Activations

Input * Weights + Bias = Activations-Pre-Activations

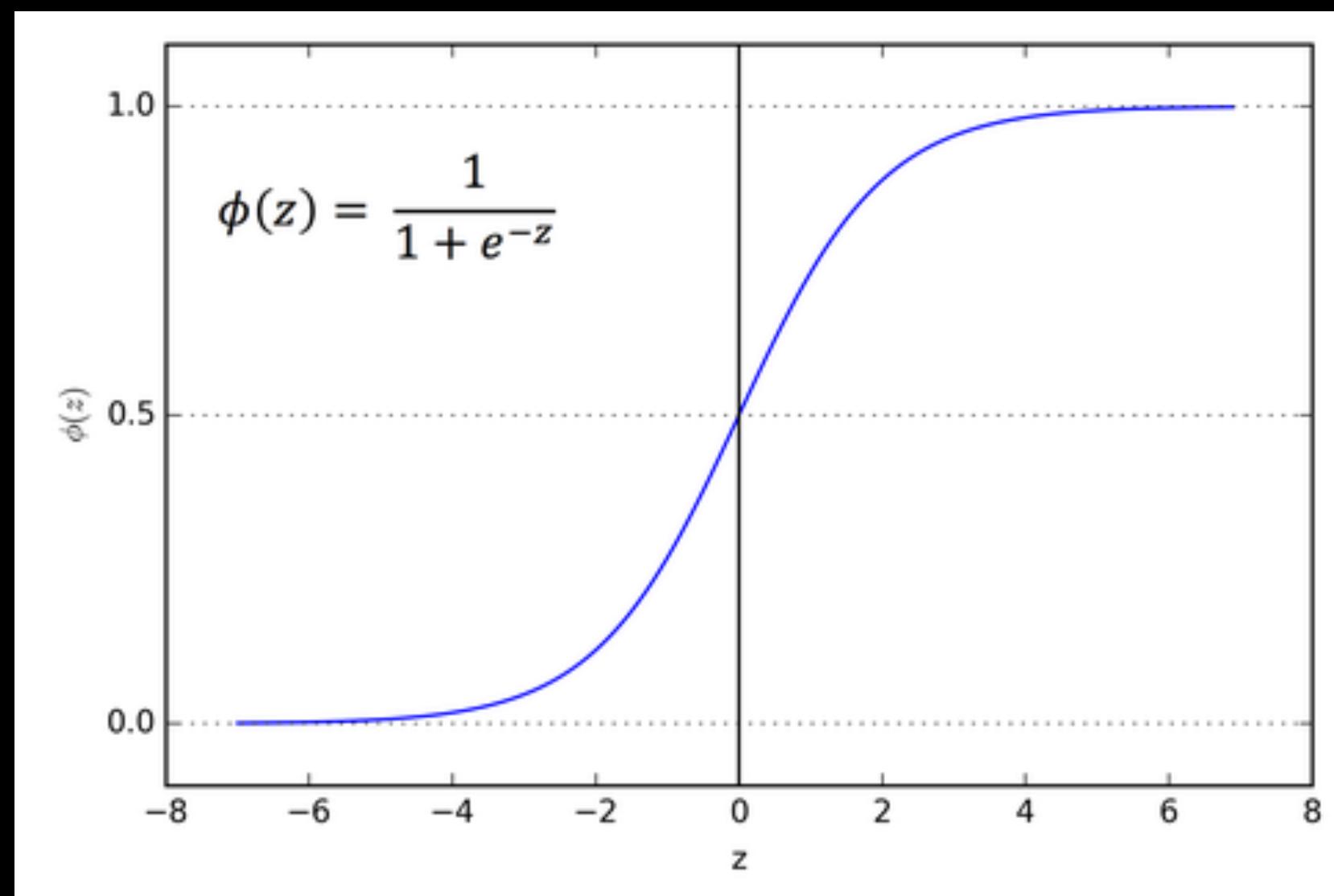
Non Linear (Pre-Activations) = Activations

Activations

Input * Weights + Bias = Activations-Pre-Activations

Non Linear (Pre-Activations) = Activations

Most commonly Used :



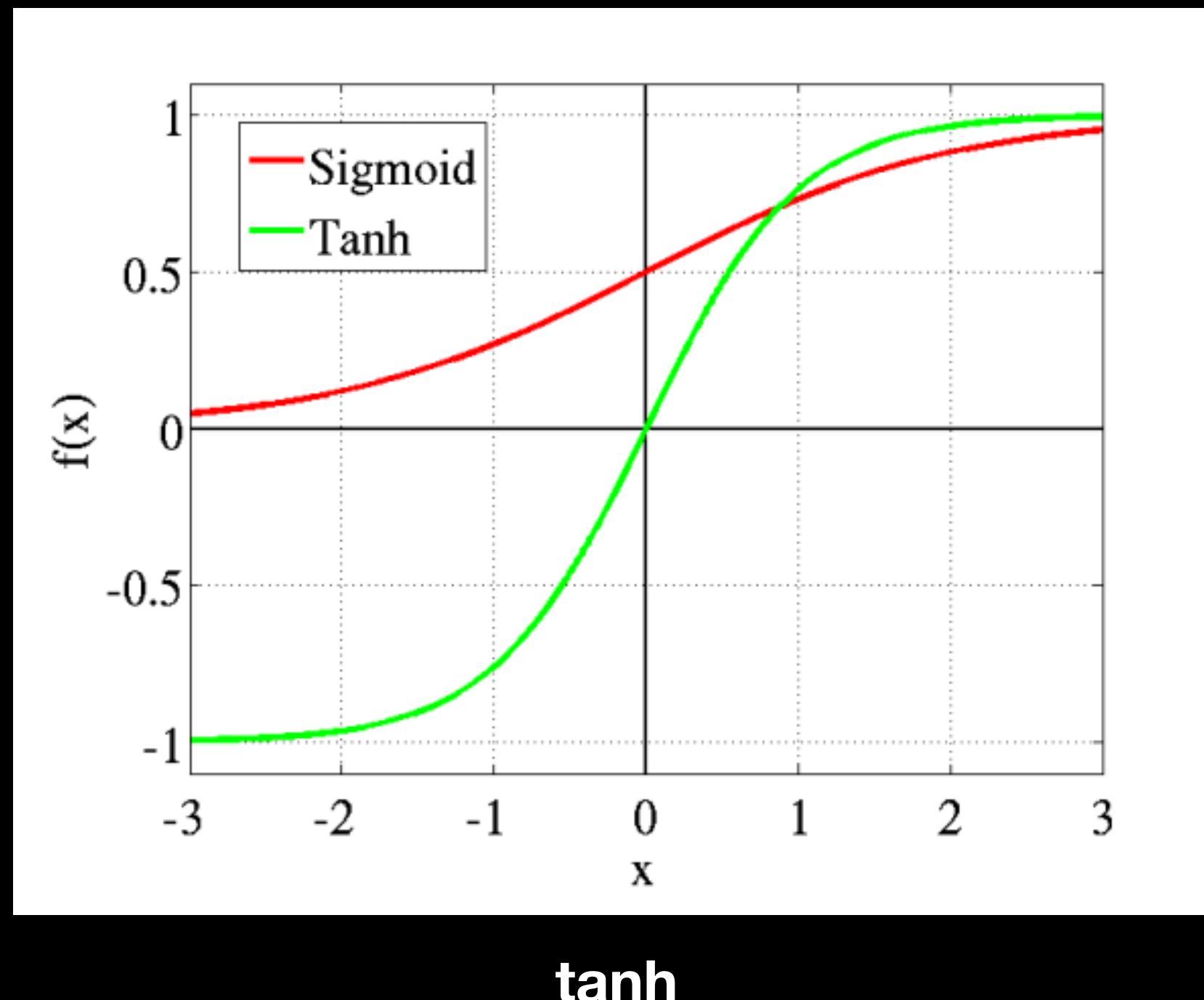
Sigmoid

Activations

Input * Weights + Bias = Activations-Pre-Activations

Non Linear (Pre-Activations) = Activations

Most commonly Used :

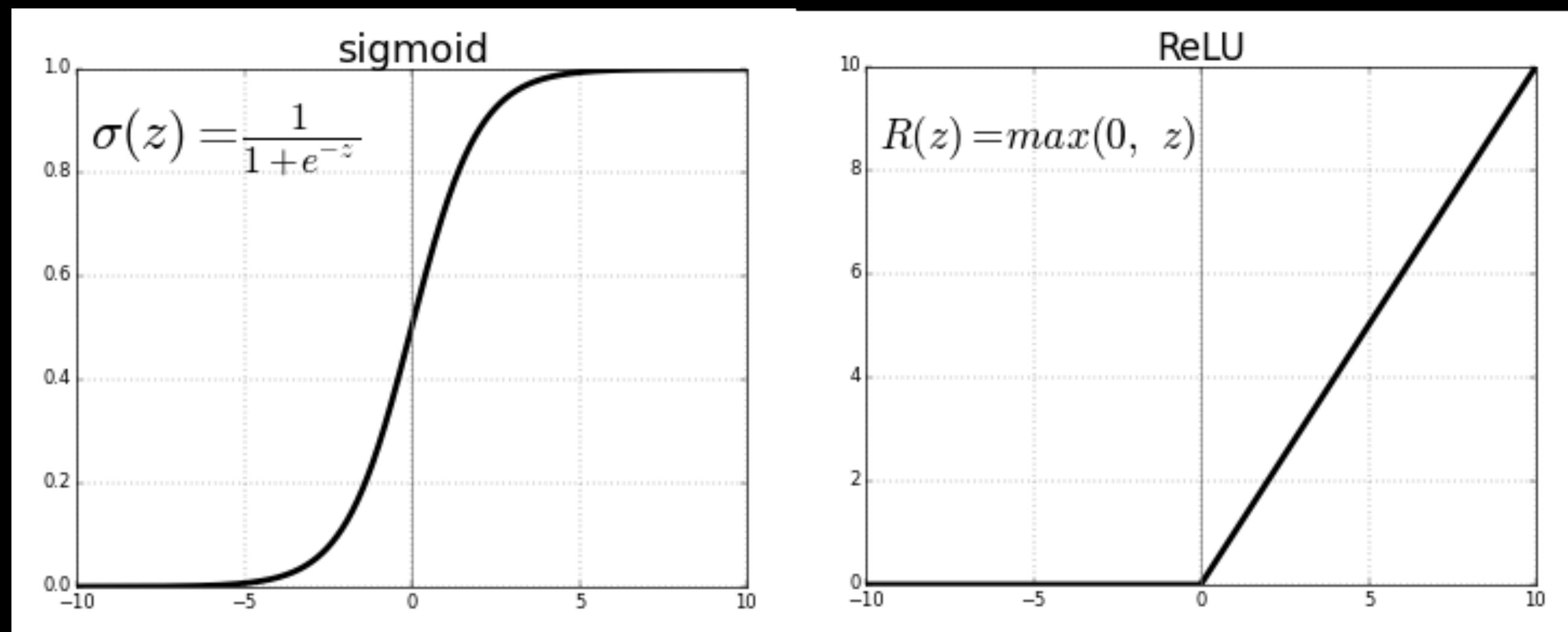


Activations

Input * Weights + Bias = Activations-Pre-Activations

Non Linear (Pre-Activations) = Activations

Most commonly Used :



Rectified Linear Unit

Activations

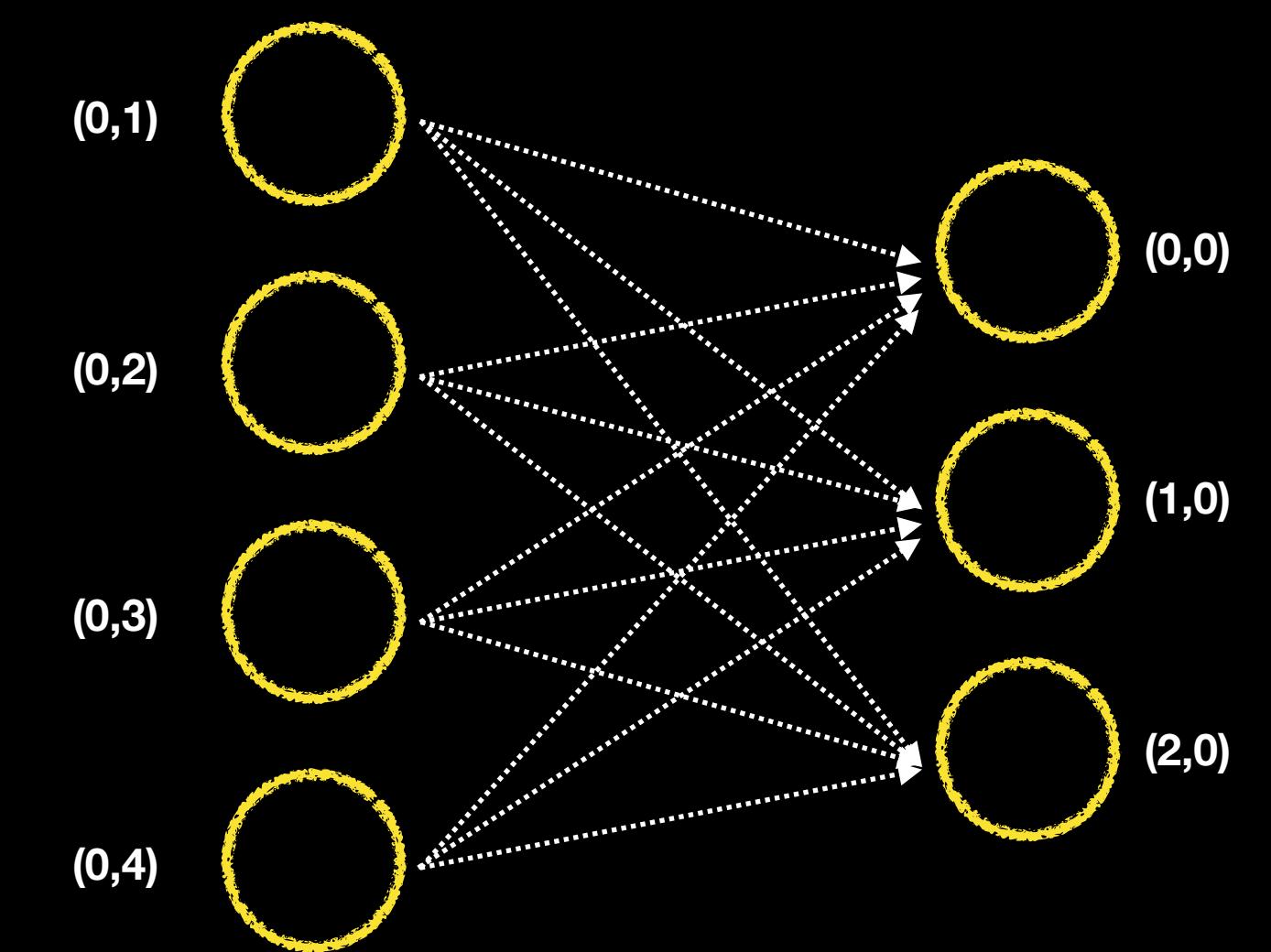
4.9		
	(0,0)	(0,1)
3.0		(0,2)
	(1,0)	(1,1)
1.4		(1,2)
	(2,0)	(2,1)
0.2		(2,2)
	(3,0)	(3,1)
		(3,2)

=

9.5
9.5
9.5

$\frac{1}{1+e^{-x}}$

0.9999251537724895
0.9999251537724895
0.9999251537724895



Input

Output

Activations

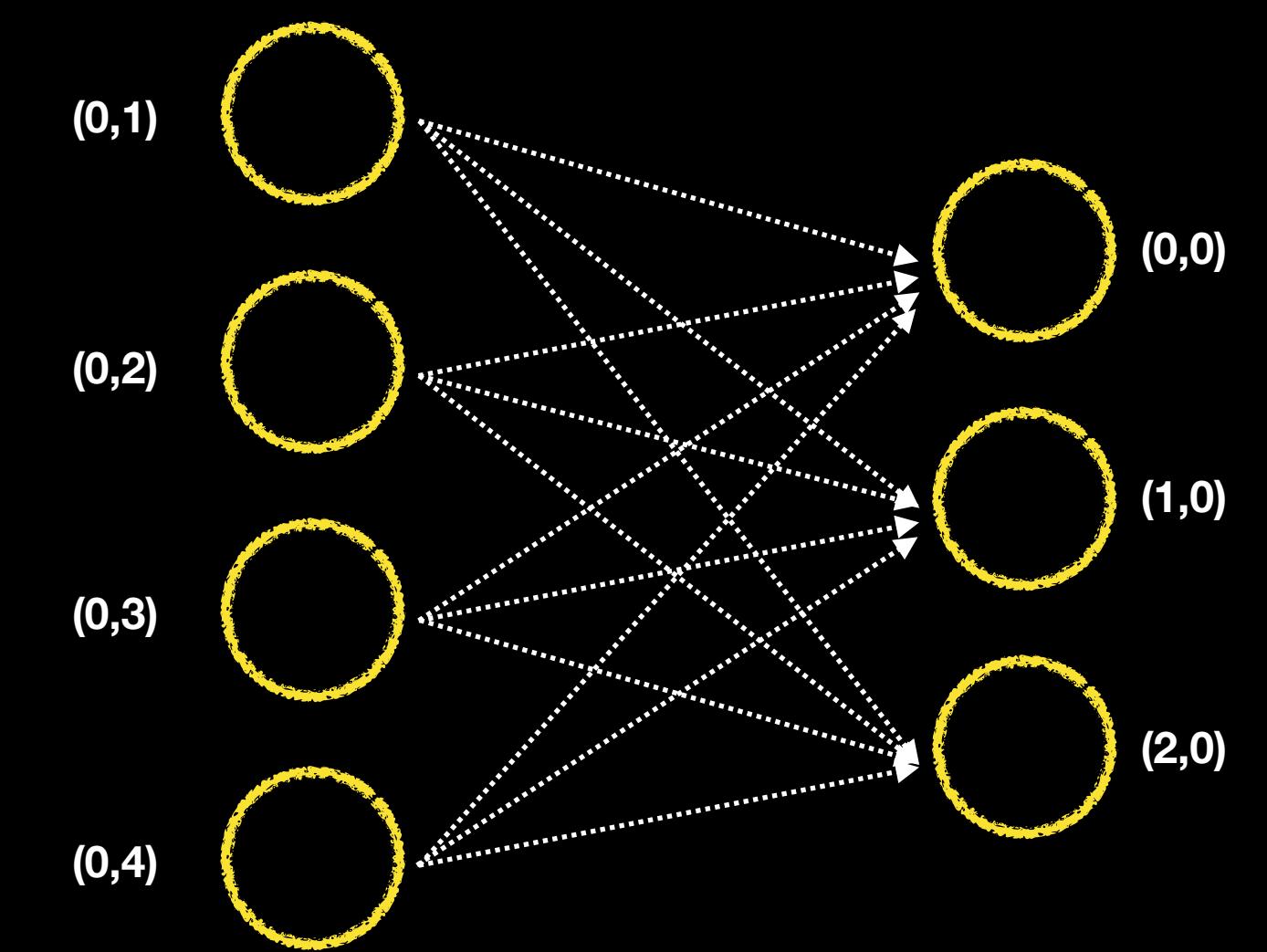
4.9		
	(0,0)	(0,1)
3.0		(0,2)
	(1,0)	(1,1)
1.4		(1,2)
	(2,0)	(2,1)
0.2		(2,2)
	(3,0)	(3,1)
		(3,2)

=

9.5
9.5
9.5

$\frac{1}{1+e^{-x}}$

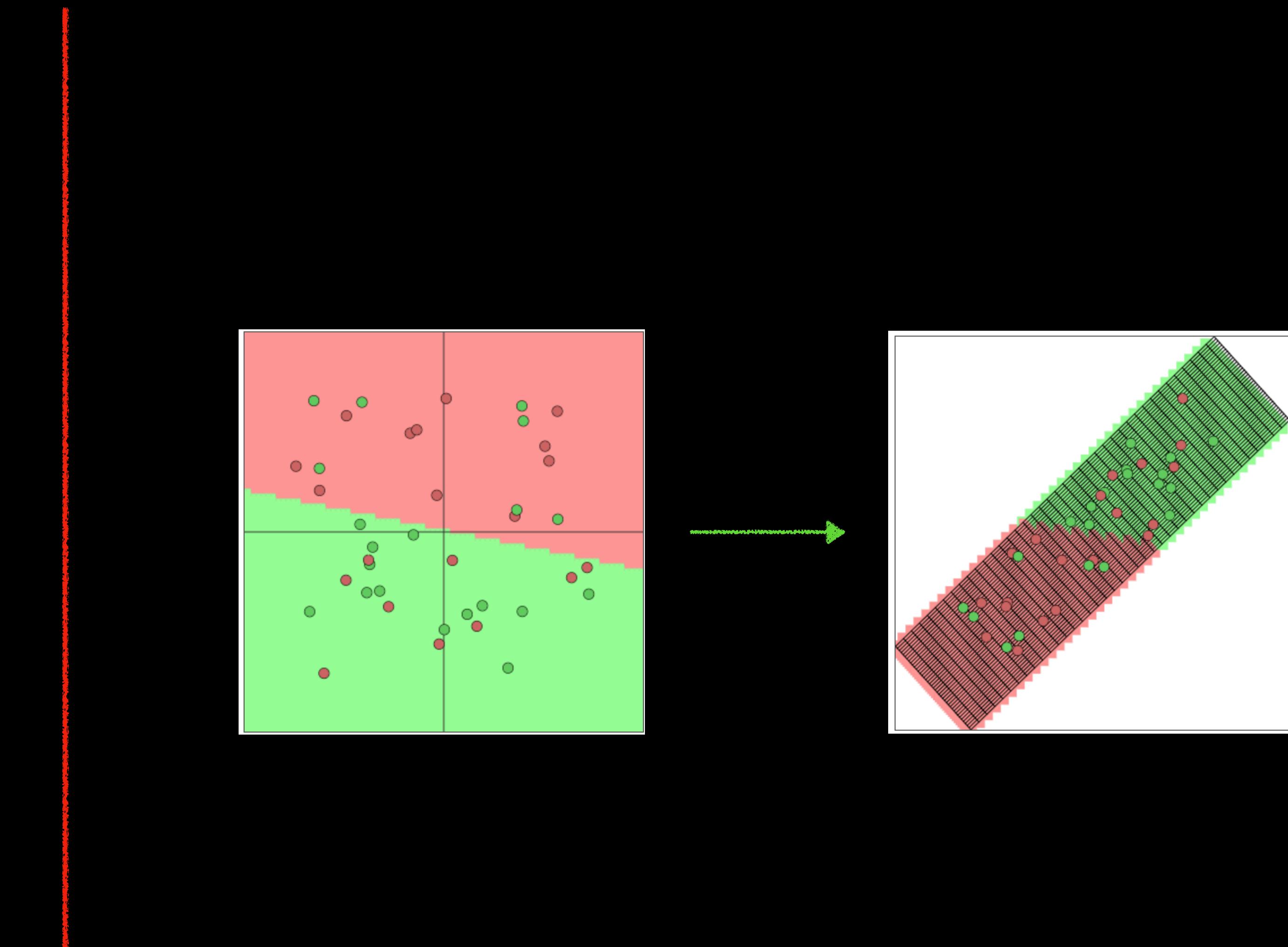
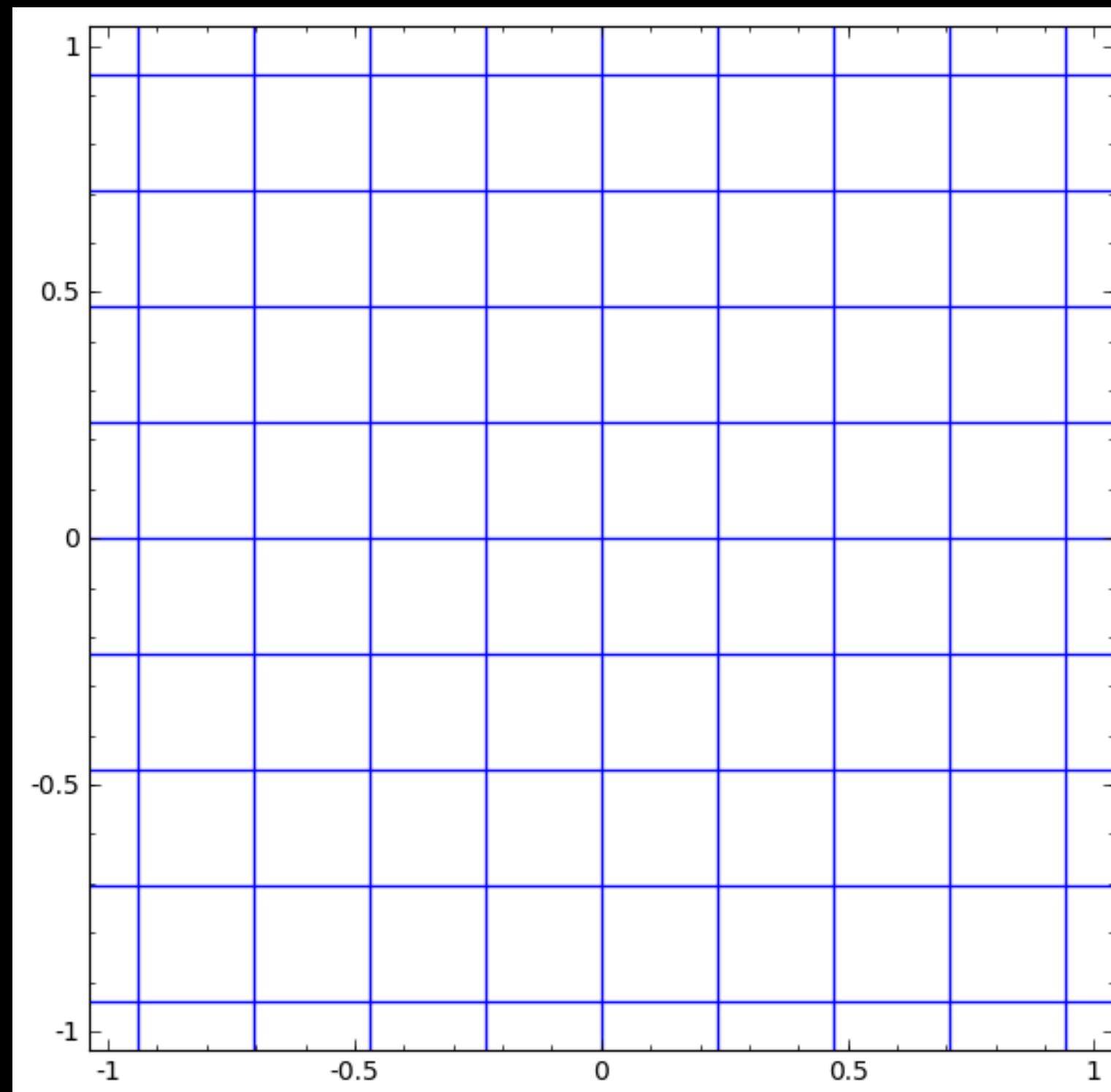
0.9999251537724895
0.9999251537724895
0.9999251537724895



Input

Output

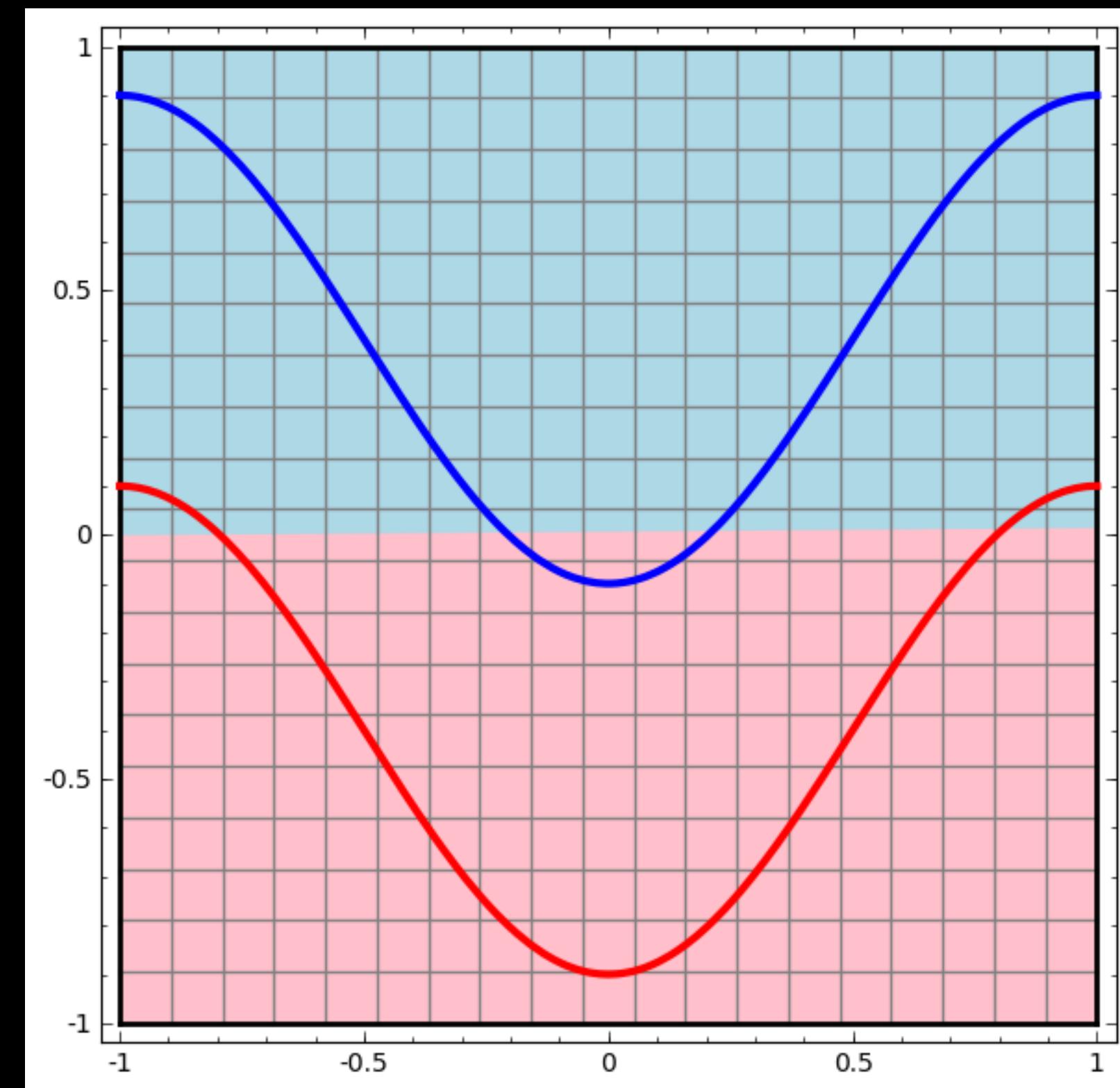
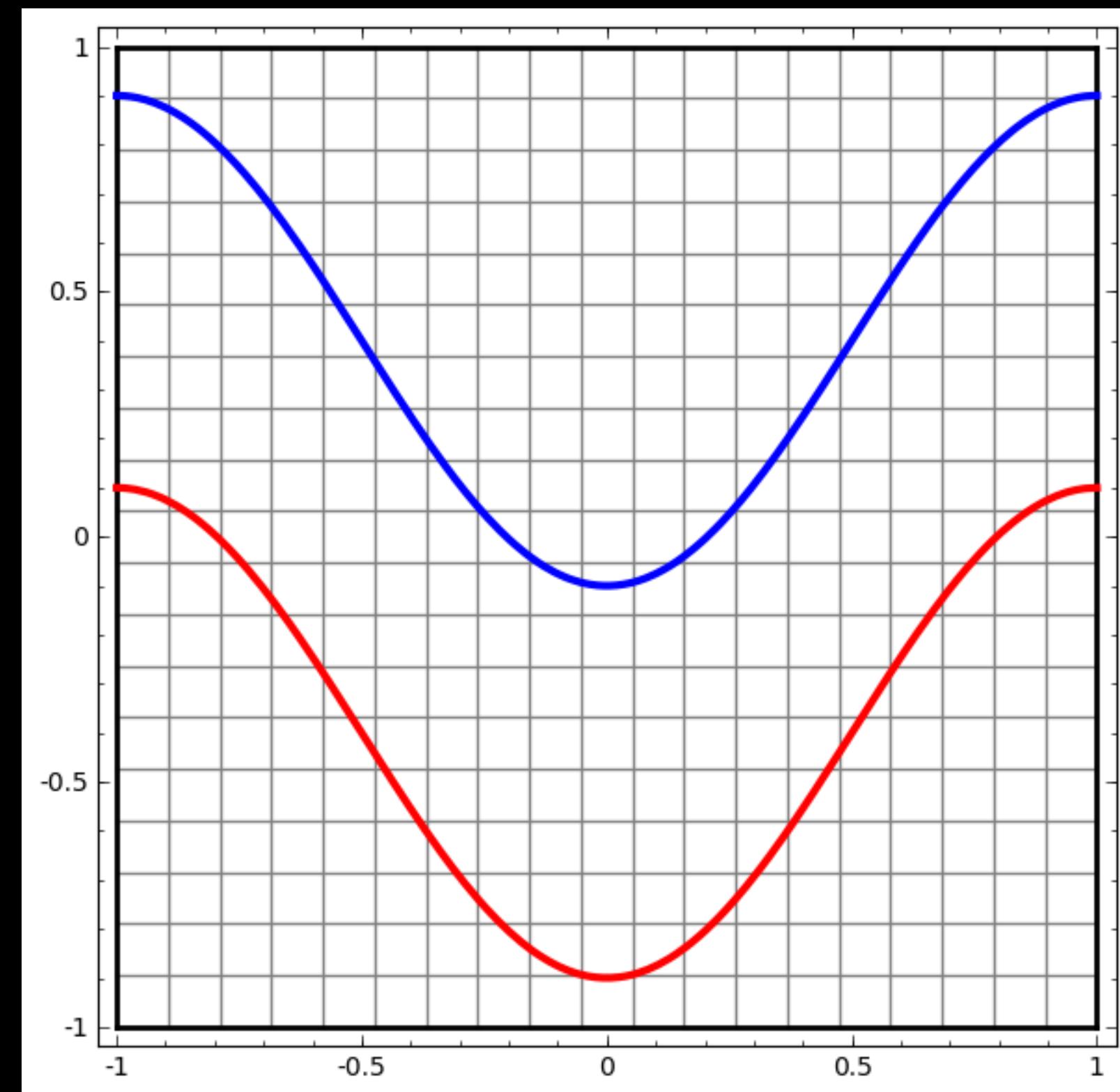
Data Space Transformation



// Chris Olah Blog (Neural Networks, Manifolds and Topology)
// Karpathy ConvNetJS <https://cs.stanford.edu/people/karpathy/convnetjs//demo/classify2d.html>

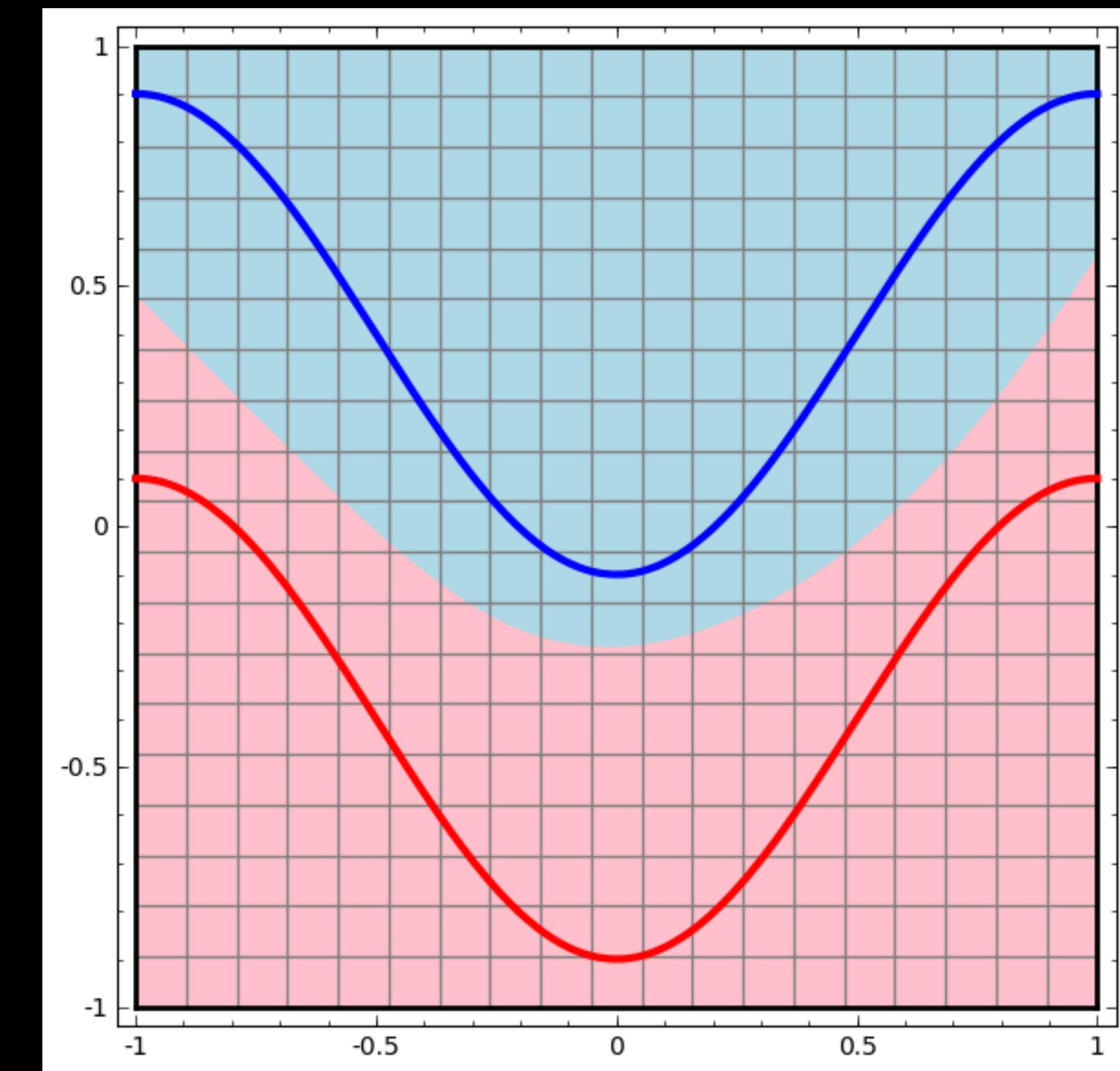
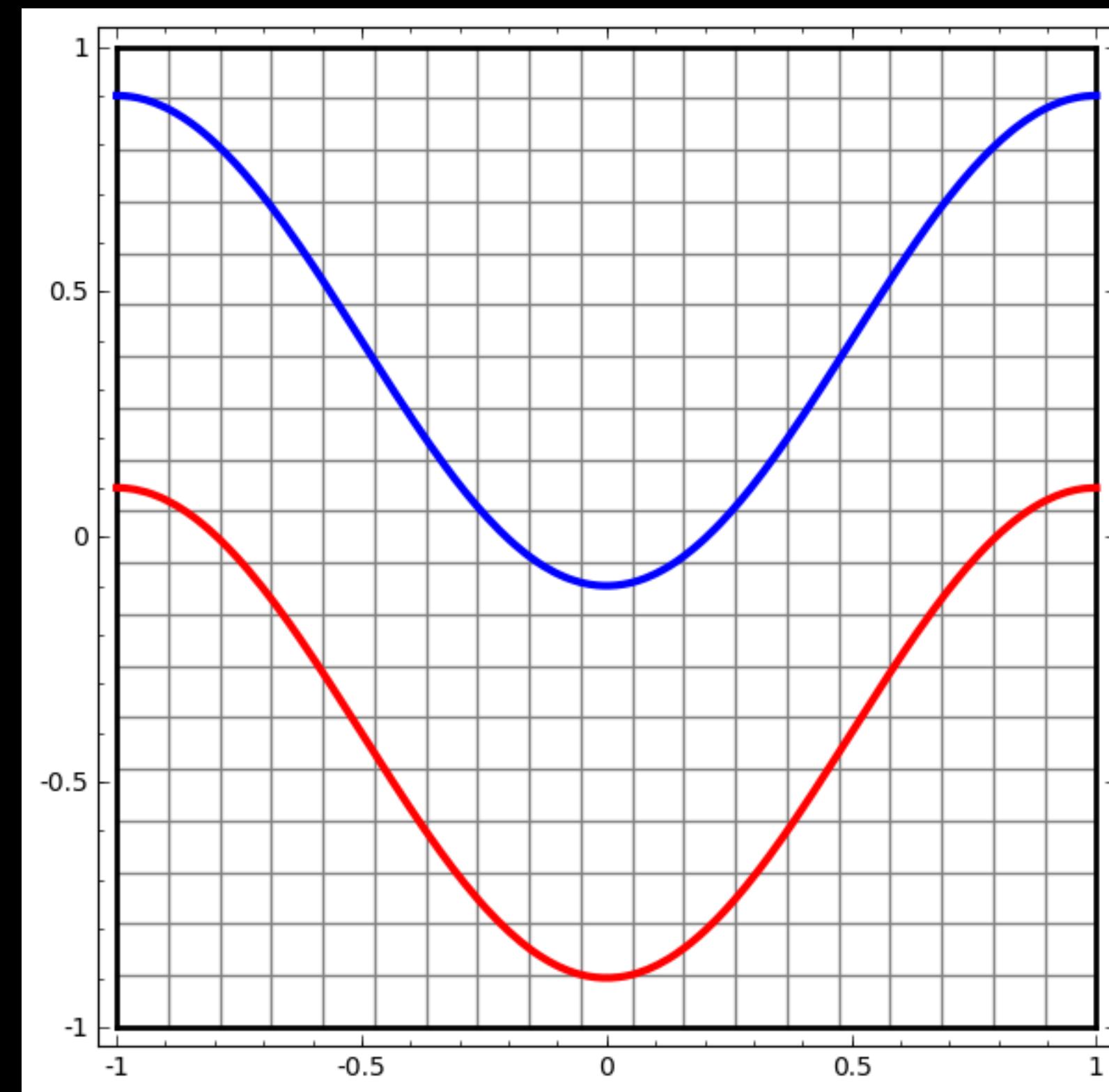
Data Space Transformation

Without the non linear function : Not Linearly Separable



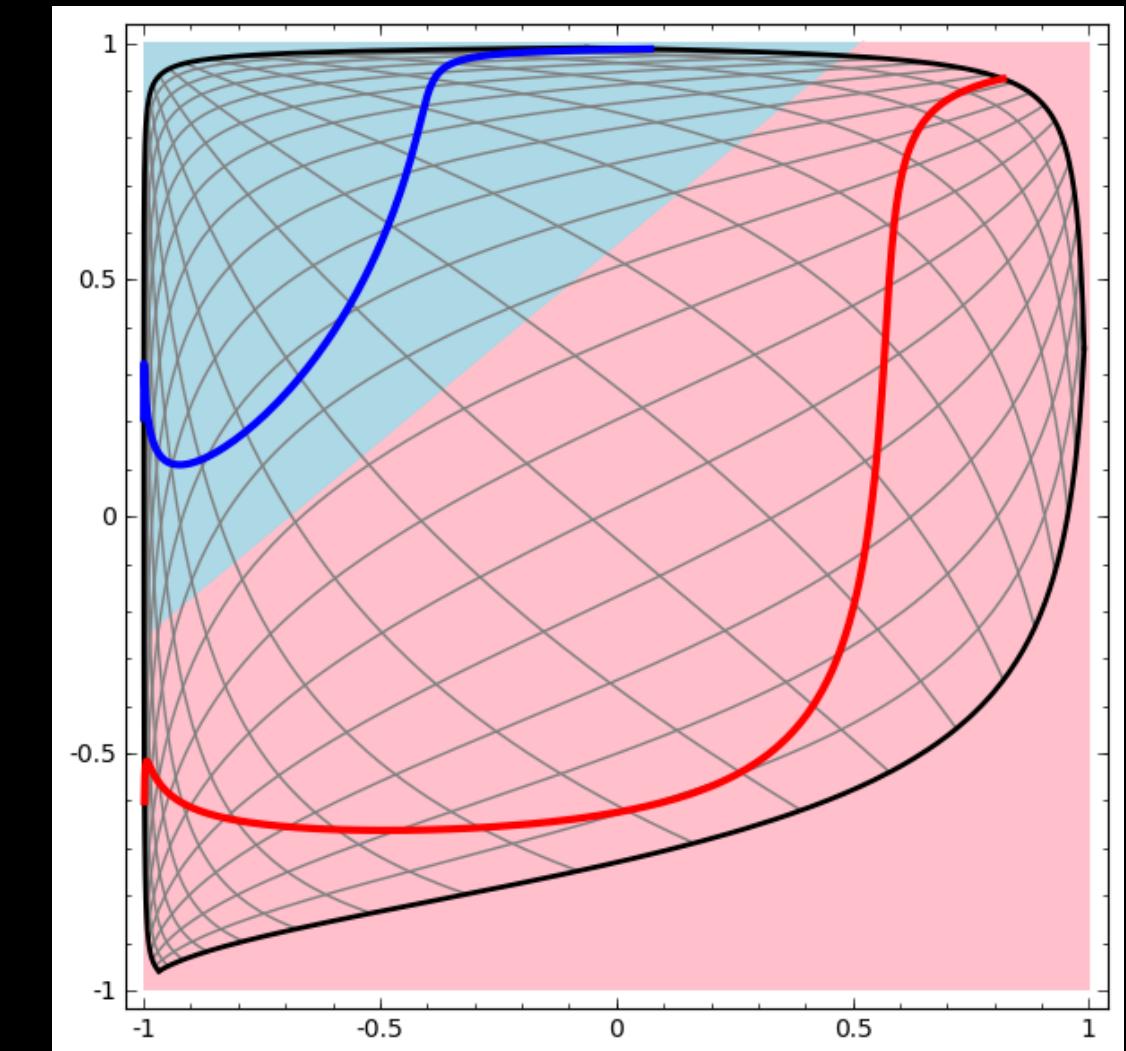
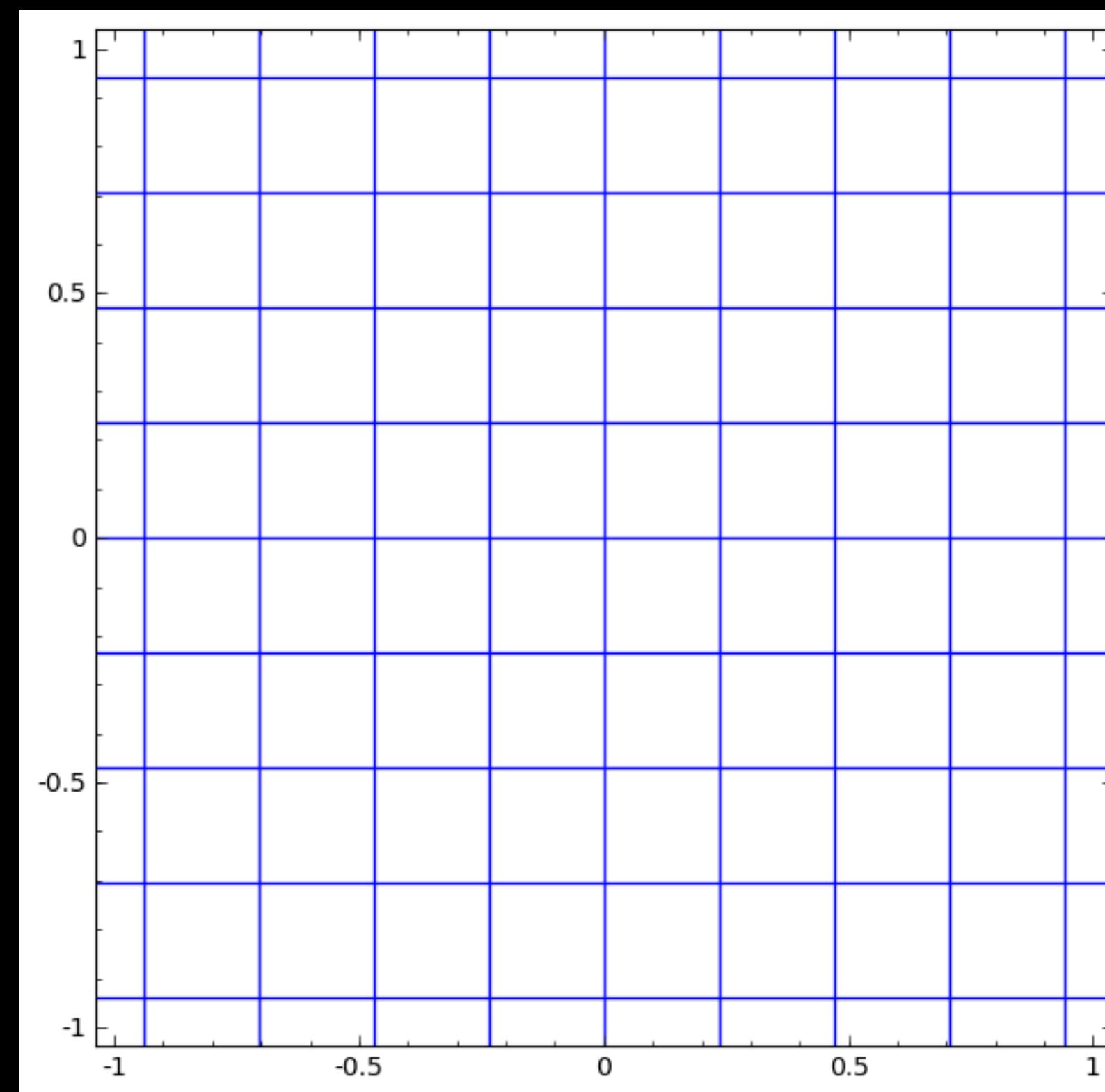
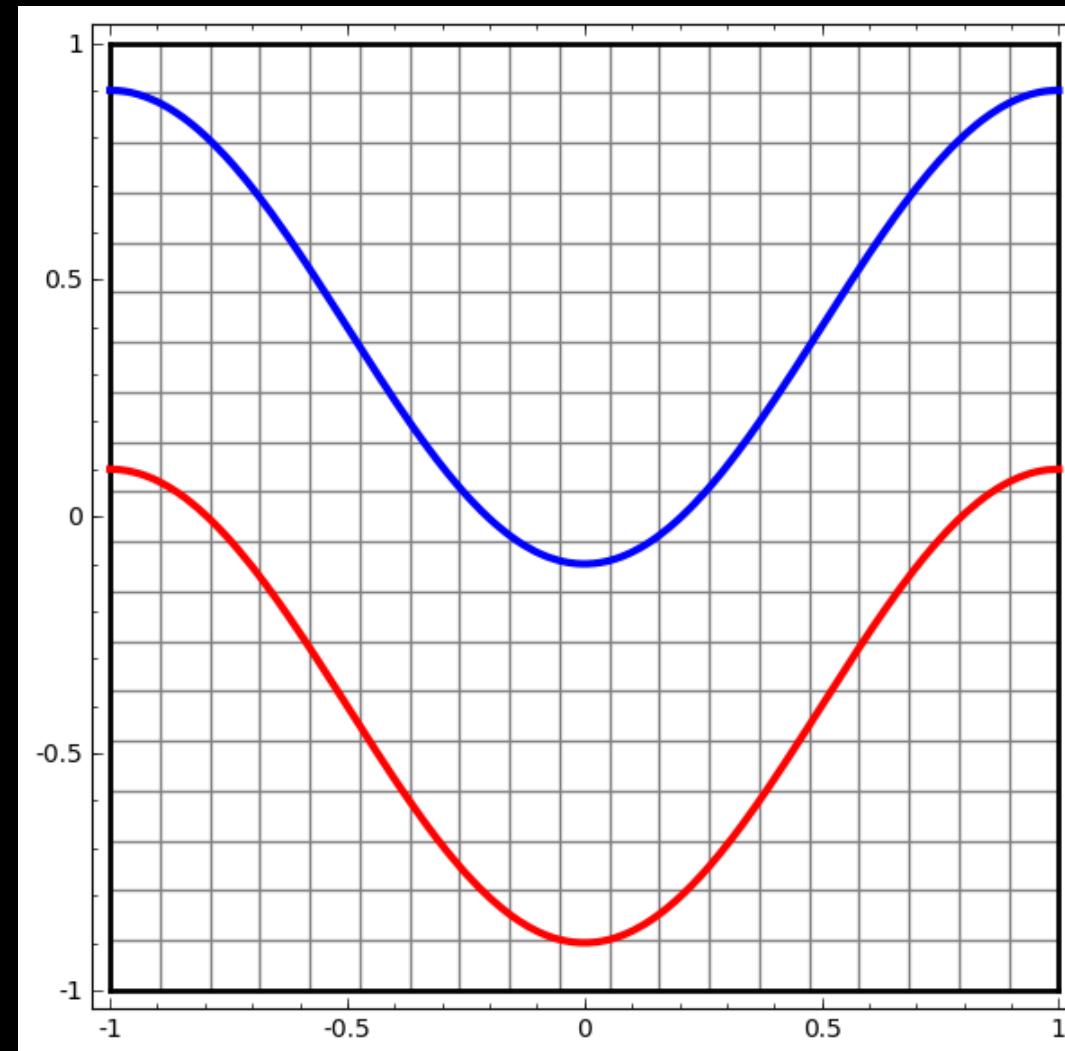
Data Space Transformation

What we want



Data Space Transformation

With the non linear function : **Linearly Separable but in a different space (After transformation)**

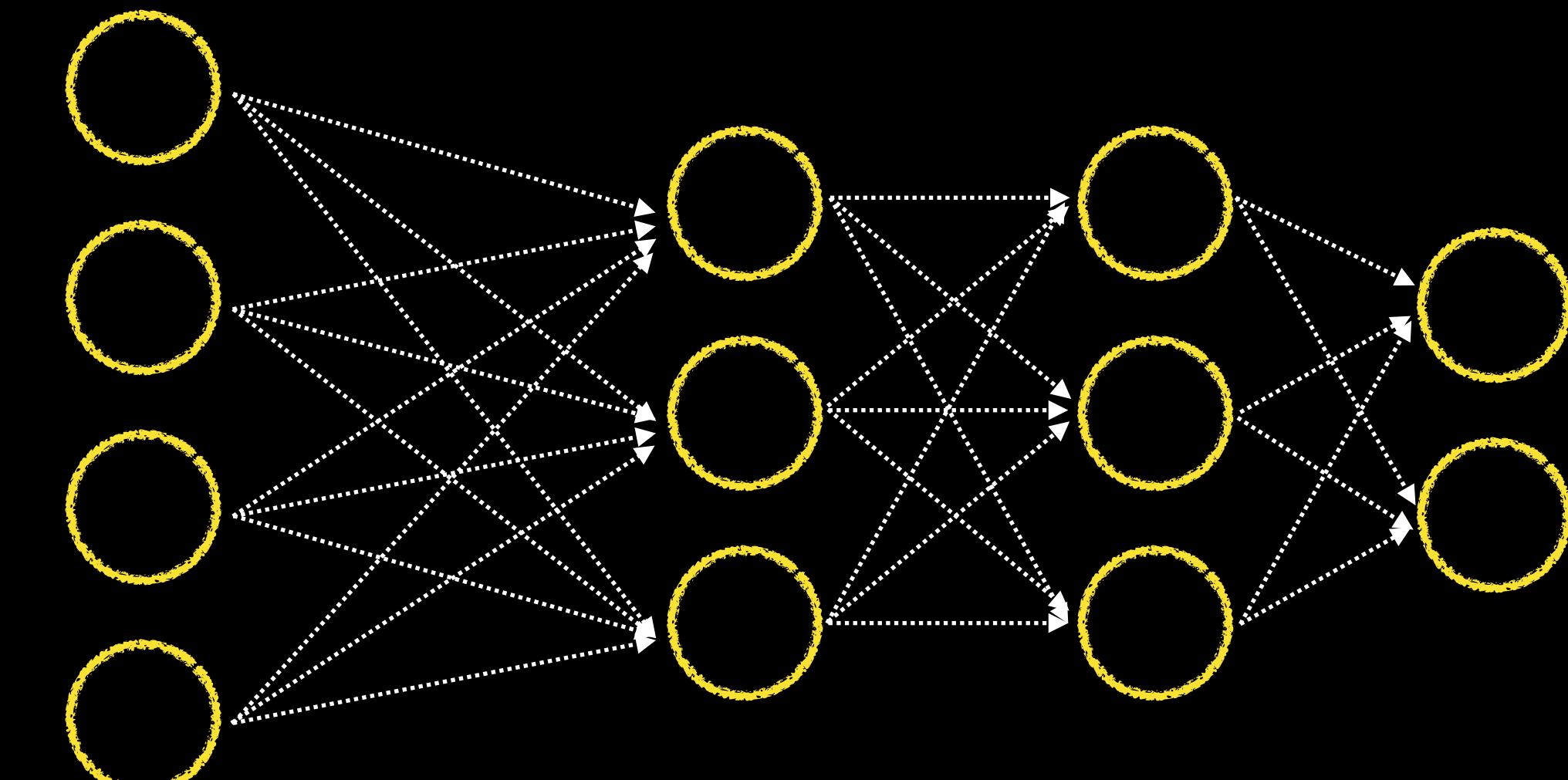


Data Space Transformation

// <https://cs.stanford.edu/people/karpathy/convnetjs//demo/classify2d.html>

Summary

- Step 1 : Initially weights and bias are randomly assigned.
- Step 2 : Vector -> Matrix Multiplication -> Some Output (Vector) -> Non-Linear Activation
- Step 3 : Calculate Loss (Mean Square Error)
- Step 4 : Backpropagate (Pass the Gradients to prior weights)
- Step 5 : Update the weights and repeat from Step 2



Keras Code

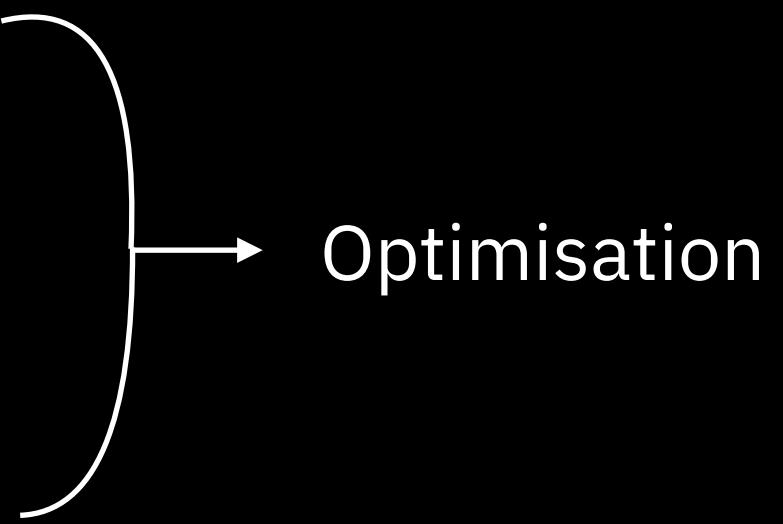
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Optimisation

Keras Code

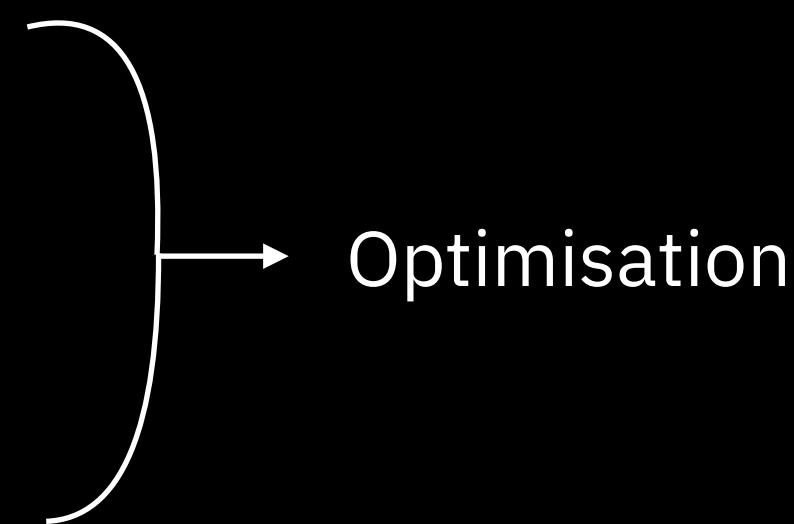
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- Packages like Tensorflow, Pytorch, Keras are all equipped with optimisers.
- There are several advanced optimisers besides what we just learnt.

```
// coding time  
// use pip or conda (if using anaconda) to install the packages  
// avoid using gpu for tutorials if you don't want to spend eternity configuring cuda.
```

Any Questions?

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