Modelling the formation of the GD-1 stellar stream inside a host with a fermionic dark matter core-halo distribution.

Martín F. Mestre[1,2,⋆], Carlos R. Argüelles[1,2], and Daniel D. Carpintero[1,2]

[1] Instituto de Astrofísica de La Plata (CONICET-UNLP), Paseo del Bosque S/N, La Plata (1900), Buenos Aires, Argentina

[2] Facultad de Ciencias Astronómicas y Geofísicas de La Plata (UNLP), Paseo del Bosque S/N, La Plata (1900), Buenos Aires, Argentina

ABSTRACT

Context. Stellar streams are a consequence of the tidal forces produced by a host galaxy on its satellites (i.e. globular clusters and dwarf spheroidals). As the self-gravity of stellar streams is almost negligible, they constitute excellent probes of the gravitational potential of the host galaxy. For this reason, some Milky Way stellar streams have been used to put constraints on the dark matter (DM) total mass and shape, under empirical DM distributions (i.e. NFW, logarithmic, etc). Within the set of non-empirical DM distributions, there exists a DM model deduced from first principles by means of the maximization of the coarse-grained entropy for self-gravitating fermions, the MEPP, after Maximum Entropy Production Principle. MEPP profiles have a rich variety of behaviours, being a large subfamily characterized by a dense core of mass $\approx 4.1 \times 10^6 M_\odot$ (that can mimick a black hole regarding the orbits of the S-stars at Sagittario A*) and an extended halo with the plasticity of a Burkert profile.

Aims. In this work we attempt to model the GD-1 stellar stream using a spherical MEPP distribution which, at the same time, is in sintony with previous fits to the S-stars.

Methods. For that purpose we used a genetic algorithm in order to fit both the stream orbit's initial conditions and the fermionic halo. We modelled the barionic potential with a bulge and two disks (thin and thick) with fixed parameters according to the recent literature. The stream observable is 6D phase-space data from the Gaia DR2 survey.

Results. We were able to find good fits for a 1D continuous subfamily of models parametrized by the fermion mass going from 56 keV/c^2 to 360 keV/c^2, respectively corresponding to core radii going from 10^3 to 10 Schwarzschild radii. For smaller and larger values of the fermion mass, there is no solution that simultaneously fits the GD-1 stream and the S-stars. The solutions have a virial radius of 28 kpc and a virial mass of $1.4 \times 10^{11} M_\odot$, the latter being at 2σ from previous Milky Way DM halo mass estimates using the Sagittarius stream. We do not assume the velocity of the local standard of rest ($v_{\rm lsr}$) and the result gives $v_{\rm lsr} \approx 244$ km/s, in agreement with recent independent estimates.

Key words. Galaxy: kinematics and dynamics – Galaxy: stellar streams – dark matter

1. Introduction

Stellar streams probe the acceleration field produced by the Milky Way (MW). This information together with barionic measurements help in making claims about the dark matter halo.

2. Methodology

In this section we explain the observables and methods used in this research.

2.1. Observables and assumed measurements

The main observables used in this project are materialized by the polynomial fits performed by Ibata et al. (2020) on the GD-1 stream using astrometry (Gaia DR2), photometry and high-precision spectroscopy datasets together with the analysis of the

---

⋆ e-mail: mmestre@fcaglp.unlp.edu.ar

STREAMFINDER algorithm. Those polinomials are the following:

$$\phi_2 = 0.008367\phi_1^3 - 0.05332\phi_1^2 - 0.07739\phi_1 - 0.02007 \quad (1)$$

$$D = -4.302\phi_1^5 - 11.54\phi_1^4 - 7.161\phi_1^3 + 5.985\phi_1^2 \\ +8.595\phi_1 + 10.36 \quad (2)$$

$$v_h = 90.68\phi_1^3 + 204.5\phi_1^2 - 254.2\phi_1 - 261.5 \quad (3)$$

$$\tilde{\mu}_\alpha = 3.794\phi_1^3 + 9.467\phi_1^2 + 1.615\phi_1 - 7.844 \quad (4)$$

$$\mu_\delta = -1.225\phi_1^3 + 8.313\phi_1^2 + 18.68\phi_1 - 3.95, \quad (5)$$

with ϕ_1 and ϕ_2 in radians, D in kpc and, $\tilde{\mu}_\alpha = \mu_\alpha cos\delta$ and μ_δ in mas/yr. These quantities correspond respectively to longitude and latitude in GD-1 celestial frame of reference (Koposov et al. 2010), heliocentric distance, proper motion in right ascension and declination. The domain of this polinomials is limited to $-90 < \phi_1[°] < 10$. To have our observable data points we sample the domain with 100 points ($\phi_1^{(i)}$ for $i = 1, ..., 100$) and evaluate the polinomials there in order to have a complete set of observables ($\phi_1^{(i)}, \phi_2^{(i)}, D^{(i)}, \tilde{\mu}_\alpha^{(i)}, \mu_\delta^{(i)}$). In one of the experiments we will consider an observable of a different nature and related to the density concentration at the origin, of size $\lesssim 1$ milliparsec, characteristic of the fermionic halo distribution. By convention, the

core radius is defined as the radius when the circular velocity reaches its first maximum. The constraint for the mass of the core of the distribution is assumed to be $m_{core} = 3.5 \times 10^6 M_\odot$ in agreement with Becerra-Vergara et al. (2020, 2021); Argüelles et al. (2022).

The assumed measurements used in this paper are the galactocentric distance of the sun $R_\odot = 8.122$ kpc (GRAVITY Collaboration et al. 2018), the sun's peculiar velocity $v_{\odot p} = (11.1, 12.24, 7.25)$ km/s (Schönrich et al. 2010) and the value of the universal constant of gravitation $G = 4.3009 \times 10^{-6}$ kpc (km/s)^2 $M_\odot^{-1}$.

2.2. The fermionic dark matter model

Our dark matter model consists of a spherical and isotropic distribution of fermions at finite temperature in hydrostatic equilibrium subject to the law of General Relativity (i.e. T.O.V equations) as defined in Argüelles et al. (2018) with the particularity that we have made a few change of variables in order to make the numerical computation more robust. The system of equations thus obtained is as follows:

$$\frac{d\nu}{d\zeta} = \frac{1}{2}\left[e^z + e^{2\zeta}\frac{P(r)}{\rho_{rel}c^2}\right][1 - e^z]^{-1}, \tag{6}$$

$$\frac{dz}{d\zeta} = -1 + e^{(2\zeta - z)}\frac{\rho(r)}{\rho_{rel}}, \tag{7}$$

where

$$\zeta = \ln(r/R), \tag{8}$$

$$z = \ln\psi, \quad \psi = \frac{M(r)}{M}\frac{R}{r}, \tag{9}$$

$$\rho(r) = \frac{4\rho_{rel}}{\sqrt{\pi}}\int_1^\infty \epsilon^2[\epsilon^2 - 1]^{1/2}f(r,\epsilon)d\epsilon, \tag{10}$$

$$P(r) = \frac{4\rho_{rel}c^2}{3\sqrt{\pi}}\int_1^\infty [\epsilon^2 - 1]^{3/2}f(r,\epsilon)d\epsilon, \tag{11}$$

$$R^2 = \frac{c^2}{8\pi G\rho_{rel}}, \tag{12}$$

$$M = 4\pi R^3\rho_{rel}, \tag{13}$$

$$\rho_{rel} = \frac{G\pi^{3/2}m^4c^3}{h^3}, \tag{14}$$

where c is the speed of light, h is the Planck constant, m is the fermion mass, $M(r)$ is the mass enclosed up to radius r, $\rho(r)$ is the density, $P(r)$ is the pressure and $\nu(r)$ is the metric exponent, i.e. we use the convention $g_{00} = e^{\nu(r)}$. The phase-space function (f) is given by a Fermi-Dirac distribution with energy cutoff:

$$f(r,\epsilon) = \begin{cases} \frac{1 - e^{[\epsilon - \epsilon_c(r)]/\beta(r)}}{1 + e^{[\epsilon - \alpha(r)]/\beta(r)}} & \epsilon \leq \epsilon_c(r) \\ 0 & \epsilon > \epsilon_c(r), \end{cases} \tag{15}$$

where $\alpha(r)$ is the chemical potential (including rest mass), $\epsilon(r)$ is the cutoff energy (including rest mass) and $\beta(r)$ is the temperature variable.

The above equations should be complemented with two thermodinamic equilibrium conditions given in Tolman (1930) and Klein (1949) together with the condition of energy conservation along the geodesic given in Merafina & Ruffini (1989):

$$\frac{1}{\alpha}\frac{d\alpha}{dr} = \frac{1}{\beta}\frac{d\beta}{dr} = \frac{1}{\epsilon_c}\frac{d\epsilon_c}{dr} = -\frac{1}{2}\frac{d\nu}{dr}. \tag{16}$$

It is not possible to integrate this equations from $r = 0$ because the right hand the change of variables $\zeta(r)$ is divergent at the origin. Nevertheless, it is possible to reach the following approximations for the initial conditions at a value $r_{min} \gtrsim 0$:

$$\nu(r_{min}) \approx \frac{1}{6}\frac{\rho_0}{\rho_{rel}}\left[\frac{r_{min}}{R}\right]^2 \equiv \tau, \tag{17}$$

$$\psi(r_{min}) \approx \frac{1}{3}\frac{\rho_0}{\rho_{rel}}\left[\frac{r_{min}}{R}\right]^2 = 2\tau, \tag{18}$$

which implies

$$\frac{r_{min}}{R} = \sqrt{6\tau\frac{\rho_{rel}}{\rho_0}}, \tag{19}$$

where $\rho_0 \equiv \rho(0)$. It can be seen that the right hand side of Eq. 6 does not depend on the metric so we can add a constant ν_0 to the solution in order to satisfy a condition of continuity with the Schwarzschild metric at the border of the fermion distribution, obtaining:

$$\nu_0 = 2\ln\left(\frac{\beta_b}{\beta_0}\sqrt{1 - \psi_b}\right),$$

where ψ_b and β_b are quantities evaluated at the border.

The system of equations thus obtained has four free parameters: m, $\alpha_0 = \alpha(0)$, $\beta_0 = \beta(0)$, and $\epsilon_{c0} = \epsilon_c(0)$ but in practice we will use the following related quantities as parameters: m, β_0, $\theta_0 = (\alpha_0 - 1)/\beta_0$, and $W_0 = (\epsilon_{c0} - 1)/\beta_0$.

Equations 6-16 are solved using a PYTHON (Van Rossum & Drake Jr 1995) script[1] that makes use of the NUMPY (Virtanen et al. 2020) and SCIPY (Harris et al. 2020) packages. We tried all the solvers available in the latter package but the only one that could properly integrate these equations was the LSODA algorithm. We used relative and absolute tolerance parameters respectively given by rtol$= 5 \times 10^{-14}$ and atol$= 0$.

2.3. Milky Way and stream models

We model our Galaxy with a combination of the fermionic dark halo recently described, whose parameters will be determined in this work, plus a barionic component fixed and identical to the one in Model I of Pouliasis et al. (2017). Here we name this full Galaxy model as Fermionic-MW.

Appart from our model, for some intermediary tasks, we will make use of the Galactic model fitted by Malhan & Ibata (2019) which consists in the MWPotential2014 model with an axisymmetric NFW profile. This fitted model corresponds to a circular velocity at the position of the sun of $v_c(R_\odot) = 244 \pm 4\,\mathrm{kms}^{-1}$ and a z−flattening of the dark matter density distribution of $q_\rho = 0.82^{+0.25}_{-0.13}$. Here we name this Galaxy model as NFW-MW.

As GD-1 is a dynamically cold stream, with its stars keeping a large degree of correlation, its almost one-dimensional distribution in phase-space can be well approximated with the orbit of its progenitor. Another facts in this direction, are that there has not been any observation of the tidal arms feature nor of the progenitor's position (Malhan & Ibata 2019; de Boer et al. 2018; Malhan et al. 2018; Price-Whelan & Bonaca 2018).

In the code, the orbit model is computed starting from initial conditions in spherical equatorial coordinates: $\alpha, \delta, D_h, \mu_\alpha\cos\delta$, μ_δ, v_h, which are respectively, RA, declination, heliocentric distance, proper motion in α times the cosine of δ, proper motion in δ and helicentric radial velocity. The code uses the ASTROPY

---

[1] https://github.com/martinmestre/stream-fit/blob/main/likelihood_grid/optim/model_def.py

ecosystem (Astropy Collaboration et al. 2022, 2018, 2013) in order to transform the initial condition to galactocentric coordinates assuming a Galactocentric reference frame with the sun at the position $\boldsymbol{x}_\odot = (-R_\odot, 0, 0)$ and a sun's velocity given by the sum of the circular velocity at the position of the sun and the sun's peculiar velocity: $\boldsymbol{v}_\odot = (11.1, v_c(R_\odot) + 12.24, 7.25)$. The circular velocity is dependent on the model and the position and given by

$$
\begin{aligned}
v_c^2(R_\odot) &= R_\odot \|\nabla \Phi(\boldsymbol{x})\|_{\boldsymbol{x}=\boldsymbol{x}_\odot} \\
&= R_\odot \|\nabla \Phi_{\mathrm{B}}(\boldsymbol{x})\|_{\boldsymbol{x}=\boldsymbol{x}_\odot} + G \frac{M_{\mathrm{DM}}(R_\odot)}{R_\odot},
\end{aligned}
\tag{20}
$$

where Φ is the total potential, Φ_B is the potential generated by the three barionic components and $M(r)$ is the enclosed dark matter mass as defined in Eq. (9). We have used the spherical symmetry property of the DM distribution in order to relate the acceleration with the enclosed mass.

A second step is to integrate the orbit forwards and backwards in time during a time interval of $\Delta t = 0.2$ Gyr, starting in both cases from a given initial condition for the progenitor. In the next sections we will explain how this initial condition was chosen for some simulations and fitted for others. The integrator used is a Runge-Kutta of order eight (DOP853 called from SciPy's solve_ivp function) with relative and absolute tolerance parameters respectively given by rtol= 5×10^{-14} and atol= 0.5×10^{-14}.

After having the stream's orbit integrated, we transform the orbit to both the ICRS and the GD-1 frame of coordinates. For the latter we used the GD1Koposov10 class defined in the Gala package (Price-Whelan 2017; Price-Whelan et al. 2020) which uses the transformation matrix defined by Koposov et al. (2010). After these two transformation we obtain the orbit expressed in the observable variables (ϕ_1, ϕ_2, D, v_h, $\tilde{\mu}_\alpha$, μ_δ) used in Eqs. (1)-(5). Finally, we build interpolators of these variables as a function of ϕ_1 that will be used, together with the observed data defined in Sec. 2.1, to evaluate the following χ^2 stream function:

$$
\chi^2_{\mathrm{stream}} = \chi^2_{\phi_2} + \chi^2_D + \chi^2_{v_h} + \chi^2_{\tilde{\mu}_\alpha} + \chi^2_{\mu_\delta},
\tag{21}
$$

$$
\chi^2_\eta = \frac{1}{\sigma_\eta^2} \sum_{i=1}^{100} (\eta^{(i)} - \eta(\phi_1^{(i)}))^2,
\tag{22}
$$

where η represents each of the four corresponding independent variables (ϕ_2, D, v_h, $\tilde{\mu}_\alpha$, μ_δ) and σ_η is the average dispersion of the stream data points taken by ocular inspection from Figs. 1,3,4 of Ibata et al. (2020): $\sigma_{\phi_2} = 0.5°$, $\sigma_D = 1.5$ kpc, $\sigma_{v_h} = 10$ km/s and $\sigma_{\tilde{\mu}_\alpha} = \sigma_\delta = 2$ mas/yr. Thus χ^2_{stream} measures the departure of the model from the stream observations. For some fits we will also consider the departure of the model from a mass constraint in the core of the distribution:

$$
\chi^2_{\mathrm{core}} = \frac{(m - m_{\mathrm{core}})^2}{\sigma_m^2},
\tag{23}
$$

where m_{core} was defined in Sec. ?? and σ_m was tested between 10% and 50% of its value, so sometimes we will use the following compound function:

$$
\chi^2_{\mathrm{full}} = \chi^2_{\mathrm{stream}} + \chi^2_{\mathrm{core}}.
\tag{24}
$$

2.4. Optimization algorithms

Our objective consists in fitting our MW model by minimizing the χ^2 function given by Eqs. (21,24). In order to do so we use two optimization algorithms for different purposes that will be explained in Sec. 3 One of them is SciPy's optimize.differential_evolution algorithm. This genetic architecture was used with metaparemeters given by strategy="best2bin", maxiter=100, popsize=50, tol=5×10^{-8} and atol=0. This method can be run in parallel with shared memory, so a convergence test was made in a cluster's node with 24 processors, varying the values of the metaparameters (e.g. doubling the values of maxiter and popsize, setting strategy="best1bin" and combinations).

The other algorithm is NLopt's LN_NELDERMEAD called from a Julia (Bezanson et al. 2017) environment. We used default values for the all its metaparameters except for reltol=5×10^{-5}.

3. Results

4. Discussion

5. Conclusions

References

Argüelles, C. R., Mestre, M. F., Becerra-Vergara, E. A., et al. 2022, MNRAS, 511, L35

Argüelles, C. R., Krut, A., Rueda, J. A., & Ruffini, R. 2018, Physics of the Dark Universe, 21

Astropy Collaboration, Price-Whelan, A. M., Lim, P. L., et al. 2022, apj, 935, 167

Astropy Collaboration, Price-Whelan, A. M., Sipőcz, B. M., et al. 2018, AJ, 156, 123

Astropy Collaboration, Robitaille, T. P., Tollerud, E. J., et al. 2013, A&A, 558, A33

Becerra-Vergara, E. A., Argüelles, C. R., Krut, A., Rueda, J. A., & Ruffini, R. 2020, A&A, 641, A34

Becerra-Vergara, E. A., Argüelles, C. R., Krut, A., Rueda, J. A., & Ruffini, R. 2021, MNRAS, 505, L64

Bezanson, J., Edelman, A., Karpinski, S., & Shah, V. B. 2017, SIAM review, 59, 65

de Boer, T. J. L., Belokurov, V., Koposov, S. E., et al. 2018, Monthly Notices of the Royal Astronomical Society, 477, 1893

GRAVITY Collaboration, Abuter, R., Amorim, A., et al. 2018, A&A, 615, L15

Harris, C. R., Millman, K. J., van der Walt, S. J., et al. 2020, Nature, 585, 357

Ibata, R., Thomas, G., Famaey, B., et al. 2020, The Astrophysical Journal, 891, 161

Klein, O. 1949, Rev. Mod. Phys., 21, 531

Koposov, S. E., Rix, H.-W., & Hogg, D. W. 2010, The Astrophysical Journal, 712, 260

Malhan, K. & Ibata, R. A. 2019, MNRAS, 486, 2995

Malhan, K., Ibata, R. A., Goldman, B., et al. 2018, Monthly Notices of the Royal Astronomical Society, 478, 3862

Merafina, M. & Ruffini, R. 1989, A&A, 221, 4

Pouliasis, E., Di Matteo, P., & Haywood, M. 2017, A&A, 598, A66

Price-Whelan, A., Sipőcz, B., Lenz, D., et al. 2020, adrn/gala: v1.3

Price-Whelan, A. M. 2017, The Journal of Open Source Software, 2

Price-Whelan, A. M. & Bonaca, A. 2018, The Astrophysical Journal Letters, 863, L20

Schönrich, R., Binney, J., & Dehnen, W. 2010, Monthly Notices of the Royal Astronomical Society, 403, 1829

Tolman, R. C. 1930, Phys. Rev., 35, 904

Van Rossum, G. & Drake Jr, F. L. 1995, Python tutorial (Centrum voor Wiskunde en Informatica Amsterdam, The Netherlands)

Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, Nature Methods, 17, 261