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Invited Review

The school bus routing problem: A review

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ABSTRACT

This paper aims to provide a comprehensive review of the school bus routing problem (SBRP). SBRP seeks to plan an efficient schedule for a fleet of school buses where each bus picks up students from various bus stops and delivers them to their designated schools while satisfying various constraints such as the maximum capacity of a bus, the maximum riding time of a student in a bus, and the time window of a school. This class of problem consists of different sub-problems involving *data preparation, bus stop selection, bus route generation, school bell time adjustment,* and *bus scheduling.* In this paper, the various assumptions, constraints, and solution methods used in the literature on SBRP are summarized. A list of issues requiring further research is also presented.

1. Introduction

In this paper, we review research works on the school bus routing problem (SBRP). SBRP has been constantly studied since the appearance of the first publication on it by Newton and Thomas (1969). However, a need for a general approach for SBRP remains as most studies on it have come about due to the appearance of real world problems which have specific assumptions and constraints. Li and Fu (2002) point out that there is no single dominant approach for the study of SBRP. Moreover, they add that many of the available approaches are problem dependent. The number of surveys on SBRP is limited (see Desrosiers et al., 1981; Braca et al., 1997). At times, SBRP is simply glossed over and briefly considered in the surveys of a larger class of problem. Examples of these are the combined problem of routing and scheduling of vehicle and crew (Bodin et al., 1983), the rural postman problem (Eiselt et al., 1995) and the multi-objective vehicle routing problem (Jozefowiez et al., 2008).

SBRP consists of smaller sub-problems. According to the decomposition of Desrosiers et al. (1981), SBRP can be solved by five steps: *data preparation*, *bus stop selection* (student assignment to stops), *bus route generation*, *school bell time adjustment*, and *route scheduling*. In the *data preparation* step, the road network consisting of home, school, bus depot, and the origin-destination (OD) matrix among them are specified. For a given network, the *bus stop selection* step determines the location of stops, and the students are assigned to them. Thereafter, the bus routes for a single school are generated in the *bus route generation* step. The *school bell time adjustment* and *route scheduling* steps are necessary for the multi-school configuration. School bus systems are usually operated by the regional board of education and not by individual schools. In addition, buses are shared by multiple schools. Therefore, the arrangements of school opening and closing hours as well as the scheduling of buses for multiple schools are required.

In most existing approaches, these sub-problems are considered separately and sequentially. Although these sub-problems are not independent but are highly interrelated, they are treated separately due to the complexity and size of the problem. Moreover, in most literature, only some parts of SBRP are considered. Especially, since the locations of bus stops and the opening and closing hours of schools are highly related to the policies of board of education, many studies assume that these information pieces are given and concentrate on the routing and scheduling of vehicles. Table 1 summarizes works on SBRP and shows the sub-problems they considered.

Although SBRP itself is a unique and independent problem, a single sub-problem or a combination of its sub-problems can be classified as a variant of existing optimization problems. By itself, the *bus route generation* sub-problem is very similar to the vehicle routing problem (VRP), which is an extensively studied application of operations research. VRP seeks to generate efficient routes for a fleet of vehicles in order to deliver (or collect) products from depots to a set of customers (Toth and Vigo, 2002). In a large number of studies, SBRP is referred to as an important real world application of VRP (Golden et al., 1984; Paessens, 1988; Ronen, 1988; Ferland and Fortin, 1989; Baker et al., 1993; Augerat et al., 1998; Russell and Chiang, 2006; Jozefowiez et al., 2008). Excellent editorial works on VRP can be found in the works of Ball et al. (1995) and Toth and Vigo (2002).

The combined problem of *bus stop selection* and *bus route generation* falls into the class of location-routing problems (LRPs). LRP includes determining the location of the facilities (in SBRP, bus stops) serving more than one customer and the optimal set of routes for a fleet of vehicles (Min et al., 1998). Inasmuch as LRP

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Table 1
Sub-problems considered in the literature.

Reference	*Bus stop selection*	*Bus route generation*	*School bell time adjustment*	*Route scheduling*
Newton and Thomas (1969)		√		
Angel et al. (1972)		√		√
Bennett and Gazis (1972)		√		
Newton and Thomas (1974)		√		√
Verderber (1974)		√		√
Gavish and Shlifer (1979)		√		
Bodin and Berman (1979)	√	√		√
Dulac et al. (1980)	√	√		
Desrosiers et al. (1981, 1986a)	√	√	√	√
Hargroves and Demetsky (1981)		√		√
Swersey and Ballard (1984)				√
Chapleau et al. (1985)	√	√		
Desrosiers et al. (1986b)				√
Graham and Nuttle (1986)				√
Russell and Morrel (1986)		√		√
Bookbinder and Edwards (1990)				√
Chen et al. (1990)		√		√
Thangiah and Nygard (1992)		√		
Bowerman et al. (1995)	√	√		
Braca et al. (1997)		√		√
Corberán et al. (2002); Pacheco and Martí (2006)		√		
Fu et al. (2005)		√		
Ripplinger (2005)		√		
Spada et al. (2005)		√		√
Schittekat et al. (2006)	√	√		
Bektaş and Elmastaş (2007)		√		
Fügenschuh (2009)			√	√

is an NP-hard problem, there are no known polynomial time algorithms that can find an optimal solution for every problem instance (Laporte, 1988). For recent developments on LRP, see the work of Nagy and Salhi (2007).

The remainder of the paper is organized as follows: Section 2 describes the details of the five sub-problems of SBRP. Thereafter, the ensuing two sections present the classification schemes for SBRP from different perspectives. Section 3 presents the classification schemes from their problem perspective. Section 4 classifies the mathematical models and heuristic solution methods in the SBRP literature. This two-way classification approach is similar to the approach used by Min et al. (1998) for LRP but differs in its detailed list. Finally, Section 5 presents the challenges and opportunities for future work and provides our concluding remarks.

2. Problem description

In this section, we present five sub-problems of SBRP and introduce the studies that worked on these sub-problems.

2.1. Data preparation

The *data preparation* sub-problem prepares the data for the other sub-problems. In this sub-problem, the road network is specified, and four types of data for SBRP are prepared: students, schools, vehicles, and OD matrix. The data for students include the location (address) of their homes, the destination school of a student, and type of student. The type of student is either general or handicapped (for details, see Section 3.5).

School data contain information about the location of the schools, the starting and ending time of schools for bus arrivals, and the maximum riding time of a student in a bus. In most studies, the starting and ending time of schools for bus arrivals are given. However, several available studies assume that the starting and ending time can be determined in *school bell time adjustment* as summarized in Section 2.4.

The data for vehicles include the origin location and bus types of the available school buses. Each bus type may have different capacities for general students and special-education students. The OD matrix stores the shortest travel times or distances between pairs of nodes (school, student location, and bus origin location). The OD matrix can be calculated using a geographic information system (GIS) and various shortest path algorithms. Kim and Jeong (2009) compared the performance of several shortest path algorithms and developed an approximation approach for OD matrix generation.

2.2. Bus stop selection

Bus stop selection seeks to select a set of bus stops and assign students to these stops. For schools in rural surroundings, the students are assumed to be picked up at their homes. However, in urban areas, the students are assumed to walk to a bus stop from their homes and take a bus at the stop.

As mentioned in Section 1, *bus stop selection* is often omitted in the literature. Many studies assume that the locations of bus stops are given. Only a few papers considered *bus stop selection*, and most of them use heuristic algorithms. Table 2 summarizes research on bus stop selection.

The heuristic solution approaches for *bus stop selection* are classified into the location-allocation-routing (LAR) strategy or the allocation-routing-location (ARL) strategy (Laporte, 1988; Bowerman et al., 1995). Figs. 1 and 2 show the concepts of these two strategies. The LAR strategy first determines a set of bus stops for a school and assigns students to these stops. Routes are generated for these selected stops. However, since the bus stops and the assignment of

Table 2
Papers on bus stop selection.

Reference	Constraints	Features
Bodin and Berman (1979)		LAR strategy
Dulac et al. (1980)		LAR strategy
Desrosiers et al. (1981, 1986a)		LAR strategy
Chapleau et al. (1985)	Maximum walking distance	ARL strategy
Bowerman et al. (1995)		ARL strategy
Schittekat et al. (2006)	Maximum walking distance	MIP model for bus stop selection

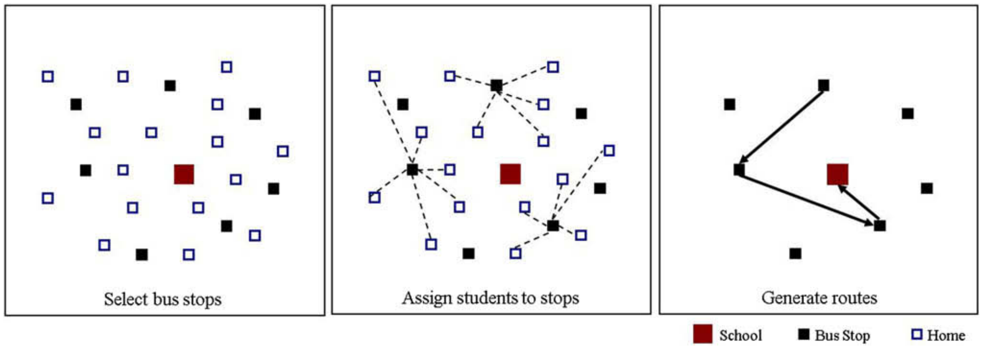

Fig. 1. Concept of LAR strategy.

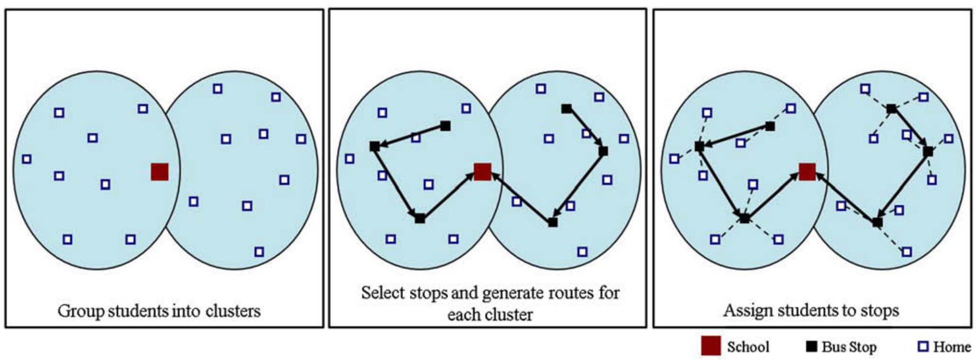

Fig. 2. Concept of ARL strategy.

the students are determined without taking into consideration their effect on generating routes, this approach tends to generate excessive routes. Bodin and Berman (1979), Dulac et al. (1980) and Desrosiers et al. (1981) proposed a heuristic approach based on the LAR strategy. In the ARL strategy, the students are allocated into clusters while satisfying vehicle capacity constraints. Subsequently, the bus stops are selected, and a route is generated for each cluster. Finally, the students in a cluster (route) are assigned to a bus stop which satisfies all the requirements given in the problem such as the maximum walking distance from home, maximum number of students that can be assigned to a bus stop, and the minimum distance apart between bus stops. Chapleau et al. (1985) and Bowerman et al. (1995) used an algorithm based on the ARL strategy.

Recently, Schittekat et al. (2006) developed a simple mathematical model for *bus stop selection* from potential stops and bus route generation for a single-school problem. However, their model was limited to a single school and was tested for small-sized random problem instances (10 bus stops and 50 students) only.

2.3. Bus route generation

In *bus route generation*, the school routes are constructed. The algorithms used in *bus route generation* can be classified into either the "route-first, cluster-second" approach or the "cluster-first, route-second" approach (Bodin and Berman, 1979). The "route-first, cluster-second" approach builds a large route by a traveling salesman problem algorithm that considers all the stops, and partitions it into smaller routes considering the constraints. Newton and Thomas (1969) and Bodin and Berman (1979) implemented this approach. The "cluster-first, route-second" approach groups the students into clusters so that each cluster can be served as a route satisfying the constraints that exist. Dulac et al. (1980), Chapleau et al. (1985) and Bowerman et al. (1995) applied this approach to SBRP. For additional information on these two approaches, see the works of Dulac et al. (1980) and Laporte and Semet (2002).

After the school routes are generated, improvement heuristics can be applied on the routes. There are plenty of improvement heuristics and metaheuristic approaches. The heuristic approach suggested by Lin (1965) dubbed as λ-opt algorithm is widely adopted for VRP studies. This algorithm deletes λ edges from the route then forms a new feasible tour by adding λ edges. The idea of λ-opt algorithm is adopted for several SBRP studies. Newton and Thomas (1969), Dulac et al. (1980), Chapleau et al. (1985) and Desrosiers et al. (1986a) applied 2-opt algorithms to improve solution. Bennett and Gazis (1972) and Bodin and Berman (1979) adopted 3-opt algorithms.

2.4. School bell time adjustment

In most studies, the starting and ending time of schools are constraints. However, there are a number of works that consider the times as decision variables and attempts to find the optimal starting and ending times to maximize the number of routes that can be served sequentially by the same bus and to reduce the number of buses used. Desrosiers et al. (1981, 1986a) determined the starting and ending times of schools using a column generation approach. Bodin et al. (1983) adopted the approach of Desrosiers et al. (1981, 1986a). They claimed that small-sized problems can be solved approximately manually. Fügenschuh (2009) considered the problem of scheduling school starting times while allowing student transshipment from a route to another. A mixed integer programming (MIP) model was developed and solved by a branch-and-cut approach with several pre-processing mechanisms and valid cuts.

2.5. Route scheduling

Route scheduling specifies the exact starting and ending time of each route and forms a chain of routes that can be executed successively by the same bus. Newton and Thomas (1974) developed a multi-school model to determine all bus routes for a school district. They assumed that there are distinguished time periods, and schools start at different time periods. Bodin (1975) and Bodin and Berman (1979) also assumed that the time windows of schools can be partitioned into distinct time periods such that the routing problem can be solved period by period, and the total route can be combined later. However, their approach may not work when the school bell times or the time periods overlap because the completion time of the previous period may be too late for the succeeding period.

Desrosiers et al. (1981, 1986a) used a heuristic method that solved a series of transportation problems. Graham and Nuttle (1986) drew comparisons of different heuristic procedures for school bus scheduling. Desrosiers et al. (1986b) developed three algorithms and tested these algorithms on eight school transportation problems with various settings such as whether the morning or afternoon problem is considered and the different interval lengths of time windows.

Braca et al. (1997) described the New York City school bus routing problem. While in most literature, the separate problem is solved for each school, and then the route schedule for each bus is determined, they solved the whole problem of all schools in one stage. Li and Fu (2002) applied a *k*th shortest path algorithm of Lawler (1972) to generate an initial route and an improvement scheme, in which the stops from the larger route are moved into smaller routes. For buses that have the same capacity, the assignment problem is formulated with the objective of minimizing bus vacant-travel (deadheading) time.

Spada et al. (2005) considered a multiple school routing problem and proposed heuristic approaches. Schools are considered in increasing order of their starting times and the routes for each school are built by using a greedy method. Thereafter, the routes are merged if possible. After which, the merged routes are improved by Simulated Annealing or a Tabu search.

Table 3
Classification based on the problem characteristics.

Criteria	Consideration
Number of schools	A. Single school
	B. Multiple schools
Surroundings of service	A. Urban
	B. Rural
Problem scope	A. Morning
	B. Afternoon
	C. Both
Mixed Loads	A. Mixed loads are allowed
	B. No mixed loads are allowed (single load problem)
Special-education students	A. Special-education students are considered
	B. Only general students are considered
Fleet mix	A. Homogeneous fleet (HO)
	B. Heterogeneous fleet (HT)
Objectives	A. Number of buses used (N)
	B. Total bus travel distance or time (TBD)
	C. Total student riding distance or time (TSD)
	D. Student walking distance (SWD)
	E. Load balancing (LB)
	F. Maximum route length (MRL)
	G. Child's time loss (TL)
Constraints	A. Vehicle capacity (C)
	B. Maximum riding time (MRT)
	C. School time window (TW)
	D. Maximum walking time or distance (MWT)
	E. Earliest pick-up time (EPT)
	F. Minimum student number to create a route (MSN)

3. Classification based on problem characteristics

In this section, we attempt to classify SBRP from its problem characteristics. Although there are a number of ways for classification, we mainly focus on the practical aspects of the problems. Table 3 shows the classification criteria from the problem perspective. The details of these criteria are discussed in the following sub-sections. Table 4 summarizes the SBRP literature with regard to the classification criteria.

3.1. Number of schools: single school or multiple schools

The number of schools in SBRP can be single or multiple. As mentioned in Section 1, the school bus system is generally a multi-school based system and not a single-school one. However, in the literature, a large number of papers focus on a single-school problem (see Table 4).

Table 4
Literature classification based on the problem characteristics.

Reference	#. Schools	Urban or rural	Mixed loads	Fleet mix	Objectives	Constraints	Problem size	Area
Newton and Thomas (1969)	Single	Urban	No	HO	Not specified	C, MRT	50–80 stops	Artifical
Angel et al. (1972)	Multiple	Suburban	No	HO	N, TBD	C, MRT	1500 students	Tippercanoe, Indiana
Bennett and Gazis (1972)	Single	Urban	No	HO	TBD, TSD	C	256 stops	Toms River, New Jersey
Newton and Thomas (1974)	Multiple	Urban	No	HO	N, TBD	C, MRT	1097 students, 76 stops	Western New York
Verderber (1974)	Multiple	Urban	No	HO	N, TBD	C, MRT	11,000 students	New York
Gavish and Shlifer (1979)	Single	Urban	No	HO	N, TBD	C, MRT	21 stops	Artificial
Bodin and Berman (1979)	Multiple	Urban	No	HO	N	C, MRT, TW	13,000 students	Brentwood, New York
Dulac et al. (1980)	Single	Urban	No	HO	N, TBD	C, MRT, MWT	585 students, 99 stops	Drummondville, Canada
Desrosiers et al. (1981, 1986a)	Multiple	Both	No	HO	N, TBD	C, MRT, MWT	About 16,000 students	Drummondville, Canada
Hargroves and Demetsky (1981)	Multiple	Suburban	Yes	HT	N, TBD	C, MRT, MSN	8537 students	Albemarle, Virginia
Swersey and Ballard (1984)	Multiple	Urban	No	HO	N	TW	37 schools	New Haven, Connecticut
Chapleau et al. (1985)	Single	Urban	No	HO	N, SWD	C, MRT, MWT	2079 students	
Russell and Morrel (1986)	Multiple	Urban	Yes	HO	TBD	C, MRT	140 students	Tulsa, Oklahoma
Chen et al. (1990)	Multiple	Rural	Yes	HO	N, TBD	C, MRT	2413 students	Choctaw, Alabama
Thangiah and Nygard (1992)	Single	Rural	No	HT	N, TBD, TSD	C, MRT	353 students	
Bowerman et al. (1995)	Single	Urban	No	HO	N, SWD, LB	C, MWT	138 students	Ontario, Canada
Braca et al. (1997)	Multiple	Urban	Yes	HO	N	C, MRT, TW, EPT, MSN	838 stops, 73 schools	Manhattan, New York
Corberán et al. (2002); Pacheco and Martí (2006)	Single	Rural	No	HO	N, MRL	C	55 stops	Burgos, Spain
Li and Fu (2002)	Single	Urban	No	HT	N, TSD, TBD, LB	C	86 students, 54 stops	Hong Kong
Ripplinger (2005)	Single	Rural	No	HT	N, TBD	C, MRT	131 students	Artificial
Spada et al. (2005)	Multiple	Rural	Yes	HT	TL	C, TW	274 students	Switzerland
Schittekat et al. (2006)	Single	Urban	No	HO	TBD	C	50 students, 10 stops	Artificial
Bektaş and Elmastaş (2007)	Single	Urban	No	HO	N, TBD	C, MRT	519 students	Ankara, Turkey
Fügenschuh (2009)	Multiple	Rural	No	HO	N, TBD	TW Range	102 schools	Germany

The solution structure of the single-school problem is similar to that obtained from traditional VRP. In traditional VRP, a route starts from a depot, traverses a number of customers, and finally returns to its starting depot. However, in SBRP, the depot is generally different from the school. In addition, in SBRP, the travel times (or distances) from the depot to the first stop and from the school to the starting depot are insignificant. Therefore, the structure of the vehicle routes of SBRP is similar to the structure from the Open VRP (Fu et al., 2005). The important feature of Open VRP is that the vehicle does not return to its starting depot after visiting the last customer on a route (Li et al., 2007). For recent advances in Open VRP, readers may refer to Repoussis et al. (in press) and Fleszar et al. (2009).

In the multi-school configuration, there are two different approaches to generate routes according to Spada et al. (2005): a school-based approach and a home-based approach. In the school-based approach, a set of routes is generated for each school, and these routes are assigned to a fleet of vehicles and scheduled with respect to the time windows of schools. The school-based approach does not allow students from different schools to travel in the same bus at the same time (no *mixed loads*). The home-based approach developed by Braca et al. (1997) inserts a stop into a route at a time. When a stop is determined to be inserted in a route, the corresponding school for this stop should also be in this route. If the school is not in the route, then the algorithm finds the best insertion point for this school. The insertion point is determined with respect to the insertion cost. The temporal precedence relationship (Bredstrom and Ronnqvist, 2008) between the student and the school should also be considered. Contrary to the school-based approach, the home-based approach allows students from different schools to be on the same bus at the same time. In other words, *mixed loads* are allowed.

3.2. Urban versus rural surroundings

The solution approach for SBRP may differ depending on whether the surroundings of the service are urban or rural. It is assumed that students in urban areas walk from their homes to the stops to take a bus. In rural areas, however, the number of students is small, and it is common to pick them up at their homes. Therefore, *bus stop selection* is not necessary for rural surroundings. Many researchers have pointed out the difference between the surroundings of urban and rural school bus systems. Bodin and Berman (1979), Chapleau et al. (1985), Bowerman et al. (1995) and Simchi-Levi et al. (2005) claimed that bus capacity is usually exhausted before the maximum riding time of a student on a bus is reached owing to the high density of students in the urban areas. Bowerman et al. (1995) even dropped the maximum riding time constraints from the problem as the constraints are not binding.

Chen et al. (1990) pointed out that the environment of a rural school is different from an urban school due to the following aspects: lower population density, greater travel distance per route, fewer number of riders per bus stop, more stops per route, fewer or virtually no one-way streets, more buses staying overnight at drivers' homes, and fewer alternative roads available for the bus stops remote from the school.

Howley et al. (2001) and Ripplinger (2005) also studied the unique attributes of the problem in rural areas. Howley et al. (2001) studied the difference between rural school bus ride and suburban school bus ride through a survey research. They verified several hypotheses from their survey such as the longer ride time and larger attendance areas in rural surroundings. Ripplinger (2005) discussed the uniqueness of rural surroundings. He pointed out that a manually generated solution could be optimal because the problem size is relatively small in rural surroundings.

3.3. Morning problem versus afternoon problem

SBRP mainly consists of morning and afternoon problems. In the morning, a bus picks up students from bus stops and then drops them off at their school. On the contrary, students are picked up at schools and delivered to their starting stops in the afternoon. Braca et al. (1997) claimed that the morning problem is more difficult than the afternoon problem for two reasons: ill-dispersed school time windows and heavier traffic. As a result, the afternoon problem receives less attention than the morning problem. Many studies are dedicated to the morning problem and the afternoon problem is only mentioned briefly. Notice that the afternoon school bus problem for a school can be converted into a morning problem with little modification (Li and Fu, 2002).

Bodin and Berman (1979) proposed a simple approach for the afternoon problem. They claimed that the sequence of stops for a school in the afternoon problem would be a replication of the morning problem or would be reversed to obtain a shorter travel time. The reversed sequence could be more intuitive. However, the replicated sequence is preferred from the perspective of equity in balancing the riding time in a bus for every student. Next, using these routes for each school, the chains of routes are constructed.

Desrosiers et al. (1981, 1986a) generated all possible schedules for a route called run combinations by considering constraints such as the upper bound on the waiting time for students between their arrival at school and the school's starting time. A run combination contains the information on which time period the route is executed in the morning and in the afternoon. By selecting an optimal set of run combinations, the schedules for the morning and afternoon are generated simultaneously.

Other than the morning and afternoon problem, Bookbinder and Edwards (1990) addressed a different type of school bus routing problem called *program scheduling problem*. Program scheduling problem seeks the best transportation plan of students across multiple schools for special school activities such as swimming and industrial art. They developed an MIP model for the problem and solved it using a modified assignment problem solution approach.

3.4. Allowance of mixed loading

Bodin and Berman (1979) indicated that the mixed loading problem can occur in the rural area. When mixed loads are allowed, students from different schools can be put on the same bus at the same time. Allowing mixed loads leads to increased flexibility and cost savings (Braca et al., 1997). When mixed loads are not allowed, the problem becomes the single load problem in which the bus can either pick up or drop off students to a single school (Bodin et al., 1983). Chen et al. (1988) pointed out that the single load assumption is over-restrictive and could use an excessive number of buses for remote children.

Although the mixed loading problem is discussed in several papers (Bodin and Berman, 1979; Bodin et al., 1983; Chen et al., 1988; Spada et al., 2005), only Braca et al. (1997) actually proposed an algorithm for mixed loads. Chen et al. (1988) developed an expert system for a rural school district and allowed mixed loads for routes. However, the routes are generated manually. Spada et al. (2005) handled mixed loads with an interactive user interface tool.

Braca et al. (1997) modified the Location Based Heuristic of Bramel and Simchi-Levi (1995) which was originally developed for capacitated VRP (CVRP). Their approach was based on a simple insertion rule. The algorithm checks two consecutive points in a route and checks whether a stop can be inserted between them. If the destined school for a stop is not in the route, then the algorithm also determines the best insertion position for the school. Subsequently, a set of routes is updated by inserting a stop into the route that results in the shortest total route length. They note the difficulty of checking feasibility when a stop is added to a route. While there is only one drop off point (the final visiting depot) in CVRP, there are more than one drop off points in the routes for multiple school configurations, and when and where the student is getting on and off is required to be determined.

3.5. Routing special-education students

The problem of routing special-education students is fundamentally different in many respects from the problem of routing general students. First, special-education students are picked up and dropped off directly at their homes and not at their bus stops. Second, there is more rigid restriction on the maximum riding time for a special-education student in a bus than for a general student. Third, each student should be served differently depending on the severity of his/her particular disability. Since some students need special equipment and help, they should be assigned to a special bus which is able to serve them. This student-bus eligibility compounds the difficulty of the problem. Only a few papers have considered the problem of routing special-education students.

Russell and Morrel (1986) addressed the special-education routing problem. They constructed an initial solution by employing a modified savings algorithm (Clarke and Wright, 1964) and improved the solution by the 3-opt algorithm and M-TOUR improvement algorithm (Russell, 1977). However, since special-education routes often have several destination schools due to the diversity of students, the number of schools to be visited by a bus tends to be excessive. To reduce the number of school visits for a bus and student riding time, they developed a shuttle system. In their shuttle system, the buses pick up the students and then travel to one of the two shuttle locations where the two schools with the most students are heading to. After which, the students belonging to the shuttle schools are unloaded, and the remaining students are reassigned to the buses. In their shuttle system, the number of school visits is limited to two schools. The shuttle system is effective in generating compact routes and reducing the number of schools visits for some of the routes.

Braca et al. (1997) briefly discussed special-education students in a school bus system. They presented the status of the school bus system in the City of New York where about 125,000 students ride school buses, and about 40,000 students are classified as special-education students. They also explained the difference between special-education and general students. However, they only focused on routing general students.

The work of Ripplinger (2005) focused on students with special needs in rural areas and two options for these students were discussed. The first option distinguishes between students with special needs and those without. Thereafter, these problems are solved separately. The second option generated a single routing plan for both types of students.

In some real world problems, special-education students and general students could ride the same bus at the same time. However, to the best of our knowledge, no studies have considered this problem.

3.6. Homogeneous or heterogeneous fleet

The problem dealt with a heterogeneous fleet of buses assumes that each bus has different characteristics such as various capacities, maximum allowable riding time, fixed cost, and per unit distance variable cost. The problem with a heterogeneous fleet of buses is similar to the heterogeneous fleet VRP (HVRP), a variant of traditional VRP.

Newton and Thomas (1974) made a unique assumption on the capacity of buses. They assumed that all buses have the same capacity. However, bus capacity changes for each school because each school has its own policy for the tolerable degree of crowding and whether standees are permitted.

Bowerman et al. (1995) studied a problem on routing a set of buses having the same capacities. In most studies, one load is identical to one student. However, Bowerman et al. (1995) assumed that each student occupies a different load (or space) for a bus. For example, if a student is in the early primary grades, this student occupies only an equivalent 2/3 load of a regular student. In this case, two buses with the same loads could have different numbers of students.

3.7. Objectives

Savas (1978) discussed three measures for evaluating the performance of public services, namely, *efficiency*, *effectiveness*, and *equity*. Using these criteria, Bowerman et al. (1995) and Li and Fu (2002) classified the objectives considered in their research.

Efficiency is defined as the ratio of the level of service to the cost of the resources required to provide such service. For a fixed level of service, the efficiency of a service can be determined by its cost. According to Bowerman et al. (1995), there are two main components in the cost of a school bus system. One is the capital cost required to run one school bus for a school year, and the other is incremental cost per unit distance travelled. Therefore, a solution that requires fewer buses and has shorter total bus travel time is preferred from an efficiency perspective.

The effectiveness of a service is measured by how well the demand is satisfied. An effective school bus system should be available to all eligible students and should have an acceptable level of service. The effectiveness of a school bus service can be determined by using measures such as the total travel time spent by students in buses and the total distance that students have to walk.

Equity considerations assess the fairness or impartiality of the provision of the service. A solution with a good efficiency measure might be inexpensive and better in cost and time perspectives, but would be unacceptable due to the lack of fairness in providing service to the students. Equity has been neglected in evaluating the performance of a school bus service as well as public services. However, its growing importance has been recognized. To improve the equity of a school bus service, balancing the loads and travel times between buses should be considered.

Table 4 shows that most SBRP studies aim to minimize the number of buses used (N) and the total bus travel distance (TBD). Only a few papers have considered the equity criteria such as load balancing (LB). Note that Spada et al. (2005) used a unique performance measure called the child's time loss (TL). The child's time loss, which can be classified into an effectiveness criterion, is measured as the difference between a child's actual travel plus waiting time and the shortest possible travel time (i.e., the travel time spent in directly driving from the child's home to the school).

3.8. Constraints

Various constraints have been considered for SBRP as shown in Table 4. The following constraints were listed by Braca et al. (1997) and Spada et al. (2005):

- *Vehicle capacity constraint:* an upper bound on the number of students in a bus.
- *Maximum riding time:* an upper bound on the travel time in a bus for each child.
- *Maximum walking distance:* a student's maximum allowable walking distance from his/her home to the bus stop.
- *School time window:* time window on the starting and ending arrival time of a bus at schools.
- *Upper bounds on the number of students at stops.*

- *Earliest pickup time for children.*
- *Minimum number of children to create a route.*

In other studies, some of these constraints are considered as objective functions. For example, Bennett and Gazis (1972) and Li and Fu (2002) considered *maximum riding time* as an objective and aimed at minimizing the total travel time spent by all children. Bowerman et al. (1995) included the total student walking distance in their objective function. Desrosiers et al. (1981, 1986a), Bodin et al. (1983) and Fügenschuh (2009) assumed the school time windows as decision variables. They adjusted the school time windows to reduce the number of needed buses.

4. Classification based on solution methods

Bowerman et al. (1995) showed that the combined problem of bus stop selection and bus route generation for a single school is an *NP*-hard problem. Even a single sub-problem such as *bus stop selection* and *bus route generation* can be shown to be *NP*-hard problems. In *bus stop selection*, each student should be assigned to one bus stop, and each stop has its capacity. Using these constraints, the *bus stop selection* problem can be transformed to the generalized assignment problem which is also known as a *NP*-hard problem (Fisher et al., 1986). The *bus route generation* problem with vehicle capacity and maximum riding time constraints corresponds to a capacitated and distance constrained Open VRP, which is known as an *NP-hard* problem (Bektaş and Elmastaş, 2007). Due to its difficulty most studies prefer heuristic approaches rather than exact approaches. Only a few papers adopted exact approaches to solve the parts dealing on SBRP.

This section consists of two parts. The mathematical formulations for SBRP are presented in Section 4.1. In Section 4.2, we will summarize the solution methods used in the literature.

4.1. Mathematical formulations

The mathematical models for SBRP are developed for various configurations. SBRP is usually formulated as mixed integer programming (MIP) or nonlinear mixed integer programming (NLMIP) models. However, most of them have not been used directly to solve the problems, and they are often not for the whole problem but for the SBRP parts. Table 5 summarizes the mathematical formulations for SBRP.

Gavish and Shlifer (1979) considered the bus route generation problem for single-school configuration and developed an NLMIP model. They generated upper bounds for the problem by solving a sequence of assignment problems and determined optimal solution using a branch and bound procedure. White (1982) improved the algorithm of Gavish and Shlifer (1979) by replacing the assignment problem with maximum matching problem. Bowerman et al. (1995) considered the *bus stop selection* and bus route generation problem simultaneously. They developed an NLMIP model but did not use the formulation to solve the problem. Li and Fu (2002) developed a multi-objective NLMIP model for the bus route generation problem and explicitly expressed various objective functions. However, the mathematical formulation is not used to solve the problem. Ripplinger (2005) developed an MIP model for the rural school vehicle routing problem. However, the routes were generated by applying a heuristic algorithm. Kara and Bektaş (2006) modeled a single-school SBRP as a multiple traveling salesman problem with multiple depots and single destination.

Schittekat et al. (2006) and Bektaş and Elmastas (2007) developed mathematical models for a single-school configuration and actually used these models to solve the problem. Schittekat et al. (2006) assumed that a school is identical to a depot, and a route starts and ends at the school. In addition, they assumed that the vehicle fleet is homogeneous and that the choice whether a student can walk to a certain stop or not is given. Under these assumptions, they developed an MIP model and solved an example with 10 stops and 50 students. However, their model was very simple and did not consider practical constraints such as the maximum riding time. Bektaş and Elmastas (2007) proposed an MIP model for the bus route generation problem. They included not only vehicle capacity but also an upper bound on the riding time in a bus for each child in their model. Using the formulation, they generated an optimal solution for an elementary school in Ankara, Turkey which has 29 pickup points. Desrosiers et al. (1981, 1986a) and Fügenschuh (2009) developed MIP models in multi-school configuration. These models aimed to schedule the school starting and ending times for a given set of routes. Swersey and Ballard (1984) developed an NLMIP model and converted it into two discretized integer programming models for the *route scheduling* problem for multiple schools.

Braca et al. (1997) and Spada et al. (2005) developed NLMIP models for a multi-school problem but did not use the models to solve the problem. Braca et al. (1997) formulated the bus route generation and bus scheduling problem as a set partitioning problem. However, it was assumed that the set of feasible routes is given. Spada et al. (2005) presented a formulation for the bus route generation and bus scheduling problem as an NLMIP model. Hanley (2007) proposed an approach to evaluate transportation cost change as school districts are redesigned. The study formulated a combined problem of school district consolidation and bus routing as a multiple objective NLMIP model.

Table 5
Mathematical formulations for SBRP.

Reference	Problem type	Mathematical model
Gavish and Shlifer (1979)	Single school	Assignment problem
White (1982)	Single school	Maximal matching problem
Desrosiers et al. (1981, 1986a)	Multi-school	MIP
Swersey and Ballard (1984)	Multi-school	NLMIP, Two discretized MIP
Bowerman et al. (1995)	Single school	NLMIP
Braca et al. (1997)	Multi-school	Set partitioning model
Li and Fu (2002)	Single school	NLMIP
Ripplinger (2005)	Single school	MIP
Spada et al. (2005)	Multi-school	NLMIP
Schittekat et al. (2006)	Single school	VRP-like model
Kara and Bektaş (2006)	Single school	Multiple traveling salesman problem
Hanley (2007)	Multi-school	NLMIP
Bektaş and Elmastaş (2007)	Single school	MIP
Fügenschuh (2009)	Multi-school	MIP

4.2. Solution methods

As mentioned in Section 4.1, relatively small-sized problems are solved by exact approaches, and heuristic approaches are preferred in most studies. Section 2 describes the detailed explanation of the solution methods used for each sub-problem. In this section, the *metaheuristics* approaches that have been applied to SBRP are presented.

In recent years, many metaheuristics have been proposed for combinatorial optimization problems: Simulated Annealing (SA), Deterministic Annealing (DA), Tabu Search (TS), Genetic Algorithms (GA), Ant Colony Optimizations (ACO), and Neural Networks (NN). These metaheuristics have been widely used for VRP (Gendreau et al., 2002; Cordeau et al., 2005; Bin et al., 2009; Belfiore and Yoshizaki, in press), but only a few papers have applied metaheuristics to SBRP.

Thangiah and Nygard (1992) developed an automated routing system of school buses called GENROUTER. GENROUTER uses a two-phase approach which is a variation of the cluster-first route-second method. In the first phase a method called genetic sectoring based on a genetic algorithm is used to cluster the student locations. For the best set of clusters obtained from genetic sectoring, the second phase improves the routes using location optimization methods.

Corberán et al. (2002) used an evolutionary method called *scatter search* to improve the initial solutions generated by two heuristics which are based on clustering mechanisms. Spada et al. (2005) applied SA and TS to improve the initial solution generated by an insertion-based heuristic. Ripplinger (2005) used a clustering based algorithm to generate a feasible solution, and TS was used as an improvement algorithm for the initial solution. Pacheco and Martí (2006) constructed a set of initial feasible solutions by using four different heuristics, two of which are from Corberán et al. (2002), one from Fisher and Jaikumar's algorithm (Fisher and Jaikumar, 1981), and an insertion mechanism. The solutions generated with these four heuristics are improved using a TS.

5. Future directions and concluding remarks

In this paper, we described SBRP and classified the literature on SBRP with regard to various aspects of the problem. As shown in Table 4, most of early research on SBRP were carried out in the USA and Canada. However, research on SBRP in the last decade has spread to countries other than the USA and Canada.

Despite the effort made for SBRP, there are still many opportunities for future research. The most important locus for future work on SBRP is the generalization of the problem. To date, existing research on it has been undertaken independently. Defining general SBRP could encourage researchers and prevent isolated research. In addition, the benchmark problem sets based on practical application should be prepared to boost cumulative and collaborative efforts among researchers.

Further study on the mixed loading problem is also needed. As discussed in Section 3.4 and shown in Table 4, most studies are restricted to a single load problem and few previous works considered the mixed loading problem. A single load restriction inevitably leads to an excessive number of buses, especially when each school has a small number of students. For example, if there are three schools which have only one student, respectively, three buses are needed for the single load case. However, the number of buses used can be reduced if mixed loads are allowed. According to Simchi-Levi et al. (2005), mixed loads are allowed for the most part in New York City and we also found that several school districts allow mixed loads in our recent research project.

In addition, as mentioned in Section 3.5, the case of allowing mixed riding of special-education and general students should be considered. According to school bus practitioners from our recent project, some of the current bus types being used have the capacity to cater to both special-education and general students. Thus, mixed riding of special-education and general students may occur. To the best of our knowledge, there is no literature considering this kind of mixed riding.

The issue of student transshipment should also be considered in future research. Most students are transported to their destination schools directly. However, some students may be transported to certain locations called transfer stations, be transferred to different buses at the stations, and then be transported to their destination schools. From our recent project, we found that many school districts in the USA are using transfer stations. Some of the schools and independent places are used as transfer stations. In a wide but sparse school district, the capacity utilization of school buses may be increased if some of the students are consolidated by student transshipment using transfer stations so the number of buses required may be reduced. It is also worth noting that although it is quite uncommon, students are transferred twice in some real-life cases. Not many previous works have considered this practical transshipment issue. Russell and Morrel (1986) dealt with the transshipment problem of special-education students using a shuttle system. Recently, Cortés et al. (2010) considered transfers in the pickup and delivery problem.

In recent years, many metaheuristics have been proposed such as Tabu Search, Variable Neighborhood Search (VNS), Very Large Neighborhood Search (VLNS), and Active Guided Evolution Strategy (AGES). Although these metaheuristics have been successfully implemented to VRP, only a few studies on SBRP have taken advantage of these. In addition, the implementation of metaheuristics is limited to route generation. In the future, metaheuristics may be extensively used to other sub-problems as well as route generation. Metaheuristics might be used to get high quality solutions for real-life large size problems in a reasonable time. One may argue that SBRP is not necessarily to be solved quickly since the school bus service plan does not need to be made on a daily basis. However, even a mathematically good quality solution can be rejected due to the various school district policies. Therefore, SBRP for a school district might be solved many times until an acceptable solution is obtained while allowing manual revisions and thus SBRP needs to be solved quickly.

Finally, research efforts should be made on exact approaches for SBRP. Although quite a few previous works proposed exact approaches for VRP as well as its variants such as CVRP and VRP with Time Windows (VRPTW), research works on exact methods on SBRP and Open VRP are very rare. Letchford et al. (2007) and Bektaş and Elmastas (2007) are among them.

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