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School bus Routing Problem: Contemporary Trends and Research Directions

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School Bus Routing Problem: Contemporary Trends and Research Directions

Highlights

- The paper categorizes 64 new SBRP publications, examining SBRPs in North America, South America, Africa, Asia, and Europe.
- SBRP researchers are exploring more complex real-world problem settings, addressing multiple sub-problem types and employing more advanced metaheuristic solution methods.
- We identify five key areas of SBRP research that offer promising opportunities for further research.

Title: School bus Routing Problem: Contemporary Trends and Research Directions

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ABSTRACT

The school bus routing problem (SBRP) is a challenging operations research problem that has been studied by researchers for almost 50 years. SBRP publications address one or more operational sub-problems, including: bus stop selection, bus route generation, bus route scheduling, school bell time adjustment, and strategic transportation policy issues. This paper reviews 64 new SBRP research publications and analyzes them by sub-problem type, problem characteristics and solution approach. The impact of key SBRP characteristics (number of schools, mixed load, fleet mix, service environment, objective and constraints) are discussed and the different solution approaches to the SBRP are summarized by sub-problem type and methodology. We found in recent years, SBRP researchers are examining more complex real-world problem settings, adopting both evolutionary-based and trajectory-based metaheuristic solution approaches, and considering ridership and travel time uncertainty. This review documents recent trends in SBRP research and highlights research gaps and promising opportunities for future SBRP research.

1. INTRODUCTION

The school bus routing problem (SBRP) is a critical, real-world transportation problem that impacts millions of families around the world on an almost daily basis. The issue that school administrators and service providers face is to develop and provide a safe, reliable and cost-effective school bus transportation system to carry students to and from schools daily. The SBRP is closely related to the vehicle routing problem (VRP) and Operations Research (OR) approaches to the SBRP date back to Newton & Thomas (1969). The subsequent thirty years of SBRP research witnessed a small and steady stream of research averaging less than one article per year and directed mainly at bus routing problems in North America (Park & Kim, 2010). However, the last decade has seen a large increase in SBRP research, with 60 publications examining SBRPs in North America, South America, Africa, Asia, and Europe (e.g. Caceres et al., 2017; Lima et al., 2017; Eldrandaly & Abdallah, 2012; Chen et al., 2015; and Kotoula et al., 2017).

Figure 1 shows the number of SBRP-related publications by decade and illustrates the recent significant increase in attention to SBRPs and related problems. Park & Kim (2010) provide a review of the twenty-nine SBRP related publications from 1969-2009. This relatively limited amount of research concerning school bus transportation prior to 2010 is surprising given that school bus transportation is the largest form of mass transit in the U.S. (American School Bus Council, 2013). This might partly be explained by the local nature of each school district, in terms of decision-making and funding, as well as problem characteristics and model features which make the SBRP challenging to address.

[Insert Figure 1 Here]

The need for an updated SBRP literature review is driven not only by the number of publications since the previous review, but importantly by the shift in the SBRP characteristics being examined and the solution approaches considered, as shown in Figure 2. Newer SBRP research gives more emphasis to real-world issues such as heterogeneous fleets, mixed loading, serving multiple schools, and ridership uncertainty, all of which not only adds complexity to the solution approach but increases the utility of the models. Along with more complex SBRP problem settings, there has been an increase in the use of metaheuristic solution approaches (from five publications in the most recent literature review by Park and Kim (2010) to 40 publications in the present literature review). Therefore, with the shift both towards more real-world and complex problem settings and to metaheuristic solution approaches, an updated review of the literature that addresses contemporary trends is presented in this paper.

[Insert Figure 2 Here]

For this literature review, SBRP publications were identified using Google Scholar searches for the following terms: “school bus routing problem”, “school bus routing”, “bus stop selection”, “bus route scheduling”, and “school bell time adjustment”. One hundred seven publications (journal articles, conference proceedings, and book chapters) were identified as potentially employing an OR methodology to examine an aspect of the SBRP. Each publication was reviewed by at least two of the authors of this literature review to verify if indeed that was the case. If these reviews disagreed concerning the classification of a publication, then it was reviewed by a third author of this review. From this process, sixty-four OR-based SBRP publications were selected for this review, of which sixty were recent SBRP publications since the Park & Kim (2010) review and four were articles prior to 2010 that were not included in the earlier review. We posit that the recent increase in SBRP research is the result of several factors. One factor in the U.S. is the recent decrease in education funding which impacts services offered by a school district such as bus transportation (Leachman et al., 2016), leading to research on innovative ways of transporting students under more severe financial constraints. Also contributing to increased research attention are algorithmic advances, including the adaptation of metaheuristics to the SBRP that allow researchers to provide solutions and scenario analysis for larger school districts in a reasonable amount of time.

To review the selected publications, we consider the complementary aspects of SBRP problem characteristics and solution methodology. Rather than summarizing both aspects of a publication together, we first discuss publications in terms of common problem characteristics (e.g., routing for a single school). Then we discuss the publications (again) in terms of solution methodologies. To provide proper context concerning the solution methods, when appropriate we mention some earlier publications (prior to 2010). A secondary objective of this review is to provide researchers who are new to SBRP with an overarching framework of the research area. To this end, we have provided brief introductions to key SBRP problem characteristics and features. The remainder of this paper is organized into three sections. Section 2 provides a classification of SBRP literature and examines the important characteristics of SBRP research. Section 3 discusses the common solution approaches to the SBRP and its sub-problems. Section 4 provides concluding remarks and a discussion concerning future research directions for the SBRP.

2. CLASSIFICATION

This section describes the classification scheme we employ to categorize the literature in terms of key problem characteristics and discusses key aspects of the problems considered. Because a school district typically uses the same set of buses for multiple trips each day (morning and afternoon), SBRPs in a general sense should consider multiple buses to transport children to and from school (i.e. between scattered residences and a small set of schools), where the schools have (often tight) time windows that are arranged in sequence. For example, a school district may include three levels of schools (elementary, middle and high schools), where high schools begin the day at 7:30 am, middle schools begin the day at 8:10 am and elementary schools begin the day at 8:50 am. Each bus may then be used for three morning trips to bring children to school, and then for afternoon trips to take children back to their homes (usually bus stops). We focus our explanations below on the morning activities to transport children to school, but afternoon activities may be treated similarly with travel in the reverse direction. However, in some settings morning and afternoon routes are not exact reflections as there are differences in before- and after-school activities (e.g., sports or arts) and daily travel patterns; e.g., parents may be available to take children to school in the morning, but not to pick up children at the end of the school day.

Our classification of SBRP research is based on seven categories shown in Table 1: Sub-problem type, Number of schools, Service environment, Load type, Fleet mix, Objective, and Constraints. This reflects the scheme from the earlier literature review of Park & Kim (2010), as well as the SBRP sub-problem types presented in Desrosiers et al. (1981). We first briefly describe each category in Table 1, and then discuss the literature using this classification.

[Insert Table 1 Here]

The first category in Table 1 identifies the five SBRP sub-problems considered in the research: bus stop selection (BSS), bus route generation (BRG), bus route scheduling (BRS), school bell time adjustment (SBA), and strategic transportation policy (STP). The bus stop selection sub-problem identifies bus stop locations for each student, generally by selecting from a set of candidate locations. The bus route generation sub-problem develops the routes for the school buses to follow, while the bus route scheduling sub-problem specifically considers time (e.g., time windows for arriving at schools). The school bell time adjustment sub-problem is to determine the start time (and end time) for each school. The strategic transportation policy sub-problem is a new sub-problem type for SBRP that includes a single article that takes a broader view than typical SBRP research to analytically assess school bus transportation routing policies.

The next four categories in Table 1 identify the scope of the research. *Number of schools* indicates whether solution approaches are for a single school only or for multiple schools. The *Service environment* category indicates whether the research addresses rural settings, urban settings or both. *Load type* distinguishes whether a bus carries students for more than one school (or destination). Mixed loads generally refer to allowing a bus to carry students for multiple schools, although another type of mixed loading called "multi-loading" (Miranda et al., 2018) is when there are multiple school sessions per day and a bus can carry students both being taken to school and being taken home from school (i.e. both pick-ups and deliveries). The *Fleet mix* category shows whether the research treats all buses as identical (homogeneous) or allows different types (or capacities) of buses (heterogeneous). The final two categories indicate the model *Objectives* and *Constraints* in the research, with the bold items being new to SBRP research. Research objectives include bus-oriented measures, such as travel distance, time or cost, as well as student-oriented measures, such as walking distance (from a residence to a bus stop). Constraints are generally based around bus/route capacities (measured in students or stops visited), or time measures such as maximum riding time or time windows (for visiting stops or schools). Some recent research also considers probabilistic aspects due to unknown ridership levels (e.g., Caceres et al., 2017).

2.1 SBRP SUB-PROBLEMS

This subsection discusses the recent SBRP publications in the context of the SBRP sub-problems considered: bus stop selection, bus route generation, bus route scheduling, school bell time adjustment, and strategic transportation policy. Perhaps the most integrated SBRP research is that of Bertsimas et al. (2019) which includes bus stop selection with bus route generation and scheduling and school bell time adjustment. Other publications addressing three of the SBRP sub-problems include de Souza & de Siqueira (2010), Bögl et al. (2015), Kang et al. (2015), and Miranda et al. (2018). We note that Desrosiers et al. (1981) also included a separate "data preparation" sub-problem, but we have incorporated data needs into the discussion of each sub-problem directly. All SBRP research relies on a variety of data on the vehicles (buses), demand (students), locations (bus stops and schools), distance matrices (travel distance or time), and school district policies (in some cases dictated by legislation or safety concerns), but specific data needs depend on the sub-problem(s) being investigated.

2.1.1 Bus stop selection

The bus stop selection sub-problem identifies a bus stop location for each student, generally based on a list of approved bus stop candidate locations. These candidate locations are provided by the district, and may be based on street-side characteristics (e.g., visibility aspects of roads that make for desirable and safe bus stop locations), bus stop characteristics (e.g., off-street features such as available space for waiting), and student route characteristics (e.g., along a student's path from their residence to a bus stop location) (see NHSTA, 2010). The data for the bus stop selection sub-problem includes policy information concerning acceptable bus stop locations, a list of potential bus stop locations, each student's residence location, each student's assigned school, a distance matrix for the walking route from each student's residence to potential bus stops, and the policy concerning the maximum student walking distance.

Seventeen publications, nearly one-quarter of the recent SBRP research, consider bus stop selection, and it is almost always examined in combination with school bus routing. Sarubbi et al. (2016) for a single school and Galdi & Thebpanya (2016) for multiple schools are exceptions that consider school bus stop selection as a stand-alone problem (e.g., to minimize the number of stops with a maximum limit on the time or distance between stops and student residences). Generally, the bus stop selection sub-problem is solved first as the location and demand for each bus stop are inputs for the bus routing problem. Bus stop selection research cuts across all categories of problems (see Tables 2 and 3), though there is limited research in each category. Kamali et al. (2013) examine a problem similar to bus stop selection concerning special needs students. However, instead of assigning students to bus stop locations, the assignment problem assigns each student to a school in the district providing the appropriate classroom setting with the objective of minimizing total distance.

2.1.2 Bus route generation

The bus route generation sub-problem is the core of the SBRP, with 60 of the 64 (new) papers in this review addressing bus route generation. The exceptions are the strategic transportation policy paper by Ellegood et al. (2015), the bus route scheduling paper of Kim et al. (2012) and the bus stop selection papers by Sarubbi et al. (2016) and Galdi & Thebpanya (2016). We distinguish between bus *trips* and bus *routes,* where a route may consist of multiple trips. A school bus *trip* begins with an empty bus at an origin point (e.g., the bus depot), adds students to the bus by visiting bus stop location(s), and finishes at a destination point where all students have been delivered to their assigned school(s) and the bus is empty. Thus, the origin and destination of a bus trip may differ. For each subsequent bus trip, the origin for the trip is the destination of the previous trip. Because a bus may make several trips in a morning (e.g., for high school, then a middle school, and then an elementary school) we define a bus *route* as either a single trip (e.g., when there is only a *single* school) or a series of trips linked together where the origin and destination of the route are the same (e.g., when *multiple* schools are involved).

Because nearly every publication considers the BRG sub-problem, we discuss salient characteristics of the BRG solution approaches in a later section. We refer the reader to Tables 2 and 3 for the details of each publication. We note that research that considers bus route generation alone predominates for single school models (69% of the publications), while 29% of multiple school models consider bus route generation alone. The bus route generation sub-problem requires relevant data concerning the available vehicles (number, capacity, operating costs, travel speed, etc.), bus stop and school locations, distance and/or time matrices for travel between bus stops, schools, and bus depot(s), the number of students at each bus stop with their assigned schools, and district routing policies (e.g., mixed loading, maximum ride time, earliest pick-up time).

2.1.3 Bus route scheduling

The bus route scheduling sub-problem includes a time dimension. This includes all multi-school BRG research where routes are constrained by time-windows or when the bus trips are linked to create bus routes of two or more trips. To accurately reflect bus arrival time, bus route scheduling solutions should consider both the travel time between stops and the load time at each stop. There is some research estimating the bus loading and unloading times using functions of the number of students to load or unload (e.g. Park et al., 2012; Minocha & Tripathi, 2014; Campbell et al., 2015, Caceres et al., 2017).

Possible factors that may impact bus loading and unloading times which have not yet been considered are the student's school level and the changing utilization level of the bus as it progresses through a route (i.e. number of students onboard).

The combination of bus route generation and bus route scheduling is common in the publications addressing multiple schools (16 of 28 publications), but rare in single school publications (only 1 of the 36 publications). The SBRP examined by Song & Kim (2013) is a special single school problem where buses are allowed to make multiple trips for a single school. The examination of both school bus routing and scheduling can be viewed as a special case of the VRP where the vehicle is capacitated, bus trips are constrained by time and/or distance, each trip must be completed within a specific time window, and multiple trips are linked together to create routes – thus similar to a capacitated, time (or distance) constrained, multi-trip VRP with time windows.

The data required for school bus route scheduling may include distance (time) matrices between stop locations, district standards for load-time and earliest pick-up time, delivery time windows for each school, the set of bus trips, and distance (time) matrices between trips. This data may be from current operations or from solving other SBRP sub-problems.

2.1.4 School bell time adjustment

The school bell time adjustment sub-problem sets the time window for morning arrivals at the school (and similarly a departure time window in the afternoon). There is clearly a linkage between both bus routing and scheduling and setting the start and end times of a school day. It is common in the United States for school districts to employ a tiered school start time system, where the number of schools with the same start time is approximately equal to the total number of schools divided by the number of tiers (Bertsimas et al., 2019). The district then attempts to have each bus route include a number of trips equal to the number of tiers. Bertsimas et al. (2019) demonstrates how this tier approach for establishing school start time windows can result in fewer trips being compatible to link, actually increasing the number of buses required for the network.

Nearly all the recent SBRP publications treat the school start and end times, or time windows, as inputs to the problem. Only two recent publications consider school bell time adjustment, Mandujano et al. (2012) and Bertsimas et al. (2019). Mandujano et al. (2012) examines a modified version of the school bell time adjustment sub-problem to determine which schools in the school district should remain open. Their solution approach is a multi-phase approach with the first phase determining from the current school locations which schools should close, expand, or continue as current and which students should be assigned to each school. The second phase of the solution approach develops routes for the schools remaining operational. Bertsimas et al. (2019) employ a multi-phase solution approach with the first phase developing multiple feasible trips for each school. The second phase adjusts school start times considering all feasible trips to maximize the combining of trips into routes, thereby minimizing the number of buses required for the transportation network. This approach found a solution that if implemented would reduce the number of buses required for the Boston, Massachusetts school district by 31%. For earlier examples of the school bell time adjustment sub-problem we refer interested readers to Desrosiers et al. (1981, 1986) and Fugenschuh (2009). The school bell time adjustment sub-problem requires route information either from the current routing plan or the bus route generation problem, district policies relating to the earliest and latest time a school delivery window can start, and the length of the delivery time window.

2.1.5 Strategic transportation policy

Historically, SBRP research has focused on solving one or more of the sub-problems for a particular setting or school district, but not on more general strategic analyses of school bus routing policies. The strategic transportation policy sub-problem examines an SBRP transportation policy issue. Ellegood et al. (2015) use continuous approximation modeling to demonstrate how a mixed load policy can reduce the total distance traveled by school buses when the distance between destination schools is less than a critical value. Their model is based on general data for a district including the number of students per school level, the number of stops per school level, proportion of stops shared, the size of the school regions, and the capacity of a school bus.

2.2 CATEGORIZATION OF SBRP RESEARCH SETTINGS

Using the scheme outlined in Table 1, we succinctly summarize the recent SBRP research for single school studies in Table 2, and for multiple school studies in Table 3. These tables also include columns on the right to indicate problem size considered in the research and the geographical region of focus. The following subsections describe relevant features of the new SBRP literature in each category from Table 1.

2.2.1 Number of schools

Although most school districts have more than one school with staggered start times to increase the utilization of buses, many researchers still present solution approaches for a *single* school. In this review, publications that examine school districts with multiple schools but utilize a methodology that considers one school at a time, are categorized as “single school”. Note that limiting the problem to a single school greatly simplifies the problem, especially the scheduling aspects. While there are more publications addressing single schools than multiple schools, there has been a significant increase in the number of multiple school models over time, with 12 multiple school publications during the first 40 years and 28 multiple school publications in the last ten years of SBRP research. It is encouraging to see this shift of focus toward the real world multiple school scenario.

The 36 single school publications are categorized in Table 2. Without multiple schools, only the bus stop selection and non-mixed load bus route generation sub-problems can be examined. Also, the complexities of time windows and multi-trip constraints are not relevant; hence, bus route generation becomes a VRP with many pick-up locations and a single destination subject to distance (time) and/or capacity constraints. The recent single school SBRP publications in Table 2 examine school districts from around the world, with 11 in Europe, 8 in Asia, 5 in South America, 3 in Africa, and 1 in North America. Additionally, the problem size considered by these publications varies from 16 stops and 33 students (Huo et al. 2014) to 87 stops and 1,169 students (de Siqueira et al. 2016).

[Insert Table 2 Here]

The 28 multiple school SBRP publications are categorized in Table 3. Note that in problems with multiple schools, there exists the option for mixed loads, where children destined for different schools are transported together on the same bus. The majority of multiple school SBRP publications examine school districts located in either North America (11 publications) or South America (6 publications). Artificial data sets are often used when examining both the multiple school setting (8 publications) and the single school setting (7 publication). Also, multiple school SBRP publications typically examine problems that are much larger than those examined by single school SBRPs, with one of the largest being 200 schools, 17,942 stops and 20,000 students (Bertsimas et al., 2019).

[Insert Table 3 Here]

2.2.2 Service environment

The service environment characteristic identifies the setting as urban, rural, or both, and this designation impacts the problem structure. The school bus stop selection sub-problem is generally considered in an urban environment (11 of the 14 bus stop selection publications with known service environments), which is characterized by a dense network of streets that provide a relatively safe environment for students to walk from their residence to a shared bus stop. Also, urban environments typically have wider streets, streets with sidewalks, and lower posted speed limits than rural environments, providing students with a safer path. In an urban environment, the density of students is such that a large number of students frequently reside within a reasonable walking distance of a shared bus stop location.

Research has shown that the service environment can have a significant impact on model development. In an urban region, the density of students is much greater than in a rural area and the service area for a school is much smaller than in rural areas. Thus, a school bus generally has to visit only a few stops before reaching capacity. Also in an urban environment, though the school district may have a policy concerning maximum ride time, this rarely impacts route development (Braca et al., 1997). Rural districts typically cover a much larger area, have longer distances between stops, and fewer students per stop than urban districts, which results in many routes being effectively constrained by a maximum ride time before the capacity of the bus is reached (Chen, 1990; Howley et al., 2001). We see from Table 3, when examining a multiple school problem setting, the school district may exhibit characteristics of both (rural and urban) service environments.

2.2.3 Mixed load routing

As noted earlier, mixed load routing is an option in settings with multiple schools. A non-mixed policy is one where all the students assigned to the same bus trip have the same destination school, and a mixed load policy is one where students on the same bus trip may not have a common destination. For example, a mixed load trip may collect students for two (or more) schools. Both mixed and non-mixed transportation policies can offer advantages to a school district, depending on the characteristics of the service region (Chen et al., 2015; Ellegood et al., 2015). The advantage of non-mixed loading is that each school bus trip visits a single school and so is always picking-up students or traveling full until all students are delivered to a school. The disadvantage of non-mixed loading is that routes are less efficient as a particular bus stop may be visited by multiple buses, and a bus may pass by a bus stop without stopping if it has students bound for a different school than those on the bus. The advantage of mixed loading is that a bus stop is usually visited only once. The disadvantages of mixed loading are the buses have extra travel between schools at the end of trips when they are only partially full, students may experience additional wait time at collection and transfer points, and the policy requires a higher degree of coordination than single school routing.

There are four mixed load transportation policies examined in the SBRP bus route generation literature: *traditional*, *interscholastic*, *transshipment* and *pickup/delivery*. A *traditional* mixed load bus trip delivers all students to their destination schools by visiting several schools during a trip. This is the most common mixed load policy in SBRP literature. Ellegood et al. (2015) show the traditional mixed loading policy is most beneficial for large school districts, service regions where bus stop locations are frequently shared between school levels, and when schools are located relatively close together. In school districts where there are several school shifts each day (e.g., morning, afternoon, and evening), then *pickup/delivery* mixed loading combines pick-ups to take students from home to school with deliveries from school to home. This may reduce travel distance and/or the number of buses required (Miranda et al., 2018). With *interscholastic* mixed loading, buses visit only one school where all students unload (even if bound for a different school). Students destined for a different school then wait at this first school until there are enough to fill a bus; and then they are shuttled directly to their destination school. Yao et al. (2016) suggest that the *interscholastic* mixed load policy can be effective in cases where there is a large number of students. Desrosiers et al. (1981, 1986) are the first to present the *interscholastic* mixed load policy in their description of mixed load rural routes.

The *transshipment* mixed load policy is where students transfer buses mid-trip, so that a *transshipment* bus trip may include a portion that is mixed load and a portion that is non-mixed load. For example, Bogl et al. (2015) consider two mixed-load trips serviced by buses A and B, where each bus carries students for both schools L and U, with bus A destined for school L and bus B destined for school U. After each bus has collected students for both schools L and U, the buses meet at a transfer point where students on the "wrong" bus transfer to the bus bound for their school. From the transfer point to the destination schools, both buses are non-mixed loads and they may make additional stops to pick up students if they have space. In Oluwadare et al. (2018) bus trips are not required to meet at a transfer point, as each bus stop is considered a pick-up and/or destination point. This allows for the possibility that a student may transfer buses mid-trip and thereby take multiple buses to reach their destination. The work presented in Tang & Yan (2010) should be considered in future research on pupil transshipment. They investigate and model both pre-distribution and post-distribution transshipment systems.

Columns 3-6 of Table 4 show the type of mixed load policy used in the mixed-load SBRP research. Note that more than half the multiple school SBRP publications consider mixed loading (Table 3), with only a few considering the *interscholastic* or *transshipment* policy. It should be noted that Desrosiers et al. (1981, 1986) and Fugenschuh (2009) are not identified as mixed load publications in Park & Kim (2010).

[Insert Table 4 Here]

2.2.4 Fleet mix

Fleet mix effectively identifies whether the set of buses under consideration have the same capacity (homogeneous), or varying capacities (heterogeneous). For many school districts where the bus capacities of their fleet may not be equal, but are similar, and the development of routes assume all eligible students are riding the bus, assuming a homogeneous bus fleet with respect to capacity is reasonable because of ridership uncertainty. Additionally, a homogeneous fleet is also a reasonable assumption in rural school districts where the capacity of the bus is usually not reached and maximum ride time is the constraining aspect of the problem. Only three of thirty-six single school publications in Table 2 consider a heterogeneous fleet (Faraj et al., 2014; de Siqueira et al., 2016; Sales et al., 2018). In contrast, nearly half of the multiple school publications (13 of 28) listed in Table 3 model a heterogeneous fleet. Additionally, eleven of these thirteen publications examine a real world problem. Heterogeneous fleets are modeled in one of two ways: either each bus has a distinct capacity (Kim et al., 2012) or the heterogeneous fleet consists of several types of buses and all buses of the same type have the equal capacity (Chen et al., 2015; Bertsimas et al. 2019). It is worth noting that recent research in multi-compartment vehicle routing problems (Alinaghian & Shokouhi 2018) could be investigated for applicability to the SBRP, particularly, in the context of heterogeneous vehicles configured to transport all or most types of pupils, including special needs students, or modeling varying levels of students in mixed load settings. The fleet type characteristic may impact the solution of both bus route generation and bus route scheduling.

2.2.5 Objectives

The majority of the recent publications that address the bus route generation sub-problem have sought to increase routing plan efficiency by minimizing cost. Many of the publications present a model to minimize a single aspect of total cost, either the fixed cost by minimizing the number of buses (N) or the variable cost by minimizing the total distance (time) traveled (TBD). Of the 64 publications, 49 include minimizing TBD as an objective and 28 include minimizing N; note that a number of publications provide multiple models so they include two or more objectives. Chalkia et al. (2016) present a model to minimize TBD where a penalty based on the safety factor (SF) of each road segment is added to the travel time of the road segment. Minocha & Tripathi (2014) seek to maximize the capacity utilization (CU) of the bus fleet, where the fleet CU is the ratio of the number of students transported to the bus fleet capacity. With a fixed number of students transported, the objective of maximizing CU is effectively the same as minimizing the number of buses. Campbell et al. (2015) considers minimizing both TBD and N sequentially for trip generation, and then employs a trip linking assignment-based model minimizing the number of buses (N). In Caceres et al. (2017), the primary objective is minimizing N followed by improvement heuristics to minimize TBD. A bi-objective total cost (TC) function is considered by 15 of the 64 publications, with the 15 split between *single* school (5) and *multiple* school (10) publications. The majority of these publications, 14 of the 15, include a fixed cost per school bus, with eleven utilizing vehicle distance or travel time (Thangiah et al., 2008; Martinez & Viegas, 2011; Mandujano et al, 2012; Kim & Park, 2013; Sghaier et al., 2013; Chen et al., 2015; Lima et al., 2016, 2017; Caceres et al., 2018; Eguizabal et al., 2018; Miranda et al., 2018), two capturing student walking distance (Riera-Ledesma & Salazar-Gonzalez, 2012, 2013) and one considering total student riding time (Bertsimas et al., 2019) as the variable cost. Shafahi et al. (2018) minimizes a TC function summarizing costs associated with bus utilization, trip compatibility and travel time. Depending on the service environment, a single aspect of a total cost function may be the primary cost driver. In a densely populated urban environment, bus trips are typically much shorter than those in a rural environment, so the fixed cost associated with the number of buses will dominate the total cost function (Braca et al., 1997). The variable cost can dominate the total cost function in a large, sparsely populated rural school district, where students may live more than 50 miles from their destination school (Lima et al., 2016). Bögl et al. (2015) employ a bi-objective function to minimize the summation of total bus travel time and a penalty for exceeding the maximum allowable number of transfers (NT). Shafahi et al. (2017) introduce the new objective of trip compatibility (TRC), where a trip is considered compatible if it can link to another trip to form a multi-trip route.

Recent publications on the bus stop selection sub-problem seek to minimize one of three objectives, total student walking distance (SWD), number of bus stops (NS), or shared bus stops (SBS). There is a service level trade-off between the SWD and NS: minimizing SWD potentially improves the effectiveness for students by reducing the average walk distance, with students being assigned to their nearest bus stop (Riera-Ledesma & Salazar-Gonzalez, 2012, 2013); however, minimizing NS potentially improves the efficiency of the bus routing plan by reducing the number of bus stops visited to reach capacity (Bögl et al., 2015; Kang et al., 2015; Galdi & Thebpanya, 2016). Bögl et al. (2015) consider the objective to minimize SBS for a school district with a transshipment mixed load policy, believing this would reduce the number of student bus transfers resulting in a more efficient transportation network. Ellegood et al. (2015) show that as the proportion of bus stops that are shared increases, the total distance traveled decreases with a traditional mixed load policy. Therefore, an objective for the bus stop selection sub-problem, not yet considered, that may improve the efficiency of a school bus network is maximizing shared bus stops.

A number of SBRP publications present models seeking to improve the equity between trips by minimizing the maximum ride length (MRL) (Serna & Bonrostro, 2001; Alabas-Uslu, 2008; Pacheco et al., 2013; Bronshtein et al., 2014). The ride length of a trip is measured as the time or distance from the first bus stop to the destination school. Therefore, minimizing the MRL seeks to improve the fairness of ride length between trips. An equity objective of load balancing has also been considered in the past

(Bowerman et al., 1995; Li & Fu, 2002). Kang et al. (2015) present a solution method to improve the effectiveness of a school bus routing plan by minimizing the total student travel distance (TSD). Xue et al. (2016) present a pickup and delivery problem (PDP) that has been augmented to consider the tradeoff between total travel distance and service quality. They employ a weighted-sum scalarization function in the objective controlling the tradeoff between routing (TBD) and loading costs (TSD).

2.2.6 Constraints

The three most common constraints are bus capacity (C), time windows (TW), and maximum ride time (MRT). Capacity constraints (C) are used in 58 of the 64 publications. Time windows (TW) are used before the start of the school day to indicate when students have to arrive (to assist school administration with managing and providing a safe environment for the student population). In some rural environments, the MRT is set very high because long bus rides are accepted given road conditions and the distance from the school to the bus stops (Lima et al., 2016).

Recent bus route generation literature has modeled five new constraints not previously considered in SBRP research: transfer time (TT), stop time window (STW), maximum stops per route (MSR), chance of overcrowding (COO), and chance of being late (COL). With transshipment mixed loading, the synchronization of routes is achieved by setting a maximum allowable waiting time or transfer time (TT) that the first bus to a transfer point is allowed to wait for the second bus (Bögl et al., 2015). The STW constraint ensures every student's ride time is no more than twice the time it would take to commute directly from the bus stop to their school (Santana & Carvajal, 2015). Riera-Ledesma & Slazar-Gonzalez (2013) consider the MSR, maximum ride time (MRT), and the minimum number of students (MSN) per route constraints. They state that these three constraints together can effectively balance the loads between routes. The potential of low capacity utilization led Caceres et al. (2017) to develop the first example of school bus route generation research considering stochastic demand with constraints that account for the probability of overcrowding (COO) and of being late to school (COL).

While minimizing the number of buses (N) is a common objective of bus route generation and bus route scheduling, researchers have also modeled the number of buses as a constraint (Kamali et al., 2013; Minocha & Tripathi, 2014; Chen et al., 2015; Shafahi et al., 2017; Eguizabal et al., 2018). With the number of buses being a constraint, school districts can evaluate the trade-off between reducing the number of buses to reduce cost and increasing the number of buses to improve service levels. With the earliest pick-up time (EPT) constraint in combination with the school delivery time window (TW) constraint, a researcher can effectively emulate the maximum ride time (MRT) constraint without having to keep track of the total time from the first stop to the destination school (Kim & Park, 2013).

A common constraint associated with the bus stop selection sub-problem relates to the school district's policy concerning the maximum walk distance or time (MWT) of students to their assigned bus stops (e.g. Silva et al., 2015; Galdi & Thebpanya, 2016; Miranda et al., 2018). The MWT policy ensures no student has to walk an unreasonable distance to their bus stop and establishes a minimum service level of the bus stop selection plan. The school district's MWT policy can be the same for all school levels or may progressively increase as students attend higher level schools (Bowerman et al., 1995; Bertsimas et al., 2019).

3. SOLUTION APPROACHES

The following section covers the solution approaches for recent SBRP research. A wide range of solution approaches have been developed for the various SBRP sub-problems, in part to reflect the varying characteristics of the school districts examined. Many SBRP publications examine multiple SBRP sub-problems. Therefore, instead of a summary of each publication's solution approach separately, we have organized this section by SBRP sub-problem. This allows us to focus the discussion on the similarities and differences of the solution approaches considered for a SBRP sub-problem. This section categorizes the solution approaches in the recent literature for the bus stop selection, bus route scheduling, and bus route generation sub-problems in turn. To provide context concerning the solution approaches, publications from Park & Kim (2010) are included when appropriate. With the concentration of SBRP research on bus route generation, these works are further categorized as classical heuristics, metaheuristics, exact methods, and methods considering uncertainty.

3.1 BUS STOP SELECTION

The fundamental version of bus stop selection is typically formulated as an assignment problem to minimize the number of stops (NS) (Kang et al., 2015; Galdi & Thebpanya, 2016), minimize the total student walking distant (SWD) (de Souza & de Siqueira, 2010; Martinez & Viegas, 2011; Riera-Ledesma & Salazar-Gonzalez, 2012, 2013) or as a bi-objective model considering both NS and SWD (Bertsimas et al., 2019). However, bus stop selection is often solved in conjunction with bus routing, for which the three general solution approaches are: location-allocation-routing (LAR), allocation-routing-location (ARL), and location-routing-allocation (LRA).

The LAR approach first locates bus stops, then assigns students to the stops to create the demand for bus route generation. This decomposition into sequential sub-problems may lead to a suboptimal routing solution, as the bus stops found in the bus stop selection sub-problem may not be best for the bus route generation sub-problem. Riera-Ledesma & Salazar-Gonzalez (2012, 2013) address this limitation with a solution method that combines the sub-problems in a bi-objective model seeking to minimize the total bus route distance and student walking distance. However, this method may result in solutions with underutilized buses. To address this issue, Riera-Ledesma & Salazar-Gonzalez (2013) include a constraint that requires a minimum number of students per route. Galdi & Thebpanya (2016) presents a solution method for the bus stop selection problem with the objective of minimizing the number of stops for each school level in Howard County, Maryland. Each school level is examined independently, because the policy concerning an acceptable bus stop location is more stringent at the elementary level than at the high school level.

The ARL approach for bus stop selection first clusters students based on the capacity of a bus, then selects bus stops within the clusters, and finally develops a route through the bus stops. For earlier examples of this approach, we refer the reader to Chapleau et al. (1995) and Bowerman et al. (2006); however, none of the recent publications examining the bus stop selection sub-problem have utilized this method. Schittekat et al. (2013) presents a third solution method, location-routing-allocation (LRA), which first identifies the set of feasible bus stops for each student, then develops routes to minimize the total travel distance considering only the feasible bus stop locations, and finally assigns students to the selected bus stop locations.

3.2 BUS ROUTE SCHEDULING

In bus route scheduling, time window constraints are key to ensuring feasible and implementable solutions. Without time window constraints, the time span from the first bus arrival to the last bus arrival at a school may be unmanageable for school administrators. Additionally, in multi-school, mixed load routing problems, a feasible solution may result in routes where a bus visits school A then school B, and another route visits school B first then school A. In this scenario, unless school A and B are located near each other, the arrival time window at each will need to be large for this solution to be implementable. A student’s ride time, including the travel time between schools, cannot exceed student ride time limits, often disallowing some schools to be serviced together on the same trip without larger arrival time windows (Kim et al., 2012).

The bus route scheduling problem frequently involves sequentially linking bus trips that are the output from the bus route generation sub-problem. Solution approaches can generally be classified as either *sequential assignment* or *integrated*. In *sequential assignment*, trips for each school or arrival time window are considered separately, generally with the objective of minimizing the number of buses (Kim et al., 2012). This approach starts with the trips for the earliest arrival time window and assigns a bus from the depot to each. The process then considers trips for the next earliest arrival time window, linking each trip to a bus (trip) assigned to the first arrival time window when feasible, otherwise assigning a new bus from the depot for the trip. This process repeats, considering each arrival time window separately until all schools have been considered. In the integrated approach, all bus trips are considered for linking into bus routes simultaneously, typically using an assignment-based mathematical programming approach. This approach conventionally seeks to minimize the number of buses required (e.g., Campbell et al. 2015), or employs a bi-objective formulation (solved lexicographically) with an emphasis on minimizing the number of buses followed by minimizing the total distance travelled (Chen et al. 2015).

In an effort to increase the proportion of bus trips that are compatible for linking with other trips to form multi-trip routes, Shafahi et al. (2017) allow the underutilization of capacity during the construction of bus trips. They further examine how the objective of the bus route generation problem impacts the performance of the bus route scheduling problem when minimizing the number of buses. They consider four objectives for the bus route generation problem: a bi-objective approach maximizing trip compatibility and minimizing travel distance; maximize trip compatibility; minimize travel distance; and minimize the number of buses. Interestingly, they found that trips developed from the bus route generation problem with an objective to minimize the number of buses were the least compatible for linking, resulting in the highest total number of buses needed to service the school district network. Eguizabal et al. (2018) demonstrates when the length of the time window increases, the number of trips that are compatible for linking will increase and the number of buses required for the network will decrease.

3.3 BUS ROUTE GENERATION

Advances in bus route generation solution methods coincide with developments in solving VRPs. Solution methods often include construction heuristics (e.g., "cluster-first, route-second", savings, etc.) combined with improvement methods (e.g., k-opt), and the complexity of the problems generally leads to the use of heuristics or combinations of exact and heuristic methods. Recent solution approaches to the bus route generation problem are categorized and discussed in the following sub-sections as classical heuristics, metaheuristics, exact methods, and uncertainty methods.

3.3.1 Classical heuristics

Classical heuristics form the foundation of many existing approaches to solving the bus route generation sub-problem. Usually, they are used in conjunction with a broader algorithmic strategy. Since bus route generation is a type of capacitated vehicle routing problem (CVRP), we will draw from its extensive literature base for categorization purposes. A review of CVRP local search can be seen in Laporte & Semet (2002), and Cordeau et al. (2007) give an overview of the VRP solution methods. The bus route generation classification supplied here is based in part on both of these articles.

In this review, classical heuristics applied to the bus route generation sub-problem of the SBRP are described as either construction or improvement heuristics. Construction heuristics are used to build initial feasible solutions, and improvement heuristics are used to perturb solutions (bus routes or trips) as driven by the problem criteria. In the bus route generation sub-problem, the savings method (based on Clark & Wright 1964), insertion methods, and two-phase approaches are used for route construction. Improvement methods will be categorized as either being intra-route or inter-route. Table 5 categorizes bus route generation literature, including articles from Park & Kim (2010), based on the construction and improvement heuristics employed.

[Insert Table 5 Here]

3.3.1.1 Construction Heuristics

As seen in Table 5, about 23% of the SBRP bus route generation papers (12 out of 53), use a savings-based classical heuristic for initial route construction. The savings method has proven to be both fast and comparably good over the years. The bus route generation savings-based methods build routes in a parallel fashion, with multiple routes constructed simultaneously. Schittekat et al. (2013) use a saving-based construction procedure with randomized selection via a GRASP algorithm. Russell et al. (1986) initially show how multiple destination schools can be considered in a Clark-Wright savings algorithm, and Campbell et al. (2015) expand upon this work by showing additional savings calculations needed when considering mixed loading. Dulac at al. (1980) suggest that many school bus routes don’t begin at a school and they provide alternative savings calculations considering this possibility.

Insertion-based methods are construction heuristics that build routes by iteratively inserting unselected bus stops into incomplete routes. Spada et al. (2005) describe a “chain construction” procedure that employs a distance evaluation function to select the index of an entering stop in a growing route. Caceres et al. (2018) use both a savings-based and a probabilistic insertion heuristic as subroutines in a column generation procedure. They seek to minimize N, with TBD being a secondary consideration. From Table 6, 30% of the SBRP bus route generation papers use insertion heuristics in route construction. Shafahi et al. (2018) provide a minimum cost matching-based insertion heuristic. Continuing their previous work Shafahi et al. (2017), they consider trip compatibility as well as compare both sequential and parallel trip construction methods.

For construction heuristics in bus route generation, about 38% used a "two-phase" method. These approaches can be either route-first, cluster-second or cluster-first, route-second, the former of which works by solving a traveling salesman problem and partitioning the resulting solution into feasible routes. Only Newton & Thomas (1969) and Bodin & Berman (1979) employ a route-first, cluster-second approach. All other two-phase approaches implemented for solving bus route generation are cluster-first, route-second, which is not surprising given that prior research has shown route-first, cluster-second approaches to CVRPs tend to generate poor solutions (Cordeau et al. 2007).

The two-phase approach in Kotoula et al. (2017) uses geo-coded addresses of student homes in a k-means clustering algorithm to group stops. Then an initial stop ordering is accomplished using an insertion-based process where the furthest non-selected stop is added to the end of the route. Next, an improvement phase within a Genetic Algorithm framework utilizes a savings-based calculation as part of the procedure. This is the only instance in the bus route generation literature where a "construction" heuristic is used in an improvement fashion. Following bus stop clustering, Santana & Carvajal (2015) utilize column generation to generate bus routes, an approach that is unusual for the SBRP bus route generation problem as the number of stops per route are relatively small when compared with many other classic vehicle routing problem types. Dulac et al. (1980) provide a comparison of multiple construction methods for the bus route generation problem. Their findings suggest that the Clark & Wright procedure tends to slightly outperform an insertion-based procedure, but ultimately no construction heuristic tested dominated the others. The popularity of two-phase methods versus savings and insertion heuristics is likely a product of the complexity and difficulty of bus route generation. Approaches that essentially break the initial construction phase of a solution method into many smaller problems seems to be a logical consequence of problem difficulty in this case.

When constructing mixed load routes, researchers have used three approaches for adding schools to a route: *insertion*, *sequential*, and *closed*. For the *insertion* method, when each bus stop is added to the trip, its corresponding school is inserted into the trip if it is not already a stop on the trip. The insertion of a new school is done so that the total increase to the objective (cost, distance, or time) is minimized;

therefore, a destination school may be inserted into the middle of a trip. New mixed load research with the insertion approach includes de Souza & de Siqueira (2010), Park et al. (2012), Ruiz et al. (2015), and Lima et al. (2016). With the *sequential* method, trips are developed so that all bus stops are visited first and then the destination schools are visited at the end of the trip, with the schools sequenced to minimize the degradation of the objective. New mixed load research with the *sequential* approach includes Campbell et al. (2015), Ellegood et al. (2015) and Kang et al. (2015). In the *insertion* and *sequential* methods, the origination and destination points of a trip may differ. However, with the *closed* method, the origination and destination point of a trip are the same. When a stop is added to a trip with students destined for a school other than the origination school, the new school is added to the trip just prior to the origination/destination school. If there are more than two schools, the schools that are not the origination/destination school are inserted into the trip just before the destination school, and are sequenced so that the impact to the objective is minimized. New mixed load research with the *closed* approach includes Silva et al. (2015) and Yao et al. (2016).

3.3.1.2 Improvement Heuristics

Table 5 shows that 53% and 55% of works in the bus route generation literature implement intra-route and inter-route improvement heuristics respectively. For this classification, we view most intra-route approaches as *k*-opt heuristics (Lin & Kernighan 1973, Helsgaun 2009), with *k* ranging from 2 to 4 (as well as unspecified or simply *k*). This is done to unify the vernacular regarding this class of heuristics, as these intra-route approaches are either a conventional or a special case implementation of the *k*-opt heuristic. In that same spirit, we classify all inter-route improvement heuristics under the unifying banner of "λ-interchange."

In Lima et al. (2017), a split move procedure is undertaken in order to create two trips from one. This work considers the criteria of TC, TBD, and LB in a multi-objective approach. A split move heuristic is then used to improve the solution. Bowerman et al. (1995) evaluate the routing implications of adding new stops and reassigning students; this is the only work to update the bus stop selection sub-problem during the bus route generation phase of the algorithm. Li & Fu (2002) incorporate an additional procedure where bus stops visited multiple times in separate routes are consolidated into a single route.

Corberan et al. (2002) provide a "combine" procedure which evaluates and compares two solutions, creating a new, unique solution within a scatter search framework. In Spada et al. (2005), the authors incorporate a 1-interchange, inter-route improvement procedure using metaheuristic search frameworks. Their implementation of the heuristic is innovative in that they relax the capacity restriction for this interchange in a temporary "solution," allowing them to effectively traverse the solution search space and identify previously unreachable solutions.

Improvement heuristics are essential components of most effective algorithms used to solve the bus route generation sub-problem of the SBRP. There has been little in the way of innovation specific to the SBRP, with a notable exception being Spada et al. (2005) and their capacity relaxing improvement procedure. More work is needed exploring the potential of this improvement procedure in heuristic algorithm frameworks.

3.3.2 Metaheuristics

The use of metaheuristics for addressing the bus route generation sub-problem of the SBRP has increased greatly, with 40 of the 64 publications in this literature review using metaheuristics. In contrast, only 5 papers in the earlier literature review use metaheuristic methods. Bus route generation researchers have utilized metaheuristics to coordinate the interaction between local improvement procedures and higher level algorithmic strategies that are capable of escaping from local optima and performing a robust search of the feasible region.

SBRP researchers have utilized 11 different metaheuristic approaches as seen in Figure 3, with the most commonly implemented being Genetic Algorithm (GA), Tabu Search (TS) and Ant Colony Optimization (ACO). Greedy Random Adaptive Search Procedure (GRASP) and Simulated Annealing were used in 4 publications as seen in Table 4. It is encouraging to see both evolutionary, population-based approaches (e.g. GA and ACO), and trajectory based local search methods (e.g. TS and GRASP) applied successfully to tackle bus route generation. Table 6 groups the metaheuristic references by their approach, and these are discussed below.

[Insert Figure 3 Here]

3.3.2.1 Population-Based Evolutionary Approaches

Population-based metaheuristic methods generate a diverse population of solutions that are iteratively improved to converge to a high-quality solution. Such evolutionary methods have a stochastic component built-in which aids the search process. GA is one of the most commonly used metaheuristics for the bus route generation sub-problem. Solutions are assigned fitness scores and highly fit solutions are combined by crossover and mutation methods. The earliest metaheuristic approach to the bus route generation problem was by Thangiah & Nygard (1992) who develop a 2-phase solution approach using GA. The approach first clusters student locations and then routes buses using an insertion method. The initial routes are then improved by GA. Diaz-Parra et al. (2013) implement GA to a simulated dataset with 200 stops. They use a *k*-means algorithm to cluster stops and generate routes using a search procedure embedded within the GA.

Sghaier et al. (2013) design a GA to minimize total costs with constraints on maximum ride times and capacity. They apply their algorithm to a simulated case study. Minocha & Tripathi (2014) implement a hybrid metaheuristic that combines GA with local search. Clustering is implemented using a location-based heuristic and the routes are generated by GA. Kang et al. (2015) solve a bus stop selection problem using a heuristic procedure and a mixed load routing problem with a GA. Chalkia et al. (2016) implement a GA emphasizing the choice of picking safe routes, with constraints for the maximum ride times and capacity. Kotoula et al. (2017) employ a GA solution method to develop school bus routes with the objective of minimizing total travel distance. Unsal & Yigit (2018) develop a GA for the dynamic SBRP where routes can be modified in real time based on traffic information. Oluwadare et al. (2018) develop routes for a school district in Akure, Nigeria using GA with an objective of minimizing the number of buses used. Sales et al (2018) develop a memetic algorithm for the SBRP with varying fixed costs and capacities. Memetic algorithms are an extension of genetic algorithms and they are used in order to reduce the likelihood of premature converge in GA and hence explore the solution space further.

[Insert Table 6 Here]

Arias-Rojas et al. (2012) implement a two-phase approach, with a clustering heuristic to group stops and an ACO metaheuristic to route each cluster. Eldrandaly & Abdallah (2012) developed a 3-phase method for route generation. The solution method first cluster stops based on distance to school and bus capacity, then an ACO metaheuristic algorithm is used to generate routes within each cluster, and the subsequent routes are improved with a k-opt improvement heuristic. Euchi & Mraihi (2012) generate initial routes using the nearest neighbor method and improvement is accomplished via an ACO metaheuristic. The routes are further improved by a variable neighborhood descent (VND) metaheuristic. Singh & Dhir (2014) utilize an ACO metaheuristic to develop both open and closed routes for a single school. Huo et al. (2014) implement an ACO metaheuristic to develop a single route with 16 stops and 33 students attending a single school in Beijing, China. Yao et al. (2016) present two mixed load, 2-phase ACO metaheuristic solution methods where both methods utilize an aggregation-based clustering algorithm to group bus stops near each school into clusters for the routing portion of the algorithm. Yigit & Unsal (2016) solve a dynamic bus route generation problem using ACO. In their approach, the bus stops to be visited daily are updated using live GPS information and mobile software from each student. Mokhtari & Ghezavati

(2018) address conflicting objectives in the form of minimizing ride times and the number of buses used. To this end, they develop bi-objective mixed integer program that is solved using a multi-objective ACO.

Fulin & Yueguang (2012) develop a particle swarm metaheuristic with the objective to minimize total cost. Corberan et al. (2002) employ a scatter search approach that uses one of two route generation procedures, the first being the nearest neighbor approach and the second being a newly developed approach by sectoring the district around the school. Perez-Rodriguez & Hernandez-Aguirre (2016) address the bus stop selection and bus route generation problems. They first assign students to bus stop locations and then develop routes using an Estimation of Distribution Algorithm (EDA), a type of evolutionary metaheuristic. Their results show that the EDA solution method is better than a GA for the addressed problem. Geem (2005) present an Harmony Search (HS) metaheuristic to minimize a total cost function using an artificial dataset. Kim & Park (2013) present a HS metaheuristic for solving the bus route generation with a single school. Five artificial data sets were developed to compare the results of the HS solution method and an exact solution method. To solve the bus stop selection and bus route generation sub-problems of the SBRP, Perez-Rodriguez & Hernandez-Aguirre (2016) use an EDA to calculate known bus stops given a set of potential nodes to visit.

3.3.2.2 Trajectory-Based Methods

The primary difference between population and trajectory-based methods is that trajectory methods use adaptive memory to store local moves and accept suboptimal moves to escape local optima, which can lead to learning effects. These methods balance both the exploitation of good solutions and the exploration of the solution space. The earliest and most popular trajectory-based method is TS designed by Glover (1986). Serna & Bonrostro (2001) utilize a 2-phase approach which first uses a local search to generate routes and then two improvement heuristics aimed at minimizing the maximum ride time. Ripplinger (2005) develop a solution method for rural school districts with the objective of minimizing student riding time. The initial routes are generated by an insertion technique and then improved using a TS algorithm. Pacheco & Marti (2006) use TS with path relinking for schools located in Burgos, Spain. Pacheco et al. (2013) give a bi-objective two phase solution method with a single school and a homogeneous bus fleet. Mushi et al. (2015) present a solution method for a single school bus route generation utilizing a TS metaheuristic. Ruiz et al. (2015) present a TS metaheuristic algorithm to develop open routes for the bus route generation with an objective to minimize the number of buses.

GRASP is a trajectory method that combines randomized construction methods with improvement procedures. Faraj et al. (2014) implement a GRASP metaheuristic that utilizes a nearest neighbor procedure to generate routes which are then improved using a 2-opt procedure. Silva et al. (2015) presented a three-phase approach to develop routes for a rural bus route generation with mixed loading for a Brazilian school district. De Siqueira et al. (2016) design and implement a GRASP metaheuristic that utilizes a construction algorithm for route generation. The authors research a heterogeneous fleet with an objective of minimizing total bus distance subject to time windows and capacity constraints. Schittekat et al. (2013) implement a hybrid GRASP and VND algorithm to minimize total bus travel distance with a constraint on capacity.

Simulated Annealing (SA), another trajectory-based metaheuristic, is utilized by Chen et al. (2015) to minimize the number of buses and total distance with a lexicographic function to solve benchmark datasets from Park & Kim (2011) and Bögl et al. (2015). Spada et al. (2005) develop a solution method for bus route generation with an objective that focuses on customer service. They implement two variants of SA and a TS metaheuristic. Lima et al. (2016) devise an innovative Iterated Local Search (ILS) random neighborhood descent metaheuristic for bus route generation with mixed loading and a heterogeneous fleet with an objective to minimize total costs. Lima et al. (2017) develop four different versions of the ILS metaheuristic to address a multi-objective SBRP with mixed loads and a heterogeneous fleet. Bögl et al. (2015) address the issue of mixed load transshipment and implement two different versions of the variable neighborhood search (VNS) metaheuristic. Miranda et al. (2018) test variants of the ILS metaheuristic combined with a VND strategy to tackle an innovative "multi-loading" problem. Shafahi et al. (2018) employ a two-step heuristic combining cost minimizing trip generation algorithm whose output is sent to a post-improvement SA algorithm.

A key issue when utilizing heuristics of any kind to solve optimization problems is that of the method's effectiveness. The quality of the decision support offered by the metaheuristic should be benchmarked by another method to instill confidence in the proposed approach. In our review of the research that proposed metaheuristic solution methods to the SBRP, we found a variety of benchmarks used. Table 7 lists the various schemes used, along with their frequency.

[Insert Table 7 Here]

One of the most commonly used benchmarking methods is comparing different versions of the proposed metaheuristic method. For instance, Silva et al. (2015) compare their mixed load GRASP metaheuristic with a single load implementation of GRASP. Bögl et al. (2015) compare the VNS implementation of the school bus transfer problem to two other VNS based implementations without bus transfers. The other commonly used technique is to implement a different metaheuristic method and compare the proposed method to it. Corberan et al. (2002) compare their scatter search implementation (an evolutionary approach) to TS (a trajectory-based method). Pacheco & Marti (2006) compare their TS implementation to scatter search, while Perez-Rodriguez & Hernandez-Aguirre (2016), Geem (2005) and Pacheco et al. (2013) use GA as their benchmark. Four of the papers used exact mixed integer programming (MIP) models to benchmarks their metaheuristics. Schittekat et al. (2013) is an excellent example of using exact MIP for smaller SBRP problems and using column generation to get lower bounds on solution quality for large problems. Mokhtari & Ghezavati (2018) use both exact methods and a GA variant to benchmark their bi-objective ACO algorithm. Seventeen publications did not use rigorous benchmarking schemes – 8 used the current practitioner's routes (typically created manually), 5 used simple heuristics (such as the Clark-Wright savings method and the sweep algorithm) and 4 did not benchmark their proposed approach.

3.3.3 Exact methods

Some bus route generation researchers have utilized exact methods such as column generation, cutting planes and integer programming models. While these methods can solve problems to optimality, the difficulty of the bus route generation sub-problem drastically increases computational times. Therefore, researchers tend to use relatively smaller data sets when applying exact methods. In Li et al. (2018), a branch-and-price-and-cut algorithm is implemented to solve pickup and delivery problems with at most 20 stores (stops) and 100 products (students).

Kumar & Jain (2015) formulate the bus route generation sub-problem with an assignment-based model and solve it using a Branch and Bound procedure. Santana et al. (2015) employ column generation to minimize TBD with stops being clustered using the algorithm in Shin & Han (2012).

Kim et al. (2012) compare three solution methods to solve the bus route generation and bus route scheduling sub-problems; MIP, Branch and Bound, and a heuristic. The algorithms found similar solutions on the smaller data sets, with the heuristic performing better as the problem instances get larger. Song & Kim (2013) use ILP for the bus route generation and an INLP model to solve the bus route scheduling problem.

Schittekat et al. (2006) present a MIP for the bus stop selection and bus route generation sub-problems to minimize TBD subject to capacity constraints. Martinez & Viegas (2011) develop a two phase MILP solution method for the bus stop selection and bus route generation sub-problems. The first phase employs a capacitated p-median problem to minimize the SWD subject to a MWT constraint. The second phase used a standard vehicle flow formulation to solve bus route generation with the objective of minimizing total fixed and variable costs. Kinable (2014) solve the bus stop selection and bus route generation problems for a single school using a column generation procedure, and computational results show an improvement over those solutions found in Schittekat et al. (2013).

Riera-Ledesma & Salazar-Gonzalez (2012) implement a cutting plane algorithm using a branch-and-cut procedure to minimize SWD and fixed vehicle costs (N) with a capacity constraint using a dataset from Baldacci et al (2007). Riera-Ledesma & Salazar-Gonzalez (2013) is an extension of this work that uses a branch-and-price algorithm with a set partitioning formulation.

3.3.4 Methods addressing uncertainty

Recently, some SBRP researchers have begun to consider the impact of uncertainty on bus route generation. Both demand (ridership) and time (travel time) uncertainty have been examined by researchers. Numerous factors can influence a student's transportation mode choice to and from school (e.g., after-school activities, inclement weather, student illness, or changing parental schedules). Yigit & Unsal (2016) and Unsal and Yigit (2018) address ridership uncertainty by dynamically updating bus stop demands using GPS software on mobile apps. Some recent work on real time production-distribution systems (Zhang et al. 2018) could be examined in the context of the SBRP, where demand for pupil transportation consists of a dynamic, online ordering system. The travel time between two points can vary depending on such factors as traffic demand and inclement weather. Sun et al. (2018) present a robust optimization approach to minimize the worst case scenario of a dynamic network. In this work, travel time uncertainty is addressed with arc travel times being calculated given both a fixed and a random component.

Levin & Boyles (2016) and Caceres et al. (2017) consider both ridership and travel time uncertainty in their research. Levin & Boyles (2016) report on a successful implementation of a decision support system to generate bus routes. Employing a Clark and Wright heuristic, they seek to minimize total bus fleet operating time for both the morning and afternoon problems while obeying capacity restrictions. They use an estimate of maximum ridership per stop in order to address ridership uncertainty and differing travel times based upon peak hour traffic congestion in their implementation to address travel time uncertainty. This approach resulted in significant cost savings for a public-school bus transportation system in Austin, TX. Caceres et al. (2017) present a chance constrained programming approach in conjunction with a column generation procedure to solve the bus route generation and bus route scheduling sub-problems of the SBRP. They seek to minimize the number of buses (N) and total distance (TBD). Caceres et al. (2017) addresses both travel time and ridership uncertainty by incorporating a possibility of arriving late while also considering the potential overbooking some buses. This is the first SBRP work addressing overbooking and their findings suggest that this should be a promising area of future SBRP research.

4. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

There has been extensive recent research on school bus routing problems, with a key trend being the consideration of more complex features of real-world settings. This especially includes modeling bus routes for multiple schools in a district and the consideration of several SBRP sub-problems together. The inclusion of multiple schools provides the opportunity for various types of mixed load routing to try and create better route plans. A key trend in solution approaches has been the application of a variety of metaheuristic algorithms, which are a natural fit for the greater model complexity.

Although a great deal of research into the five SBRP sub-problems have been conducted over the last fifty years there are several keys areas that offer promising opportunities for further research: 1) solution approaches, 2) analysis of mixed loading, 3) incorporating ridership uncertainty, 4) broader models to incorporate social and cultural factors, and 5) implementation of SBRP solution approaches.

1) *Solution approaches*: A wide variety of solution approaches have been developed for SBRPs in various settings. Yet, there is no consensus on the best approach for the different sub-problems in different settings. Thus, research to identify the best solution approach for each problem type subject to school district characteristics would be very welcome. The increase in the design and implementation of innovative metaheuristic approaches for SBRP is particularly encouraging, and these may be extended to multi-objective problems. Metaheuristics are natural candidates for SBRPs where service levels are typically reconciled with operational costs. Another area in need of research is combining aspects of evolutionary and trajectory methods to create hybrid metaheuristics that exploit features of both classes (cf. Euchi & Mraihi, 2012; Pacheco et al., 2013; Schittekat et al., 2013; Chen et al., 2015). There is also much room to develop metaheuristic methods for mixed loading SBRP problems, as currently only a few studies have done so (Kang et al., 2015; Ruiz et al., 2015; Silva et al., 2015; Lima et al., 2016, 2017; Yao et al., 2016). We also encourage SBRP researchers to better benchmark their metaheuristic approaches with exact solutions or lower bounds to increase the confidence in the proposed methods. Currently, only about 50% of the metaheuristic SBRP publications have benchmarked their methods.

2) *Analysis of mixed loading:* While there has been an increase in the number publications examining mixed loading (thirteen included in this review compared to five identified in Park & Kim (2010)); this research is largely limited to traditional mixed loading. Only Yao et al. (2016), Chen et al. (1990), Russell & Morrel (1986), and Desrosiers et al. (1981, 1986) consider *interscholastic* mixed loading and only

Fugenschuh (2009) and Bögl et al. (2015) consider *transshipment* mixed loading. Further analysis of mixed loading options at the policy level could include time constraints and other real-world complexities. For example, in the U.S. there are unwritten policies concerning the school levels that are acceptable to use for mixed loads, as it is generally unacceptable to mix only elementary and middle school students on a bus, while other combinations with high school students are deemed acceptable (State Directors for Public Transportation Services 2012).

3) *Ridership uncertainty*: Research incorporating ridership uncertainty is an area we believe offers opportunities for significant cost savings. Only three papers thus far have considered this in their research (Levin & Boyles, 2016; Yigit & Unsal, 2016, and Caceres et al. 2017). Further work is needed analyzing overbooking policies and its potential cost savings, especially given the promising findings in Caceres (2017). A related area of research that can significantly impact school bus routing is the role of mobile computing for better understanding and predicting ridership (see Yigit & Unsal, 2016).

4) *Broader models to incorporate social and cultural factors*: Social and cultural factors can influence school bus transportation policies and these factors might be regional, national, or global. The use of dedicated school buses varies around the world and the propensity for students to ride school buses changes with the age of students. Further, note that bus capacities may depend on the age of students as many transportation directors assume that a bus seat can hold either three lower elementary grade level students or two students from all other grade levels (Bowerman et al. 1995). Recent research by Kamali et al. (2013) and Caceres et al. (2018) has also looked at busing requirements of students with special needs where traditional routing methods may not suffice. To improve the generalizability of SBRP research findings, it important that the relevant social and cultural factors are captured within models.

5) *Implementation of SBRP solution approaches:* In spite of a good deal of excellent SBRP research, there remains a large gap between school district practices and academic modeling. Other large VRP problem areas have witnessed the implementation of advanced solution methods that have resulted in millions of dollars of savings for firms within the industries (e.g. waste management, small package delivery; see for example, Holland et al., 2017; Nguyen-Trong et al., 2017 and Stenger et al., 2013). As SBRPs are one of the largest vehicle routing problem worldwide, impacting millions of children and their families on an almost daily basis, there is great potential for benefits from improved routing and scheduling of school buses. The work of Bertsimas et al. (2019) highlights not only the potential financial benefits, but also the societal benefits that can be achieved by applying operations research methods to SBRP. With a bi-objective (number of buses and average riding time) solution approach, Bertsimas et al.

(2019) reduced the Boston Public School district transportation cost by $5 million annually, with no increase to average ride time. One major challenge to widespread implementation of advanced solution methods is that each school district tends to focus only on developing the best bus transportation system for their district, and there are no large national firms that provide school bus transportation with the resources to invest in the development of state-of-the-art school bus transportation system software. Thus, more studies are needed to quantify the benefits in practice from optimizing the SBRP, as well as to document how costs (Baykasoğlu and Özbel, 2016) and service levels are impacted when demand is aggregated across a region. Further work is needed to incorporate metaheuristic solution methods into software tools that practitioners can use to solve SBRPs. Most metaheuristic methods have parameters that need to be fine-tuned and hence cannot be handed off as a standalone mechanism (see Schittekat et al. 2013).

In summary, it is encouraging to see the considerable recent and ongoing SBRP research. Ideally, the benefits of this research would be not only the development of more efficient, safer and more equitable school district transportation plans, but also the societal and educational benefits from reducing the "wasted" time children spend riding buses.

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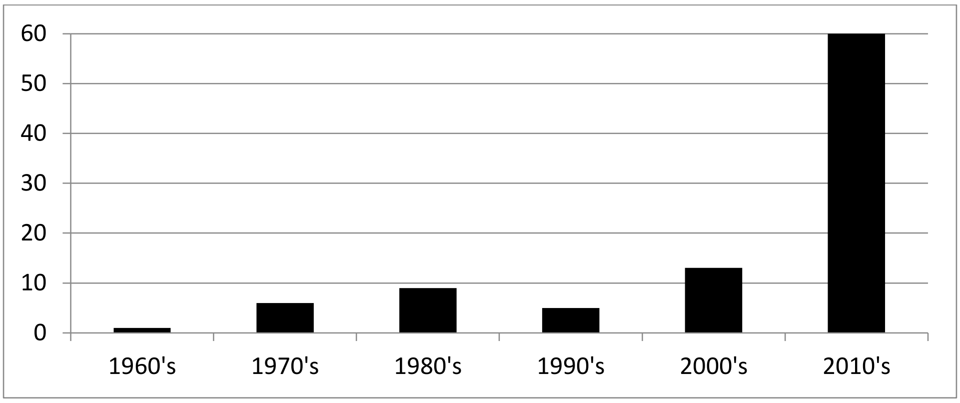

Figure 1: Frequency of SBRP publications by decade

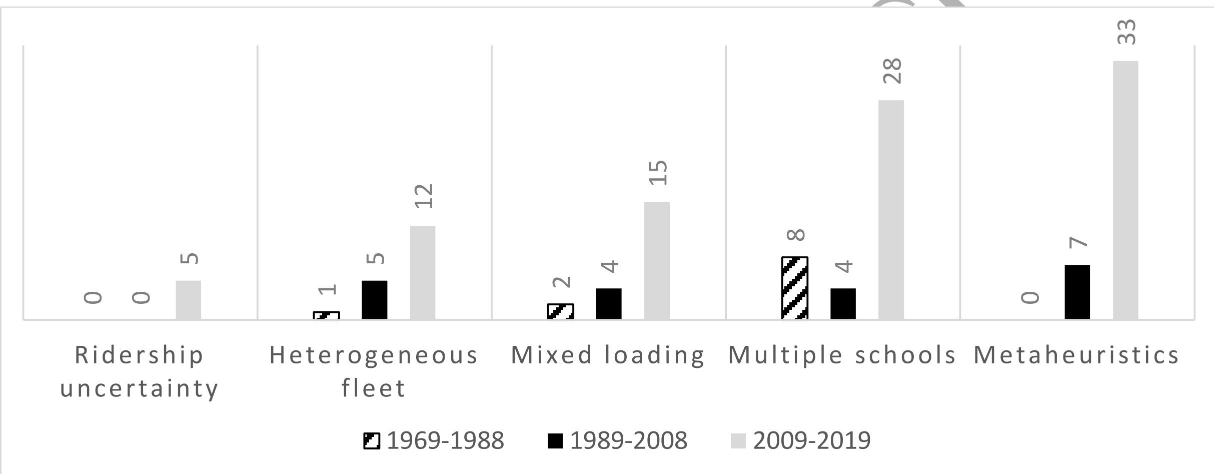

Figure 2: Frequency of SBRP characteristics and metaheuristic solution approach

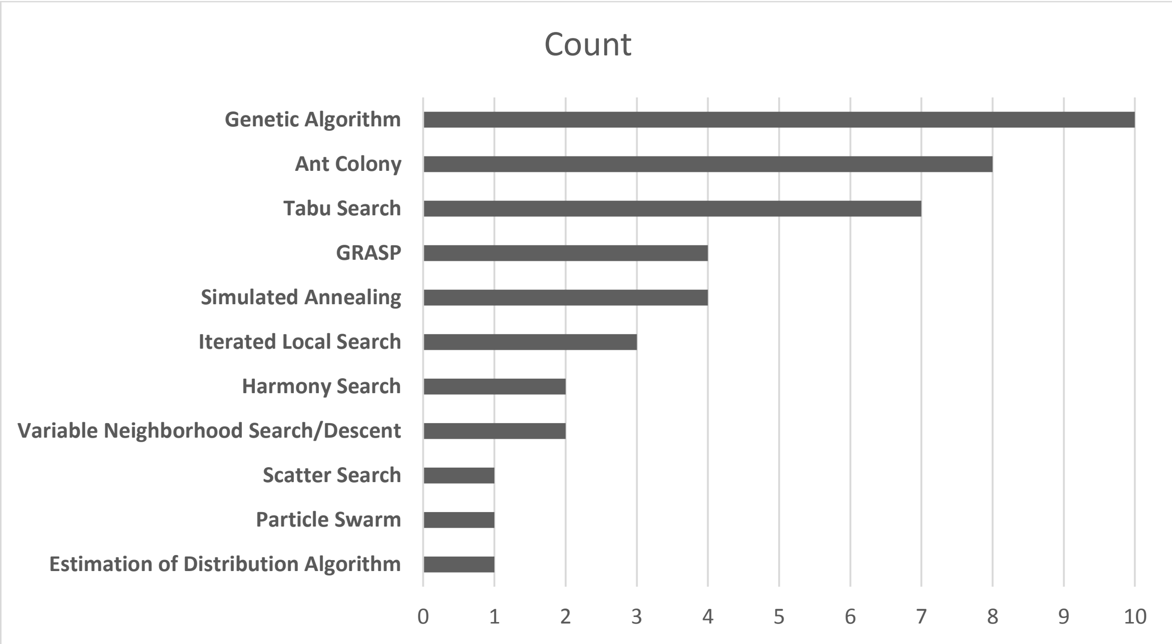

Figure 3 – SBRP bus route generation metaheuristic implementation count.

Table 1: Classification scheme for SBRP literature

Category	**Characteristics**	**Abbreviation**
Sub-problem type	Bus stop selection	BSS
	Bus route generation	BRG
	Bus route scheduling	BRS
	School bell time adjustment	SBA
	Strategic transportation policy	STP
Number of schools	Single school	Single
	Multiple schools	Multiple
Service environment	Urban	Urban
	Rural	Rural
	Urban and rural	Both
Load type	Mixed loads allowed	Yes
	Mixed loads not allowed	No
Fleet Mix	Homogenous fleet	HO
	Heterogeneous fleet	HT
Objectives	Number of buses used	N
	Total bus travel distance or time	TBD
	Total student riding distance or time	TSD
	Total student walking distance	SWD
	Maximum route length	MRL
	Load or ride time balance	LB
	Shared bus stop	**SBS**
	Capacity utilization	**CU**
	Total cost	**TC**
	Trip compatibility	**TRC**
	Safety factor	**SF**
	Number of transfers	**NT**
	Number of stops	**NS**
Constraints	Vehicle capacity	C
	Maximum riding time	MRT
	School time window	TW
	Maximum walking time or distance	MWT
	Earliest pick-up time	EPT
	Minimum student number to create a route	MSN
	Transfer time	**TT**
	Stop time window	**STW**
	Maximum stops per route	**MSR**
	Chance of overcrowding	**COO**
	Chance of being late	**COL**

Table 2: Sub-problem type and characteristics of recent *single* school SBRP publications.

Reference	Year	Sub-problem type	Service Environment	Fleet Mix	Obj.	Con.	Problem Size	Area
Serna & Bonrostro	2001	BRG	Urban	HO	MRL	C, MRT	57 stops, 429 students	Spain
Geem	2005	BRG	Urban	HO	N, TBD	C, MRT	10 stops, 130 students	Artificial
Alabas-Uslu	2008	BRG	Rural	HO	N, MRL	C	55 stops	Spain
Martinez & Viegas	2011	BSS, BRG	Urban	HO	SWD, TC (N, TBD)	C, MWT, MRT	13 stops, 118 students	Portugal
Arias-Rojas et al.	2012	BRG	Urban	HO	TBD	C	398 stops, 598 students	Colombia
Eldrandaly & Abdallah	2012	BRG	Urban	HO	TBD	C	13 stops, 120 students	Egypt
Euchi & Mraihi	2012	BRG	Urban	HO	TBD	C	200 stops	Tunisia
Fulin & Yueguang	2012	BRG	Urban	HO	TBD	C	25 stops	China
Riera-Ledesma & Salazar-Gonzalez	2012	BSS, BRG	Urban	HO	TC (N, SWD)	C, MWT	188 stops, 187 students	Artifical
Diaz-Parra et al.	2013	BRG	Urban	HO	N, TBD	C	200 stops	Mexico
Kim & Park	2013	BRG	Urban	HO	TC (N, TBD)	C, TW, EPT	18 stops	Artificial
Pacheco et al.	2013	BRG	Urban	HO	TBD, MRL	C,	57 stops, 429 students	Spain
Riera-Ledesma & Salazar-Gonzalez	2013	BSS, BRG	Urban	HO	TC (N, SWD)	C, MRT, MWT, MSR, MSN	26 stops	Artificial
Schittekat et al.	2013	BSS, BRG	Urban	HO	TBD	C	80 stops, 800 students	Belgium
Sghaier et al.	2013	BRG	Urban	HO	TC (N, TBD)	MRT, C	30 stops, 519 students	Unknown
Song & Kim	2013	BRG, BRS	Urban	HO	TBD	MRT, C	54 bus stops, 109 students	South Korea
Bronshtein et al.	2014	BRG	N/A	HO	MRL	C	45 buses, 620 stops	Artificial
Faraj et al.	2014	BSS, BRG	Rural	HT	TBD	C, MRT, MWT	23 schools, 67 stops, 221 students	Brazil
Huo et al.	2014	BSS, BRG	Urban	HO	TBD	C	16 stops, 33 students	China
Kinable	2014	BSS, BRG	Urban	HO	TBD	C, N	40 stops, 800 students	Artificial
Minocha & Tripathi	2014	BRG	Urban	HO	CU	C	50 stops, 286 students	India
Singh & Dhir	2014	BRG	Rural	HO	TBD	C	1000 students	India
Worwa	2014	BRG	Urban	HO	TBD	C, MRT	6 stops, 50 students	Poland
Mushi et al.	2015	BRG	Urban	HO	TBD	C	58 stops, 456 students	Tanzania
Santana & Carvajal	2015	BRG	Both	HO	TBD	C, STW	440 stops, 600 students	Colombia
Baykasoglu & Ozbel	2016	BRG	Unknown	HO	TBD	Unknown	Unknown	Turkey
Chalkia et al.	2016	BRG	Urban	HO	TBD, SF	MRT, C	29 bus stops, 60 students	Greece

de Siqueira et al.	2016	BRG	Both	HT	TBD	C, TW	87 stops, 1169 students	Brazil
Hashi et al.	2016	BRG	Urban	HO	N, TBD, MRL	C	11 stops, 180 students	Bangladesh
Perez-Rodriguez & Hernandez-Aguirre	2016	BSS, BRG	Both	HO	NS, TBD	MWT, C	80 stops, 800 students	Belgium
Sarubbi et al.	2016	BSS	Rural	n/a	NS	MWT	23 schools, 67 stops, 221 students	Brazil
Yigit & Unsal	2016	BRG	Urban	HO	TBD	Unknown	78 stops	Turkey
Kotoula et al.	2017	BRG	Urban	HO	TBD	C, MRT	198 students	Greece
Sales et al.	2018	BSS, BRG	Unknown	HT	TBD, N	MWT, C	5,250 students, 250 stops	Artificial
Sun et al.	2018	BRG	Rural	HO	TBD	TW	8 stops	China
Unsal & Yigit	2018	BRG	Urban	HO	TBD	Unknown	131 stops	Turkey

Obj. – Objectives, Con. – Constraints

Table 3: Sub-problem type and characteristics of recent *multiple* school SBRP publications.

Reference	Year	Sub-problem type	Mixed loads	Fleet Mix	Service Environment	Obj.	Con.	Problem Size	Area
Mandujano et al.	2012	BRG, SBA, BRS	No	HO	Rural	TC (N, TBD)	C	88 stops, 2,052 students	Brazil
Kamali et al.	2013	BSS, BRG			Both	TBD	C, N	111 students, 111 stops, 14 schools	United States
Kumar & Jain	2015	BRG, BRS			Urban	N	C, TW	40 schools, 11,600 students	Artificial
Levin & Boyles	2016	BRG			Urban	TBD	C, MRT	2 schools, 62 stops, 1,578 students	United States
Caceres et al.	2017	BRG, BRS			Urban	N, TBD	MRT, C, COO, COL	177 stops, 1237 students	United States
Shafahi et al.	2017	BRG, BRS			Unknown	TRC+TBD, TRC, TBD, N	C, TW	20 schools, 200 stops, 1,822 students	Artificial
Eguizabal et al.	2018	BRG, BRS			Both	TC (N, TBD), LB	C, TW, MRT, N, MSN	244 students	Spain
Shafahi et al.	2018	BRG, BRS			Both	TC	C, TW, MRT	100 schools, 2000 stops	Artificial
Kim et al.	2012	BRS		HT	Urban	N	C, TW	110 schools, 515 stops, 13,817 students	United States
Chen et al.	2015	BRG, BRS			Both	N, TBD, TC	TW, C, N	100 schools, 562 buses, 28175 students	China
Galdi & Thebpanya	2016	BSS			Both	NS	MWT	72 schools, 6360 stops, 38839 students	United States
Bertsimas et al.	2019	BSS, BRG, BRS, SBA			Urban	NS, SWD TC (N, TSD)	C, TW, MRT	200 schools, 17,942 stops, 20,000 students	United States
Park et al.	2012	BRG	Yes	HO	Urban	N	MRT, TW, C	90 schools, 2000 stops	United States
Bögl et al.	2015	BSS, BRG, BRS			Unknown	TBD, NT, SWD, NS, SBS	C, TW, TT, MWT	8 schools, 500 students	Artificial
Campbell et al.	2015	BRG, BRS			Both	TBD	C, MRT, TW	22 buses, 364 stops, 2301 students	United States
Ellegood et al.	2015	STP			Both	TBD	C	22 buses, 364 stops, 2301 students	United States
Ruiz et al.	2015	BRG, BRS			Urban	N	C	2 schools, 21 stops	Artificial
Yao et al.	2016	BRG, BRS			Urban	TBD	C	2 schools, 96 stops, 1088 students	Artificial
Oluwadare et al.	2018	BRG			Both	TBD, N	C	Unknown	Nigeria
Thangiah et al.	2008	BRG		HT	Rural	TC (N, TBD)	C, MRT	71 stops, 5 schools	United States

de Souza & de Siqueira	2010	BSS, BRG, BRS			Urban	SWD, TBD	C, MWT	49 schools, 2,711 stops, 4,501 students	Brazil
Kang et al.	2015	BSS, BRG, BRS			Unknown	NS, TBD, N, TSD	C, MRT, TW	10 schools, 50 stops, 500 students	Artificial
Silva et al.	2015	BRG			Rural	TBD	C, MRT, MWT	23 schools, 716 students	Brazil
Lima et al	2016	BRG			Rural	TC (N, TBD)	C	2000 stops, 27594 students	Brazil
Lima et al.	2017	BRG, BRS			Both	TC, TBD, LB	C	20 schools, 500 students	Brazil
Caceres et al.	2018	BRG			Urban	TC (N, TBD)	C, MRT, TW	39 schools	United States
Miranda et al.	2018	BSS, BRG, BRS			Rural	TC (N, TBD)	C, TW, MWT	65 schools, 2,774 students	Brazil
Mokhtari & Ghezavati	2018	BRG			Rural	N, LB	C, MRT, TW	50 schools, 250 stops, 5,906 students	Artificial

Obj. = Objectives, Con. = Constraints

Table 4: Mixed load policy considered: Trad = Traditional; Inter = Interscholastic; Trans = Transshipment; P/D = pickup/delivery.

		Mixed Load Policy			
Reference	**Year**	**Trad**	**Inter**	**Trans**	**P/D**
Hargroves & Demetsky*	1981	X			
Desrosiers et al.*	1981 1986		X		
Russell & Morrel*	1986	X	X		
Chen et al. *	1990		X		
Braca et al. *	1997	X			
Spada et al. *	2005	X			
Thangiah et al.	2008	X			
Fugenschuh*	2009			X	
de Souza & de Siqueira	2010	X			
Park et al.	2012	X			
Bögl et al.	2015			X	
Campbell et al.	2015	X			
Ellegood et al.	2015	X			
Kang et al.	2015	X			
Ruiz et al.	2015	X			
Silva et al.	2015	X			
Lima et al	2016	X			
Yao et al.	2016		X		
Lima et al.	2017	X			
Caceres et al.	2018	X			
Miranda et al.	2018				X
Mokhtari & Ghezavati	2018	X			
Oluwadare et al.	2018			X	

* - Denotes publications from the Park & Kim (2010).

Table 5: Construction and improvement heuristics of SBRP bus route generation research.

Reference	Year	Construction			Improvement	
		Savings	**Insertion**	**Two-phase**	**Intra-route (k-opt)**	**Inter-route (λ-interchange)**
Newton & Thomas*	1969		X		*k*	
Angel et al.*	1972			X		
Bennett & Gazis*	1972	X			3	
Newton & Thomas*	1974		X		*k*	
Verderber*	1974	X				
Bodin & Berman*	1979		X			
Gavish & Shlifer*	1979	X				
Dulac et al.*	1980	X	X			2
Desrosiers et al.*	1981			X	2	2
Chapleu et al.*	1985			X	2	1, 2
Desrosiers et al.*	1986			X	2	2
Russell et al.*	1986	X			3	λ
Thangiah & Nygard*	1992	X	X	X	*k*	λ
Bowerman et al.*	1995			X	2	
Braca et al.*	1997			X		1
Serna & Bonrostro	2001				*k*	λ
Corberan et al.*	2002		X		restricted 3&4	1
Li & Fu*	2002			X		1
Ripplinger*	2005		X			1
Spada et al.*	2005		X			1, λ
Pacheco & Marti*	2006			X		λ
Alabas-Uslu	2008			X	2	1
Thangiah et al.	2008		X		2	λ
de Souza & Siqueira	2010			X		
Arias-Rojas et al.	2012			X		
Eldrandaly & Abdallah	2012			X	*k*	
Euchi & Mraihi	2012				2	
Kim et al.	2012					1 & 2
Mandujano et al.	2012			X		
Park et al.	2012		X			1
Diaz-Parra et al.	2013			X		2
Kamali et al.	2013				2	
Pacheco et al.	2013		X		restricted 3 & 4	λ
Schittekat et al.	2013	X			*k*	
Sghaier et al.	2013				*k*	λ
Song, S-M., Kim, T.	2013		X			
Faraj et al.	2014				2 & restricted 3&4	
Minocha & Tripathi	2014					λ
Campbell et al.	2015	X			2	λ
Chen et al.	2015	X			2	1 & 2
Kang et al.	2015			X	*k*	1 & 2
Mushi et al.	2015				2	1, 2

Ruiz et al.	2015		X			1
Santana & Carvajal	2015			X		
Silva et al.	2015			X	2 & restricted 3&4	
de Souza & Siqueira	2016				2	
Yao et al.	2016			X	2	2
Kotoula et al.	2017	X		X		
Lima et al.	2017	X			2 & split move	λ
Caceres et al.	2018	X	X			
Sales et al.	2018					λ
Shafahi et al.	2018		X			
Bertsimas et al.	2019		X			

* - Denotes publications from Park & Kim (2010).

Table 6: SBRP bus route generation metaheuristic references.

Evolutionary Metaheuristics	Reference	Year
Ant Colony Optimization	Arias-Rojas et al.	2012
	Eldrandaly & Abdallah	2012
	Euchi & Mraihi	2012
	GurpreetSingh & Dhir	2014
	Huo et al.	2014
	Yao et al.	2016
	Yigit & Unsal	2016
	Mokhtari & Ghezavati	2018
Genetic Algorithm	Thangiah & Nygard*	1992
	Diaz-Parra et al.	2013
	Sghaier et al.	2013
	Minocha & Tripathi	2014
	Kang et al.	2015
	Chalkia et al.	2016
	Kotoula et al.	2017
	Oluwadare & Nwaiwu	2018
	Sales et al	2018
	Unsal & Yigit	2018
Harmony Search	Geem	2005
	Kim & Park	2013
Estimation of Distribution Algorithm	Perez-Rodriguez & Hernandez-Aguirre	2016
Particle Swarm	Fulin & Yueguang	2012
Scatter Search	Corberan et al.*	2002

Trajectory Based Metaheuristics	Reference	Year
GRASP	Schittekat et al.	2013
	Faraj et al.	2014
	Silva et al.	2015
	de Siqueira et al.	2016
Variable Neighborhood Search/Descent	Bögl et al.	2015
	Lima et al.	2016
Simulated Annealing	Spada et al.*	2005
	Chen et al.	2015
	Lima et al.	2016
	Shafahi et al.	2018
Tabu Search	Serna & Bonrostro	2001
	Ripplinger*	2005
	Spada et al.*	2005
	Pacheco & Marti*	2006
	Pacheco et al.	2013
	Mushi et al.	2015
	Ruiz et al.	2015
Iterated Local Search	Lima et al.	2016
	Lima et al.	2017
	Miranda et al.	2018

* - Denotes publications from Park & Kim (2010).

Table 7 – Benchmarks used to Measure Metaheuristic Effectiveness

Benchmark Used	Count	Reference
Different Metaheuristics	9	Corberan et al. (2002)*, Geem (2005), Spada et al. (2005)*, Pacheco & Marti (2006)*, Pacheco et al. (2013), Perez-Rodriguez & Hernandez-Aguirre (2016), Lima et al. (2016), Oluwadare & Nwaiwu (2018), Mokhtari & Ghezavati (2018)
Version Comparisons	8	Euchi & Mraihi (2012), GurpreetSingh & Dhir (2014), Bögl et al. (2015), Silva et al. (2015), Yao et al. (2016), Yigit & Unsal (2016), Lima et al. (2017), Miranda et al. (2018)
Exact Methods	7	Kim & Park (2013), Schittekat et al. (2013), Faraj et al. (2014), Chen et al. (2015), Sales et al (2018), Mokhtari & Ghezavati (2018), Shafahi et al. (2018)

* - Denotes publications from Park & Kim (2010).