Exact exponential and FPT ex. class

May 19, 2014

Ex.1

Recall that independent set is the set of vertices in a graph, no two of which are adjacent. That is, it is a set I of vertices such that for every two vertices in I, there is no edge connecting the two. Equivalently, each edge in the graph has at most one endpoint in I. The size of an independent set is the number of vertices it contains.

Lemma 0.1. *Let $G = (V, E)$ be a graph and let v be a vertex of G. If no maximum independent set of G contains v then every maximum independent set of G contains at least two vertices of $N(v)$.*

1. Prove the Lemma.
2. Improve upon the branching algorithm *mis1* of Chap. 1 by the use of the Lemma

Ex.2

For the next ex. you need to recall the definitions of FPT and kernelization. Let $\mathcal{A}$ be a decidable parameterized problem. We will show that $\mathcal{A}$ is fixed-parameter tractable if and only if there exists a kernelization algorithm for it.

- Show that if we have a kernelization algorithm, the problem is in FPT.

- Assume the problem is in FPT and runs in $f(k) \cdot n^d$ steps. We claim that by running the algorithm n^{d+1} times, we can show that the problem has a kernelization. How can we do that?
  Hint: We can produce trivial instances that reply yes and no in polynomial time.

Ex.3

1. Show that any full k-ary tree has a vertex cover of size $\leq \sum_{i=1}^{\lfloor \frac{\log_k n-1}{2} \rfloor} \frac{n}{k^{2i}}$.

2. In this ex. We start by mentioning that Vertex cover is solvable in polynomial time on trees.

The algorithm:
while $L \neq \emptyset$
  do f $\leftarrow$ remove first leaf from L
    if $f \in T$
      if $mark[f] == FALSE$ and $parent[f]$ is $null$
          then mark[f] $\leftarrow$ TRUE
      else if $mark[f] == FALSE$ and $parent[f] \neq null$
          then $mark[parent[f]] \leftarrow TRUE$
        remove f from T
        (this implicitly removes leaf-parent edge if it exists)
        if $parent[f] \neq null$ and $parent[f] \neq root[T]$
          then if children[parent[f]] is null
            then append parent[f] to L

You may use the algorithm as black box.
Consider a graph that can be transformed into a tree by deleting k edges. Assume that this set of edges is given. Give a fixed-parameter algorithm for Vertex Cover restricted to this class of graphs, parameterized by k.

Ex.4

We would like to parameterize the problem SAT

- What can we say about the following parameterization?
  $kSAT = \{(F,k)|F$ is a satisfiable $k - CNF$ formula$\}$

- Can we say something better about the suggested parameterizaion?
  $CNF?SAT = \{(F,k)|F\}$ is a satisfiable with k varaibles$\}$

- Come up with additional parameter that keeps the problem in FPT.

Ex. 5

A question from the lecture concerns the following algorithm for the $Vertex-Cover$ problem: choose an arbitrary edge $e = \{u, v\}$ that has not yet been covered and branch on the two sub-cases; on one branch include u in the solution and in the other include v in the solution. Return the smaller of the two solutions. We want to prove that this algorithm does not run in $FPT - time$ if parameterized by the size of the Vertex-Cover.

Prove that a graph composed of the collection of star graphs $S_{r1} \ldots S_{rl}$ is a counter example.