```
> restart; interface(showassumed = 1) :
> with(plots) : with(plottools) :
> Cj := Kc·sqrt( 1 / (Vb + V(t)) )
```

$$Cj := Kc \sqrt{\frac{1}{Vb + V(t)}} \tag{1}$$

```
> DE := Cj·diff(V(t), t) + Cfixed·diff(V(t), t) = Ihat·sin(omega·t)
```

$$DE := Kc \sqrt{\frac{1}{Vb + V(t)}} \left(\frac{d}{dt} V(t) \right) + Cfixed \left(\frac{d}{dt} V(t) \right) = Ihat \sin(\omega t) \tag{2}$$

```
> solution := dsolve(DE)
```

$$solution := \frac{1}{Ihat\, \omega} \left(-2\, Kc \sqrt{\frac{1}{Vb + V(t)}}\, V(t)\, \omega - 2\, Kc \sqrt{\frac{1}{Vb + V(t)}}\, Vb\, \omega - Cfixed\, V(t)\, \omega + \_C1\, Ihat\, \omega - \cos(\omega t)\, Ihat \right) = 0 \tag{3}$$

```
> solution := solve(dsolve(DE), V(t)) :
> solution := V(t) = solution[1], V(t) = solution[2] :
> solution[1]
```

$$V(t) = \left(-\frac{1}{Cfixed\, Vb\, \omega + \_C1\, Ihat\, \omega - \cos(\omega t)\, Ihat} \left(2\, Kc\, Vb\, \omega \left(Kc\, \omega + \sqrt{Cfixed^2\, Vb\, \omega^2 + Cfixed\, Ihat\, \_C1\, \omega^2 - Cfixed\, Ihat \cos(\omega t)\, \omega + Kc^2\, \omega^2} \right) \right) + \_C1\, Ihat\, \omega - \cos(\omega t)\, Ihat \right) \Bigg/ \left(\omega \left(\frac{1}{Cfixed\, Vb\, \omega + \_C1\, Ihat\, \omega - \cos(\omega t)\, Ihat} \left(2\, Kc \left(Kc\, \omega + \sqrt{Cfixed^2\, Vb\, \omega^2 + Cfixed\, Ihat\, \_C1\, \omega^2 - Cfixed\, Ihat \cos(\omega t)\, \omega + Kc^2\, \omega^2} \right) \right) + Cfixed \right) \right) \tag{4}$$

```
> solve(subs(cos(omega·t) = 0, rhs(solution[1])) = Vdc, _C1)
```

$$\frac{Cfixed\, Vdc + 2\sqrt{Kc^2\, Vb + Kc^2\, Vdc}}{Ihat}, \frac{Cfixed\, Vdc - 2\sqrt{Kc^2\, Vb + Kc^2\, Vdc}}{Ihat} \tag{5}$$

```
> solution := simplify(subs(_C1 = solve(subs(cos(omega·t) = 0, rhs(solution[1])) = Vdc,
    _C1)[1], solution[1]))
```

$$solution := V(t) = \left(2 \left(-2 \cos(\omega t)\, Ihat + \omega\, Cfixed\, (Vb + 2\, Vdc) \right) \omega \sqrt{Kc^2\, (Vb + Vdc)} + Ihat^2 \cos(\omega t)^2 - Ihat\, \omega\, Cfixed\, (Vb + 2\, Vdc) \cos(\omega t) - \omega \left(\left(-Vdc\, (Vb + Vdc)\, Cfixed^2 - 2\, Kc^2\, (Vb + 2\, Vdc) \right) \omega + 2 \left(\omega \left(2\, Cfixed \sqrt{Kc^2\, (Vb + Vdc)}\, \omega - Cfixed\, Ihat \cos(\omega t) + \omega \left((Vb \right. \right. \right. \right. \tag{6}$$

$$+ Vdc)\ Cfixed^2 + Kc^2)\Big)\Big)^{1/2}\ Kc\ Vb\Big)\Big)\Big/\Big(\Big(2\ Cfixed\sqrt{Kc^2\ (Vb + Vdc)}\ \omega$$

$$- Cfixed\ Ihat\cos(\omega\, t) + \big((Vb + Vdc)\ Cfixed^2 + 2\ Kc^2\big)\ \omega$$

$$+ 2\ Kc\ \Big(\omega\ \Big(2\ Cfixed\sqrt{Kc^2\ (Vb + Vdc)}\ \omega - Cfixed\ Ihat\cos(\omega\, t) + \omega\ \big((Vb$$

$$+ Vdc)\ Cfixed^2 + Kc^2\big)\Big)\Big)^{1/2}\Big)\ \omega\Big)$$

```
> taylor_0 := simplify(coeftayl(subs(cos(omega·t) = x, rhs(solution)), x = 0, 0)):
> simplify(taylor_0 - eval(subs(cos(omega·t) = x, rhs(solution)), x = 0))
```

$$0 \tag{7}$$

```
> taylor_1 := simplify(coeftayl(subs(cos(omega·t) = x, rhs(solution)), x = 0, 1)):
> taylor_2 := simplify(coeftayl(subs(cos(omega·t) = x, rhs(solution)), x = 0, 2)):
> ac_resp := simplify(taylor_0 + taylor_1·cos(omega·t) + taylor_2
      · (cos(2 omega·t) + 1)/2):
> amplitude1 := taylor_1:
> amplitude2 := taylor_2/2:
> HD2 := simplify(amplitude2/amplitude1)
```

$$HD2 := -\Big(Ihat\ Kc\ \Big(\Big(2\ Cfixed\ \Big(\Big(\frac{3\ Vb}{8} + \frac{3\ Vdc}{8}\Big)\ Cfixed^2 + Kc^2\Big)\sqrt{Kc^2\ (Vb + Vdc)} \tag{8}$$

$$+ \frac{3\ (Vb + Vdc)^2\ Cfixed^4}{16} + \frac{7\ Kc^2\ (Vb + Vdc)\ Cfixed^2}{4} + Kc^4\Big)$$

$$Kc\sqrt{\omega^2\ \big((Vb + Vdc)\ Cfixed^2 + 2\ Cfixed\sqrt{Kc^2\ (Vb + Vdc)} + Kc^2\big)} + \omega\ \Big(3\ \Big(\Big(\frac{Vb}{12}$$

$$+ \frac{Vdc}{12}\Big)\ Cfixed^2 + Kc^2\Big)\ Cfixed\ \Big(\Big(\frac{3\ Vb}{4} + \frac{3\ Vdc}{4}\Big)\ Cfixed^2$$

$$+ Kc^2\Big)\sqrt{Kc^2\ (Vb + Vdc)} + \frac{(Vb + Vdc)^3\ Cfixed^6}{32} + \frac{15\ Kc^2\ (Vb + Vdc)^2\ Cfixed^4}{16}$$

$$+ \frac{15\ Kc^4\ (Vb + Vdc)\ Cfixed^2}{4} + Kc^6\Big)\Big)\Big)\Big/$$

$$\Big(8\ \Big(Kc\sqrt{\omega^2\ \big((Vb + Vdc)\ Cfixed^2 + 2\ Cfixed\sqrt{Kc^2\ (Vb + Vdc)} + Kc^2\big)}$$

$$+ \omega\ \Big(Cfixed\sqrt{Kc^2\ (Vb + Vdc)} + \Big(\frac{Vb}{2} + \frac{Vdc}{2}\Big)\ Cfixed^2 + Kc^2\Big)\Big)\ \big((Vb$$

$$+ Vdc)\ Cfixed^2 + 2\ Cfixed\sqrt{Kc^2\ (Vb + Vdc)} + Kc^2\big)\ \Big(\Big(\Big(\Big(\frac{Vb}{4} + \frac{Vdc}{4}\Big)\ Cfixed^2$$

$$+ \frac{Kc^2}{2}\Big)\sqrt{Kc^2\ (Vb + Vdc)}$$

$$+\frac{(Vb+Vdc)\,Cfixed\left(\left(\frac{Vb}{8}+\frac{Vdc}{8}\right)Cfixed^2+Kc^2\right)}{2}\Bigg)$$

$$\sqrt{\omega^2\left((Vb+Vdc)\,Cfixed^2+2\,Cfixed\sqrt{Kc^2\,(Vb+Vdc)}+Kc^2\right)}+Kc\left(\left(\left(\frac{3\,Vb}{4}+\frac{3\,Vdc}{4}\right)Cfixed^2+\frac{Kc^2}{2}\right)\sqrt{Kc^2\,(Vb+Vdc)}+(Vb+Vdc)\left(\left(\frac{3\,Vb}{16}+\frac{3\,Vdc}{16}\right)Cfixed^2+Kc^2\right)Cfixed\right)\omega\right)\Bigg)$$

> $HD2 := simplify\left(\frac{amplitude2}{amplitude1}, assume = positive\right)$ assuming $Vdc :: real$, $Vb \geq 0$, $Vb :: real$, $Cj \geq 0$, $Cj :: real$, $Cfixed \geq 0$, $Cfixed :: real$, $t \in \mathbb{R}$, $t :: real$, $omega \geq 0$, $omega :: real$, $Ihat \geq 0$, $Ihat :: real$, $Kc \geq 0$, $Kc :: real$

$$HD2 := -\Bigg(Ihat\,Kc\left(Kc\left(2\,Kc\left(\left(\frac{3\,Vb}{8}+\frac{3\,Vdc}{8}\right)Cfixed^2+Kc^2\right)Cfixed\sqrt{Vb+Vdc}+\frac{3\,(Vb+Vdc)^2\,Cfixed^4}{16}+\frac{7\,Kc^2\,(Vb+Vdc)\,Cfixed^2}{4}+Kc^4\right)\sqrt{2\,Cfixed\,Kc\sqrt{Vb+Vdc}+(Vb+Vdc)\,Cfixed^2+Kc^2}+3\left(\left(\frac{3\,Vb}{4}+\frac{3\,Vdc}{4}\right)Cfixed^2+Kc^2\right)Kc\,Cfixed\left(\left(\frac{Vb}{12}+\frac{Vdc}{12}\right)Cfixed^2+Kc^2\right)\sqrt{Vb+Vdc}+\frac{(Vb+Vdc)^3\,Cfixed^6}{32}+\frac{15\,Kc^2\,(Vb+Vdc)^2\,Cfixed^4}{16}+\frac{15\,Kc^4\,(Vb+Vdc)\,Cfixed^2}{4}+Kc^6\right)\Bigg)\Bigg/\Bigg(4\left(Kc\sqrt{2\,Cfixed\,Kc\sqrt{Vb+Vdc}+(Vb+Vdc)\,Cfixed^2+Kc^2}+Cfixed\,Kc\sqrt{Vb+Vdc}+\left(\frac{Vb}{2}+\frac{Vdc}{2}\right)Cfixed^2+Kc^2\right)(2\,Cfixed\,Kc\sqrt{Vb+Vdc}+(Vb+Vdc)\,Cfixed^2+Kc^2)\,\omega\left(\left(\left(\left(\frac{Vb}{2}+\frac{Vdc}{2}\right)Cfixed^2+Kc^2\right)Kc\sqrt{Vb+Vdc}+(Vb+Vdc)\,Cfixed\left(\left(\frac{Vb}{8}+\frac{Vdc}{8}\right)Cfixed^2+Kc^2\right)\right)\sqrt{2\,Cfixed\,Kc\sqrt{Vb+Vdc}+(Vb+Vdc)\,Cfixed^2+Kc^2}+Kc\left(Kc\left(\left(\frac{3\,Vb}{2}+\frac{3\,Vdc}{2}\right)Cfixed^2+Kc^2\right)\sqrt{Vb+Vdc}+2\,(Vb+Vdc)\left(\left(\frac{3\,Vb}{16}+\frac{3\,Vdc}{16}\right)Cfixed^2+Kc^2\right)Cfixed\right)\right)\Bigg)\tag{9}$$

> $simplify(algsubs(Vb+Vdc=Vbias, HD2), assume=positive)$

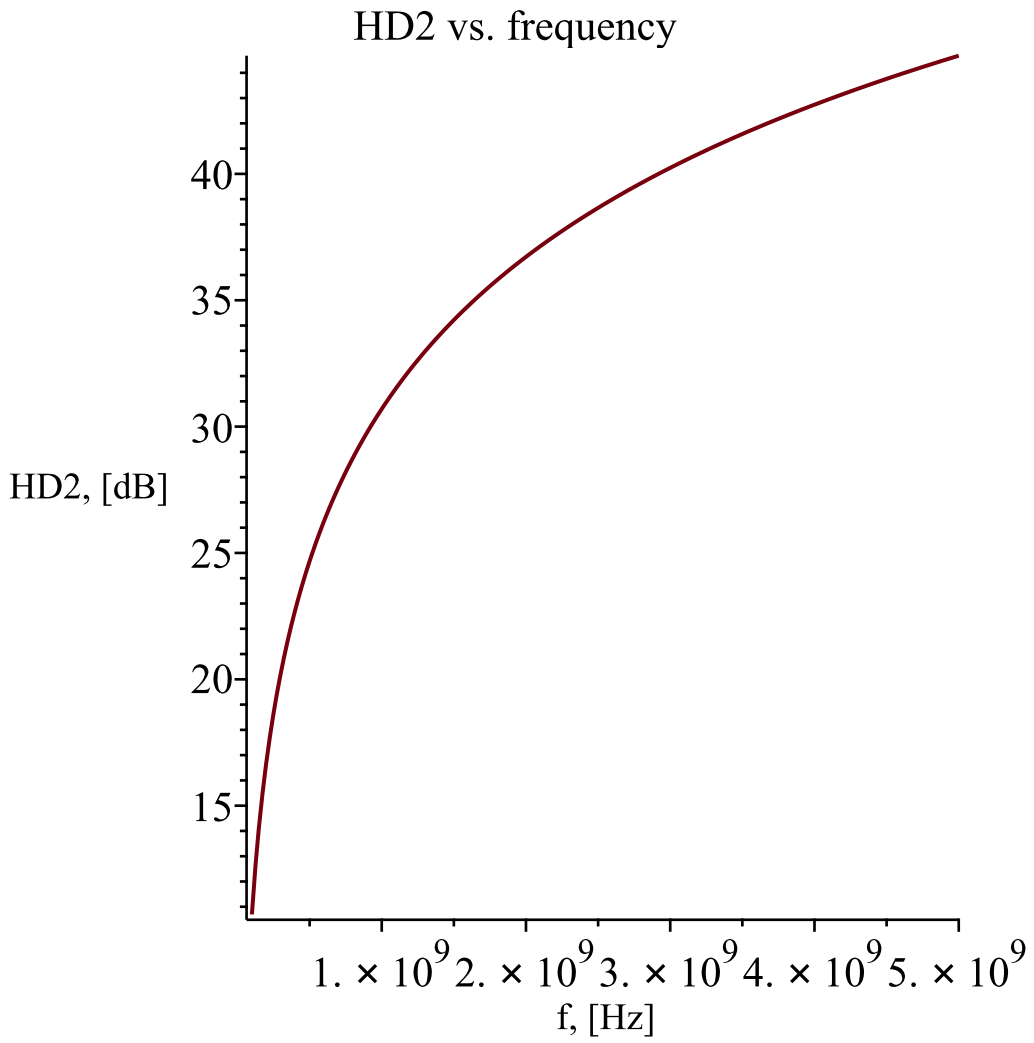
$$-\Big(Ihat\,Kc\left(Cfixed^6\,Vbias^3+12\,Kc\,Cfixed^5\,Vbias^{5/2}+60\,Kc^2\,Cfixed^4\,Vbias^2+160\,Cfixed^3\,Kc^3\,Vbias^{3/2}+240\,Kc^4\,Cfixed^2\,Vbias+192\,Cfixed\,Kc^5\sqrt{Vbias}+64\,Kc^6\right)\Big)\Big/\Big(8\left(Cfixed^2\,Vbias+4\,Cfixed\,Kc\sqrt{Vbias}+4\,Kc^2\right)\left(Cfixed^2\,Vbias\right.\tag{10}$$

$$+ 2\,Cfixed\,Kc\sqrt{Vbias} + Kc^2)\,\omega\,(Cfixed^4\,Vbias^{5/2} + 8\,Cfixed^3\,Kc\,Vbias^2 + 24\,Cfixed^2\,Kc^2\,Vbias^{3/2} + 32\,Cfixed\,Kc^3\,Vbias + 16\,Kc^4\sqrt{Vbias})\Big)$$

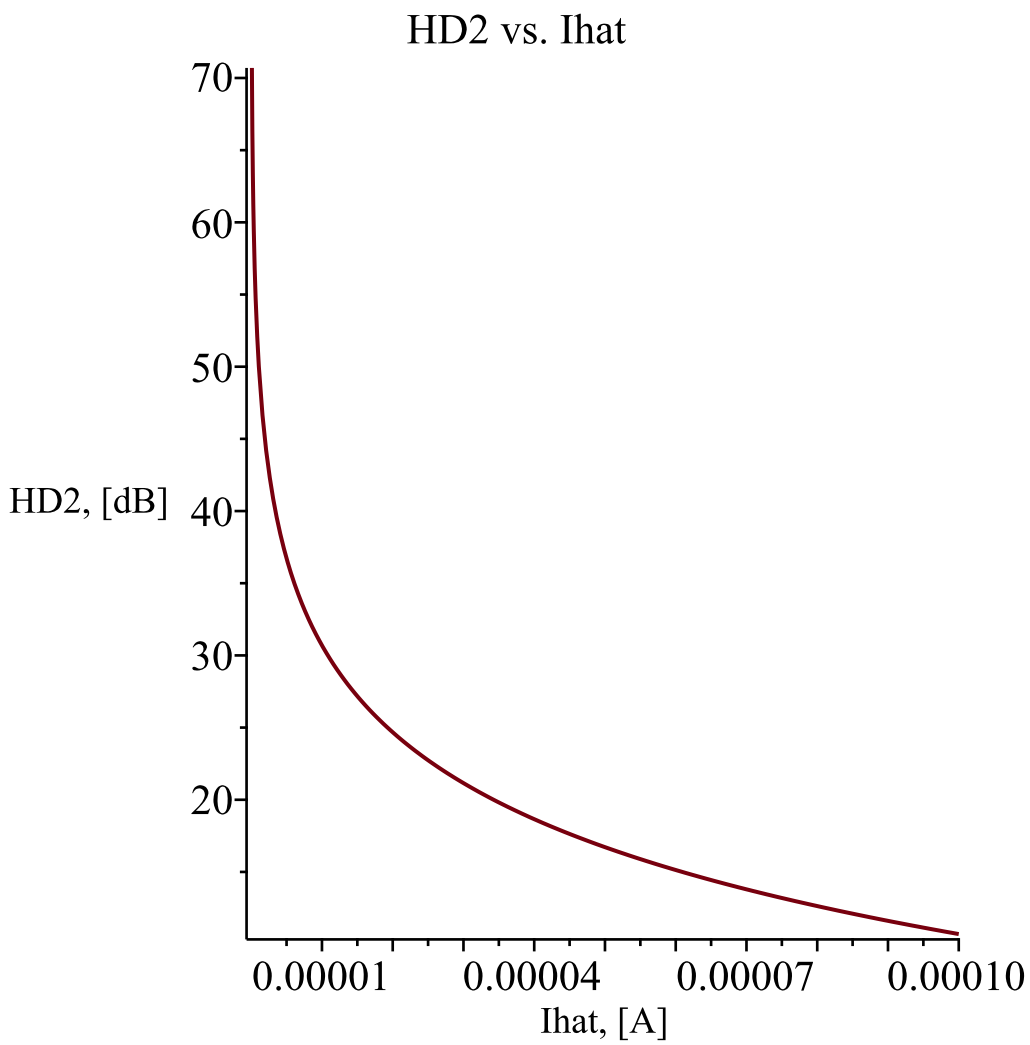
```
> subs(Vbias = Vb + Vdc, simplify(algsubs(Vb + Vdc = Vbias, HD2), assume = positive))
```

$$-\Big(Ihat\,Kc\,\big((Vb+Vdc)^3\,Cfixed^6 + 12\,Kc\,Cfixed^5\,(Vb+Vdc)^{5/2} + 60\,Kc^2\,(Vb+Vdc)^2\,Cfixed^4 + 160\,Cfixed^3\,Kc^3\,(Vb+Vdc)^{3/2} + 240\,Kc^4\,(Vb+Vdc)\,Cfixed^2 + 192\,Cfixed\,Kc^5\sqrt{Vb+Vdc} + 64\,Kc^6\big)\Big) \Big/ \Big(8\,\big((Vb+Vdc)\,Cfixed^2 + 4\,Cfixed\,Kc\sqrt{Vb+Vdc} + 4\,Kc^2\big)\,\big(2\,Cfixed\,Kc\sqrt{Vb+Vdc} + (Vb+Vdc)\,Cfixed^2 + Kc^2\big)\,\omega\,\big(Cfixed^4\,(Vb+Vdc)^{5/2} + 8\,Cfixed^3\,Kc\,(Vb+Vdc)^2 + 24\,Cfixed^2\,Kc^2\,(Vb+Vdc)^{3/2} + 32\,Cfixed\,Kc^3\,(Vb+Vdc) + 16\,Kc^4\sqrt{Vb+Vdc}\big)\Big) \tag{11}$$

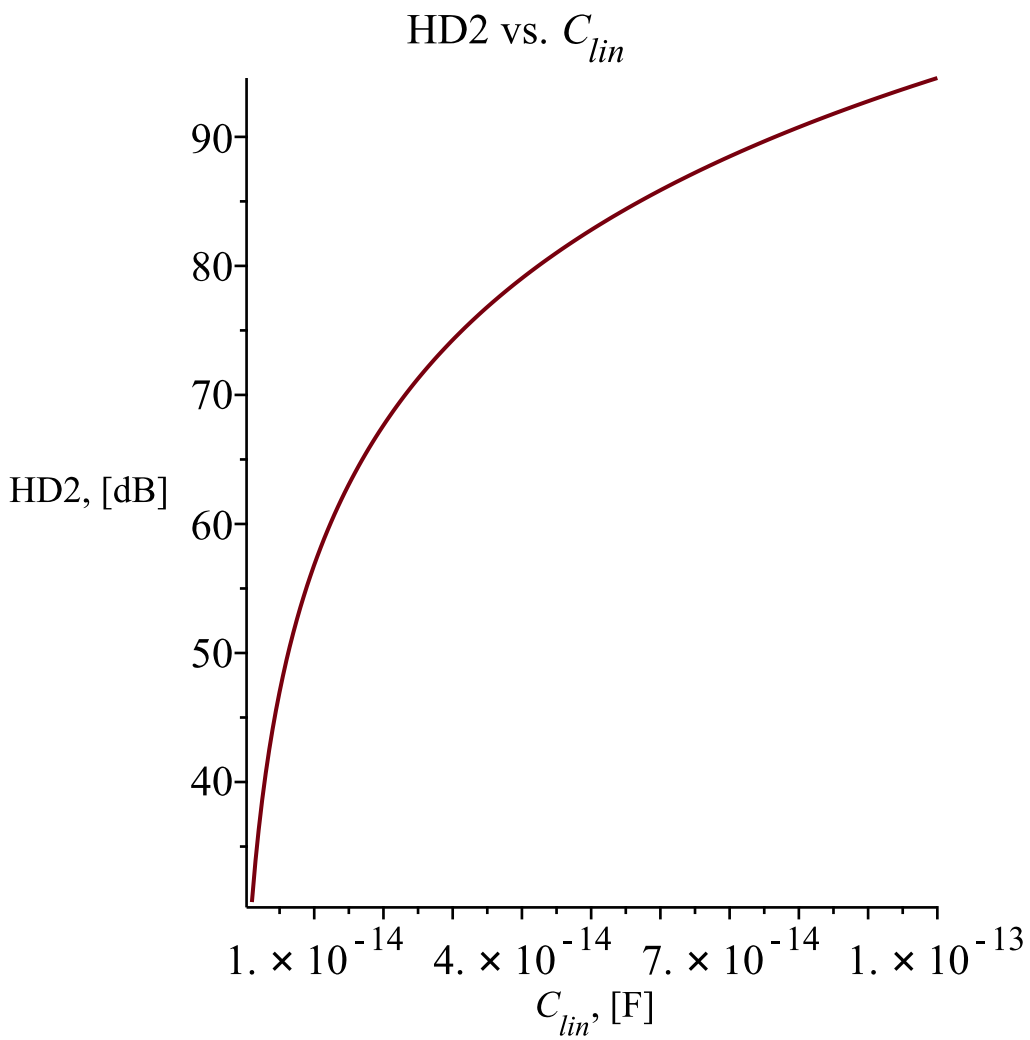
```
> P1 := plot(evalf((eval(simplify(-20·log10(-HD2)), [omega = 2·Pi·freq, Cfixed = 1e-15,
      Ihat = 10e-6, Kc = 2e-15, Vb = 0.62, Vdc = 1]))), freq = 0.1e9..5e9, title
      = "HD2 vs. frequency", labels = ["f, [Hz]", "HD2, [dB]"], size = [400, 400])
```



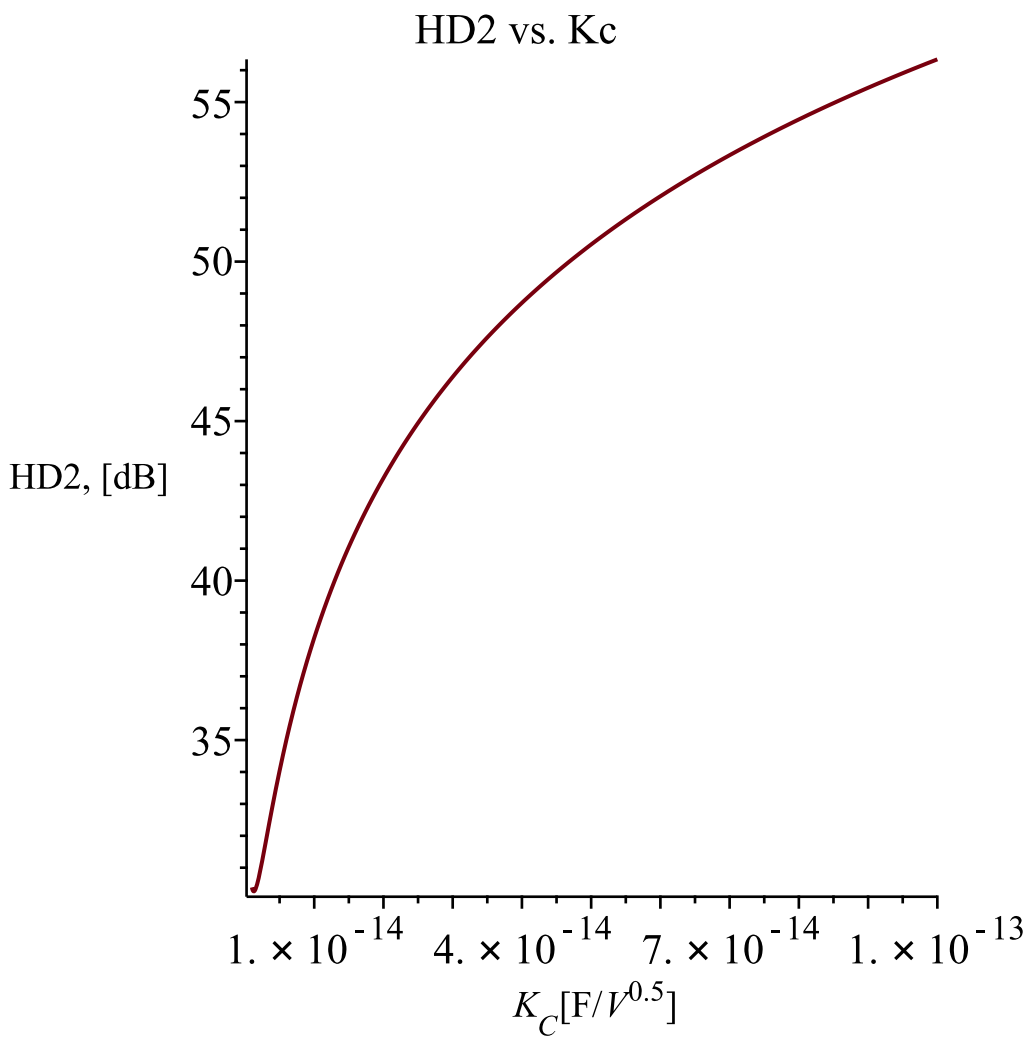

```
> P2 := plot(evalf((eval(simplify(-20·log10(-HD2)), [omega = 2·Pi·1e9, Cfixed = 1e-15,
      Kc = 2e-15, Vb = 0.62, Vdc = 1]))), Ihat = 0.1e-6..100e-6, title
      = typeset("HD2 vs. Ihat"), labels = ["Ihat, [A]","HD2, [dB]"], size = [400, 400])
```



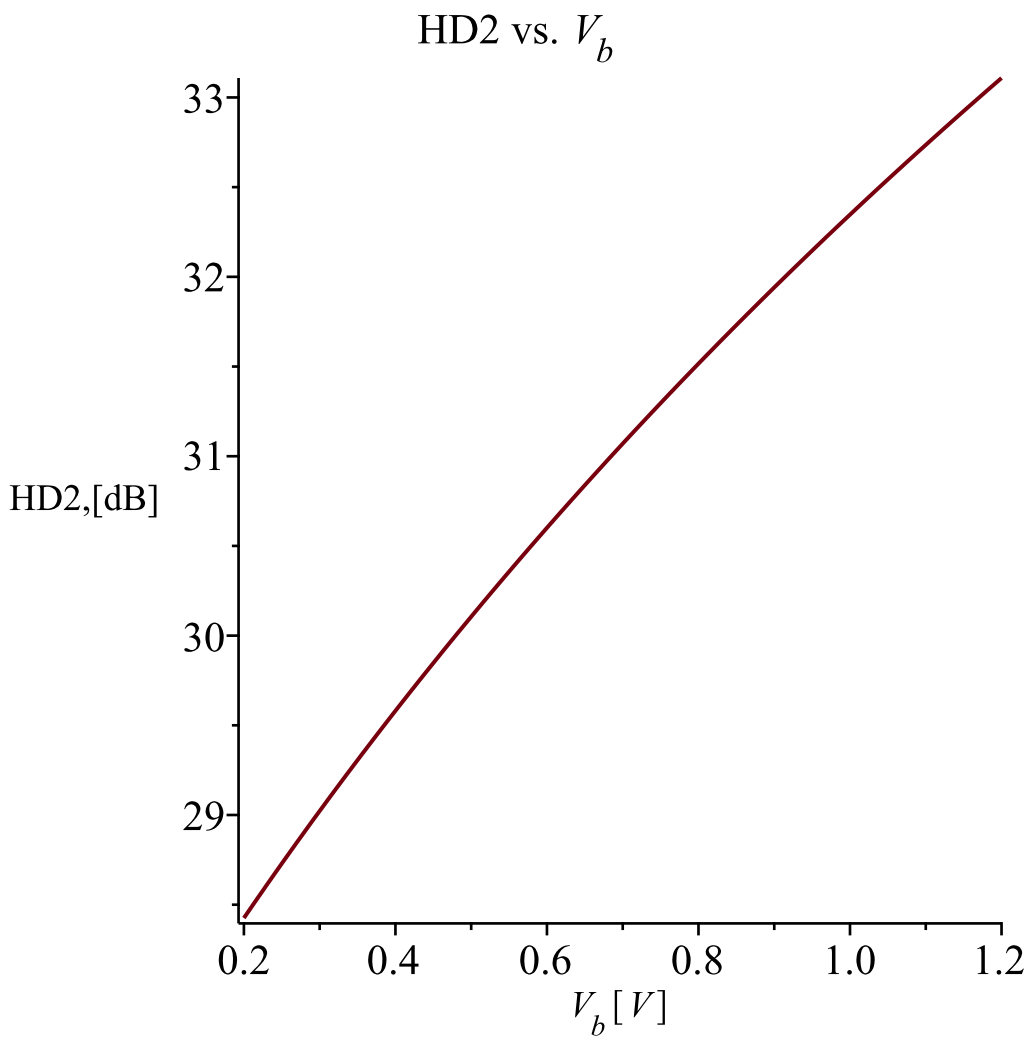

```
> P3 := plot(evalf((eval(simplify(-20·log10(-HD2)), [omega = 2·Pi·1e9, Ihat = 10e-6,
      Kc = 2e-15, Vb = 0.62, Vdc = 1]))), Cfixed = 1e-15..100e-15, title
      = typeset("HD2 vs. ", C[lin]), labels = [typeset(C[lin], ", [F]"), "HD2, [dB]"], size
      = [400, 400])
```



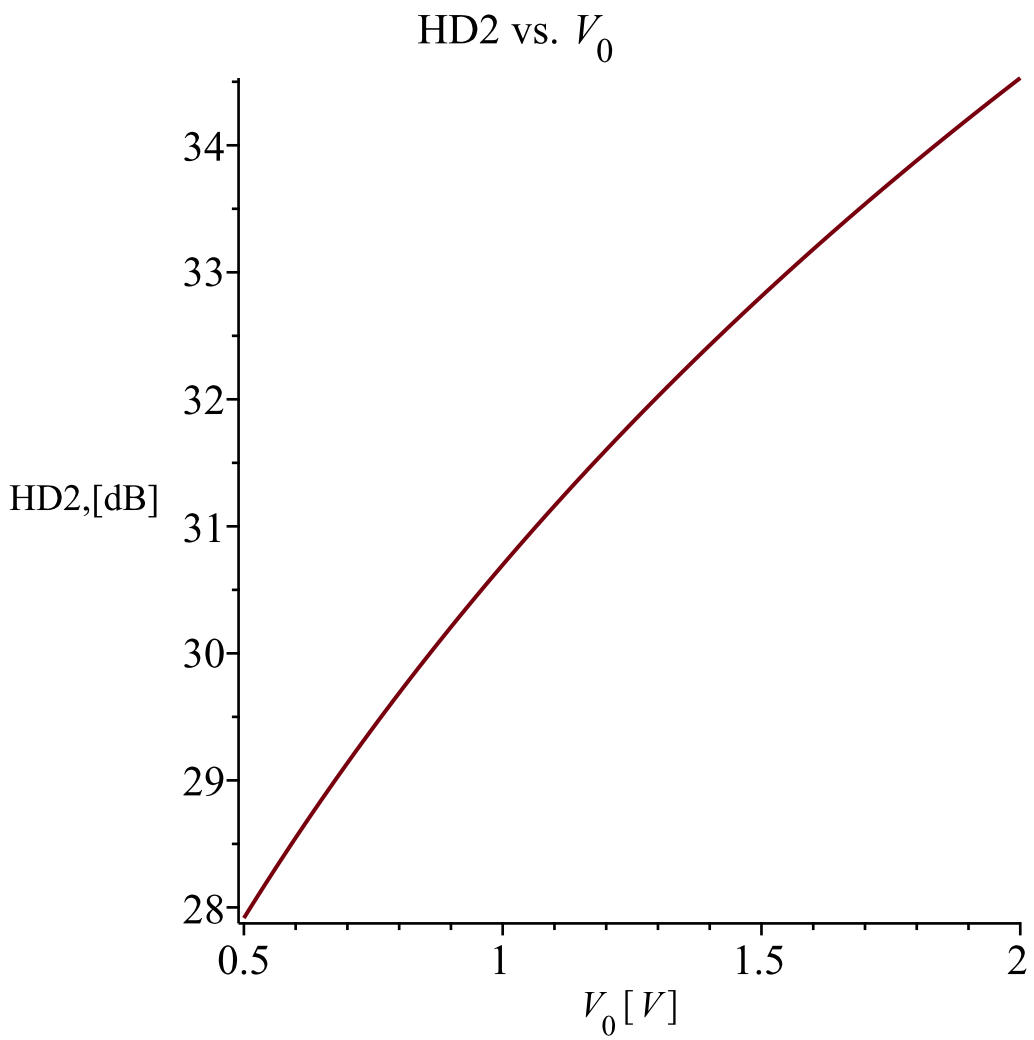

```
> P4 := plot(evalf((eval(simplify(-20·log10(-HD2)), [omega=2·Pi·1e9, Cfixed=1e−15,
     Ihat=10e−6, Vb=0.62, Vdc=1]))), Kc=1e−15..100e−15, title="HD2 vs. Kc",
     labels=[typeset(K[C], "[F/", V^0.5, "]"), "HD2, [dB]"], size=[400, 400])
```



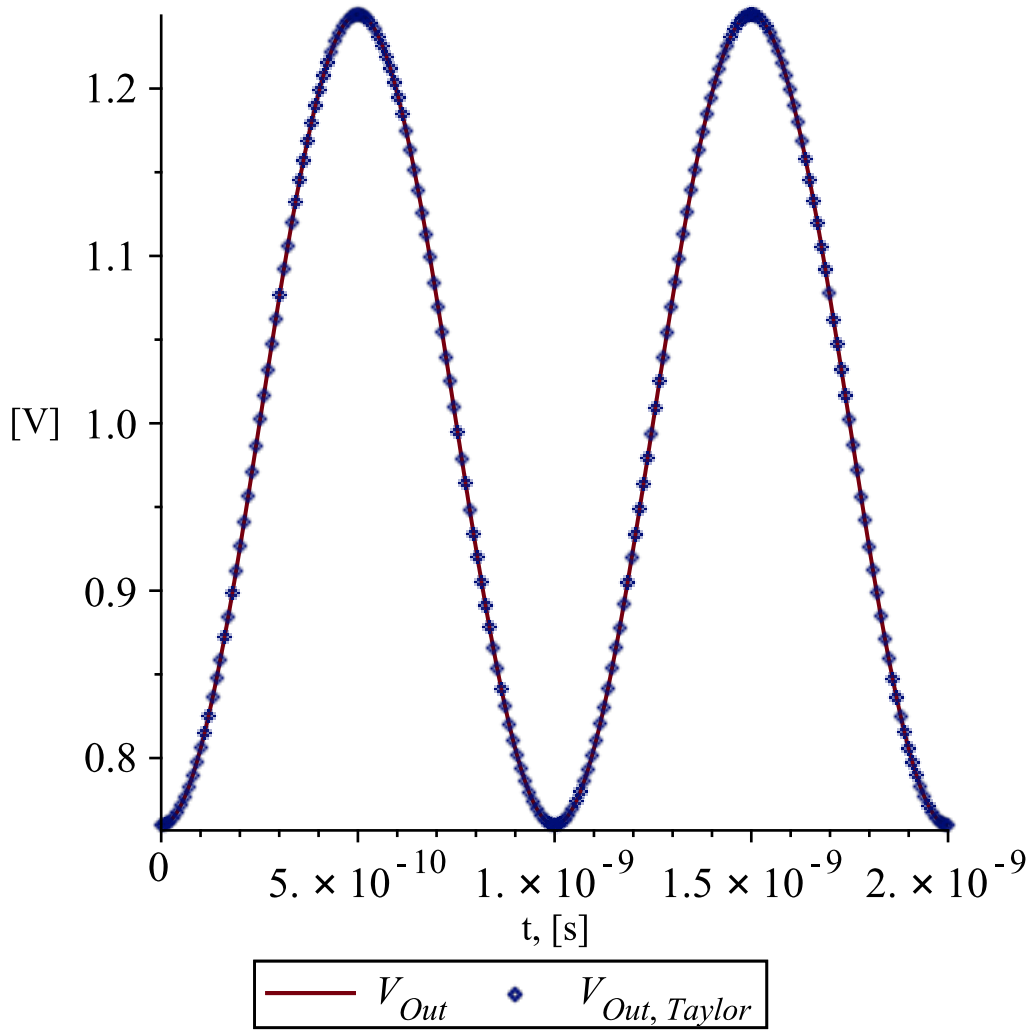

```
> P5 := plot(evalf((eval(simplify(-20·log10(-HD2)), [omega=2·Pi·1e9, Cfixed=1e-15,
      Kc=2e-15, Ihat=10e-6, Vdc=1]))), Vb=0.2..1.2, title=[typeset("HD2 vs. ",
      V[b])], labels=[typeset(V[b], [V]), "HD2,[dB]"], size=[400, 400])
```



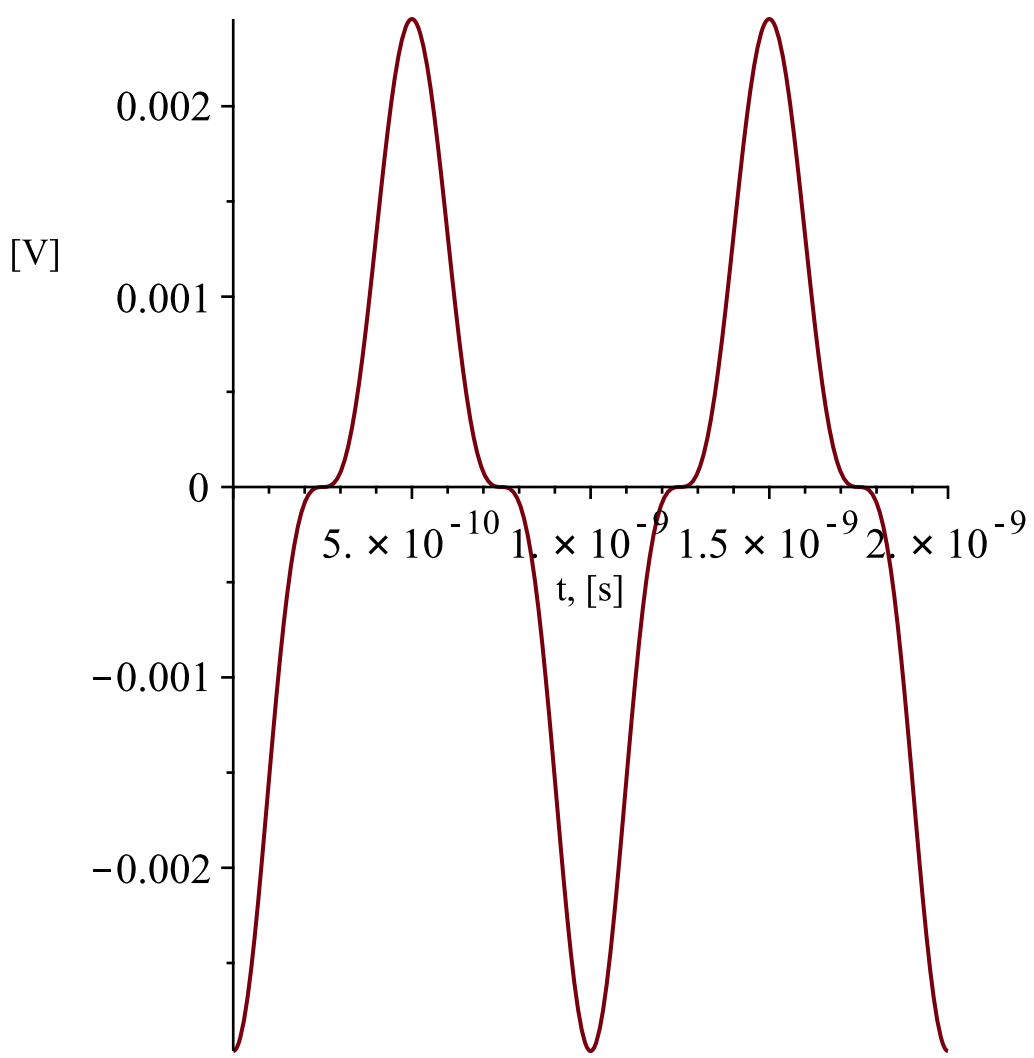

```
> P6 := plot(evalf((eval(simplify(-20·log10(-HD2)), [omega = 2·Pi·1e9, Cfixed = 1e-15,
      Kc = 2e-15, Ihat = 10e-6, Vb = 0.62]))), Vdc = 0.5..2, title
      = "HD2 vs. "typeset("HD2 vs. ", V[0])", labels = [typeset(V[0], [V]), "HD2,[dB]"],
      size = [400, 400])
```



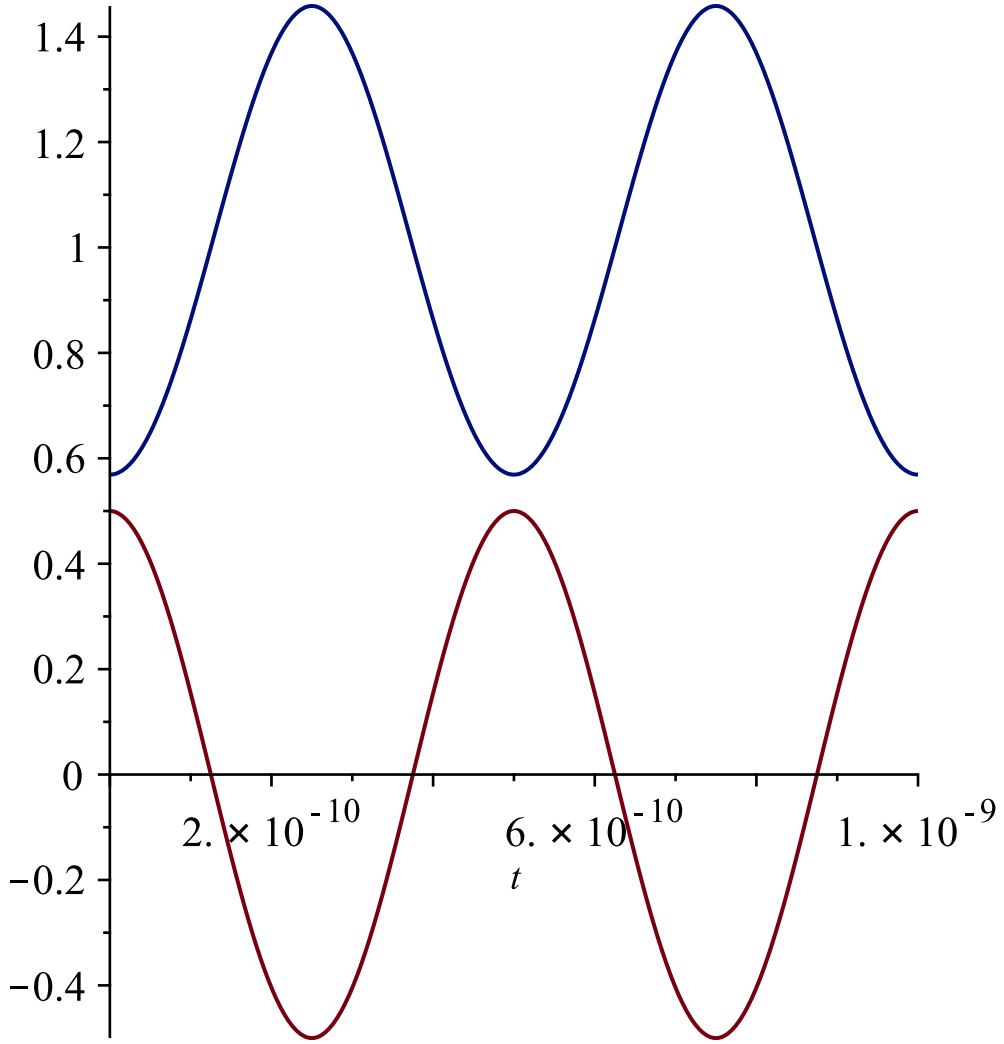

```
> plot(eval([rhs(solution), ac_resp], [omega = 2·Pi·1e9, Cfixed = 10e−15, Ihat = 20e−6, Kc
    = 4e−15, Vb = 0.62, Vdc = 1]), t = 0 ...2e−8, style = [line, point], labels = ["t, [s]",
    "[V]"], legend = [typeset(V[Out]), typeset(V[Out, Taylor])], size = [400, 400])
```



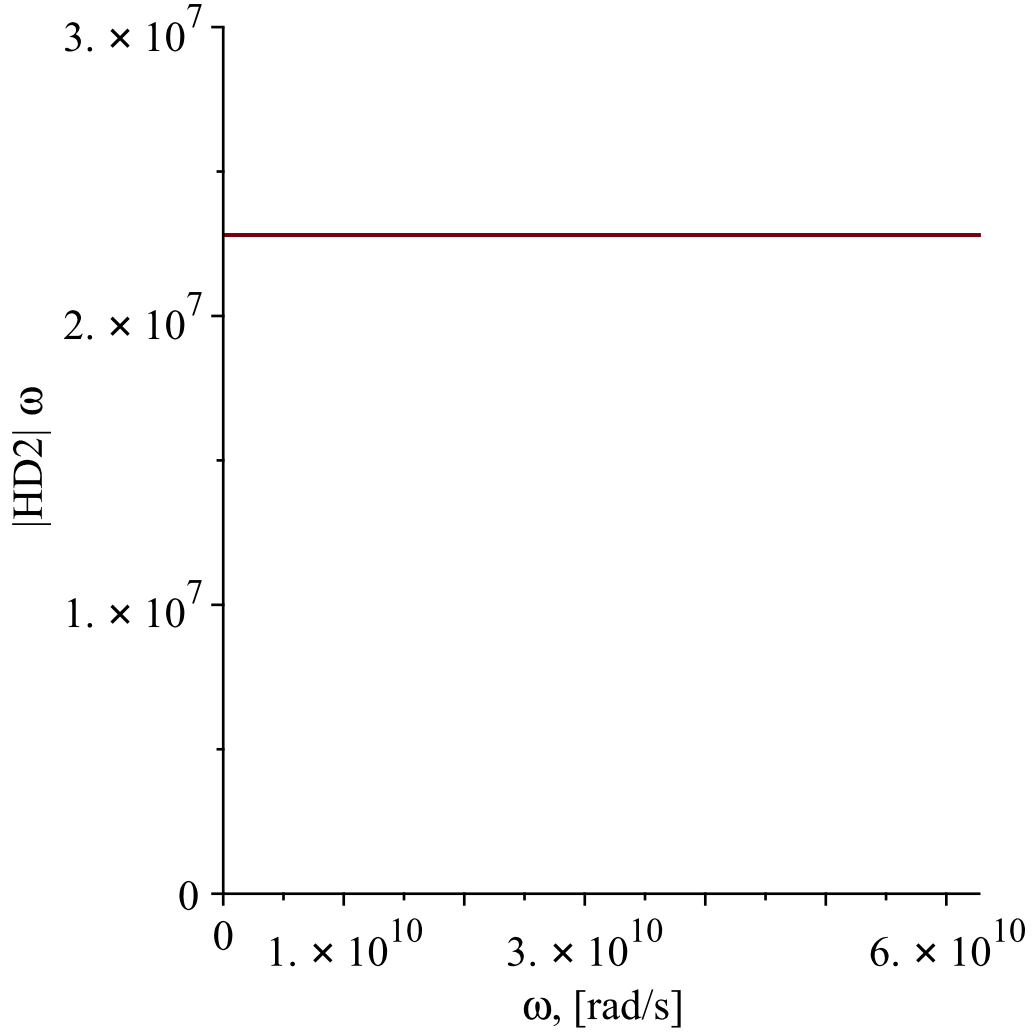

```
> plot(eval([ac_resp-rhs(solution)], [omega=2·Pi·1e9, Cfixed=1e−15, Ihat=10e−6, Kc
  =2e−15, Vb=0.62, Vdc=1]), t=0...2e−8, labels=["t, [s]", "[V]"], size=[400, 400])
```




```
> plot( eval( [ 0.5 cos(omega·t), rhs(solution) ], [omega = 2·Pi·2e9, Cfixed = 1e−15, Ihat
      = 10e−6, Kc = 1e−15, Vb = 0.62, Vdc = 1 ]), t = 0 ...1e−8 )
```

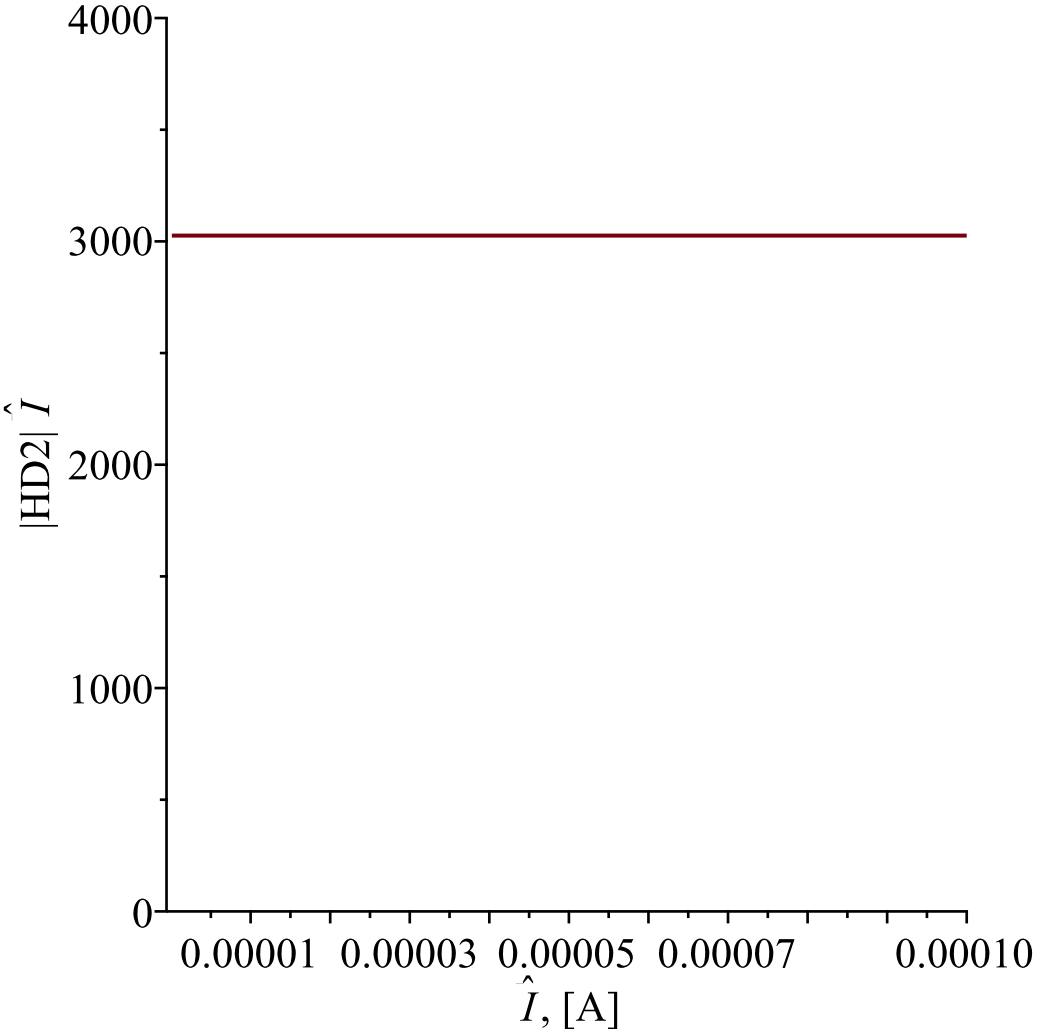



```
> R1 :=
    plot( subs( [Cfixed = 1e−15, Ihat = 1.2e−6, Kc = 1e−15, Vb = 0.62, Vdc = 1 ], |HD2| )
          / ω^-1 ,
    omega = 2·Pi·1e2 ..2·Pi·1e10, y = 0 ..3e7, size = [400, 400 ], labels = [ typeset( ω,
    ", [rad/s]" ), typeset( "|HD2| ", ω^1 ) ], labeldirections = [ "horizontal", "vertical" ] )
```



> $R2 :=$

$$plot\left(\frac{1}{Ihat^{1}} (subs([\text{omega} = 2 \cdot \text{Pi} \cdot 1e9,\ Cfixed = 1e{-}15,\ Kc = 1e{-}15,\ Vb = 0.62,\ Vdc = 1],\ |HD2|)),\ Ihat = 0.1e\text{-}6\ ..100e{-}6,\ y = 0\ ..4e3,\ size = [400, 400],\ labels = \left[typeset(\hat{I}, \text{", [A]"}), typeset(\text{"|HD2| "}, \hat{I}^{1}) \right],\ labeldirections = [\text{"horizontal", "vertical"}] \right)$$



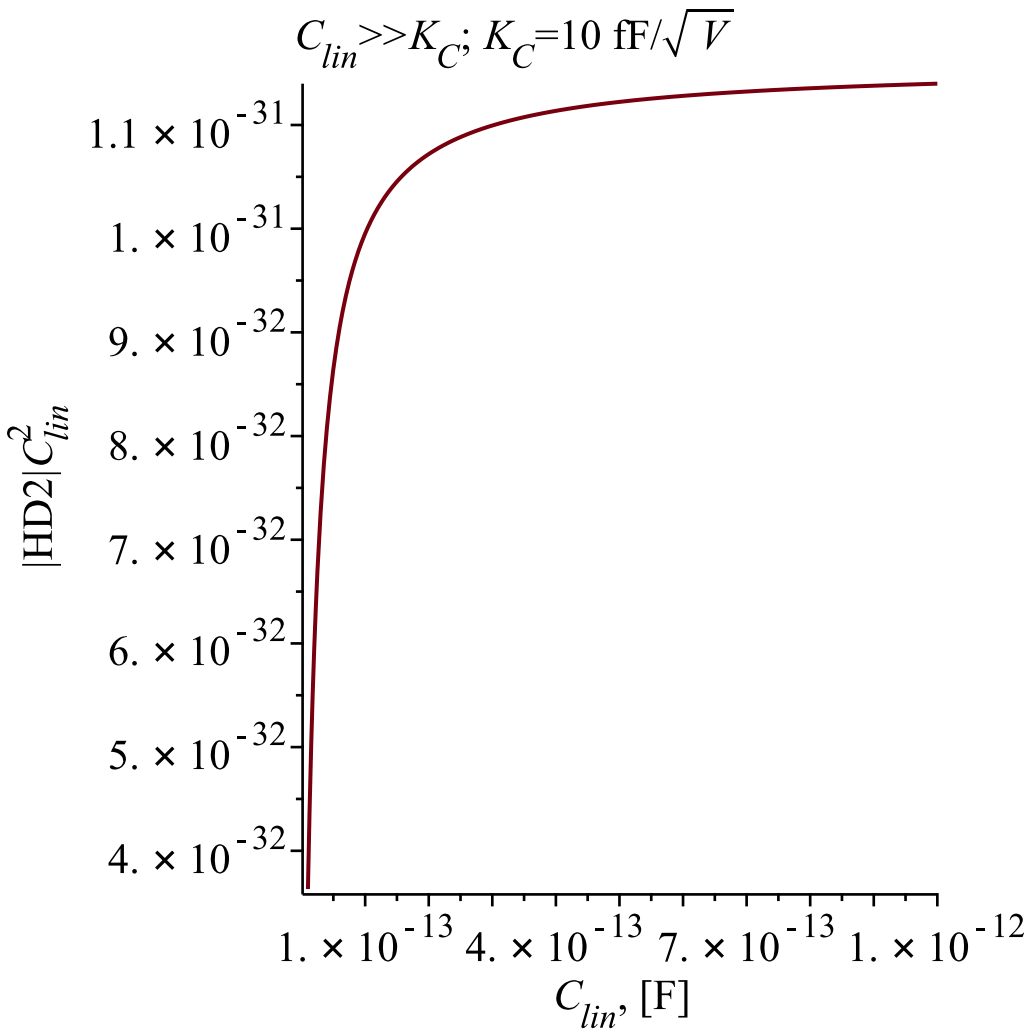
> *R3_Kc_small* := *plot*

$$\left(\frac{subs([\text{omega} = 2\cdot \text{Pi}\cdot 1e9,\ Ihat = 1.2e{-}6,\ Kc = 10e{-}15,\ Vb = 0.62,\ Vdc = 1], |HD2|)}{Cfixed^{-2}}, \right.$$

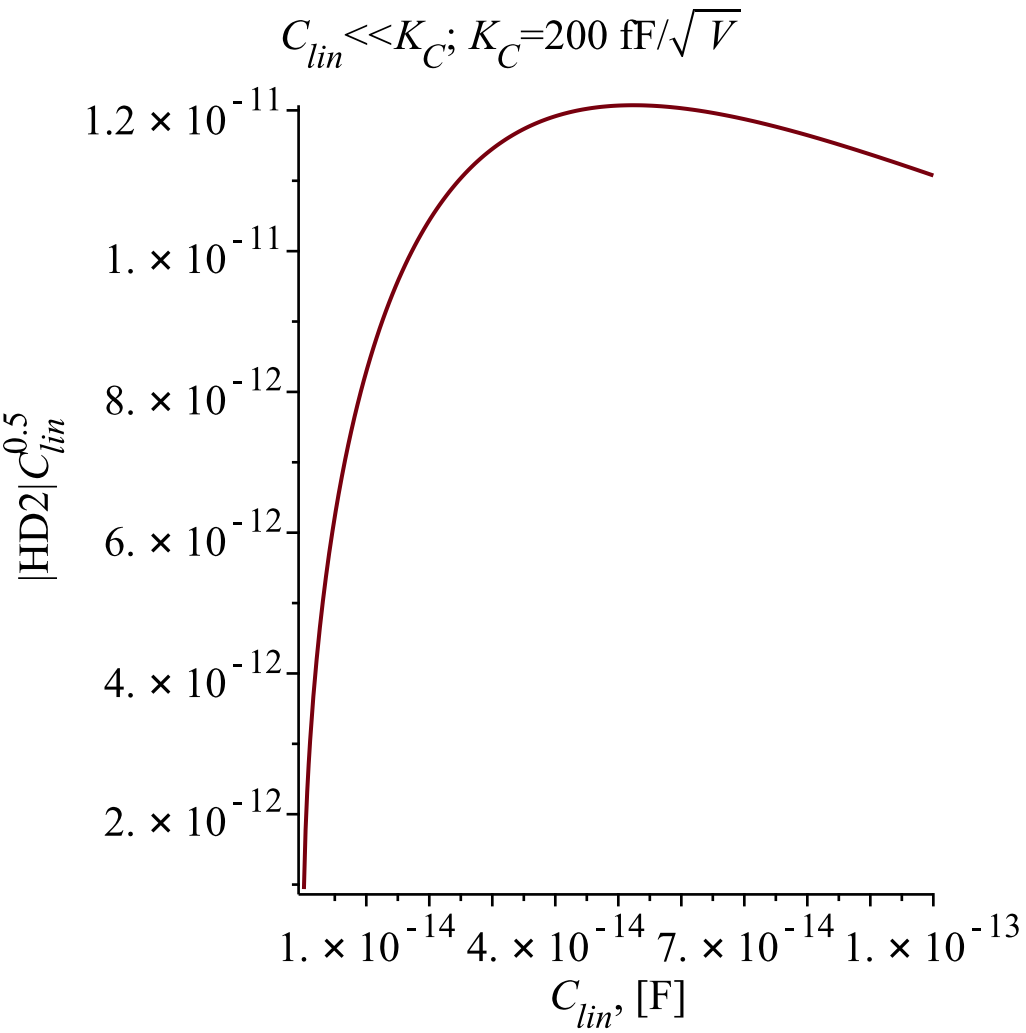
$$Cfixed = 10e{-}15 \,..\, 1000e{-}15,\ size = [400, 400],\ labels = [typeset(C[lin], \text{", [F]" }),$$

$$typeset(\text{"|HD2|"}, C[lin]^2)],\ labeldirections = [\text{"horizontal", "vertical"}],\ title$$

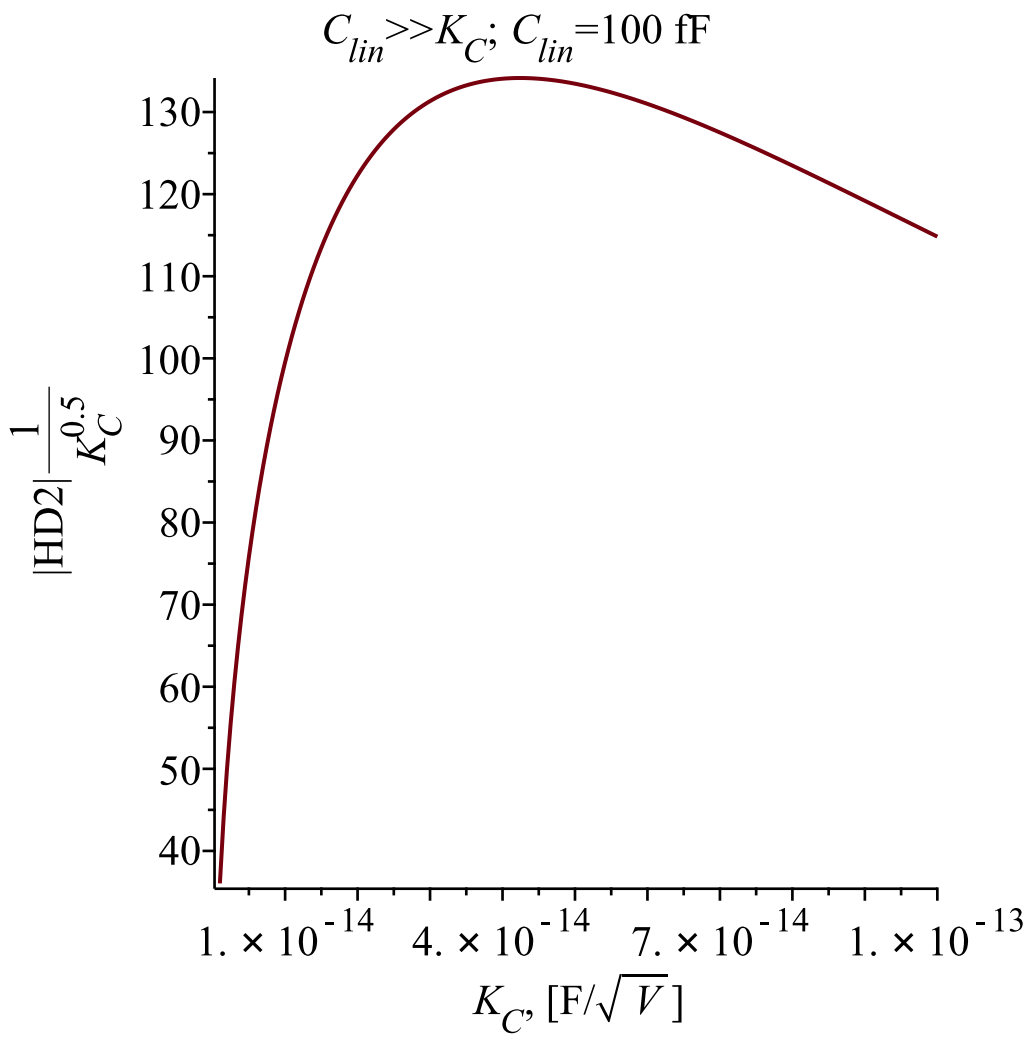
$$\left. = [typeset(C[lin], \text{"OO" }, K[C], \text{"; "}, K[C], \text{"=10 fF/"}, \text{sqrt}(V))] \right)$$



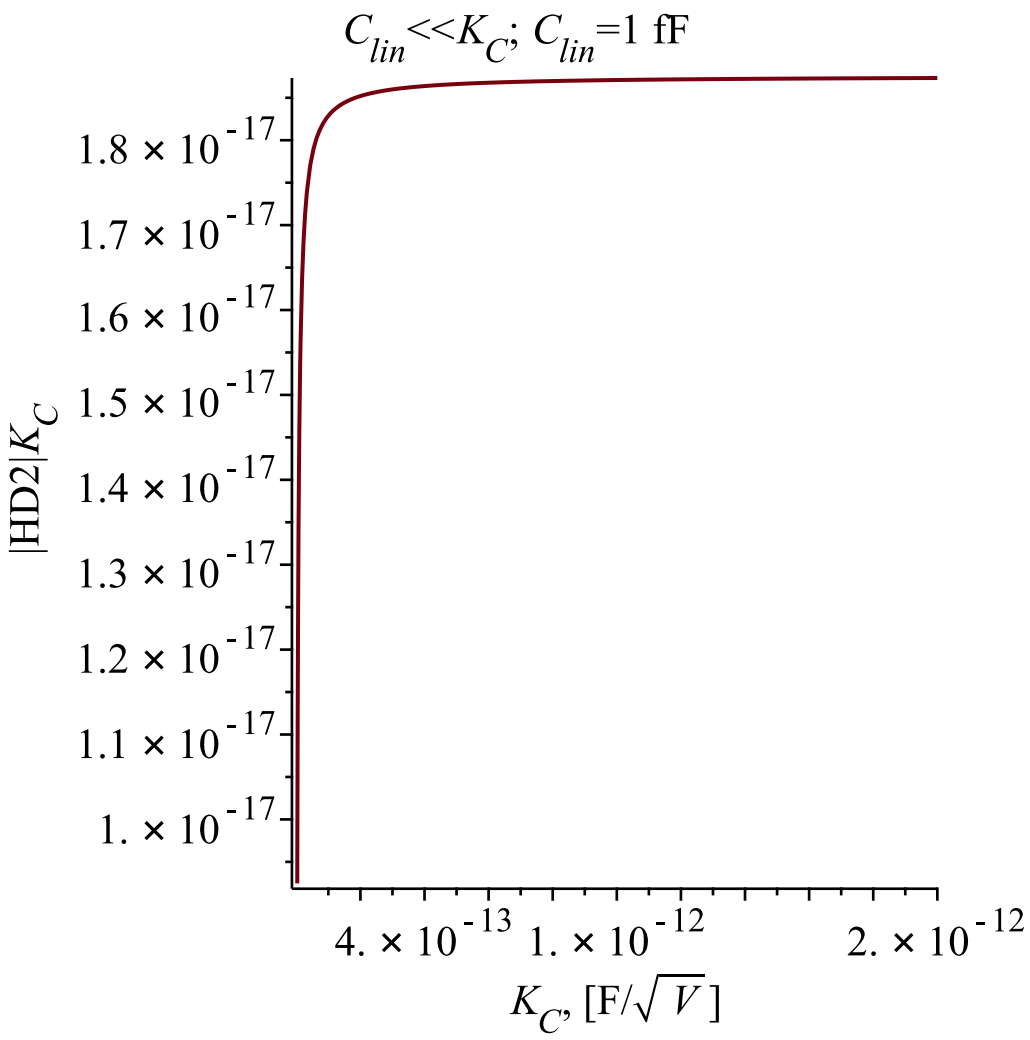

```
> R3_Kc_big :=
    plot( 1/Cfixed^-0.5 (subs( [omega = 2·Pi·1e9, Ihat = 1.2e−6, Kc = 200e−15, Vb
    = 0.62, Vdc = 1 ], |HD2| ) ), Cfixed = 0.1e−15 ..100e−15, size = [400, 400 ], labels
    = [ typeset( C[ lin ], ", [F]" ), typeset( "|HD2|", C[ lin ]^0.5 ) ], labeldirections = [ "horizontal",
    "vertical" ], title = [ typeset( C[ lin ], "!!", K[ C ], "; ", K[ C ], "=200 fF/", sqrt( V ) ) ] )
```




```
> R4_Kc_small := plot
    ( 1/Kc^0.5 (subs( [omega = 2·Pi·1e9, Ihat = 1.2e−6, Cfixed = 100e−15, Vb = 0.62,
    Vdc = 1 ], |HD2| ) ), Kc = 1e−15 ..100e−15, size = [400, 400 ], labels = [ typeset(K[C],
    ", [F/", sqrt(V), "]" ), typeset( "|HD2|", K[C]^-0.5 ) ], labeldirections = ["horizontal",
    "vertical"], title = [typeset( C[lin], "OO" , K[C], "; ", C[lin], "=100 fF" ) ])
```

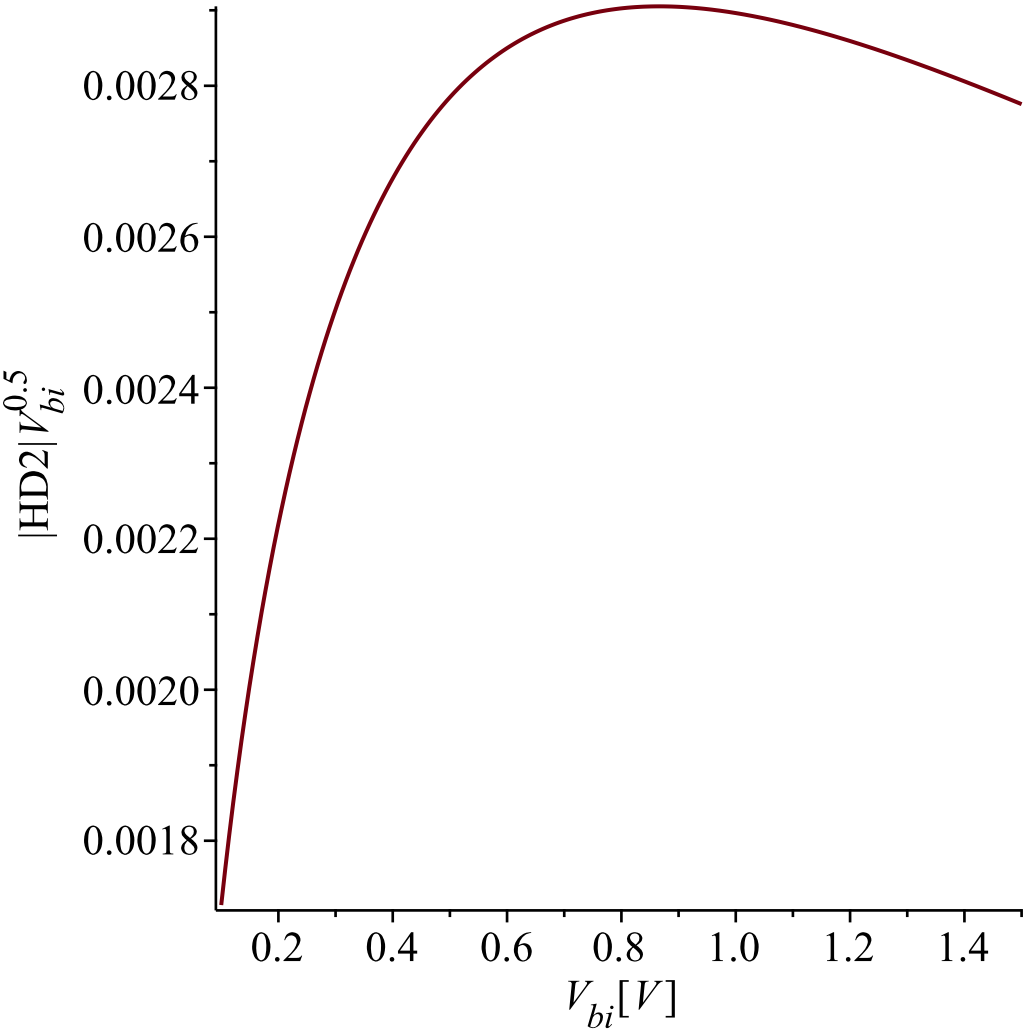



```
> R4_Kc_big :=
  plot( 1/(Kc^-1) (subs( [omega = 2·Pi·1e9, Ihat = 1.2e−6, Cfixed = 1e−15, Vb = 0.62,
  Vdc = 1 ], |HD2| ) ), Kc = 3e−15 ..2000e−15, size = [400, 400 ], labels = [ typeset( K[ C ],
  ", [F/", sqrt( V ), "]" ), typeset( "|HD2|", K[ C ]^1 ) ], labeldirections = [ "horizontal",
  "vertical" ], title = [ typeset( C[ lin ], "!!", K[ C ], "; ", C[ lin ], "=1 fF" ) ] )
```

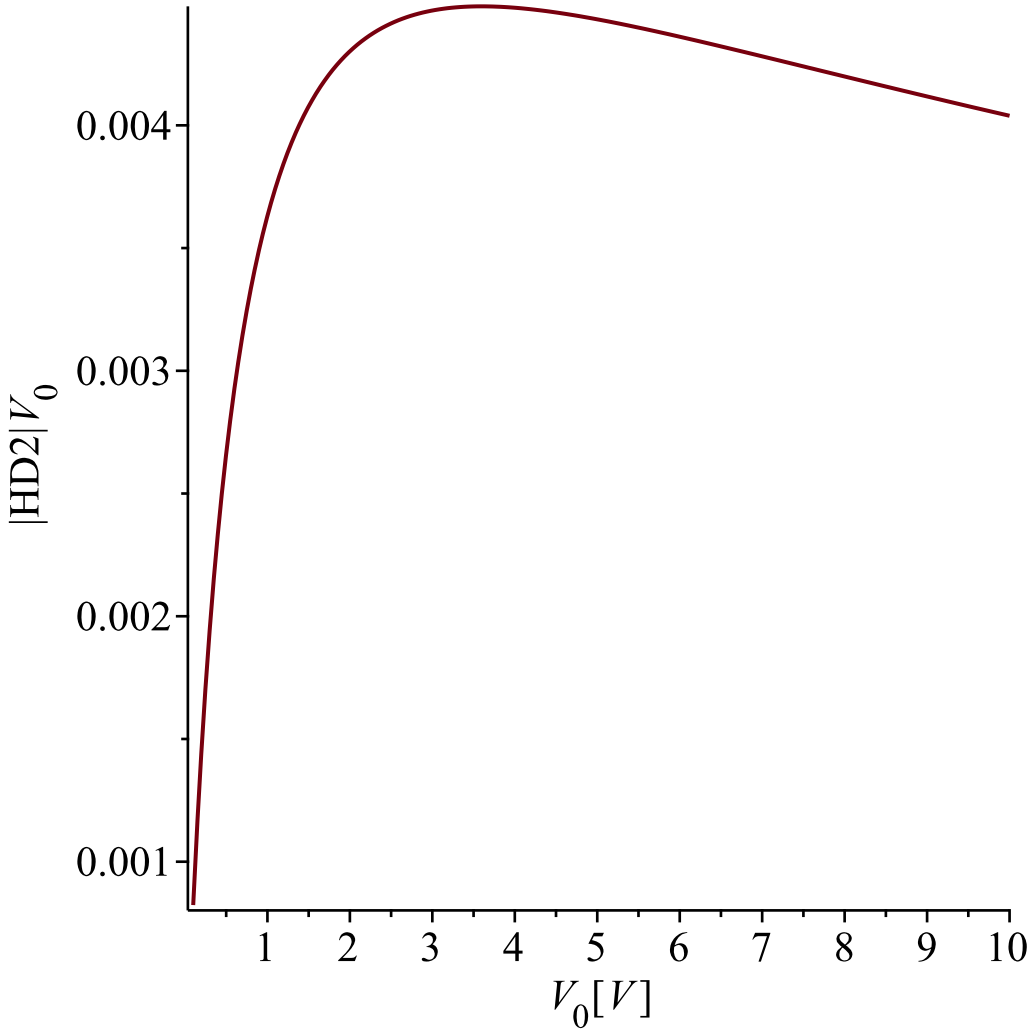


> *R5* :=

plot($\frac{1}{Vb^{-0.5}}$ (*subs*([omega = 2·Pi·1e9, *Ihat* = 1.2e−6, *Cfixed* = 1e−15, *Kc* = 1e−15, *Vdc* = 1], |*HD2*|)), *Vb* = 0.1 ..1.5, *size* = [400, 400], *labels* = [*typeset*(*V*[*bi*], "[", *V*, "]"), *typeset*("|HD2|", *V*[*bi*]^0.5)], *labeldirections* = ["horizontal", "vertical"])

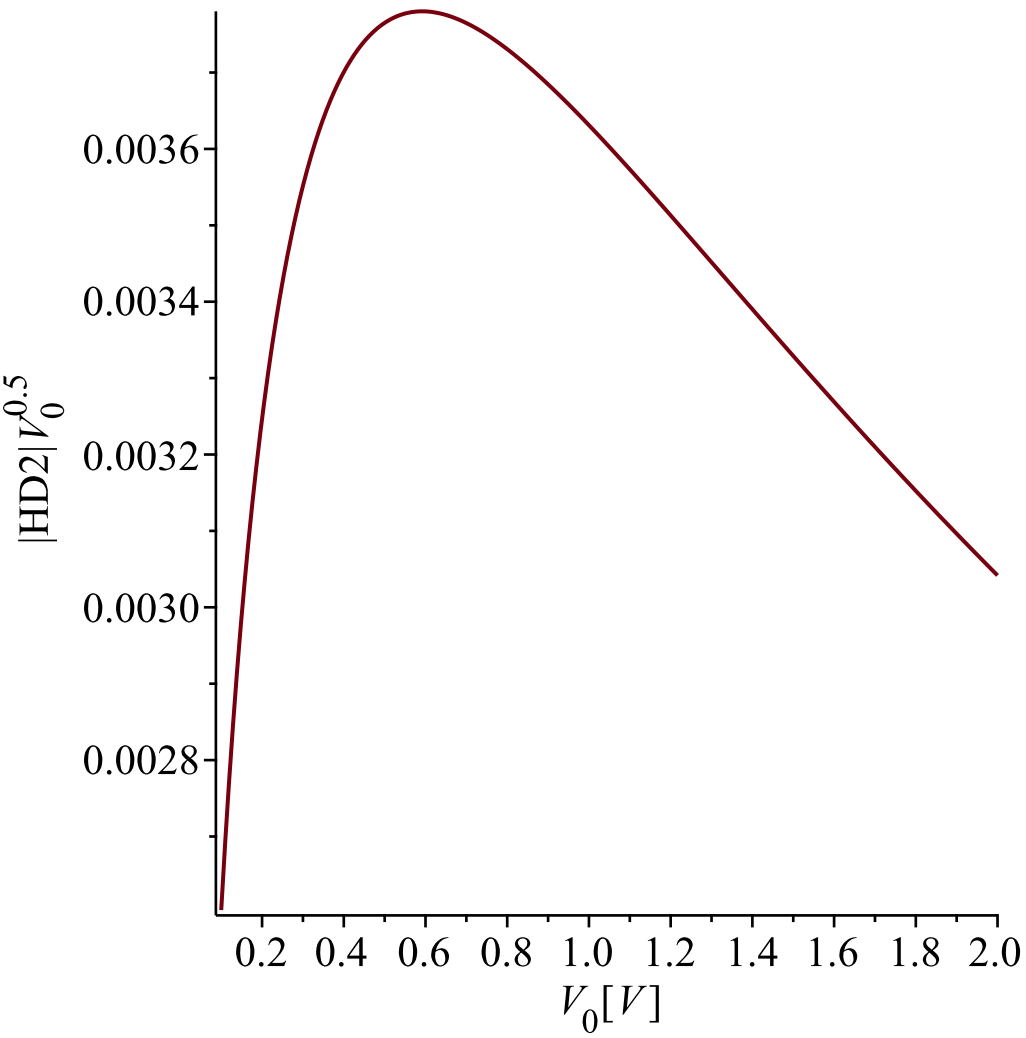



```
> R6 := plot
     ( 1/Vdc^-1 (subs( [omega = 2·Pi·1e9, Ihat = 1.2e−6, Cfixed = 1e−15, Kc = 1e−15, Vb
     = 0.62 ], |HD2| )), Vdc = 0.1 ..10, size = [400, 400 ], labels = [typeset( V[0], "[", V, "]" ),
     typeset( "|HD2|", V[0]^1 ) ], labeldirections = [ "horizontal", "vertical"] )
```

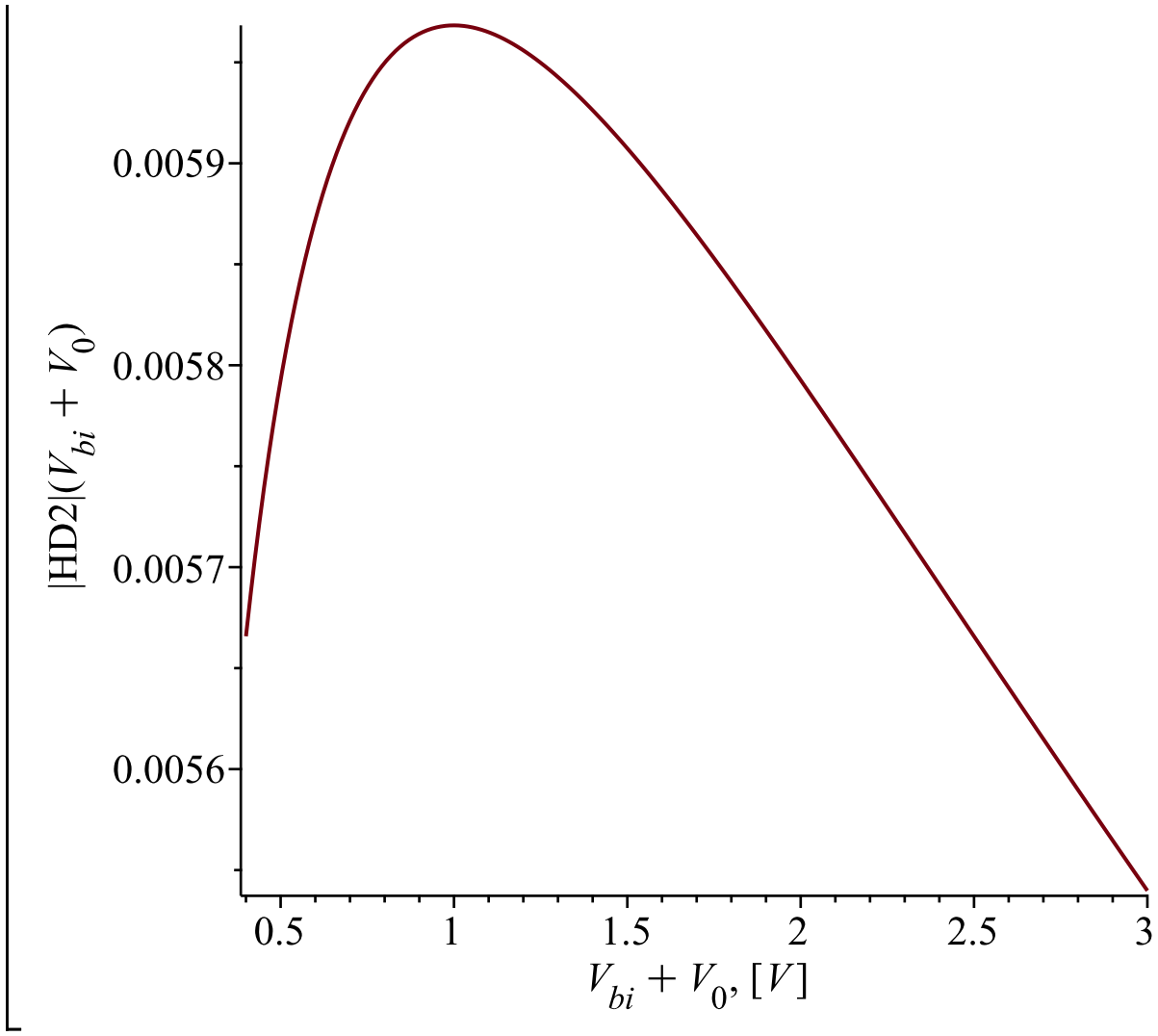



```
> plot
```

$$\left(\frac{1}{Vdc^{-0.5}} \left(subs\left([\text{omega} = 2 \cdot \text{Pi} \cdot 1e9,\ Ihat = 1.2e{-}6,\ Cfixed = 1e{-}15,\ Kc = 1e{-}15,\ Vb = 0.62],\ |HD2| \right) \right),\ Vdc = 0.1\ ..2,\ size = [400, 400],\ labels = \left[typeset\left(V[0], "[", V, "]" \right), typeset\left("|HD2|", V[0]^{0.5} \right) \right],\ labeldirections = [\text{"horizontal"}, \text{"vertical"}] \right)$$




```
> R6 := plot( 1/V0^-1 (subs( [omega = 2·Pi·1e9, Ihat = 1.2e−6, Cfixed = 1e−15, Kc = 1e−15],
  algsubs( Vb + Vdc = V0, |HD2| ) ) ), V0 = 0.4 ..3, size = [800, 400], labels = [typeset( V[bi]
  + V[0], '', [', V, ']' ), typeset( '|HD2|(', V[bi] + V[0], ')' ) ], labeldirections
  = ["horizontal", "vertical"] )
```


0.0059
0.0058
0.0057
0.0056
|HD2|(V_bi + V_0)
0.5
1
1.5
2
2.5
3
V_bi + V_0, [V]