Project

(Estimation risk on Regulatory Capital)

Financial Engineering AY 2018-2019

Consider the Vasicek model on a homogeneous portfolio to quantify portfolio credit risk and consider an additional estimation risk, i.e. the risk arising from errors in model parameters.

It is required to quantify the estimation risk following a Bayesian approach (as discussed in [1]), i.e. to evaluate the impact of the uncertainty of each parameter of the model on the capital requirement.

The dataset, with Moody's public data, includes for the years 1983-2017 the following quantities (computed as a mean on all Moody's rated corporate issuers):

a. the one-year Default Rate (DR) for only *speculative grade* issuers and for *all rated* issuers
b. the one-year Recovery Rate (RR).

Initially consider the *speculative grade* issuers Default Rates. Always consider

- two confidence levels: α=99% and α=99.9% (CL)
- two homogeneous portfolios: an asymptotic one (LHP) and a small one composed of N=50 obligors (HP).

It is required to reply to the following points.

A. Compute the capital requirements in the nominal model for the two portfolios LHP and HP, for the two confidence levels CL considered. As regards the correlation parameter, consider the following two cases

    i. the mean correlation between obligors as in [1], p.6, Table 1

    ii. the correlation as a function of DR as chosen by the Basel Committee (see e.g. [1] eq.2 or [4] p.13).

B. Consider the frequentist inference (see e.g. [1], Section 3.1, and [2]).

    1. Check if the data provided satisfy the assumptions on parameters distributions, as discussed in [1], Section 3.1 and 4.2.

    2. Compute the regulatory capital and the AddOn (percentage increment w.r.t. the nominal value) in the alternative model for the two confidence levels CL, in the LHP and HP cases, introducing uncertainty on one parameter at a time and on all parameters together (suppose all parameters to be independent). In particular, choose the correlation distributed as a Beta (see [1], p.6, eq. 8).

3. Repeat the same analysis as in point 2. considering now the correlation as a deterministic function of DR [1], [4].

C. Consider the Bayesian inference (see e.g. [1], Section 3.2, and [3]).

1. Find:

   i. the posterior distribution for the default threshold d, considering the empirical variance for the distribution of the observed data given the unknown parameters

   ii. the empirical posterior distribution for the correlation ρ (see [1], eq. 9), considering the empirical variance for the distribution of the observed data given the unknown parameters and the mean value for ρ as in [1], p.6, Table 1.

   Hint: To compute numerically the integral at denominator of the posterior distribution ([1], eq.9), consider ρ defined in a range [0+ε,1-ε], with e.g. ε=0.005.

2. Compute the regulatory capital and the AddOn in the alternative model for the two confidence levels CL, in the LHP and HP cases, introducing uncertainty on one parameter at a time and on both parameters together (in this case the parameters are not independent [3]).

   Hint: To simulate values from an empirical distribution, consider the standard approach via Cumulative Distribution Function.

   Hint: Since the parameters are not independent, perform nested simulations, i.e., for each simulated correlation value, generate a vector of random default threshold values.

3. Find:

   i. the posterior distribution for the default threshold d, considering the Cramer-Rao lower bound for the distribution of the observed data given the unknown parameters, in the HP case.

      Hint: compute the Cramer-Rao lower bound numerically. In particular, consider: i) as likelihood the probability that the number of defaulted obligors is equal to m, i.e.

      $$P(Ndef = m) = \int_{-\infty}^{+\infty} \binom{N}{m} p(y)^m \big(1 - p(y)\big)^{N-m} \phi(y)dy$$

      where $p(y)$ is the probability of default given the market variable y and $\phi(y)$ is the probability density function of a standard Gaussian random variable; ii) the partial derivative with respect to the considered parameter (i.e. d); iii) the expected value on the variable m.

      Hint: pay attention to the number of simulations that are used when debugging the code since it can have high computational cost.

   ii. the empirical posterior distribution for the correlation ρ (see [1], eq. 9), considering the Cramer-Rao lower bound for the distribution of the observed data given the unknown parameters and the mean value for ρ as in [1], p.6, Table 1. Consider only the HP case.

4. With the distributions found in point 3., repeat the same analysis as in point 2 for the HP case.

D. [Facultative] Repeat what has been done in previous points for the *all rated* issuers Default Rate and compare the results with the ones obtained for the *speculative grade* issuers Default Rate.

E. Discuss the results.

Realize a library in Matlab. Optional Python.

[1] Baviera, R., Bianchi, G., and Canevisio, A. (2019). Parameters uncertainty impact on capital requirements. Internal Report. Politecnico di Milano.

[2] Löffler, G. (2003). The effects of estimation error on measures of portfolio credit risk. Journal of Banking & Finance, 27(8), 1427-1453.

[3] Tarashev, N. (2010). Measuring portfolio credit risk correctly: Why parameter uncertainty matters. Journal of Banking & Finance, 34(9), 2065-2076.

[4] BIS, (2005). An explanatory note on the Basel II IRB risk weight functions. Bank for International Settlements.

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