NUMERICAL ANALYSIS FOR PARTIAL DIFFERENTIAL EQUATIONS
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LAB 1 - INTRODUCTION TO MATLAB®

VECTORS AND ARRAYS

1. Define the following vectors

$$v_1 = [1, 2, 4, \dots, 1024],$$
$$v_2 = [\cos(\pi), \cos(\pi/2), \dots, \cos(\pi/10)]^T,$$
$$v_3 = [0.1, 0.05, 0.025, \dots, 0.003125],$$

2. Find a short Matlab® expression to build the matrix

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 7 & 5 & 3 & 1 & -1 & -3 \\ 4 & 8 & 16 & 32 & 64 & 128 & 256 \end{pmatrix}$$

3. Give a Matlab® expression that uses only a single matrix multiplication with B to obtain
   - the sum of columns 5 and 7 of B
   - the last row of B
   - a version of B with rows 2 and 3 swapped

PLOTS

1. Define the following functions
   - $f(x) = x\sin(x) + \left(\frac{1}{2}\right)^{\sqrt{x}}$
   - $f(x) = x^4 + \log(x^3 + 1)$

   using the Matlab® commands `@` (handle function) and `inline`. Evaluate both functions on the vector `x=[0 1 2 3]`.

2. Plot the function
$$f(x) = 2 + (x - 3)\sin(5(x - 3))$$
   for $0 \leq x \leq 6$ together with the lines $y = -x + 5$ and $y = x - 1$.

3. Plot in y-logarthmic scale the functions $y = e^x$ e $y = e^{2x}$ on the interval $0 \leq x \leq 10$.

Scripts and Loops

1. Write a Matlab® script `mat_hilbert.m` to compute the Hilbert whose elements $a_{i\,j}$ are defined by:
$$a_{i\,j} = \frac{1}{i+j-1} \quad i,j = 1,...,5.$$
Compare the result with the one obtained uisng the Matlab® command `hilb(5)`.

2. Use a `while` loop to determine the number of years needed to accumulate one million euros in a bank account supposing to start with 10k euros, to deposit 10k euros at the end of each year and if the bank recognizes an annual interest 2% per year on bank accounts.

3. Determine the first integer number n such that $\sum_{k=1}^{n} k \geq 88$, where $\sum_{k=1}^{n} k = 1+2+3+ .. + n$.

Functions and outputs

1. Write a Matlab® function that takes in input the edges a, b, c and check whether a generic triangle is rectangle or not.

2. Write a Matlab® fucntion that takes in input an integer n and computes the matrix $T \in \mathbb{R}^{x \times n}$ defined as
$$T = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots \\ 0 & 1 & 0 & 1 & \dots \\ 1 & 0 & 1 & 0 & \dots \\ 0 & 1 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

3. Show that the sequence
$$\begin{cases} a_0 = 1 \\ a_n = \dfrac{(a_{n-1})^2 + 2}{2a_{n-1}} \quad n = 1, 2, \dots \end{cases}$$
converges to $\sqrt{2}$. Plot the values of the sequence as a function of n.