NUMERICAL ANALYSIS FOR PARTIAL DIFFERENTIAL EQUATIONS
A.A. 2019/2020

LECTURER: Prof. A. Quarteroni TEACHING ASSISTANT AND TUTOR: Dr. F. Regazzoni

LAB 4
DISCONTINUOUS GALERKIN-FINITE ELEMENT METHODS FOR ELLIPTIC PROBLEMS

# EXERCISE 1

In the unit square $\Omega \equiv (0,1) \times (0,1)$ we consider the following problem:

$$\begin{cases} -\Delta u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $f \in L^2(\Omega)$ is a given function.

1. Introduce the SIPG formulation for problem (1).
2. Compute $f$ supposing that the analytical solution of (1) is given by $u_{ex} = (x-x^2)e^{3x}\sin(2\pi y)$.
3. Using the provided Matlab code compute the errors in the $\|\cdot\|_{L^2(\Omega)}$, $\|\cdot\|_{H^1(\Omega)}$ and $\|\cdot\|_{DG}$ norm on a sequence of triangular grids. Consider linear finite elements ($k = 1$) and quadratic finite elements ($k = 2$). Choose the penalty constant $\gamma = \alpha k^2/h$ with $\alpha = 10$. Plot the condition number of the stiffness matrix as a function of the mesh size $h$. Give a comment on the results, moving from the theoretical knowledge.
4. Repeat the previous point choosing $k = 2$ and $\alpha = 1$ for the SIPG and the NIPG methods.