Topological Vector Spaces

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Contents

1 Seminorms, function spaces, and convergence

Definition 1.1. Let V be $\mathbb{K}$-vector space, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$. By a *seminorm* on V one then understands a map $p : V \to \mathbb{R}_{\geq 0}$ with the following properties:

(N1) The map p is absolute homogeneous that means

$$p(rv) = |r|\, p(v) \quad \text{for all } v \in V \text{ and } r \in \mathbb{K}.$$

(N2) The map p is subadditive or in other words satisfies the triangle inequality which means that

$$p(v + w) \leq p(v) + p(w) \quad \text{for all } v, w \in V.$$

A seminorm is called a *norm*, if in addition the following axiom is satisfied:

(N3) For all $v \in V$ the relation $p(v) = 0$ holds true if and only if $v = 0$.

A $\mathbb{K}$-vector space together with a norm $\| \ \| : V \to \mathbb{R}_{\geq 0}$, $v \mapsto \|v\|$ is called a *normed vector space*.

2 Other chapters

1. Preliminaries
2. Topological Vector Spaces
3. Hilbert Spaces

Miscellany

2. GNU Free Documentation License
3. Auto Generated Index