Convex Optimization: Homework 2

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Exercise 1.

Calculate the minimum over $\mathbb{R}^2_{++}$ of the function $f(x,y) = \frac{1}{xy} + x + y$.

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Minimizing a quadratic-over-linear fractional function. Consider the proble

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Let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be a function such that $|f(x)| \leq \|x\|^2$ for every $x \in \mathbb{R}^n$. Show that f is Fréchet differentiable in 0.

Solution: By hypothesis, $f(\vec{0}) = 0$

$$\begin{aligned}
\lim_{\|x\|\to 0} \frac{f(x) - f(\vec{0})}{\|x\|} &\leq \lim_{\|x\|\to 0} \frac{\|x\|^2}{\|x\|} \\
&= 0.
\end{aligned}$$

Thus, the linear function $\ell(x) = 0$ is the Fréchet derivative of f at 0.

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