Stochastic Processes: Homework 0

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Exercise 1

Consider a sequence of i.i.d. random variables $(X_i)_{i\in\mathbb{N}}$ with $\mathbf{E}\, X_i = 0$ and $\mathbf{Var}\, X_i = 1$ for every $i \in \mathbb{N}$.

1. Show with th Law of Large Numbers that,

$$\lim_{n\to\infty} \|X_1, \ldots, X_n\|_2 - \sqrt{n} \to 0$$

   (a) in $\mathbb{P}$,

   (b) a.e.,

   (c) in distribution,

   (d) Show that if $X_i \in L^p$ for some $p > 1$, then it converges in L^q for every $q \in [1 \leq p)$.

2.