# Proper Orthogonal Decomposition Exercises

## Morning Exercises

Go to `https://github.com/martinschiodt/Summerschool_exercises` and download the code needed for the exercises. Open the `Driver.m` script in `MATLAB` and augment the script and functions with your code.

### Exercise 1

Plot the first and last time instance of `U` and `V` in a contour plot. *Note: The data has virtually no visible temporal evolution, however, this is besides the point for the current exercise.*

### Exercise 2

Restructure `U` and `V` to produce two separate POD ensemble matrices (one for each coordinate direction).

### Exercise 3

Construct the POD kernel within the `POD.m` function and run the routine for both `U` and `V`.

### Exercise 4

Plot the POD eigenvalues using `loglog` - what do you see?

### Exercise 5

Plot the first 8 POD modes - what do you see? How do the modes correspond to the eigenvalues?

### Exercise 6

Project any realization onto the discovered POD basis. This requires an update of the code within the `Projection.m` function. Consider the projection error as a function of `n_modes`. *Note: any realization, $x_i$, may be expanded in the POD basis, $\{\varphi_n\}_{n=1}^N$, using*

$$x_i = \sum_{n=1}^{N} c_{in}\varphi_n. \tag{1}$$

*It is this expansion/projection you need to implement. Since our basis is orthonormal, the projection coefficients are found through*

$$c_{in} = x_i \cdot \varphi_n. \tag{2}$$

## Afternoon Exercises

### Exercise 1

Restructure `U` and `V` to produce two separate POD ensembles, appropriate for finding temporal features within the dataset.

### Exercise 2

Update your `POD.m` function to apply the method of snapshots (MOS) if `n_ensemble < n_data`. Run the POD routine to produce POD eigenvalues and modes. *Note: You may use the code snippet below as a guide*:

```
if n_ensemble < n_data
    % Build MOS kernel and decompose it
    K = ??
    [A,S,~] = svd(K,'econ');
    S = diag(S);
    % Compute POD modes
    Phi = POD_modes_from_A(U,Um,A,S,n_ensemble);
else
    % Build classic kernel and decompose it
    K = ??
    [Phi, S, ~] = svd(K,'econ');
    S = diag(S);
end
```

### Exercise 3

Plot the POD spectra.

### Exercise 4

Plot the first 6 POD modes.