Brake Heat Energy Dissipation Calculations

One of the main requirements for the GDM project was to increase bike reliability. Specifically, the brakes which have been underperforming. Apart from the safety issue this raises, good brake performance is essential for a good lap time as the rider needs to 'feel' in complete control of the bike to optimise his/her performance. Biggest challenge at the moment is that there are no brakes in the market that suit our specific needs, (a 270 kilogram bike going over 280 km/h) hence coming up with at least a crude model to estimate the braking power needed is essential.

Disc Brake Heat Energy Dissipation Equation

The approach chosen after extensive research to model the Disc Brake Heat Energy Dissipation is based on a paper by (Talati and Jalafar 2009) *Analysis of Heat Conduction in a Brake System.* The equation given on the paper, (Equation 8 specifically) used for this calculation is:

$$\mathrm{d}\dot{E}_d = \sigma \mathrm{d}P = \sigma \mu p \omega \phi_0 r^2 \mathrm{d}r \qquad (1)$$

Where E_d is the energy dissipated by the disc (J), σ is the heat partition coefficient (proportion of heat absorbed by the disc, dimensionless between 0-1), μ is the friction coefficient between the pad and the disc (dimensionless), *p* is the pressure (Pascals) at effective radius *r*. ω is the angular velocity of the disc (radians/second) φ_0 is the pad contact angle (radians), r is the effective radius (metres).

However, considering this equation is a differential, due to the logarithmic nature of braking, a simplified version will be utilised in which heat loss is assumed happens at a constant rate. Additionally, in *Analysis of Heat Conduction in a Brake* Hence the actual equation used to model the heat energy dissipation is:

$$E_d = \sigma \mu p \omega \phi_o r^2 \qquad (2)$$

All calculations will be done from values for corner 1.

Corner Velocity Loss

These velocity values are obtained from raw data and processed by the C++ program.

$$v_{loss} = v_{initial} - v_{final} \qquad (3)$$

$$v_{loss} = 75.2415 - 55.8779 = 19.3637\ m/s$$

Where *v* stands for bike velocity in metres per second.

Kinetic Energy Loss per corner

$$Ke = \left[\left(\frac{1}{2} \cdot m \cdot v_{initial}^2\right) - \left(\frac{1}{2} \cdot m \cdot v_{final}^2\right)\right] \qquad (4)$$

Ke is kinetic energy loss (Joules), *m* is the mass of the bike (and the rider) which is known to be 380 kilos. Velocity *v* is obtained by raw data in program.

$$Ke = \left[\left(\frac{1}{2}\cdot 380\cdot 75.2415^2\right) - \left(\frac{1}{2}\cdot 380\cdot 55.8779^2\right)\right] = 482401\,Joules$$

(De)Acceleration Rate per corner

$$Acceleration\,rate = \frac{distance}{velocity\,loss} \qquad (5)$$

Since distance and velocity have the sampling rate due to the datalogger, acceleration rate in each corner can be found by a simple calculation. Corner length provided by program.

$$Acceleration\,rate = \frac{257.872}{-19.3637} = -13.3173\,m/s^2$$

Angular Speed at the Front Tyre

The Revolutions per Minute (RPM) of the motor at each corner is provided by the program. From that value and knowing that the **gear ratio (N)** between the motor and the front wheel is N = 83/17 (measured by counting teeth on workshop), the angular speed can be calculated.

$$Revolutions\,per\,second = \frac{RPM}{60} \quad (6)$$

Plugging in values:

$$Revolutions\,per\,second = \frac{110204.3}{60} = 1836.74$$

From the RPS, the angular speed can be calculated:

$$\omega = (rev\,per\,second) * 2\pi \quad (7)$$

Plugging in values:

$$\omega_{motor} = (1836.74) * 2\pi = 11540.58\,radians/s$$

However, angular speed at the front tyre is needed, applying gear ratio (N):

$$\omega_{front} = \frac{\omega_{motor}}{N} \qquad (8)$$

Plugging values:

$$\omega_{front} = \frac{11540.58}{\left(\frac{83}{17}\right)} = \ 2363\,.73\,radians/s$$

Hence the initial angular speed for each corner can be calculated.

Force driving the Front Wheel

The force driving the front wheel can be easily calculated using the gear ratio, the motor torque (T) at that instant (obtained from program) and the radius of the front wheel. Calculating the torque at the wheel:

$$T_{wheel} = \frac{T_{motor}}{N} \quad (9)$$

Plugging in values:

$$T_{wheel} = \frac{1115.453}{(\frac{83}{17})} = 228.5\ Nm$$

Knowing the radius of the front wheel (measured at the workshop) it is possible to calculate the force F driving the front wheel.

$$T = F \cdot r_{wheel} \quad (10)$$

$$F = \frac{T}{r_{wheel}}$$

Plugging values:

$$F = \frac{228.5}{0.6} = 380.83\ N$$

This calculated force is the sliding force needed to be stopped, which can be modelled as a reactionary negative force to the force applied by the brake callipers by Newton's Second Law.

Pressure applied on brake disc

The brake piston diameter was found to be 20mm. There is a total of 6 of them, hence the total area (A) is:

$$A = 6(\pi \cdot r_{piston}^2) \quad (11)$$

Plugging in values:

$$A = 6\left(\pi \cdot (\frac{10}{1000})^2\right) = 0.1885 m^2$$

Considering that:

$$P = \frac{F}{A} \qquad (12)$$

For Corner 1 the pressure needed is:

$$P = \frac{380.83}{0.1885} = 2020.32\ Pa$$

Pad Contact Angle per Corner

This is an essential parameter for the calculation. The first step is knowing how many revolutions is done by the tyre in each corner, to know that an estimate for the time spent in the corner must be found:

$$Time\ (avg) = \frac{s_{corner}}{v_{average}} \quad (13)$$

Where S is corner length and v is velocity. Getting values from program:

$$Time\ (avg) = \frac{257.872}{65.56} = \quad 3.93\ seconds$$

Knowing the average time, and the already calculated angular speed the pad contact angle in radians can be calculated:

$$\phi_0(radians) = \ \omega \cdot t \quad (14)$$

Placing values:

$$\phi_0(radians) = \ 2363.73 \cdot 3.93 = \quad 9289.5\ radians$$

Hence the pad contact angle is calculated.

Brake Heat Energy Dissipation Calculation

The effective radius *r*, is the distance between the centre of the brake disc to the centre of the braking area in the disc. It was calculated by repeated measurement in the workshop, and then averaged for best results. The measured value was 0.106412 metres. The friction coefficient is known to be 0.41 and the heat partition coefficient for a brake disc was estimated to be 0.9. Hence all the parameters needed are calculated, and the brake heat energy dissipation can be calculated:

$$E_d = \ \sigma \mu p \omega \phi_o r^2$$

Plugging in values for corner 1:

$$E_d = \ 0.9 \cdot 0.41 \cdot 2020.32 \cdot 2363.73 \cdot 9289.5 \cdot 0.106412^2 = 299340J$$

With the heat energy dissipation calculated, knowing the material properties of the brake disc, fatigue, fade and other failure conditions can be tested, ensuring the brakes are suitable for racing.