

## Formulário

## Antenas e Guias de Onda

### Revisões

$$Z_{IN} = \frac{Z_0^2}{Z_L}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$VSWR = \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$$

### Parâmetros S e T

$$\begin{bmatrix} v_1^{ref} \\ v_2^{ref} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} v_1^{inc} \\ v_2^{inc} \end{bmatrix}$$

$$\begin{bmatrix} v_1^{inc} \\ v_1^{ref} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} v_2^{ref} \\ v_2^{inc} \end{bmatrix}$$

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11}S_{22}}{S_{21}} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{T_{21}}{T_{11}} & T_{22} - \frac{T_{21}T_{12}}{T_{11}} \\ \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}} \end{bmatrix}$$

### Amplificadores

$$\rho_{IN} = S_{11} + \frac{S_{12}S_{21}\rho_L}{1-S_{22}\rho_L}$$

$$\rho_{OUT} = S_{22} + \frac{S_{12}S_{21}\rho_S}{1-S_{11}\rho_S}$$

$$G_T = \frac{P_L}{P_{AVS}} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{IN}\Gamma_S|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_A = \frac{P_{AVL}}{P_{AVS}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1}{|1-\Gamma_{OUT}|^2}$$

$$G_P = \frac{P_L}{P_{IN}} = \frac{1}{|1-\Gamma_{IN}|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_{TU,max} = \frac{1}{|1-S_{11}|^2} |S_{21}|^2 \frac{1}{|1-S_{22}|^2}$$

$$G_T = \frac{P_L}{P_{AVS}} = \frac{P_{AVL}}{P_{AVS}} \frac{P_L}{P_{AVL}} = G_A L_L$$

$$G_T = \frac{P_L}{P_{AVS}} = \frac{P_{IN}}{P_{AVS}} \frac{P_L}{P_{IN}} = L_S G_P$$

### Estabilidade

$$|\Gamma_S| < 1 \wedge |\Gamma_L| < 1 \wedge |\Gamma_{IN}| < 1 \wedge |\Gamma_{OUT}| < 1$$

## Filters

$$L = \frac{g_k Z_0}{2\pi f_c}$$

$$C = \frac{g_k}{2\pi Z_0 f_c}$$

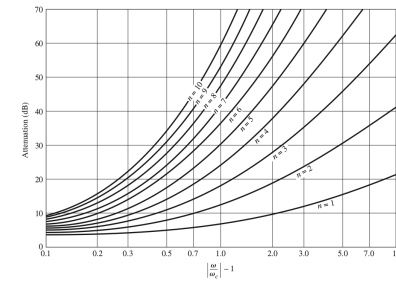
### Transformada de Richards

$$\omega_c = \tan(\beta l)$$

$$Z = jg_k \omega_c \Rightarrow Z = jZ_0 \tan(\beta l) \quad \wedge Z_0 = g_k$$

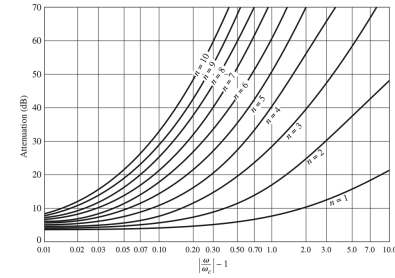
$$Y = jg_k \omega_c \Rightarrow Y = \frac{j \tan(\beta l)}{Z_0} \quad \wedge Z_0 = \frac{1}{g_k}$$

### Maximum Flat (3dB)



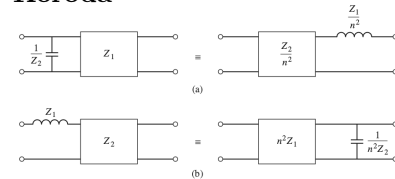
N	g <sub>1</sub>	g <sub>2</sub>	g <sub>3</sub>	g <sub>4</sub>	g <sub>5</sub>	g <sub>6</sub>	g <sub>7</sub>	g <sub>8</sub>	g <sub>9</sub>	g <sub>10</sub>	g <sub>11</sub>
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.4142	0.5176	1.0000					
7	0.4450	1.2470	1.8019	1.2470	0.4450	1.0000					
8	0.3902	1.1111	1.6629	1.1111	0.3902	1.0000					
9	0.3473	1.0000	1.5321	1.0000	0.3473	1.0000					
10	0.3129	0.9080	1.4142	0.9080	0.3129	1.0000					

### Equal Ripple (3dB)



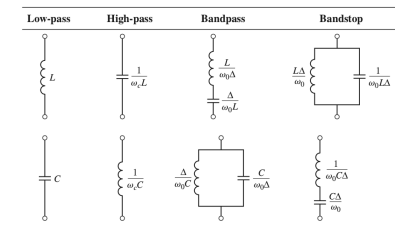
N	g <sub>1</sub>	g <sub>2</sub>	g <sub>3</sub>	g <sub>4</sub>	g <sub>5</sub>	g <sub>6</sub>	g <sub>7</sub>	g <sub>8</sub>	g <sub>9</sub>	g <sub>10</sub>	g <sub>11</sub>
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8136	4.7260	0.8051	4.5142	0.6091	5.8095

### Koroda



$$n^2 = 1 + \frac{Z_2}{Z_1}$$

### Frequency Transformations



$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

### Stepped Impedance

$$\text{Bobine} \rightarrow \beta l = \frac{L Z_0}{Z_{high}}$$

$$\text{Condensador} \rightarrow \beta l = \frac{C Z_{low}}{Z_0}$$

### Guias de Onda

$$\begin{array}{lll} E_z = 0 & H_z = 0 & \text{Modos TEM} \\ E_z = 0 & H_z \neq 0 & \text{Modos TE ou H} \\ E_z \neq 0 & H_z = 0 & \text{Modos TM ou E} \\ E_z \neq 0 & H_z \neq 0 & \text{Modos Híbridos} \end{array}$$

$$\begin{array}{l|l} \epsilon_0 & 8.854187817 \times 10^{-12} F \cdot m^{-1} \\ \mu_0 & 4\pi \times 10^{-7} H \cdot m^{-1} \end{array}$$

$$k_c^2 = k^2 - \beta^2$$

$$\eta = 120\pi \rightarrow \text{Impedância Característica do ar}$$

### Retangulares

O modo TM não se propaga se  $n = 0$  ou  $m = 0$

$\lambda_0 \rightarrow$  Comprimento de onda em meio livre

$\lambda_{0c} \rightarrow$  Comprimento de onda de corte no meio

$$k = \omega \sqrt{\mu\epsilon} = \frac{2\pi}{\lambda}$$

$$f_{cmn} = \frac{v}{2} \sqrt{\left(\frac{n}{y_1}\right)^2 + \left(\frac{m}{z_1}\right)^2}$$

$$\lambda_{cmn} = \frac{2}{\sqrt{\left(\frac{n}{y_1}\right)^2 + \left(\frac{m}{z_1}\right)^2}}$$

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{n}{y_1}\right)^2 - \left(\frac{m}{z_1}\right)^2}$$

$$k_c^2 = \left(\frac{n}{y_1}\right)^2 + \left(\frac{m}{z_1}\right)^2$$

$$\gamma = \alpha_d + j\beta = \sqrt{k_c^2 - k^2}$$

$$\gamma = \sqrt{\left(\frac{n}{y_1}\right)^2 + \left(\frac{m}{z_1}\right)^2 - \omega^2 \mu\epsilon} = \sqrt{k_c^2 - \beta_0^2}$$

$$v_p = \frac{v}{\beta} = \frac{v_0}{\sqrt{1 - \left(\frac{n\lambda_0}{2y_1}\right)^2 - \left(\frac{m\lambda_0}{2z_1}\right)^2}} = \frac{v_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_{0c}}\right)^2}}$$

$$Z_{TE} = \frac{k\eta}{\beta} \text{ e se } f > f_c, Z_{TE} = \frac{Z_d}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_{0c}}\right)^2}}$$

$$v = \frac{c}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

### Cilíndrico

$n \rightarrow$  ordem da função de Bessel

$k \rightarrow$  raiz da ordem  $n$  da função de Bessel

$k' \rightarrow$  raiz da ordem  $n$  da derivada da função de Bessel

$a \rightarrow$  raio do guia de onda

$p_{nm} \rightarrow$  raiz de ordem  $m$  da função de Bessel de ordem  $n$

$p'_{nm} \rightarrow$  raiz de ordem  $m$  da derivada da função de Bessel de ordem  $n$

Os modos TE e TM obrigam a  $m \neq 0$

Modo  $TE_{nm}$

$$k_{c_{nm}} = \frac{p'_{nm}}{a}$$
$$\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p'_{nm}}{r}\right)^2}$$
$$f_{c_{nm}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p'_{nm}}{2\pi a\sqrt{\mu\epsilon}}$$

$$f_c = \frac{v}{2\pi} \frac{k'_{nm}}{a}$$
$$\lambda_c = \frac{k'_{nm}}{2\pi a}$$
$$Z_{TE} = \frac{\eta^k}{\beta}$$

k	$J_0'(x)$	$J_1'(x)$	$J_2'(x)$	$J_3'(x)$	$J_4'(x)$	$J_5'(x)$
1	3.8317	1.8412	3.0542	4.2012	5.3175	6.4156
2	7.0156	5.3314	6.7061	8.0152	9.2824	10.5199
3	10.1735	8.5363	9.9695	11.3459	12.6819	13.9872
4	13.3237	11.7060	13.1704	14.5858	15.9641	17.3128
5	16.4706	14.8636	16.3475	17.7887	19.1960	20.5755

Modo TM

$$k_{cm} = \frac{p_{nm}}{a}$$
$$\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p'_{nm}}{r}\right)^2}$$
$$f_{c_{nm}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p_{nm}}{2\pi a\sqrt{\mu\epsilon}}$$

$$f_c = \frac{v}{2\pi} \frac{k_{nm}}{a}$$
$$\lambda_c = \frac{2\pi a}{k_{nm}}$$
$$Z_{TM} = \frac{\eta\beta}{k}$$

k	$J_0(x)$	$J_1(x)$	$J_2(x)$	$J_3(x)$	$J_4(x)$	$J_5(x)$
1	2.4048	3.8317	5.1356	6.3802	7.5883	8.7715
2	5.5201	7.0156	8.4172	9.7610	11.0647	12.3386
3	8.6537	10.1735	11.6198	13.0152	14.3725	15.7002
4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801
5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178

Perdas

Dielétrico  $\rightarrow \alpha_d = \omega\epsilon \tan(\delta)$

$$\alpha_{d_{mn}} = \frac{\omega \tan(\delta)}{2c\sqrt{1 - \left(\frac{fc_{mn}}{f}\right)^2}} = \frac{\omega\sqrt{\mu\epsilon} \tan(\delta)}{2\beta} =$$
$$\frac{k^2 \tan(\delta)}{2\beta}$$

Não há propagação ( $f < f_c$ ):

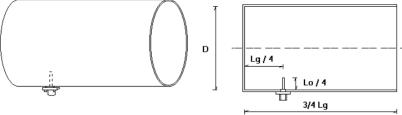
$$\alpha = \frac{2\pi}{\lambda} \sqrt{\left(\frac{\lambda}{\lambda_c}\right)^2 - 1}$$

Há propagação ( $f > f_c$ ):

$$\alpha_d = \frac{\pi \tan(\delta)}{\lambda\sqrt{1 - \frac{\lambda}{\lambda_c}}}$$
$$\alpha_c = \frac{Re[Z_c]}{Z_d y_1} \frac{1 + 2\frac{y_1}{x_1} \left(\frac{\lambda}{\lambda_c}\right)^2}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$
$$P_{max} = \frac{E^2 y_1 z_1}{4Z_{TE}}$$
$$R_S = \frac{1}{\sigma\delta} = \sqrt{\frac{\omega\mu}{2\sigma}} \rightarrow \text{Efeito de Skin}$$

Propagação ( $f > f_c$ )

$$\gamma = \pm \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 - \omega^2 \mu\epsilon}$$
$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \frac{p'_{nm}}{a}$$
$$\lambda_{0c} = \frac{2\pi a}{p'_{nm}}$$
$$\beta = \sqrt{\omega^2 \mu\epsilon - \left(\frac{p'_{nm}}{a}\right)^2}$$
$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - (\lambda_0/\lambda_{0c})^2}}$$
$$v_p = \frac{\omega}{\beta} = \frac{v_0}{\sqrt{1 - (\lambda_0/\lambda_{0c})^2}}, \quad v_0 = 1/\sqrt{\mu\epsilon}$$



Fibras

$$\sin(\theta_2) = \left(\frac{\mu_1}{\mu_2}\right) \sin(\theta_2)$$
$$\theta_1 = \theta_c \Rightarrow \theta_2 = 90^\circ$$
$$\sin(\theta_c) = \frac{n_2}{n_1}$$

$$\sin[\theta_{i_{max}}] = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

Fractional Difference  $\rightarrow \Delta = \frac{n_1 - n_2}{n_1}$

Guide light effective  $\Rightarrow \Delta << 1$

$$NA = n_1 \sqrt{2\Delta}$$

$\theta$  close to  $\theta_{critical} \Rightarrow$  Higher order modes

$\theta$  larger than  $\theta_{critical} \Rightarrow$  Lower order modes

$$v = \frac{2\pi d}{\lambda} \sqrt{\mu_1^2 - \mu_2^2} = \frac{2\pi d}{\lambda} NA$$
$$M = \frac{V^2}{2}$$

Velocidade de Fase  $\rightarrow v_f = \frac{\omega}{k} = \frac{c}{n}$

Velocidade de Grupo  $\rightarrow v_g = \frac{d\omega}{dk} = \frac{c}{m}$

$$m = n - \lambda_0 \frac{dn}{d\lambda_0}$$

Antenas

Diretividade  $\rightarrow D(\theta, \Phi) = \frac{U(\theta, \Phi)}{U_0}$

Na direção de máximo  $\rightarrow D(\theta, \Phi) = 4\pi \frac{U_{max}}{P_{rad}}$

$$G(\theta, \Phi) = \frac{U(\theta, \Phi)}{U_0} = 4\pi \frac{U_{max}}{P_{in}}$$

Densidade de uma antena isotrópica sem perdas:

$$U_0 = \frac{P_{rad}}{4\pi} = \frac{P_{in}}{4\pi}$$
$$U(\theta, \Phi) = R^2 S_{rad} = R^2 \frac{1}{2} \frac{|E_\theta|^2}{\eta}$$
$$G(\theta, \Phi) = e_t D(\theta, \Phi)$$
$$\rho = \frac{P_{rad}}{P_{IN}}$$

**Potência**

$$P_{in} = P_{rad} + P_D$$
$$P_D = P_{ref} + P_c + P_d$$

$P_{ref} \rightarrow$  Potência refletida por desaptação

$P_c \rightarrow$  Potência dissipada nas paredes condutoras da antena

$P_d \rightarrow$  Potência dissipada no dielétrico

Eficiência

eficiência  $\rightarrow e_t = \frac{P_{rad}}{P_{in}} = e_r e_c e_d$

Eficiência de Radiação  $\rightarrow e_{cd} = e_c e_d$

$$S_{rad} = \frac{1}{2} (E \times H^*)$$
$$e_r = \frac{P_{tran}}{P_{in}} = (1 - |\rho|^2)$$

**Impedância**

$$Z_A = R_A + jX_A \Rightarrow R_A = R_r + R_p$$

Efeito de Skin  $\rightarrow \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

Effective Area

$$A_e = \frac{P_r}{S_i} =$$
$$S_{isotropica} = \frac{P_{rad1}}{4\pi r^2}$$
$$S_1 = D_1 \frac{P_{rad1}}{4\pi r^2}$$

$S_i \rightarrow$  Densidade de Potência

$$A_e = \frac{\lambda^2}{4\pi} G$$

Transmissão

$$P_{rad} = e_{t1} P_{in}$$
$$G_1 = e_{t1} D_1$$
$$P_T = A_{e2} S_1 = A_{e2} G_1 \frac{P_{in}}{4\pi r^2}$$
$$|E_\theta| = \frac{\eta I \Delta Z}{2\pi R \lambda} \sin(\theta)$$

Fórmula de Friis

$$P_T = A_{e2} \frac{P_{in}}{4\pi r^2} G_1 = P_{in} \left(\frac{\lambda}{4\pi r}\right)^2 G_t G_r$$
$$\left(\frac{\lambda}{4\pi r}\right)^2 \rightarrow \text{Perdas em espaço livre}$$

Radar

$$P_{ref} = S_i \sigma$$
$$P = A_e S_A = P_t \sigma G^2 \frac{1}{4\pi} \left(\frac{\lambda}{4\pi r^2}\right)^2$$

Antenas Filiformes

$$H_\Phi = j \frac{\beta I \Delta z}{4\pi r} e^{-j\beta r} \sin(\theta)$$
$$E_\theta = j\eta \frac{\beta I \Delta z}{4\pi r} e^{-j\beta r} \sin(\theta)$$

Dipolo

$$P = P_{rad} + jQ = \eta \frac{\pi}{3} \left(\frac{I \Delta z}{\lambda}\right)^2 \left[1 - j \frac{1}{(\beta r)^3}\right]$$
$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

Em campo distante ( $f \gg \lambda$ ), Q é desprezável face a  $P_{rad}$

Resistência de Radiação  $\rightarrow R_r = \eta \frac{2\pi}{3} \left(\frac{\Delta z}{\lambda}\right)^2$

$$U(\theta, \Phi) = r^2 S = r^2 \frac{1}{2} \left(\frac{|E|^2}{\eta}\right)$$
$$U(\theta) = r^2 S_{rad} = \frac{\eta}{2} \left(\frac{\beta I \Delta z}{4\pi}\right)^2 \sin^2(\theta)$$
$$D(\theta, \Phi) = 4\pi \frac{U(\theta, \Phi)}{P_{rad}}$$
$$A_e = \frac{\lambda^2}{4\pi} D$$

Num dipolo curto,  $U_{max}$ ,  $P_{rad}$  e  $R_r$  passa para  $1/4$ .

$$U(\theta) = R^2 S_{rad}(\theta, \Phi, R)$$
$$S_{rad} = \frac{\eta |I_0|^2}{8\pi^2 R^2} \left[ \frac{\cos\left(\frac{p_i}{2} \cos(\theta)\right)}{\sin(\theta)} \right]^2$$
$$U_n(\theta, \Phi) = \left( \frac{\cos\left(\frac{p_i}{2} \cos(\theta)\right)}{\sin(\theta)} \right)^2$$

Monopolo

$$E_2^{mono} = E_2^{di} \Rightarrow S_\theta^{mono} = S_\theta^{di} \Rightarrow U_\theta^{mono} = U_\theta^{di}, se \theta \leq \frac{\pi}{2}$$
$$P_{rad}^{mono} = \frac{1}{2} P_{rad}^{di}$$
$$D^{mono} = 2D^{di}$$
$$|E_g| = \frac{\eta I_0}{2\pi R}$$

Antenas Microstrip

$$W = \frac{1}{2f_r \cdot \sqrt{\epsilon_0 \mu_0}} \sqrt{\frac{2}{\epsilon_r + 1}}$$
$$\epsilon_{reff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2 \cdot \sqrt{1 + \frac{12d}{W}}}$$
$$\Delta L =$$
$$0.412d \cdot (\epsilon_{reff} + 0.3) \frac{\frac{W}{d} + 0.264}{(\epsilon_{reff} - 0.258)(\frac{W}{d} + 0.8)}$$
$$L = \frac{1}{2f_r \sqrt{\epsilon_{reff} \cdot \sqrt{\epsilon_0 \mu_0}}} - 2\Delta L$$
$$Z_a = 90 \frac{\epsilon_r}{\epsilon_r - 1} \left(\frac{L}{W}\right)^2$$

Agregados de Antenas

$$\phi = \beta d \cos(\theta) + \alpha$$
$$E_{\theta T} =$$
$$j\eta \frac{I_0 e^{-j\beta R}}{2\pi R} \left[ \frac{\cos\left(\frac{p_i}{2} \cos(\theta)\right)}{\sin(\theta)} \right] \left[ 2\cos\left(\frac{\beta d \cos(\theta) + \alpha}{2}\right) \right]$$

$$FA = \frac{\sin(N \frac{\phi}{2})}{\sin(\frac{\phi}{2})}$$

