Optics

Geometrical optics

Snell's law

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Refraction plane surfaces

$$s' = s\frac{n_2}{n_1}$$

Refraction spherical surfaces

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

$$m = -\frac{n_1 s'}{n_2 s}$$

Thin lens

$$
\begin{aligned}
\frac{1}{f} &= \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\
&= \frac{1}{s} + \frac{1}{s'}
\end{aligned}
$$

Power

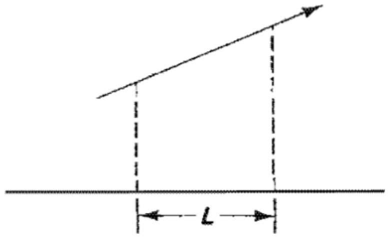
$$
\begin{aligned}
P &= \frac{1}{f} \\
&= \sum_{i=1}^{n} \frac{1}{f_i}
\end{aligned}
$$

Matrix methods

Ray-transfer matrices

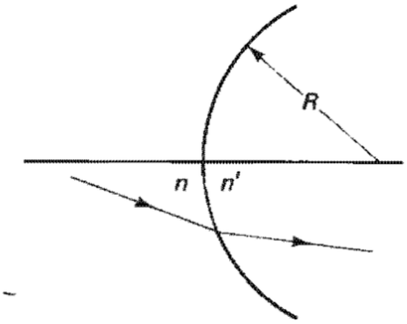
Translation matrix:

$$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$



Refraction matrix,
spherical interface:

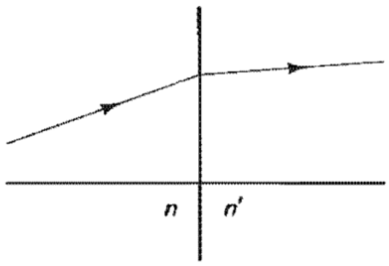
$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{n - n'}{Rn'} & \dfrac{n}{n'} \end{bmatrix}$$



$(+R)$: convex
$(-R)$: concave

Refraction matrix,
plane interface:

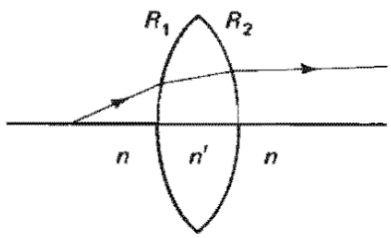
$$M = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n}{n'} \end{bmatrix}$$



Thin-lens matrix:

$$M = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

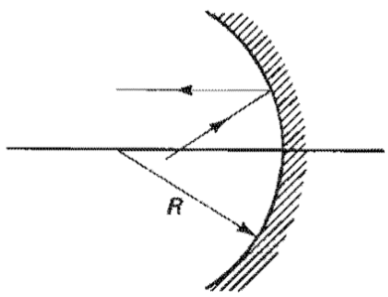
$$\frac{1}{f} = \frac{n' - n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



$(+f)$: convex
$(-f)$: concave

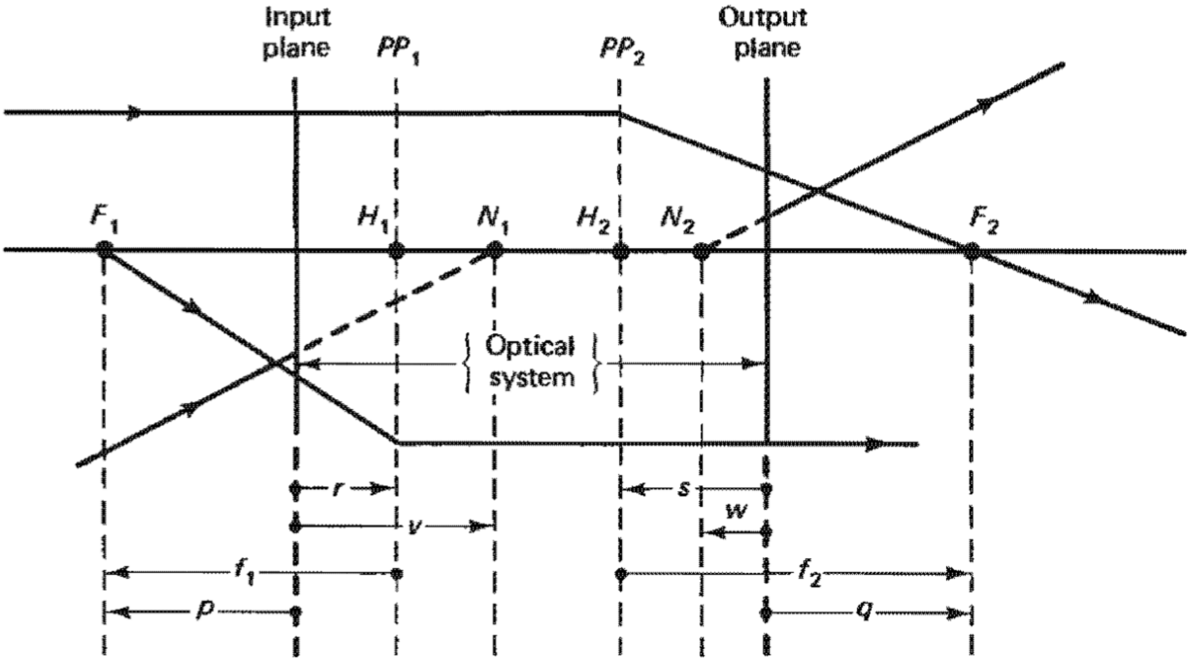
Spherical mirror
matrix:

$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{2}{R} & 1 \end{bmatrix}$$



$(+R)$: convex
$(-R)$: concave

Cardinal points



$$p = \frac{D}{C} \qquad F_1$$

$$q = -\frac{A}{C} \qquad F_2$$

$$r = \frac{D - n_0/n_f}{C} \qquad H_1$$

$$s = \frac{1 - A}{C} \qquad H_2$$

$$v = \frac{D - 1}{C} \qquad N_1$$

$$w = \frac{n_0/n_f - A}{C} \qquad N_2$$

Located relative to input (1) and output (2) reference planes

$$f_1 = p - r = \frac{n_0/n_f}{C} \qquad F_1$$

$$f_2 = q - s = -\frac{1}{C} \qquad F_2$$

Located relative to principal planes

$$f_1 = \frac{-y_f}{\alpha_0} = \frac{-(Ay_0 + B\alpha_0)}{\alpha_0} = \frac{AD}{C} - B$$

$$f_1 = \frac{AD - BC}{C} = \frac{\text{Det}(M)}{C} = \left(\frac{n_0}{n_f}\right)\frac{1}{C} \tag{4-25}$$

Optical instrumentation (ork)

Camera

Circular opening with diameter D, image radius d, and focal length f:

$$\begin{aligned} E_e &\propto \frac{D^2}{d^2} \\ &\propto \frac{1}{A^2}, \qquad A \equiv \frac{f}{D} \text{ relative opening} \end{aligned}$$

Magnifying glass

Near point of normal eye is 25 cm.

Angular magnification at infinity:

$$M = \frac{25}{f}$$

At near point:

$$M = \frac{25}{f} + 1$$

Two lenses at a distance L from each other:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

If the system is independent of the refracting index:

$$L = \frac{1}{2}\left(f_1 + f_2\right)$$

Microscopes

Objective and eyepiece focal lenghts f_o and f_e, with a distance d between the lenses.

$$M = 25\,\frac{f_e + f_o - d}{f_o f_e}$$

Telescopes

Exit and objective diameter D_{ex} and D_{obj}

$$\begin{aligned} M &= -\frac{f_o}{f_e} \\ &= \frac{D_{obj}}{D_{ex}} \end{aligned}$$

¿Holography?

Wave optics

$E = cB$

$u_E = \frac{1}{2}\epsilon E^2$

$u_B = \frac{1}{2}\frac{1}{\mu}B^2$

$u_E = E_B$

$u = u_E + u_B = \epsilon c E B$

$\vec{S} = \epsilon_0 c^2 \vec{E} \times \vec{B}$

$E_e = \langle \|\vec{S}\| \rangle = \frac{1}{2}\epsilon c^2 E_0 B_0$

Fresnell Equations

$$\begin{aligned} r_{TE} &= \frac{\cos\theta - n\cos\theta_t}{\cos\theta + n\cos\theta_t} \\ &= \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \end{aligned} \qquad t_{TE} = 1 + r_{TE}$$

$$\begin{aligned} r_{TM} &= \frac{\cos\theta_t - n\cos\theta}{\cos\theta_t + n\cos\theta} \\ &= -\frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \end{aligned} \qquad n\,t_{TM} = 1 - r_{TM}$$

$r_{TM} = 0 \Rightarrow \theta = \theta_p = \arctan n$

External reflection

$n = \frac{n_2}{n_1} > 1$

$\phi_{TE} = \pi \ \phi_{TM} = -\pi, \quad \theta < \theta_p \ \phi_{TM} = 0, \qquad \theta > \theta_p$

Internal reflection

$\theta_c = \arcsin n$

$\theta > \theta_c$:

$$\tan \frac{\phi_{TE}}{2} = -\frac{\sqrt{\sin^2 \theta - n^2}}{\cos \theta}$$
$$\tan \frac{\phi_{TM}}{2} = -\frac{\sqrt{\sin^2 \theta - n^2}}{n^2 \cos \theta}$$

$\theta \in (\theta_p', \theta_c)$:

$$\phi_{TM} = \pi$$

Otherwise:

$$\phi = 0$$

Reflectance and Transmittance

$R = \frac{P_r}{P_i} = r^2 = \frac{E_r^2}{E^2} \ T = \frac{P_t}{P_i} = n\frac{\cos \theta_t}{\cos \theta} t^2 \ T + R = 1$

Interference

$I = \epsilon_0 c \left\langle \vec{E}^2 \right\rangle = \frac{1}{2}\epsilon_0 c E_0^2$

$\overrightarrow{E_i} = \overrightarrow{E_{0i}} \cos(ks_i - \omega t + \phi_i)$

$\overrightarrow{E_p} = \overrightarrow{E_1} + \overrightarrow{E_2}$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle \cos \delta \rangle, \quad \delta = k(s_2 - s_2) + \phi_2 - \phi_1$

Fringe contrast/visibility

$\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Young's double slit

Point source ⇒ same phase, propagation, etc.

$I = 4I_0 \cos^2 \frac{\delta}{2}$

Constructive:

$\Delta s = s_2 - s_1 = m\lambda \approx a \sin\theta$

Destructive:

$\Delta s = \left(\frac{1}{2} + m\right)\lambda \approx a \sin\theta$

Resultant waves

$E_i = E_{0i} \cos(\alpha_i - \omega t)$

$$\begin{aligned} E_r &= \sum_i E_i = E_0 \cos(\alpha - \omega t), \\ \tan\alpha &= \frac{\sum_i E_{0i} \sin\alpha_i}{\sum_i E_{0i} \cos\alpha_i} \\ E_0^2 &= \left(\sum_i E_{0i} \sin\alpha_i\right)^2 + \left(\sum_i E_{0i} \cos\alpha_i\right)^2 \\ &= \sum_i E_{0i}^2 + 2\sum_i \sum_{j>i} E_{0i}E_{0j} \cos(\alpha_i - \alpha_j) \end{aligned}$$

Random sources

N randomly phased sources of equal amplitude, N grows large:

$$E_0^2 = \sum_{i=1}^{N} E_{0i}^2 = NE_{01}^2$$

Coherent sources

N coherent sources:

$$E_0^2 = \sum_{i=1}^{N} E_{0i}^2 + 2\sum_{i=1}^{N} \sum_{j>i}^{N} E_{0i}E_{0j}$$

N coherent sourcces of equal amplitude:

$$E_0^2 = N^2 E_{01}^2$$

Standing waves

$E_1 = E_0 \sin(\omega t + kx)$

$E_2 = E_0 \sin(\omega t - kx - \phi_r)$

$$\begin{aligned} E_R &= E_1 + E_2 \\ &= 2E_0 \cos\left(kx + \frac{\phi}{2}\right) \sin\left(\omega t - \frac{\phi_R}{2}\right) \end{aligned}$$

Beat frequency $\omega_b = 2\ \omega_g$

Beats

$E_i = E_0 \cos(k_i x - \omega_i t)$

$$\begin{aligned} E_R &= E_1 + E_2 \\ &= 2E_0 \cos(k_p x + \omega_p t) \cos(k_g x - \omega_g t) \end{aligned}$$

$$\begin{aligned} \omega_p &= \frac{\omega_1 + \omega_2}{2}, \qquad & k_p &= \frac{k_1 + k_2}{2} \\ \omega_g &= \frac{\omega_1 - \omega_2}{2}, \qquad & k_g &= \frac{k_1 - k_2}{2} \end{aligned}$$

Dispersion

$$\begin{aligned} v_p &= \frac{c}{n} \\ v_g = \frac{dw}{dk} &= v_p\left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right) \\ &= v_p - \lambda \frac{dv_p}{d\lambda} \\ &= v_p\left(1 - \frac{\omega}{n}\frac{dn}{d\omega}\right) \end{aligned}$$

Medium of normal dispersion ⇒ $\frac{dn}{d\lambda} < 0$

Michelson interferometer

$2\,d \cos\theta = m\,\lambda$

Mirror translation Δd and Δm fringes passing a point:

$$\Delta m = \frac{2\Delta d}{\lambda}$$

Center dark spot:

$$m_{max} = \frac{2d}{\lambda}$$

Dielectric films

Equal internal and external reflection:

$\frac{n_f}{n_0} = \frac{n_s}{n_f} \Rightarrow n_f = \sqrt{n_0 n_s}$

Fiber optics

$$\vec{E} = \vec{E}(x)e^{\beta z - \omega t}$$
$$\vec{B} = \vec{B}(x)e^{\beta z - \omega t}$$

Satisfying maxwell's equations:

$$\begin{aligned} \text{TE:} \qquad & \frac{\partial^2 E_y}{\partial x^2}(x) = \left[\beta^2 - k^2(x)\right]E_y(x) \\ \text{TM:} \qquad & \frac{\partial^2 B_y}{\partial x^2}(x) = \left[\beta^2 - k^2(x)\right]B_y(x) \end{aligned}$$

Implies relative refractive index N satisfying

$$\begin{aligned} \text{TE:} \qquad & \frac{\omega b}{c}\sqrt{n_1^2 - N^2} = 2\arctan\sqrt{\frac{N^2 - n_2^2}{n_1^2 - N^2}} + m\pi \\ \text{TM:} \qquad & \frac{\omega b}{c}\sqrt{n_1^2 - N^2} = 2\arctan\left(\frac{n_1^2}{n_2^2}\sqrt{\frac{N^2 - n_2^2}{n_1^2 - N^2}}\right) + m\pi \end{aligned}$$

for $m \in \mathbb{N}$.

Evanescent Waves

Light penetration of y into medium when $\theta > \theta_c$:

$$E_t = E_{0t}\exp\left(ixk_t\frac{\sin\theta}{n}\right)\exp\left(-i\omega t\right)\exp\left(-\alpha y\right)$$
$$\alpha \equiv k_t\sqrt{\frac{\sin^2\theta}{n^2} - 1}$$

Coherence

Temporal/longitudinal coherence lenght

$$l_c = \frac{2\pi c}{\Delta f}$$

Coherent time and frequency bandwidth:

$$\tau_c = \frac{l_c}{c}, \qquad \Delta f = \frac{1}{\tau_c}$$

Uniform wavelength distribution around λ with linewidth $\Delta\lambda$:

$$l_c = \frac{\lambda^2}{\Delta\lambda}$$

Spacial coherence

Rectangular source of width s and angluar width ϕ:

$$l_s = \frac{r\lambda}{s} \approx \frac{\lambda}{\phi}$$

Circular source of diameter s:

$$l_s = 1.22\frac{r\lambda}{s} \approx 1.22\frac{\lambda}{\phi}$$

Visibility

The intensity at a point P

$$I_P = I_{1P} + I_{2P} + 2\sqrt{I_{1P}I_{2P}}\mathfrak{R}(\gamma(\tau))$$

with the normalized correlation function

$$\gamma(\tau) = \left(1 - \frac{\tau}{\tau_0}\right)e^{i\omega\tau}$$

With equal beams, we get

$$\begin{array}{ccc} \text{Complete incoherence} & \tau \to \tau_0 & V = 0 \\ \text{Complete coherence} & \tau = 0 & V = 1 \\ \text{Partial coherence} & 0 < \tau < \tau_0 & V = |\gamma(\tau)| \end{array}$$

Diffraction

Fraunhoffer diffraction

Single rectangular slit, **length much larger than width**:

$$dE_p = \frac{E_L ds}{r} e^{i(kr-\omega t)} \approx \frac{E_L ds}{r_0} e^{i(kr_0-\omega t)} e^{ik\Delta r}, \qquad \Delta r = s \sin\theta$$

$$E_p = \frac{E_l b}{r_0} \text{sinc}\beta \, e^{i(kr_0-\omega t)}, \qquad \beta = \frac{1}{2} kb \sin\theta$$

$$I = I_0 \text{sinc}^2 \beta, \qquad I_0 = \frac{1}{2}\epsilon_0 c \left(\frac{E_l b}{r_0}\right)^2$$

Many slits:

$$I = I_0 \text{sinc}^2 \beta \, \frac{\sin^2(N\alpha)}{\sin^2 \alpha}$$

Rectangular slit:

$$I = I_0 \text{sinc}^2 \beta \, \text{sinc}^2 \alpha$$

Circular slit

$$I = I_0 \left(\frac{2J_1(\gamma)}{\gamma}\right)^2, \qquad \gamma = \frac{1}{2} k \, D \sin\theta$$

$$J_1(\gamma) = \sum_{l=0}^{\infty} \frac{(-1)^l}{2^{l+1} l!(l+1)!} \gamma^{2l+1}$$

Zeros of bessel functions and their derivatives

Beam spreading

Width W of central maximum at a distance L:

$$W = L\Delta\theta = \frac{2L\lambda}{b}$$

Resolution

(Sircular aperture)
Rayleigh's criterion:

$$(\Delta\theta)_{min} = \frac{1.22\lambda}{D}$$

Diffraction grating

Grating width a

Optical path difference:

$$\Delta d = a(\sin\theta_i + \sin\theta_m)$$