CoCoA$^+$: Adding vs. Averaging in Distributed Optimization

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joint work with

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Outline

- Problem Formulation
- Serial and Parallel Coordinate Descent Method (CDM)
- Distributed CDM
- Original CoCoA Framework
- CoCoA$^+$ Framework
- Computation vs. Communication Trade-off
- Spark
- Some Numerical Experiments

The Problem - Regularized Empirical Loss Minimization

Let $\{(x_i, y_i)\}_{i=1}^n$ be our training data data, $x_i \in \mathbf{R}^d$ and $y_i \in \mathbf{R}$.

$$\min_{w \in \mathbf{R}^d} \left[P(w) := \frac{1}{n} \sum_{i=1}^n \ell_i(w^T x_i) + \frac{\lambda}{2} \|w\|^2 \right] \tag{P}$$

where

- $\lambda > 0$ is a regularization parameter
- $\ell_i(\cdot)$ is convex loss function which can depend on the label y_i
  Examples:
  - Logistic loss: $\ell_i(\zeta) = \log(1 + \exp(-y_i \zeta))$
  - Hinge loss: $\ell_i(\zeta) = \max\{0, 1 - y_i \zeta\}$

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The dual problem

$$\max_{\alpha \in \mathbf{R}^n} \left[D(\alpha) := -\frac{\lambda}{2} \|A\alpha\|^2 - \frac{1}{n} \sum_{i=1}^n \ell_i^*(-\alpha_i) \right] \tag{D}$$

where $A = \frac{1}{\lambda n} X^T$ and $X^T = [x_1, x_2, \ldots, x_n] \in \mathbf{R}^{d \times n}$

- ℓ_i^* is convex conjugate of ℓ_i
- wlog $\|x_i\| \leq 1$

Duality

Primal-Dual mapping

For any $\alpha \in \mathrm{dom}(D)$ we can define

$$w(\alpha) := A\alpha \tag{1}$$

From strong duality we have that $w^* = w(\alpha^*)$ is optimal to (P) if α^* is optimal solution to (D).

Gap function

$$G(\alpha) = P(w(\alpha)) - D(\alpha)$$

The Setting & Challenges

- The size of matrix A is huge (e.g. TBs of data)
- We want to use many nodes of computer cluster (or cloud) to speed-up the computation

Challenges

- **distributed data:** no single machine can load the whole instance
- **expensive communication:**

	latency
RAM	100 nanoseconds
standard network connection	250,000 nanoseconds

- **unreliable nodes:** we assume that the node can die at any point during the computation (we want to have fault tolerant solution)

The Serial/Parallel/Distributed SDCA Algorithm

Serial **S**tochastic **D**ual **C**oordinate **A**scent

```
choose $\alpha^{(0)} \in \mathbf{R}^n$
repeat
    $\alpha^{(t+1)} = \alpha^{(t)}$
    pick a random coordinate $i \in \{1, \dots, n\}$
    compute the update: $h_t^i(\alpha^{(t)}) := \arg\max_h D(\alpha^{(t)} + he_i)$
    apply the update: $\alpha_i^{(t+1)} = \alpha_i^{(t+1)} + h_t^i(\alpha^{(t)})e_i$
```

The Serial/Parallel/Distributed SDCA Algorithm

Parallel Stochastic Dual Coordinate Ascent

```
choose $\alpha^{(0)} \in \mathbf{R}^n$
repeat
    $\alpha^{(t+1)} = \alpha^{(t)}$
    pick a random coordinate $i \in \{1, \dots, n\}$
    pick a random subset $S \subset \{1, \dots, n\}$ with $|S| = H$
    for each $i \in S$ in **parallel** do
        compute the update: $h_t^i(\alpha^{(t)}) := \arg\max_h D(\alpha^{(t)} + he_i)$
    apply the update: $\alpha_i^{(t+1)} = \alpha_i^{(t+1)} + h_t^i(\alpha^{(t)})e_i$
    apply the update: $\alpha_i^{(t+1)} = \alpha_i^{(t+1)} + \frac{1}{H} \sum_{i \in S} h_t^i(\alpha^{(t)})e_i$
```

The Serial/Parallel/Distributed SDCA Algorithm

- assume we have K nodes (computers) each with parallel processing power
- we **partition** the coordinates $\{1,2,\dots,n\}$ into K **balanced** sets $\mathcal{P}_1,\dots,\mathcal{P}_K$ $\forall k \in \{1,\dots,K\}$ we have $|\mathcal{P}_k| = \frac{n}{K}$

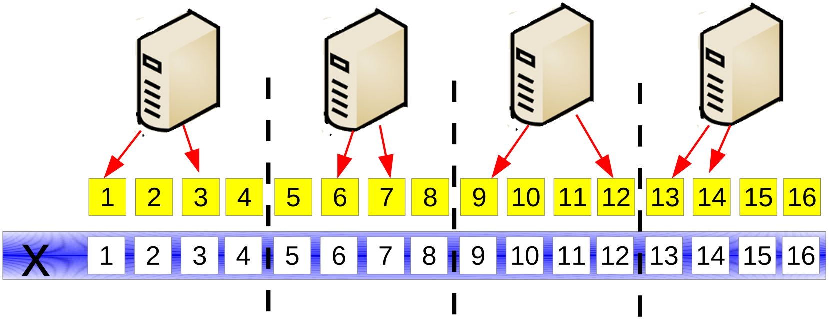
Distributed Stochastic Dual Coordinate Ascent

```
choose $\alpha^{(0)} \in \mathbf{R}^n$
repeat
    $\alpha^{(t+1)} = \alpha^{(t)}$
    for each computer $k \in \{1,\dots,K\}$ in parallel do
        pick a random subset $S \subset \{1,\dots,n\}$ with $|S| = H$
        pick a random subset $S_k \subset \mathcal{P}_k$ with $|S| = H \le \frac{n}{K}$
        for each $i \in S_k$ in parallel do
            compute the update: $h_t^i(\alpha^{(t)}) := \arg\max_h D(\alpha^{(t)} + he_i)$
    apply the update: $\alpha_i^{(t+1)} = \alpha_i^{(t+1)} + \frac{1}{H}\sum_{i\in S} h_t^i(\alpha^{(t)})e_i$
    apply the update: $\alpha_i^{(t+1)} = \alpha_i^{(t+1)} + \frac{1}{KH}\sum_{k\in\{1,\dots,K\}}\sum_{i\in S_k} h_t^i(\alpha^{(t)})e_i$
```

The distributed algorithm can need (in the worst case) the **same number of iterations as a serial one!**

Distributed CDM

Illustration: $K = 4$ and $H = 2$



Distributed CDM

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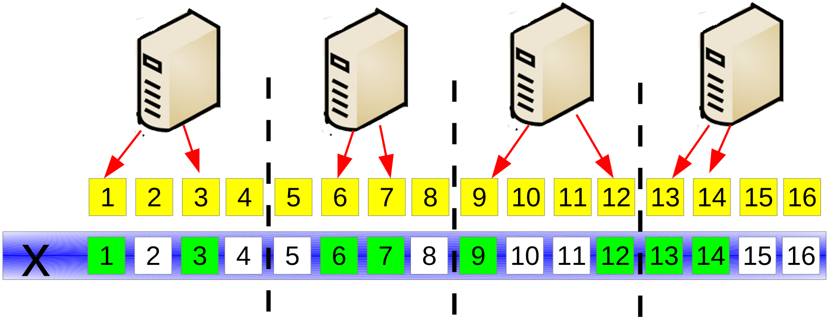

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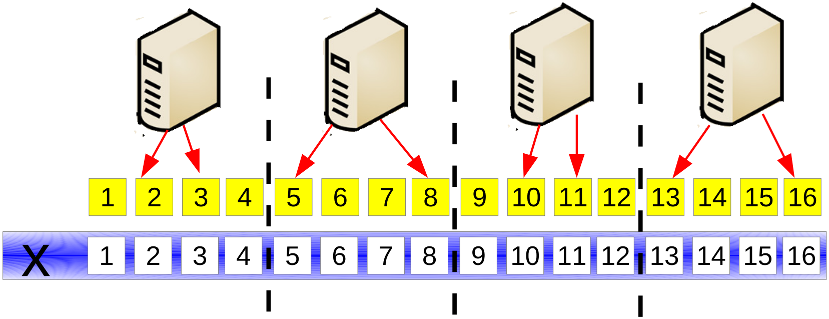
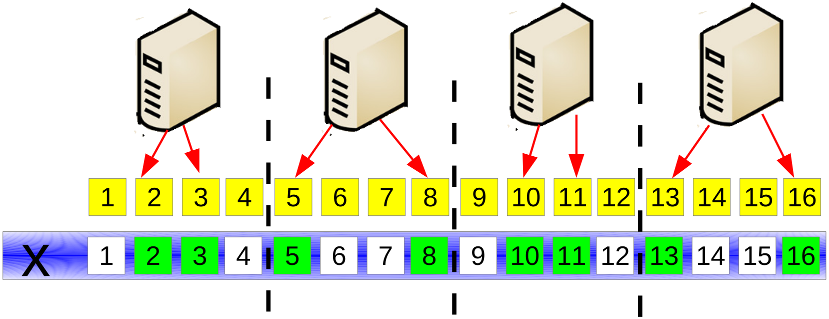


Distributed CDM

Illustration: $K = 4$ and $H = 2$

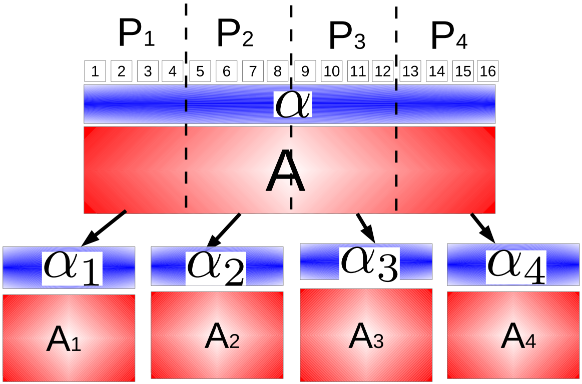


Disadvantages of Distributed CDM

- we cannot choose $H > |\mathcal{P}_k|$!
- the computation of step is very easy (usually close form or a bit complicated 1D problem)
- after taking H steps, usually the objective function doesn't change much
- it is almost **impossible** to balance computation and communication

Data Distribution

Vector α and columns of matrix A are partitioned according $\{\mathcal{P}_k\}_{k=1}^K$.



Notation: For $k \in \{1, 2, \dots, K\}$ we use $\alpha_k \in \mathbf{R}^{|\mathcal{P}_k|}$ is a subvector of α.
Vector $\alpha_{[k]} \in \mathbf{R}^n$ is a vector obtained from vector α by setting all coordinates $\notin \mathcal{P}_k$ to zero.
Example: $\alpha_1 = (*, *, *, *)^T$, $\alpha_{[1]} = (*, *, *, *, 0, 0, \dots, 0)^T$.

Local Problem

CoCoA subproblem

At iteration t at node k

$$\begin{aligned}(\Delta\alpha^*)_{[k]}^{(t)} &= \arg\max_{\Delta\alpha_{[k]}\in\mathbf{R}^n} D(\alpha^{(t)} + \Delta\alpha_{[k]}) \\ &= \arg\max_{\Delta\alpha_{[k]}\in\mathbf{R}^n} \left(-\frac{\lambda}{2}\|A(\alpha^{(t)} + \Delta\alpha_{[k]})\|^2 - \frac{1}{n}\sum_{i=1}^{n} \ell_i^*(-(\alpha^{(t)} + \Delta\alpha_{[k]})_i)\right)\end{aligned}$$

- we cannot solve the subproblem as it depends on $\alpha^{(t)}$ and A

Local Problem

CoCoA subproblem

At iteration t at node k

$$(\Delta\alpha^*)^{(t)}_{[k]} = \arg\max_{\Delta\alpha_{[k]} \in \mathbf{R}^n} D(\alpha^{(t)} + \Delta\alpha_{[k]})$$

$$= \arg\max_{\Delta\alpha_{[k]} \in \mathbf{R}^n} \left(-\frac{\lambda}{2} \|A(\alpha^{(t)} + \Delta\alpha_{[k]})\|^2 - \frac{1}{n} \sum_{i=1}^{n} \ell_i^*(-(\alpha^{(t)} + \Delta\alpha_{[k]})_i) \right)$$

- we cannot solve the subproblem as it depends on $\alpha^{(t)}$ and A
- if we know $w^{(t)} = A\alpha^{(t)}$ then

$$(\Delta\alpha^*)^{(t)}_{[k]} = \arg\max_{\Delta\alpha_{[k]} \in \mathbf{R}^n} \left(-\frac{\lambda}{2} \|w^{(t)} + A\Delta\alpha_{[k]}\|^2 - \frac{1}{n} \sum_{i \in \mathcal{P}_k} \ell_i^*(-(\alpha^{(t)} + \Delta\alpha_{[k]})_i) \right)$$

- if we know $w^{(t)}$ we can compute $(\Delta\alpha^*)^{(t)}_{[k]}$

The CoCoA Framework

Communication-Efficient Distributed Dual Coordinate Ascent

```
**Input:** $T \geq 1$
**Data:** $\{(x_i, y_i)\}_{i=1}^n$ distributed over $K$ machines
**Initialize:** $\alpha_{[k]}^{(0)} \leftarrow 0$ for all machines $k$, and $w^{(0)} \leftarrow 0$
for $t = 1, 2, \dots, T$
    for all machines $k = 1, 2, \dots, K$ in parallel
        Solve local problem approximately to obtain $\Delta\alpha_{[k]}$        computation
        $\alpha_{[k]}^{(t)} \leftarrow \alpha_{[k]}^{(t-1)} + \frac{1}{K}\Delta\alpha_{[k]}$
        $\Delta w_k \leftarrow \frac{1}{K} A \Delta\alpha_{[k]}$
    reduce $w^{(t)} \leftarrow w^{(t-1)} + \sum_{k=1}^K \Delta w_k$        communication
```

- The performance of this methods (in worst case) can be the same as if we **randomly pick** k and solve corresponding subproblem and replace $\frac{1}{K}$ by 1
- How accurately do we need to solve the local sub-problem?
- How to change the local problem to avoid averaging (e.g. just to add local solutions)?
- Can we prove it will be better?

Smarter Subproblem

Local Subproblem for CoCoA$^+$

$$\max_{\Delta\alpha_{[k]}\in\mathbf{R}^n} \mathcal{G}_k^{\sigma'}(\Delta\alpha_{[k]}; w^{(t)}) \tag{2}$$

where

$$\begin{aligned}\mathcal{G}_k^{\sigma'}(\Delta\alpha_{[k]}; w^{(t)}) = -\frac{1}{n}\sum_{i\in\mathcal{P}_k} \ell_i^*(-(\alpha_{[k]}^{(t)} + \Delta\alpha_{[k]})_i) \\ -\frac{1}{K}\frac{\lambda}{2}\|w^{(t)}\|^2 - \lambda(w^{(t)})^T A\Delta\alpha_{[k]} \\ -\frac{\lambda}{2}\sigma'\left\|A\Delta\alpha_{[k]}\right\|^2.\end{aligned}$$

Compare with:

$$\max_{\Delta\alpha_{[k]}\in\mathbf{R}^n}\left(-\frac{\lambda}{2}\|w^{(t)} + A\Delta\alpha_{[k]}\|^2 - \frac{1}{n}\sum_{i\in\mathcal{P}_k}\ell_i^*(-(\alpha^{(t)} + \Delta\alpha_{[k]})_i)\right) \tag{3}$$

If $\sigma' = 1$ then the optimal solutions of (2) and (3) coincides.

The CoCoA$^+$ Framework

Communication-Efficient Distributed Dual Coordinate Ascent

```
Input: $T \geq 1$, $\gamma \in [\frac{1}{K}, 1]$, $\sigma' \in [1, \infty)$
Data: $\{(x_i, y_i)\}_{i=1}^n$ distributed over $K$ machines
Initialize: $\alpha_{[k]}^{(0)} \leftarrow 0$ for all machines $k$, and $w^{(0)} \leftarrow 0$
for $t = 1, 2, \dots, T$
    for all machines $k = 1, 2, \dots, K$ in parallel
        approximately max $\mathcal{G}^{\sigma'}(\Delta\alpha_{[k]}; w^{(t)})$ to obtain $\Delta\alpha_{[k]}$        computation
        $\alpha_{[k]}^{(t)} \leftarrow \alpha_{[k]}^{(t-1)} + \gamma\Delta\alpha_{[k]}$
        $\Delta w_k \leftarrow \gamma A \Delta\alpha_{[k]}$
    reduce $w^{(t)} \leftarrow w^{(t-1)} + \sum_{k=1}^K \Delta w_k$        communication
```

- If $\gamma = \frac{1}{K}$ we obtain CoCoA
- If $\gamma = \frac{1}{K}$ then $\sigma' = 1$ is "safe" value
- What about another values of γ? (we want $\gamma = 1$)

CoCoA$^+$ Parameters - σ' and γ

- σ' measures the difficulty of the given data partition
- it must be chosen not smaller than

$$\sigma' \geq \sigma'_{min} \overset{\text{def}}{=} \gamma \max_{\alpha \in \mathbf{R}^n} \frac{\|A\alpha\|^2}{\sum_{k=1}^{K} \|A\alpha_{[k]}\|^2} \tag{4}$$

Lemma

For any $\alpha \in \mathbf{R}^n$ $(\alpha \neq \mathbf{0})$ we have

$$\frac{\|A\alpha\|^2}{\sum_{k=1}^{K} \|A\alpha_{[k]}\|^2} \leq K$$

- We can take the safe value $\sigma' = K \cdot \gamma$
  Again: if $\gamma = \frac{1}{K}$ then $\sigma' = K \cdot \frac{1}{K} = 1$ is a safe value
- New: if $\gamma = 1$ then $\sigma' = K \cdot 1 = K$ is a safe value

How Accurately?

Assumption: Θ-approximate solution

We assume that there exists $\Theta \in [0, 1)$ such that $\forall k \in [K]$, the local solver at any iteration t produces a **(possibly) randomized** approximate solution $\Delta\alpha_{[k]}$, which satisfies

$$\mathbf{E}\left[\mathcal{G}_k^{\sigma'}(\Delta\alpha^*_{[k]}, w) - \mathcal{G}_k^{\sigma'}(\Delta\alpha_{[k]}, w)\right] \le \Theta \left(\mathcal{G}_k^{\sigma'}(\Delta\alpha^*_{[k]}, w) - \mathcal{G}_k^{\sigma'}(\mathbf{0}, w)\right), \qquad (5)$$

where

$$\Delta\alpha^* \in \arg\min_{\Delta\alpha \in \mathbf{R}^n} \sum_{k=1}^{K} \mathcal{G}_k^{\sigma'}(\Delta\alpha_{[k]}, w). \qquad (6)$$

- because the subproblem is **not really** what one wants to solve, therefore in practise $\Theta \approx 0.9$ (depending on the cluster and problem)
- what about convergence guarantees?
- how to get Θ approximate solution?

Iteration Complexity - Smooth Loss

Theorem

Assume the loss functions functions ℓ_i are $(1/\mu)$-smooth, for $i \in \{1, 2, \dots, n\}$. We define

$$\sigma_k \overset{def}{=} \max_{\alpha_{[k]} \in \mathbf{R}^n} \frac{\|A\alpha_{[k]}\|^2}{\|\alpha_{[k]}\|^2} \leq |\mathcal{P}_k| \tag{7}$$

and $\sigma_{\max} = \max_{k \in [K]} \sigma_k$.
Then after T iterations of CoCoA$^+$, with

$$T \geq \frac{1}{\gamma(1-\Theta)} \frac{\lambda\mu n + \sigma_{\max}\sigma'}{\lambda\mu n} \log \frac{1}{\epsilon},$$

it holds that $\mathbf{E}[D(\alpha^*) - D(\alpha^T)] \leq \epsilon$.
Furthermore, after T iterations with

$$T \geq \frac{1}{\gamma(1-\Theta)} \frac{\lambda\mu n + \sigma_{\max}\sigma'}{\lambda\mu n} \log \left(\frac{1}{\gamma(1-\Theta)} \frac{\lambda\mu n + \sigma_{\max}\sigma'}{\lambda\mu n} \frac{1}{\epsilon} \right),$$

we have the expected duality gap

$$\mathbf{E}[P(w(\alpha^{(T)})) - D(\alpha^{(T)})] \leq \epsilon.$$

Averaging vs. Adding

The leading term is $\frac{1}{\gamma(1-\Theta)} \frac{\lambda\mu n + \sigma_{\max}\sigma'}{\lambda\mu n}$. Let us assume that $\forall k : |\mathcal{P}_k| = \frac{n}{K}$

Averaging

$\gamma = \frac{1}{K}$
$\sigma' = 1$

$$\frac{K}{1-\Theta} \frac{\lambda\mu n + \frac{n}{K}}{\lambda\mu n}$$

$$\frac{1}{1-\Theta} \frac{\lambda\mu K + 1}{\lambda\mu}$$

Adding

$\gamma = 1$
$\sigma' = K$

$$\frac{1}{1-\Theta} \frac{\lambda\mu n + \frac{n}{K} K}{\lambda\mu n}$$

$$\frac{1}{1-\Theta} \frac{\lambda\mu + 1}{\lambda\mu}$$

Note: this is in the worst case (for the worst case example)

Iteration Complexity - General Convex Loss

Theorem

Consider CoCoA$^+$ starting with $\alpha^0 = \mathbf{0} \in \mathbf{R}^n$ and $\forall i \in \{1, 2, \dots, n\} : \ell_i(\cdot)$ be L-Lipschitz continuous and $\epsilon > 0$ be the desired duality gap. Then after T iterations, where

$$T \geq T_0 + \max\{\left\lceil \frac{1}{\gamma(1-\Theta)} \right\rceil, \frac{4L^2\sigma\sigma'}{\lambda n^2 \epsilon \gamma(1-\Theta)}\},$$

$$T_0 \geq t_0 + \left(\frac{2}{\gamma(1-\Theta)} \left(\frac{8L^2\sigma\sigma'}{\lambda n^2 \epsilon} - 1 \right) \right)_+,$$

$$t_0 \geq \max(0, \left\lceil \tfrac{1}{\gamma(1-\Theta)} \log(\tfrac{2\lambda n^2 (D(\alpha^*) - D(\alpha^0))}{4L^2\sigma\sigma'}) \right\rceil),$$

we have that the expected duality gap satisfies $\mathbf{E}[P(w(\overline{\alpha})) - D(\overline{\alpha})] \leq \epsilon$, at the averaged iterate

$$\overline{\alpha} := \tfrac{1}{T-T_0} \sum_{t=T_0+1}^{T-1} \alpha^{(t)},$$

where $\sigma = \sum_{k=1}^{K} |\mathcal{P}_k| \sigma_k$.

SDCA as a Local Solver

SDCA

```
1: Input: $\alpha_{[k]}, w = w(\alpha)$
2: Data: Local $\{(x_i, y_i)\}_{i \in \mathcal{P}_k}$
3: Initialize: $\Delta\alpha_{[k]}^0 = 0 \in \mathbb{R}^n$
4: for $h = 0, 1, \ldots, H-1$ do
5:     choose $i \in \mathcal{P}_k$ uniformly at random
6:     $\delta_i^* = \arg\max_{\delta_i \in \mathbf{R}} \mathcal{G}_k^{\sigma'}(\Delta\alpha_{[k]}^h + \delta_i e_i, w)$
7:     $\Delta\alpha_{[k]}^{(h+1)} = \Delta\alpha_{[k]}^{(h)} + \delta_i^* e_i$
8: end for
9: Output: $\Delta\alpha_{[k]}^{(H)}$
```

Theorem

Assume the functions ℓ_i are $(1/\mu)-$smooth for $i \in \{1, 2, \ldots, n\}$. If

$$H \geq n_k \frac{\sigma' + \lambda n \mu}{\lambda n \mu} \log \frac{1}{\Theta} \tag{8}$$

then SDCA will produce a Θ-approximate solution.

Total Runtime

- To get ϵ accuracy we need

$$\mathcal{O}\left(\frac{1}{1-\Theta}\log\frac{1}{\epsilon}\right)$$

- Recall $\Theta = \left(1 - \frac{\lambda n\gamma}{1+\lambda n\gamma}\frac{K}{n}\right)^{H}$

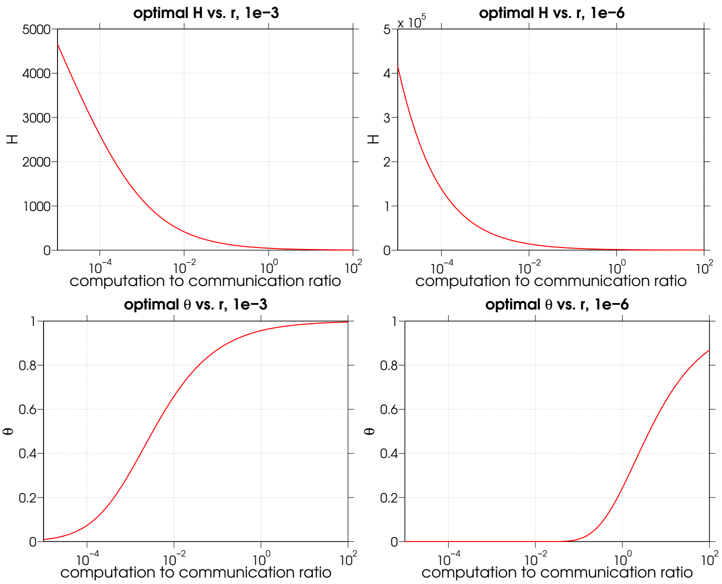
Let

- τ_o be the duration of communication per iteration
- τ_c be the duration of **ONE** coordinate update during the inner iteration

Total runtime

$$\mathcal{O}\left(\frac{1}{1-\Theta}(\tau_O + H\tau_c)\right) = \mathcal{O}\left(\frac{1}{1-\Theta}\left(1 + H\underbrace{\frac{\tau_c}{\tau_o}}_{r_{c/o}}\right)\right)$$

$H(\tau_c/\tau_o)$, $\Theta(\tau_c/\tau_o)$



Apache Spark

Apache Spark

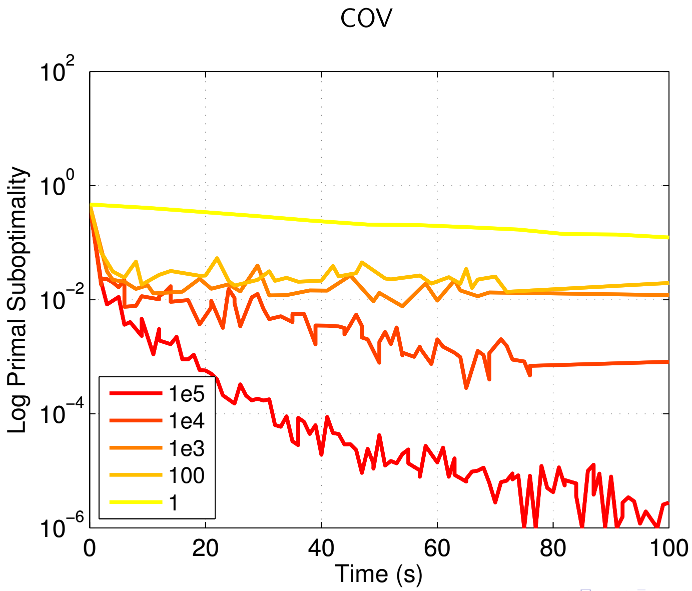
- is a fast and general engine for large-scale data processing
- runs on Hadoop, Mesos, standalone, or in the cloud. It can access diverse data sources including HDFS, Cassandra, HBase, S3
- **is slower than our C++ code** for CoCoA$^+$
- we run it on Amazon Elastic Compute Cloud (Amazon EC2)

Numerical Experiments

Datasets

Dataset	**Training** (n)	**Features** (d)	$nnz/(n \cdot d)$	λ	**Workers** (K)
cov	522,911	54	22.22%	1e-6	4
rcv1	677,399	47,236	0.16%	1e-6	8
imagenet	32,751	160,000	100%	1e-5	32

Dependence of Primal Suboptimality on H



Comparison with Different Algorithms

- COCOA

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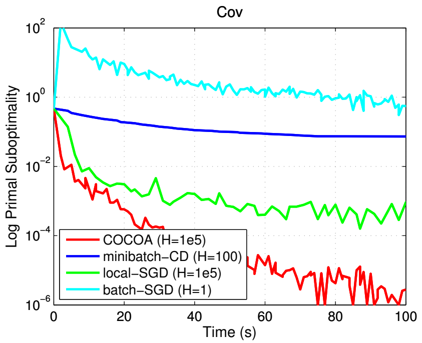
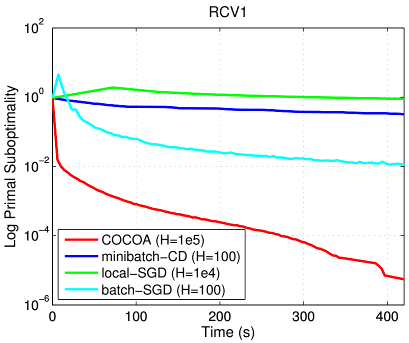
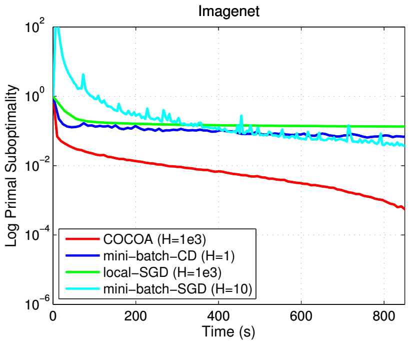
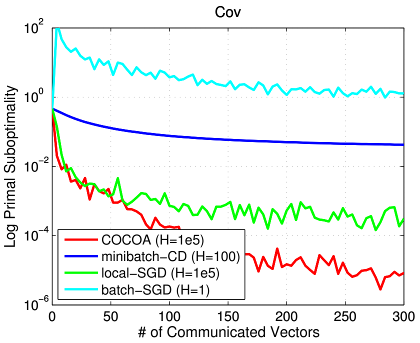
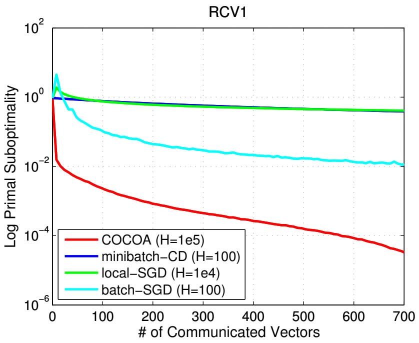

Cov
Log Primal Suboptimality
Time (s)
COCOA (H=1e5)
minibatch−CD (H=100)
local−SGD (H=1e5)
batch−SGD (H=1)


RCV1
Log Primal Suboptimality
Time (s)
COCOA (H=1e5)
minibatch−CD (H=100)
local−SGD (H=1e4)
batch−SGD (H=100)

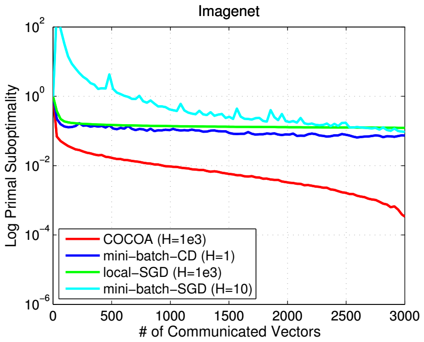

Imagenet
Log Primal Suboptimality
Time (s)
COCOA (H=1e3)
mini−batch−CD (H=1)
local−SGD (H=1e3)
mini−batch−SGD (H=10)



Cov
Log Primal Suboptimality
# of Communicated Vectors
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
50
100
150
200
250
300
COCOA (H=1e5)
minibatch−CD (H=100)
local−SGD (H=1e5)
batch−SGD (H=1)

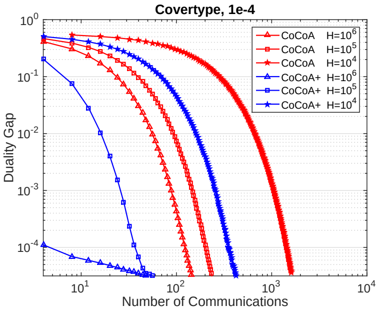


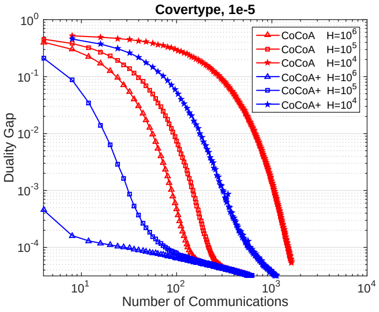
RCV1
Log Primal Suboptimality
# of Communicated Vectors
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
100
200
300
400
500
600
700
COCOA (H=1e5)
minibatch−CD (H=100)
local−SGD (H=1e4)
batch−SGD (H=100)

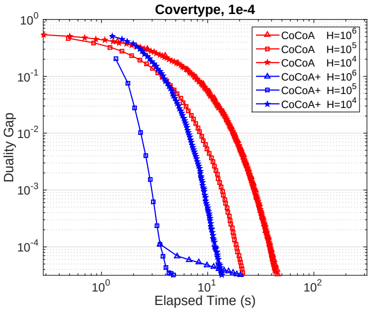


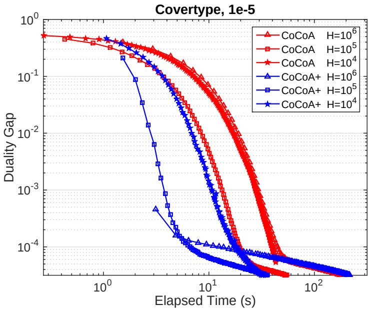
Imagenet
Log Primal Suboptimality
# of Communicated Vectors
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
500
1000
1500
2000
2500
3000
COCOA (H=1e3)
mini−batch−CD (H=1)
local−SGD (H=1e3)
mini−batch−SGD (H=10)

CoCoA vs. CoCoA$^+$

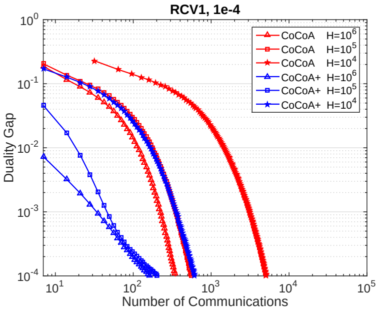


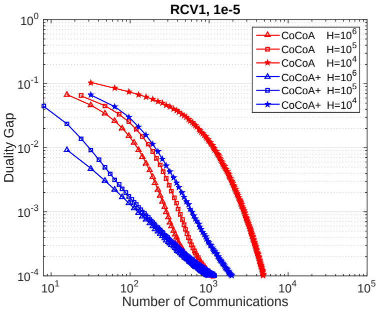


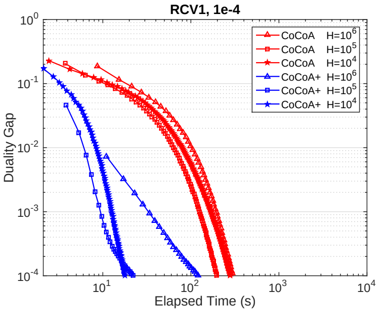


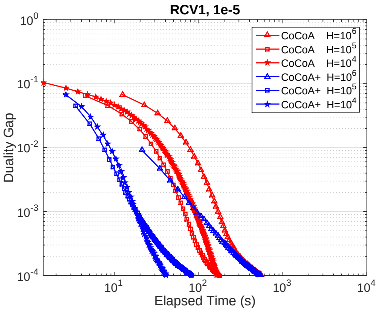


CoCoA vs. CoCoA$^+$

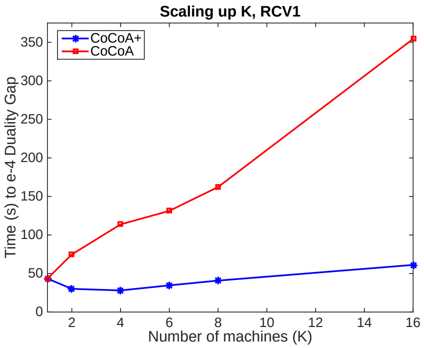




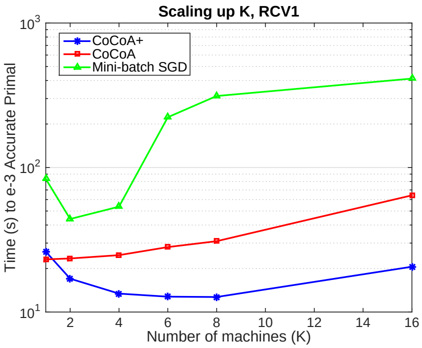




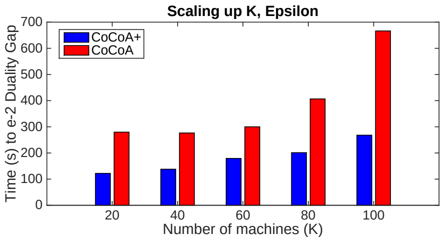
Scaling up



Scaling up K, RCV1
Time (s) to e-4 Duality Gap
Number of machines (K)
CoCoA+
CoCoA

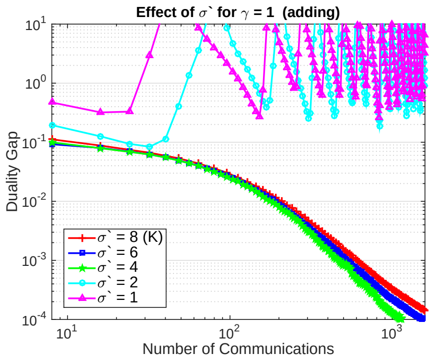


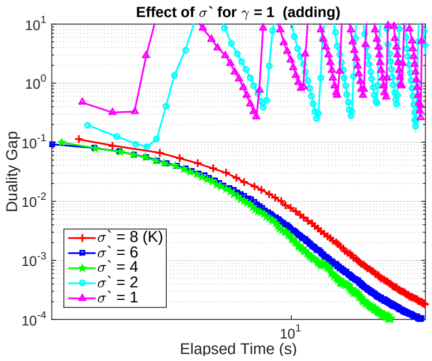
Scaling up K, RCV1
Time (s) to e-3 Accurate Primal
Number of machines (K)
CoCoA+
CoCoA
Mini-batch SGD



Scaling up K, Epsilon
Time (s) to e-2 Duality Gap
Number of machines (K)
CoCoA+
CoCoA

Effect of σ'





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