Exercise 1

```
clear
syms x1 x2 xe
g = @(x1,x2,x3) 4*x1 + 2*x2 + x3
```

```
g = function_handle with value:
    @(x1,x2,x3)4*x1+2*x2+x3
```

```

A = [   1 0 0 1 0 0 0;
        4 1 0 0 1 0 0;
        8 4 1 0 0 1 0];
b = [ 1;4;16];
c = -[4;2;1; 0; 0; 0; -1 ];
v = [4;5;6];
simplex(c,A,b,v,1)
```

```
Initial tableau:
  Columns 1 through 2

       1              0
       4              1
       8              4
      -4             -2

  Columns 3 through 4

       0              1
       0              0
       1              0
      -1              0

  Columns 5 through 6

       0              0
       1              0
       0              1
       0              0

  Columns 7 through 8

       0              1
       0              4
       0             16
       1              0

Pivot point:
       1              1

New tableau:
  Columns 1 through 2

       1              0
       0              1
       0              4
       0             -2

  Columns 3 through 4

       0              1
```

```
       0              -4
       1              -8
      -1               4

  Columns 5 through 6

       0               0
       1               0
       0               1
       0               0

  Columns 7 through 8

       0               1
       0               0
       0               8
       1               4

Pivot point:
       2               2

New tableau:
  Columns 1 through 2

       1               0
       0               1
       0               0
       0               0

  Columns 3 through 4

       0               1
       0              -4
       1               8
      -1              -4

  Columns 5 through 6

       0               0
       1               0
      -4               1
       2               0

  Columns 7 through 8

       0               1
       0               0
       0               8
       1               4

Pivot point:
       1               4

New tableau:
  Columns 1 through 2

       1               0
       4               1
      -8               0
       4               0

  Columns 3 through 4

       0               1
       0               0
```

```
       1              0
      -1              0

  Columns 5 through 6

       0              0
       1              0
      -4              1
       2              0

  Columns 7 through 8

       0              1
       0              4
       0              0
       1              8

Pivot point:
       3              3

New tableau:
  Columns 1 through 2

       1              0
       4              1
      -8              0
      -4              0

  Columns 3 through 4

       0              1
       0              0
       1              0
       0              0

  Columns 5 through 6

       0              0
       1              0
      -4              1
      -2              1

  Columns 7 through 8

       0              1
       0              4
       0              0
       1              8

Pivot point:
       1              1

New tableau:
  Columns 1 through 2

       1              0
       0              1
       0              0
       0              0

  Columns 3 through 4

       0              1
       0             -4
       1              8
```

```
       0              4

  Columns 5 through 6

       0              0
       1              0
      -4              1
      -2              1

  Columns 7 through 8

       0              1
       0              0
       0              8
       1             12

Pivot point:
       2              5

New tableau:
  Columns 1 through 2

       1              0
       0              1
       0              4
       0              2

  Columns 3 through 4

       0              1
       0             -4
       1             -8
       0             -4

  Columns 5 through 6

       0              0
       1              0
       0              1
       0              1

  Columns 7 through 8

       0              1
       0              0
       0              8
       1             12

Pivot point:
       1              4

New tableau:
  Columns 1 through 2

       1              0
       4              1
       8              4
       4              2

  Columns 3 through 4

       0              1
       0              0
       1              0
       0              0
```

```
  Columns 5 through 6

       0              0
       1              0
       0              1
       0              1

  Columns 7 through 8

       0              1
       0              4
       0             16
       1             16
ans = 7×1
       0
       0
      16
       1
       4
       0
       0
```

a)

Initial table is:

Initial tableau:

1	0	0	1	0	0	0	1
4	1	0	0	1	0	0	4
8	4	1	0	0	1	0	16
-4	-2	-1	0	0	0	1	0

From last collumn we pick most negative value (In that case it is -4 which is first collumn)

We calculare ratio for this collumns so:

```
1/1
```

```
ans = 
       1
```

```
4/4
```

```
ans = 
       1
```

```
16/8
```

```
ans = 
       2
```

From results we pick lowest value which has row 1 and 2 (value = 1) we pick one and that will be our pivot.

First operation is to change our pivot (first row) to value 1 which is already done we can skip

Second operation is to make Row 2 column 1 - 0 so: -4R1 + R2 -> R2

Third operation is to make Row 3 column 2 - 0 so: -8R1 + R3 -> R3

Forth Operation is to make our Last row collumn 1 = 0 so: 4R1 + R4 -> R4

Result table:

1	0	0	1	0	0	0	1
0	1	0	-4	1	0	0	0
0	4	1	-8	0	1	0	8
0	-2	-1	4	0	0	1	4

Maximum is 16 !

b)

Dual problem from Maximization will be minimization

```
c = -[1;4;16];
```

Dual problem:

Minimize y1 + 4y2 + 16y3

Subject to: y1 + 4y2 + 8y3 => 4

y2 + 4y3 => 2

y3 =>1

Exercise 2

```
clear
syms x1 x2
g = @(x1,x2) x1^3 - 3*x1 -x2^2
```

```
g = function_handle with value:
    @(x1,x2)x1^3-3*x1-x2^2
```

```
%%a

df1_x1 = diff(g,x1);
df1_x2 = diff(g,x2);
Df = [df1_x1 df1_x2]
```

Df = $\begin{pmatrix} 3x_1^2 - 3 & -2x_2 \end{pmatrix}$

```
gradient = Df'
```

gradient =

$$\begin{pmatrix} 3\,\overline{x_1}^2 - 3 \\ -2\,\overline{x_2} \end{pmatrix}$$

```
gradient = matlabFunction(gradient);

[x1,x2] = solve(gradient(x1,x2)==0,x1,x2)
```

x1 =

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

x2 =

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Critical points are [-1 1] and [0 0] with values 1 and 0

b)

```
g(-1,1)
```

```
ans =
      1
```

```
g(0,0)
```

```
ans =
      0
```

```
clear;
syms x1 x2;
%c)
f = @(x1,x2) (x1 - 2)^2 + (x2 - 1)^2;
h = @(x1,x2) (x1 - 1)^2 + (x2 + 2)^2 -1;

%Lagrange

syms lambda
l = f(x1,x2)+lambda*h;

%Calculate derivatives
dlx1 = diff(l,x1);
dlx2 = diff(l,x2);
dll = diff(l,lambda);

%Main function to get points
syms x1 x2 lambda
[x1, x2, lambda] = solve(dlx1==0,dlx2==0,dll==0, x1, x2, lambda)
```

x1 =

$$\begin{pmatrix} \frac{\sqrt{10}}{10} + 1 \\ 1 - \frac{\sqrt{10}}{10} \end{pmatrix}$$

```
x2 =
```

$$\begin{pmatrix} \frac{3\sqrt{10}}{10} - 2 \\ -\frac{3\sqrt{10}}{10} - 2 \end{pmatrix}$$

```
lambda =
```

$$\begin{pmatrix} \sqrt{10} - 1 \\ -\sqrt{10} - 1 \end{pmatrix}$$

```
%Save points to variables
x1 = [sqrt(10)/10+1 ; 3*sqrt(10)/10 - 2 ]
```

```
x1 = 2×1
     949/721
    -758/721
```

```
x2 = [1-sqrt(10)/10 ; -3*sqrt(10)/10 - 2 ]
```

```
x2 = 2×1
     493/721
   -2126/721
```

```
%Insert points into function to get values
f(x1(1),x1(2))
```

```
ans =
    6569/1405
```

```
f(x2(1),x2(2))
```

```
ans =
    1975/114
```

Point x2 is minimum with value of 1975/114

Point x1 is maximum with value of 6569/1405

Exercise 3

a) No

b) No

c)

d) Searching for optimal solution in a finite set of potentional solutions.

Examples: Sellers problem (From internet)

A sales rep sells 12 types of vacuum cleaners. During one day he has to visit 9 customers by car to demonstrate his products.

Problem 1: What is the shortest route? ("Traveling Salesman problem")

Problem 2: All the 12 models do not fit in the car. What models to take? ("Knapsack Problem") Given data:

- distances between locations

- trunk size of the car and sizes of models

- expected profit / utility for demonstrating each model

e) quasi-newton uses approximate of hessian which is "cheaper" to caluclate than Newton which use exact hessian.

f) (i)

Best way to develop representation us by use of bits because the ycan be very easily mutated by just switching them. we have integers to 7 which we can fit into 3 bits but we have also "+/-" mark hre so we can add bit for that (first bit) so we have total of 4 bits

for example: (3,-2) => [0011,1001] where first but is 0 for + and 1 for - other 3 bits are actual number

(ii)

Matrix of current grid with kinda simulated heat values could be created for this exercise but no time. same as for exercise (f-iii)

Goal is to maximize heat for givent 2 points from all possible points.

o = @(x1,x2) ....

(iii)

Function should return heat of current position (Higher == better)

f = @(x1,x2) return current heat

(IV)

Parent 1: [0011,1001]

Parrent 2: [0001,1111]

Crossover in the middle:

Child: [0011,1111]

Exercise 4

```
% Given:
clear;
c1 = [  0 1;
        1 1]
```

```
c1 = 2×2
       0
       1
```

```
[k,NC(1)] = size(c1);

c2 = [ 3 3 4;
       1 2 2]
```

```
c2 = 2×3
       3              3
       1              2
```

```
[k,NC(2)] = size(c2);

X = [1 3 2;
    2 0 1]
```

```
X = 2×3
       1              3
       2              0
```

```
[k,NX] = size(X);
[k,NX] = size(X);
K=2
```

```
K =
       2
```

```




%% a) NC
% Calculate class means along each dimension
mean1 = mean(c1,2)
```

```
mean1 = 2×1
       1/2
       1
```

```
mean2 = mean(c2,2)
```

```
mean2 = 2×1
      10/3
       5/3
```

```
% Calculate euclidean distances from class means
x_c1_dist = sqrt(sum((X-mean1)'.^2, 2))';    % Norm of each column
x_c2_dist = sqrt(sum((X-mean2)'.^2, 2))';    % Norm of each column
nc_dist = [x_c1_dist; x_c2_dist];

% Classify samples to the nearest centroid
[nc_min_dist,nc_classification] = min(nc_dist);
```

```
nc_classification
```

```
nc_classification = 1×3
      1            2
```

```
%% b) NNC
% Calculate the distance to all class samples
nn_dist = zeros(2,NX);
for i=1:NX
    % Calculate distance to samples of class 1
    dist_c1 = [];
    for j=1:NC(1)
       dist_c1(j) = norm(X(:,i)-c1(:,j));
    end
    % Find the nearest class 1 neighbor
    nn_dist(1,i) = min(dist_c1);

    % Calculate distance to samples of class 2
    dist_c2 = [];
    for j=1:NC(2)
       dist_c2(j) = norm(X(:,i)-c2(:,j));
    end
    % Find the nearest class 2 neighbor
    nn_dist(2,i) = min(dist_c2);
end
nn_dist;

% Classify samples to the nearest neighbor
[nn_min_dist,nn_classification] = min(nn_dist);
nn_classification
```

```
nn_classification = 1×3
      1            2
```

c) byas-based

```
[k,N(1)] = size(c1);
[k,N(2)] = size(c2);
a_priori = [];
for i=1:K
   a_priori_prob(i) = N(i)/sum(N);
end
a_priori_prob;
% Calculate class means along each dimension
mean1 = [mean(c1(1,:));
        mean(c1(2,:))];

mean2 = [mean(c2(1,:));
        mean(c2(2,:))];
% Calculate euclidean distances from class means
x_c1_dist = vecnorm(X-mean1);
x_c2_dist = vecnorm(X-mean2);
```

```
% Calculate the class-conditional probability density
p_x_given_c1 = x_c1_dist ./ (x_c1_dist + x_c2_dist);
p_x_given_c2 = x_c2_dist ./ (x_c1_dist + x_c2_dist);

p_x_given_c = [p_x_given_c1;
               p_x_given_c2];
% Multiplying the a priori probability and the posteriori distribution element wise
P_c_given_x = a_priori_prob' .* p_x_given_c;

% Compare probabilties by evaluating difference
diff = P_c_given_x(1,:) - P_c_given_x(2,:);

class = zeros(1,NX);
for i=1:NX
    if diff(i) > 0
       class(i) = 1;
    elseif diff(i) < 0
        class(i) = 2;
    else
        class(i) = 0;   % Classes are equally likely
    end
end
class
```

```
class = 1×3
     2            1
```

nc_classification = 1×3

1 2 2

nn_classification = 1×3

1 2 1

class = 1×3

2 1 2

d)

From output classification we can see that classifications are different for each of the classifier.

NC and NN classifiers missmatched only in one of the test point while bayes based classifier classified data odifferently.

This can be also caused because of small train data-set

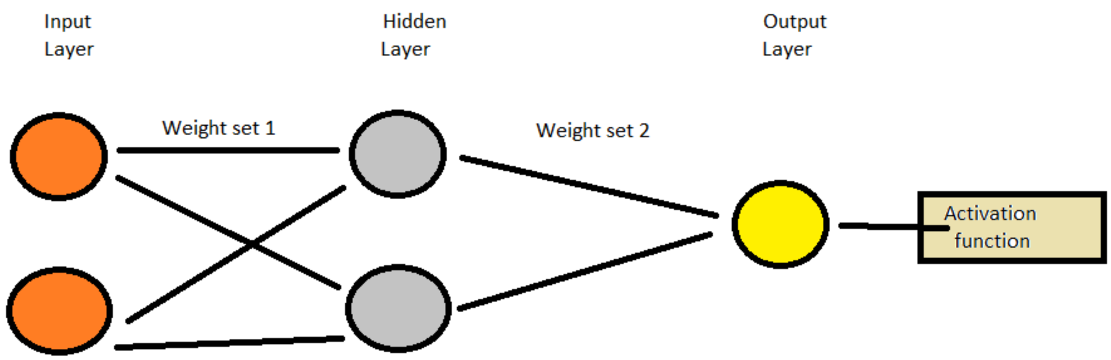
Exercise 5

a)

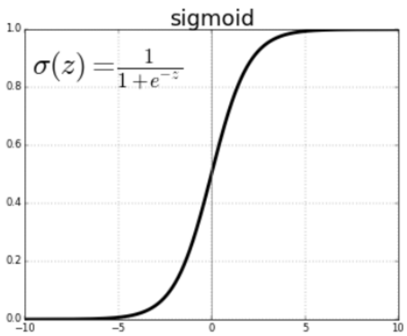
NO - LDA is preffered to use for more than 2 classes.

b)

c)



sigmoid activation function:



d) Cluster is not linear

Exercise 6

```
%a) no train data set ?

X = [3;7;8;9]; %test data



%b) but not trained classifier (example how to calculate risk)
loss = [0 0.3;
        0.2 0]
```

```
loss = 2×2
       0
```

```
    1/5
```

```
risk = loss*P_c_given_x
```

```
risk = 2×3
     152/1245       589/8456       199/
     237/9208       169/3446       155/
```

```
class = zeros(1,NX);
for i=1:NX
    if risk(1,i) < risk(2,i)
       class(i) = 1;
    elseif risk(1,i) > risk(2,i)
        class(i) = 2;
    else
        class(i) = 0;   % Classes are equally likely
    end
end
class
```

```
class = 1×3
       2            2
```