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LECTURE 7

ADVANCED CODING FOR WIRELESS CHANNELS - CONVOLUTIONAL CODING

JOHN ROHDE (JR@IHA.DK)
M.SC.EE, PHD

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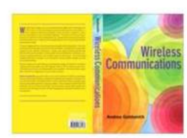

Review of Last Lecture

- Equalization – a powerful tool to cope with ISI and noise. For situations with delay spread large causing self-interference.
- Linear equalizers
  - ZF: Removes all ISI (with long enough filter), but can introduce severe noise
  - MMSE: Compromise where both ISI and noise is equalized out. Most popular of linear types.
- Non-linear:
  - MLSE: The optimum, based on estimation on sequence of data, the complixity is very high – can be complicated in real life.
  - DFE: Based on feedback principles. Complexity much lower than MLSE, solve the issue of noise much better than the linear equalizers. The drawback it that the feedback error can propagate. This cannot be corrected through channel coding.
- Choice of equalization is a compromise between complexity and speed (convergence and tracking).

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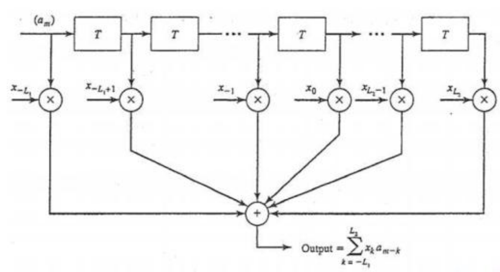
Traversal Structure - Equivalent Discrete-time Channel Model

- In practice ISI is assumed to span a finite number of symbols
- Consequently ISI can be viewed as passing the data sequence $\{a_m\}$ through a FIR filter with coefficients $\{x_n, -L_1 < n < L_2\}$
- This filter is the equivalent discrete-time channel filter



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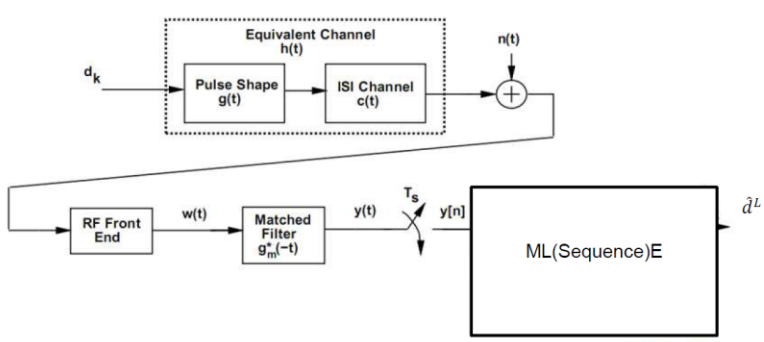
Non-linear Equalizers - MLSE (I)

- Maximum-likelihood-sequence-estimation avoids the problem of noise enhancements since it does not use equalization filter.
- It estimates <u>the sequence</u> of transmitted symbols.



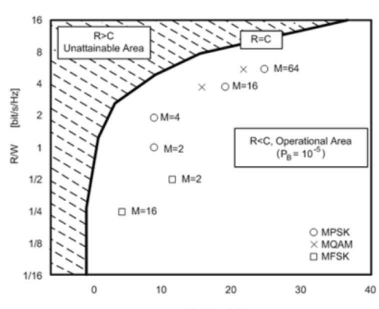
Topics of Todays Lecture – Convolutional Codes

- Coding Overview, short review of block codes
- Convolutional Codes
  - Code Characterization – Encoding
  - Maximum Likelihood Decoding
  - Viterbi Algorithm
  - Distance Properties
  - State Diagrams and Transfer Functions
  - Error Probability for Convolutional Codes

Channel Coding - a feature that improves the bit estimation

Theoretical Channel Capacity

- Shannon, 1948:



$$C = W \cdot \log_2\left(1 + \frac{S}{N}\right) \qquad [\text{bits/s}]$$

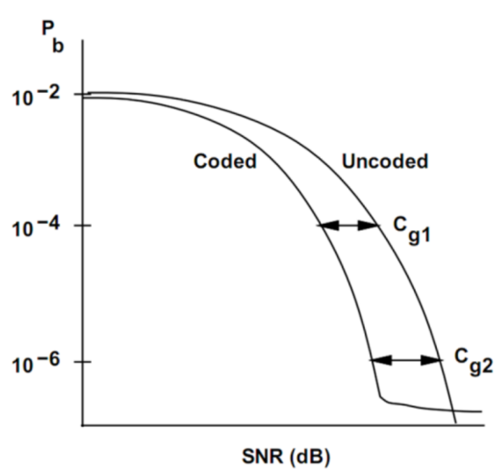
C = theoretical channel capacity
W = bandwidth in Hz
S = signal strength
N = noise

- Theoretically possible to transmit information at any rate R, where $R \le C$, with an arbitrary small error probability by using a sufficiently complicated coding scheme.

Coding Gain

- Coding allows errors introduced by transmission of wireless signal to be detected and maybe even corrected.
- We deal with codes for AWGN as well as codes for fading channels
  - AWGN codes typically does not work for fading channels, where errors happen in bursts !
- Link Capacity: You will learn codes that can go very close to the theoretical Shannon capacity limit with a reasonable level of complexity
- Performance Measures: The amount of error reduction of a specific code is typically given by its coding gain in AWGN and its diversity gain in a fading environment.
  - Coding gain in AWGN:
    - Definition: The amount that the bit energy or signal-to-noise power ratio can be reduced for a given BER or FER probability.
    - Generally the gain is a function of the Euclidean distance of the code, which equals the minimum distance in signal space between codewords or error events. Codes for AWGN maximize the Euclidian distance.

Coding Gain in AWGN Channels



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Ultra Short Review - Linear Block Codes

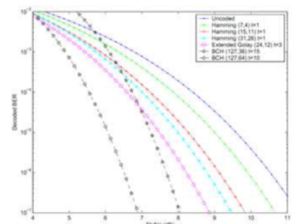

Figure 8.4

You should all be familiar with these:

- Hamming Distance
- d_{min}
- Generator Matrix
- Parity-check
- Syndrome Testing
- Cyclic Codes
- Hard Decision Decoding
- Soft Decision Decoding
- Linear Block Codes (Hamming, Golay, BCH)
- Non-binary Block Codes: Reed Solomon Code

YOU CAN READ PAGE 230-246 !

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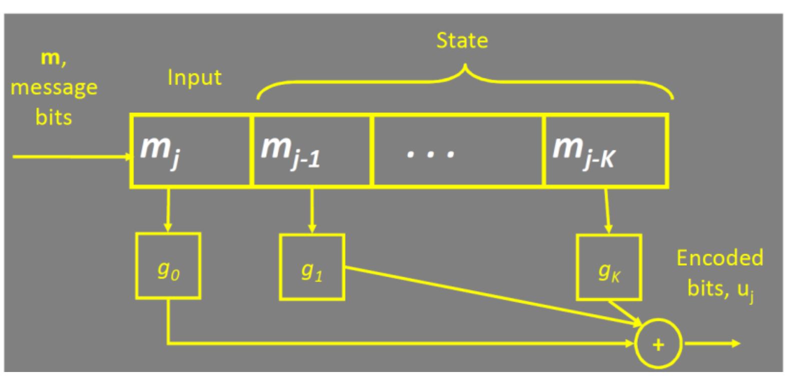
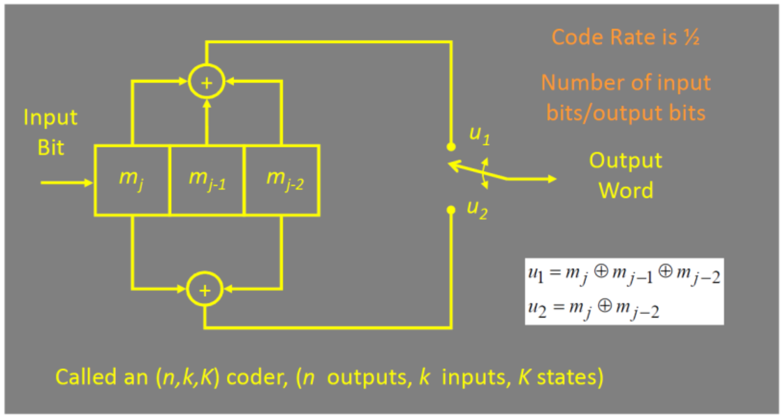
Structure of the Convolutional Coder

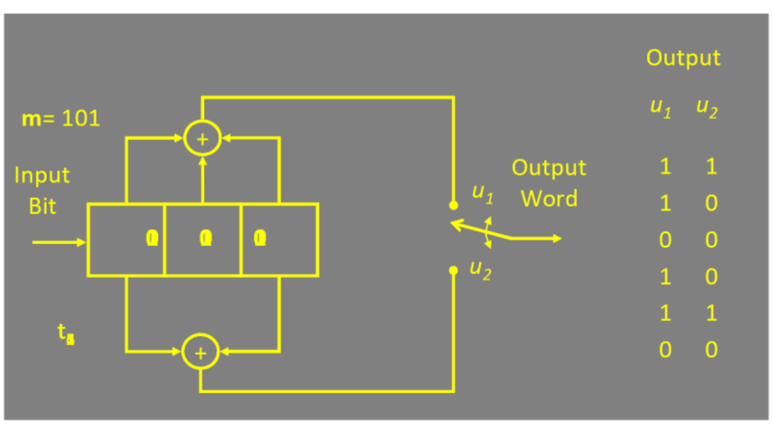


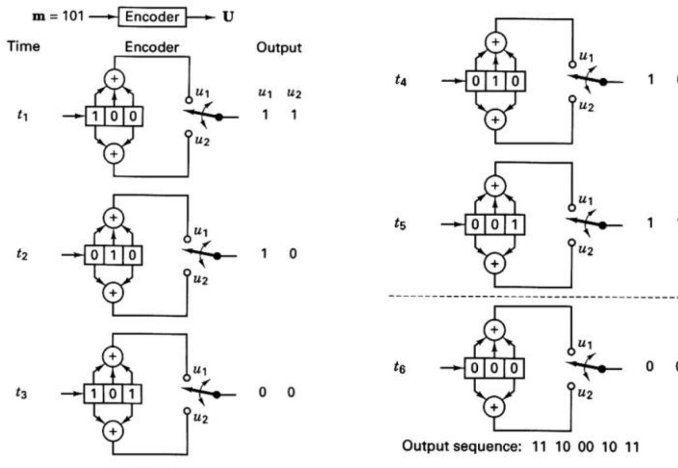
$$x_j = m_{j-K} g_K \oplus \cdots \oplus m_{j-1} g_1 \oplus m_j g_0$$
$$= \sum_{i=0}^{K} m_{j-i} g_i \quad \text{(modulo 2)}$$

Convolutional Coder - Example



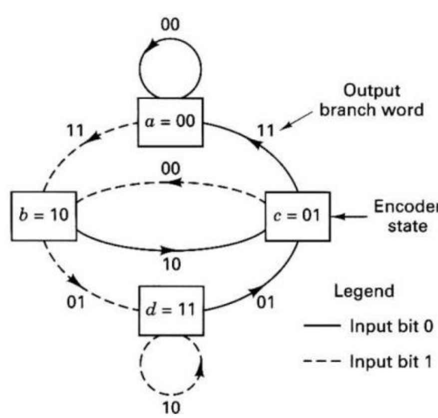
Convolutional Code - Example





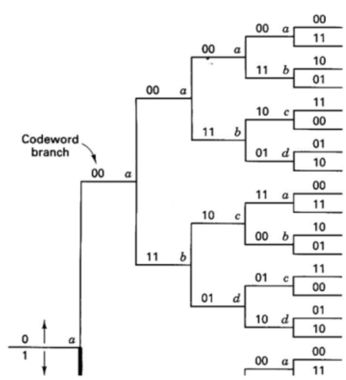
The Finite State Machine Notation

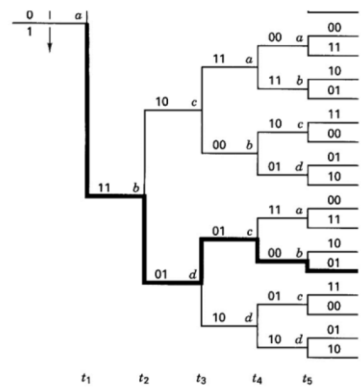
- Each possible state is viewed as a node
- Input bit determines the next state



The Code Tree

Beacuse of finite length of shift register the tree starts repeating itself (length 3)





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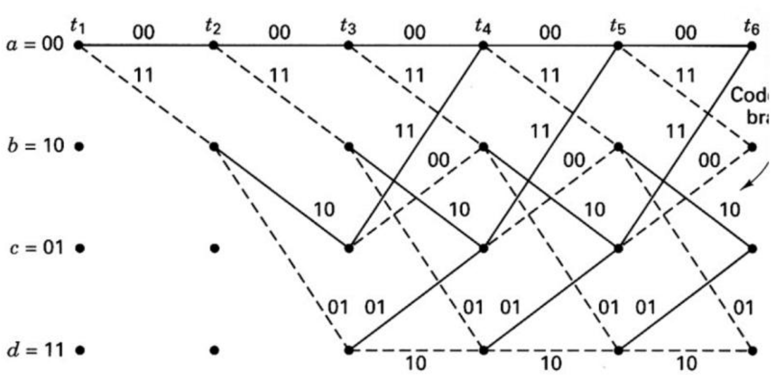
The Trellis Diagram



0 input ———— 1 input - - - - - -

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Exercise

- Consider a (3,1,3) convolutional code
- List the possible states, determine the state transitions produced by m_j=0 and m_j=1.
- Construct and label the state diagram taking the encoded output bits to be:

$$u_1 = m_j$$

$$u_2 = m_{j-2} \oplus m_j$$

$$u_3 = m_{j-3} \oplus m_{j-1}$$

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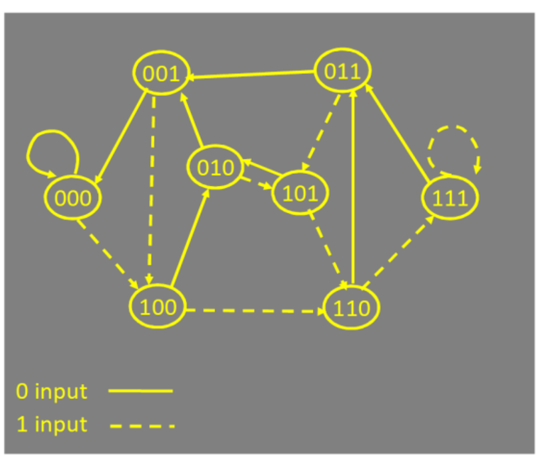
Exercise Solution



Encoding exercise / solution :

Encode the sequence: ‘1 1 0 1’

Use the encoder from the last exercise :

$$u_1 = m_j$$
$$u_2 = m_{j-2} \oplus m_j$$
$$u_3 = m_{j-3} \oplus m_{j-1}$$

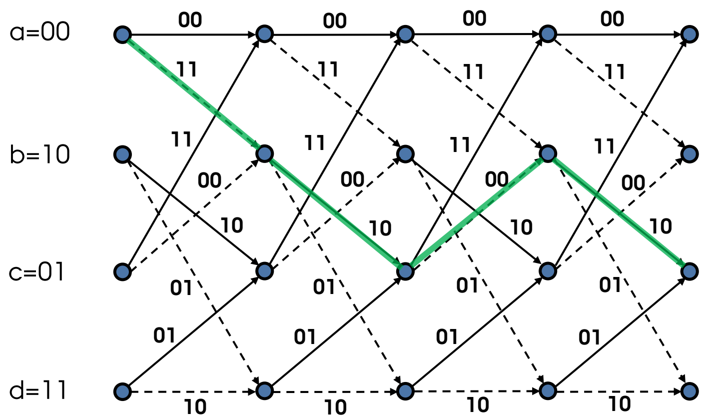
Message Bit	Codeword
1	110
1	111
0	011
1	101
0	000
0	010
0	001
0	000

Free Distance and Coding Gain

- Recall, error control ability of block codes depends on its minimum distance from a codeword
- Convolutional codes do not divide up neatly,
  - Can compute distance of <u>entire sequence</u>, *X*, from possible sequences
  - Or simplify the problem by using the zeros appended at the end of a message to limit the number of path possibilities.

Encoding - example

Original sequence: 1010
Encoded sequence: 11100010



Encoding - once again ! (1)

- Design the coder to correct the most likely error patterns (i.e. the patterns with the fewest errors)
- This strategy is optimum in the sense that it minimizes word-error probability.
- Corresponds to choosing the code word with the smallest Hamming distance from the received vector

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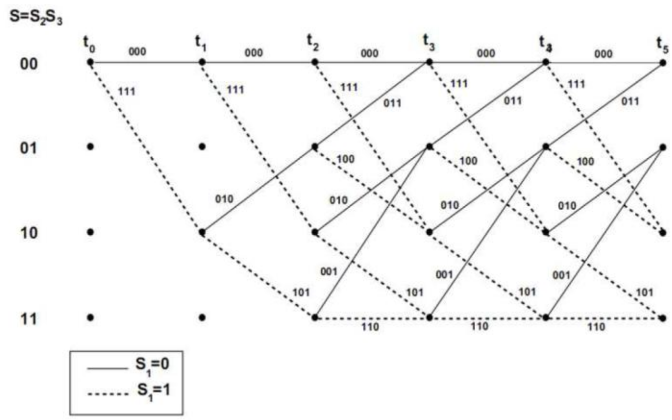
Encoding - once again ! (2)

- Example from book:



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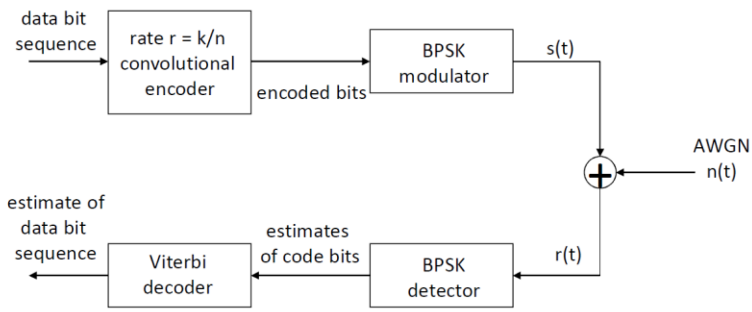
Encoding - once again ! (3)

- Encoding example from book:

Example 8.7: Consider the convolution code represented by the trellis in Figure 8.7. For an initial state $S = S_2S_3 = 01$, find the state sequence S and the encoder output C for input bit sequence $\mathbf{U} = 011$.

Simplified Model Example – working on sequences



Decoding – Maximum-Likelihood Principles (1)

- Unlike for and (n,k) block code, where Maximum-likelihood detection is equivalent to finding the length-n codeword closest to the received length-n codeword, convolutional codes is about finding the most likely sequence of coded symbols (**C**) given the received sequence (**R**).
- In our context this is equivalent to finding the shortest path (or route) through the trellis using the likelihood - log(probability(**R**|**C**)) - function. Each branch is calculates and aggregated over the complete length of the received sequence.
- Both hard and soft decision decoding principles can be used.

Decoding – Maximum-Likelihood Principles (2)

- Problem: ML decoders need to examine entire received sequence of N bits to find a valid path (of 2^N options) close to that sequence

- Viterbi Algorithm is the most widespread solution !
  - It is a ML method, optimum but computationally and storage-wise intense

Decoding using the Viterbi Algorithm

Remember: An (n,k,K) coder = (n outputs, k inputs, K states)

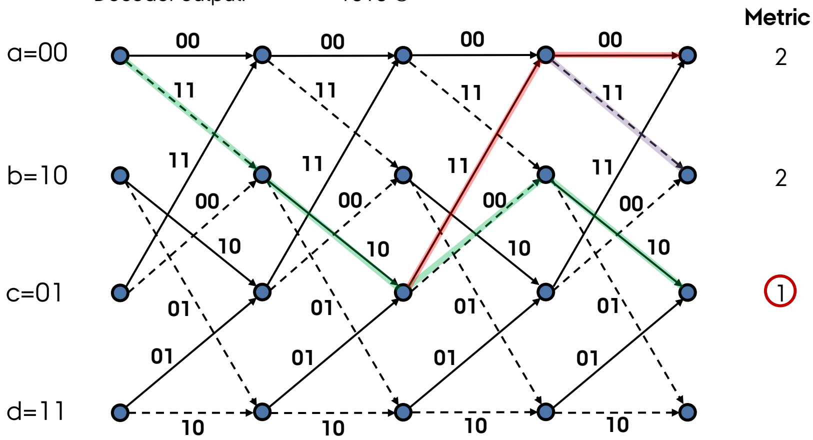
- Exhaustive search is just silly (to many possibilities)

- Viterbi algorithm applies ML principles to limit to the comparison of $2^{k(K-1)}$ *surviving paths*

- Each path has a metric based on Hamming distance between received sequence and that path (the code output).

- The concept is, by systematically removing the paths from the pool of potential solutions that cannot achieve the highest log-likelihood (the path metric), to narrow down the number of possibilities to select from when finding the ML solution.

Original sequence: 1010
Encoded sequence: 11100010
Received : 11101010
Decoder output: 1010 ☺



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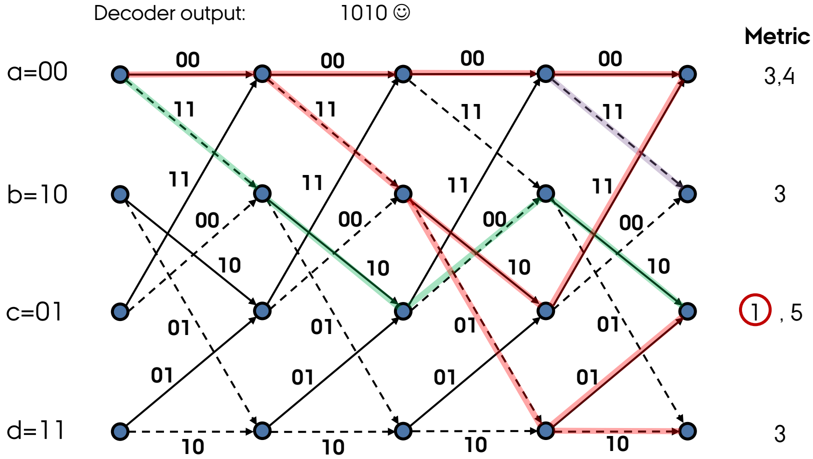
Original sequence: 1010
Encoded sequence: 11100010
Received : 10100010
Decoder output: 1010 ☺



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Viterbi Decoder Issues

- Computationally expensive, increases exponentially in (K-1) and linearly with *N*
  - Two paths/node
  - Store $2^{k(K-1)}$ surviving paths with *N* branches
- Delay in decoding until the ML path is found
- For N>>1, can eliminate paths with same origin and different metrics (metric divergence)
- Can read *k* bits and delete paths as soon as *5(K-1)n* message bits are processed

Rule of thumb !

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Error Probability for Convolutional Codes

- Convolutional codes are linear codes.
- Therefore, error probability can be found by first assuming that the all-zero sequence is transmitted and then determining the probability that the decoder decides in favor of a difference sequence.
- A.G. presents formulas for soft decision (8.71) and hard decision decoding (8.76).

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GSM Channel Encoding
– in principle simple and old (but not that simple) !

- Speech or user data bits are protected using two concatenated codes. GSM has blocks of 260 bits, containing 50 class Ia bits, 132 class Ib bits and 78 Class II bits. The first step involves block coding. Three parity bits are added to allow error detection for the first 50 class Ia bits. 4 bits are added to the Class Ib bits. The 189 Class Ia and Ib bits are then convolutionally encoded at rate 1/2, i.e., the generate 378 code bits. The coder has constraint length K = 5. Five consecutive bits are used to create the transmit bits. Every time one bit is fed into the encoder, two bit appear at the output.

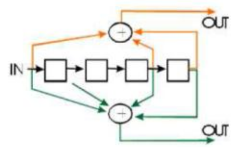

Figure: Convolutional encoder for transmission in GSM. Each input bit generates two output bits, determined by the input bit and four previous bits.

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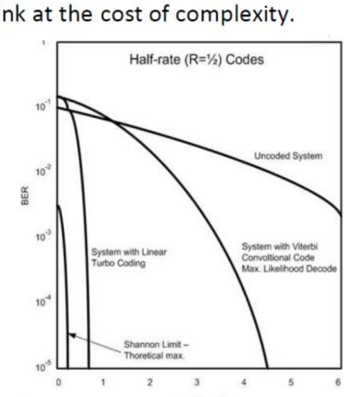
Close to the Shannon Limit

- Prime wireless engineering research targets has for the last decades been do create a code as close to the Shannon limit as possible.
- To increase the transmission eff. of the link at the cost of complexity.

Year	Rate ½ Code	SNR Required for BER < 10^{-5}
1948	Shannon theoretical	0 dB
1967	BCH (255,123) code	5.4 dB
1977	Convolutional Code	4.5 dB
1993	Iterative Turbo Code	0.7 dB
2001	Iterative LDPC Code (block size of 10^7)	0.0245 dB

2008 : Using non-linear Turbo codes (for certain scenarios with high order modulation) can get similar performance and reduce implementation complexity !



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Main Points

- Coding Overview – linear block codes
- Convolutional Codes
  - Sequential Encoding
  - State-diagrams
  - Code Tree and Trellis Diagrams
  - Maximum Likelihood Decoding
  - Viterbi Algorithm
  - Distance Properties
- Performance
  - Trade-off towards complexity and memory storage.