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DEPARTMENT OF ENGINEERING

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LECTURE 5

PERFORMANCE OF DIGITAL MODULATION OVER WIRELESS CHANNELS

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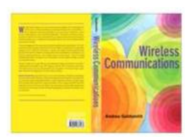

Review of Last Lectures (1)

STATISTICAL model of a Narrow-band fading Channel:

- Definition of a narrowband fading channel : $T_m \ll T$
- Statistical multipath model leads to a time-varying channel impulse response
- Narrowband model has in-phase and quadrature components that are zero-mean stationary Gaussian processes
- Auto and cross correlation depends on angle-of-arrival of multipath
- Uniform scattering makes autocorrelation of in-phase and quadrature follow Bessel function
- Signal components de-correlate over half wavelength
- Cross correlation is zero (in-phase/quadrature independent)
- PSD of received signal has bowel shape centered at carrier frequency: useful for simulations

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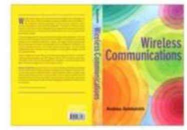
Review of Last Lectures (2)



STATISTICAL model of a Wideband fading Channel:

- Definition of a wideband fading channel : $T_m >> T$
- Wideband fading channels are much more complicated to deal with than narrowband fading channels (e.g. 4G/LTE vs. WSN).
- Scattering function characterizes the r.m.s. delay and Doppler spread. Key parameters for system design.
- Power delay profile (PDP) is the average power associated with a given multipath delay.
- Delay spread defines maximum delay of significant multipath components (time domain). Inverse is coherence bandwidth of channel (frequency domain), describing the frequency span (Δf) with a constant channel.
- Doppler spread defines maximum non-zero Doppler (frequency domain), its inverse is coherence time (time domain) which is the time duration over which the channel impulse response is considered to be NOT varying

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Topics of Todays Lecture

Describe the impact on digital modulation performance of noise, flat fading, frequency-selective fading and Doppler

- Performance measures can here be either :
  - Probability of error (defined relative to either symbols or bits)
  - Outage probability (defined as the probability that the instantaneous SNR falls below a given threshold)
- Flat fading can cause dramatic increase in either the BER or the outage probability
- Frequency-selective fading gives rise to ISI, which cause an irreducible error-floor in the received signal.
- Doppler cause spectral broadening, which leads to adjacent channel interference (small at low speeds) and an irreducible error-floor in systems with differential phase encoding (e.g. DPSK), since the relative phase between symbols de-correlates over time.

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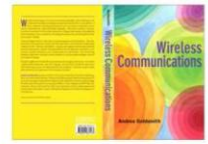

Goal of this Lecture

- Performance in the AWGN Channel
- Performance in a Fading Channel
- Performance Impact of the Doppler Spread
- Inter-Symbol-Interference

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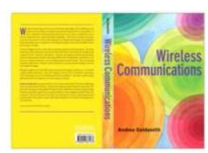


Goal of this Lecture – detailed !

- Performance in the AWGN Channel
  - Signal-to-noise Power Ratio and Bit/Symbol Energy
  - Error probability Approximation for Coherent Modulations (BPSK/QPSK, MPSK, MPAM/MQAM, FSK/CPFSK)
  - Error probability for Differential Modulation
- Performance in a Fading Channel
  - Outage Probability
  - Average Probability of Error
  - Combined Outage and Average Error Probability
- Performance Impact of the Doppler Spread
- Inter-Symbol-Interference (ISI)

Essential Assumption I have made !

- You are familiar with the content of Chapter 4:
  - Capacity of Wireless AWGN channels, Capacity flat fading channels including Shannon capacity, Capacity of frequency-selective channels

- You are familiar with the content of Chapter 5:
  - Signal space analysis and decision regions, Classic modulation techniques, Pulse shaping, Symbol synchronization and Phase estimation.

A bit of history !

- What is information ? - somewhat unclear until 1948

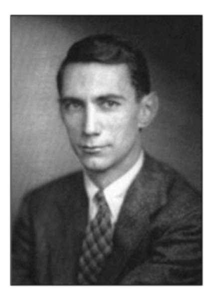
- INFORMATION THEORY is generally considered to have been founded in 1948 by Claude E. Shannon

- Shannon presents a formal way of defining *information* using the term Entropy [bit]. In principle equally relevant as having *energy* as a physical term

- Shannon creates a number of interrelated major result with respect to information theory. Primarily within storage, coding and **capacity**.

- From the applied engineering point of view:

  The **noisy-channel capacity** result is vital !



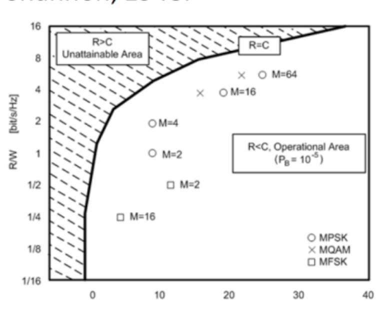
Simplified Communication Model

For wireless the radio link is extremely critical. The system bandwidth is limited, noise and interference is added, the coverage area is limited. Because of this we have a lower throughput, transmission errors and increased complexity & cost.

Definition:	Channel capacity is the max. data rate at which error-free communication over the channel is performed.

Theoretical Channel Capacity

- Shannon, 1948:



$$C = W \cdot \log_2\left(1 + \frac{S}{N}\right) \qquad [\text{bits/s}]$$

$C =$ theoretical channel capacity
$W =$ bandwidth in Hz
$S =$ signal strangth
$N =$ noise

- Theoretically possible to transmit information at any rate R, where $R \leq C$, with an arbitrary small error probability by using a sufficiently complicated coding scheme.

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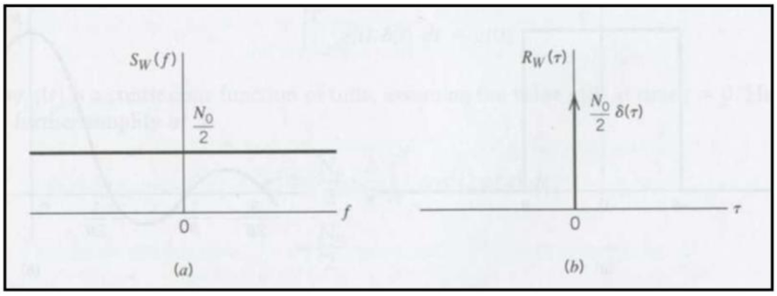
Additive White Gaussian Noise - Characteristics



A nice and not too impossible channel to deal with !

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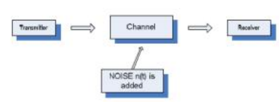
AWGN Channels – SNR and SINR (1)



- Definition of Signal-to-noise-ratio (SNR):

$$SNR = \frac{P_r}{N_0 B}$$

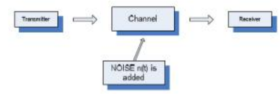
  - ...where P_r is the received signal power,
  - B is the bandwidth of the complex envelope u(t) of the transmitted signal s(t)
  - n(t) is the noise added in the channel, which can be described by a white Gaussian random process with mean zero and power spectral density (PSD) equal to $N_0/2$.

- In systems with interference (undesired signals in same frequency band) we use the signal-to-interference-plus-noise-power-ratio (SINR).
- With the assumption that the interference is distributed Gaussian we can approximate the SINR to:

$$SNR = \frac{P_r}{N_0 B + P_I}$$

  - where P_I is the av. power of the interference.

AWGN Channels – SNR and SINR (2)



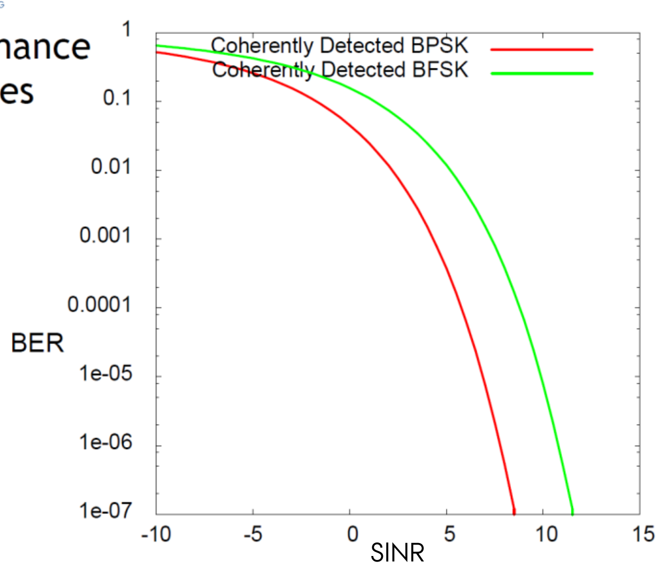
- SNR expressed via signal energy per bit (E_b) or the per symbol (E_s)

$$SNR = \frac{P_r}{N_0 B} = \frac{E_s}{N_0 B T_s} = \frac{E_b}{N_0 B T_b}$$

- The book also uses notation: $\gamma_s = \frac{E_s}{N_0}$ and $\gamma_b = \frac{E_b}{N_0}$ for the SNR/symbol and SNR/bit.

- For performance measures we are normally interested in the bit-error-probability (P_b) as a function of γ_b

Performance Examples



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BER as a function of E_b/N_0 BPSK and QPSK

Still for the AWGN only channel !

For BPSK: $P_b = Q(\sqrt{2\gamma_b})$ (6.6)

For QPSK: $P_s = 2Q(\sqrt{\gamma_b})$ (6.12)

...where Q() is defined as the probability that a Gaussian random variable (X) with mean 0 and variance 1 is greater than z:

$$Q(z) = p(X > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \qquad (5.39)$$

READ APPENDIX B again for a better understanding of random variables if necessary !

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Performance Approximations – coherent modulation

Modulation	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BFSK:		$P_b = Q\left(\sqrt{\gamma_b}\right)$
BPSK:		$P_b = Q\left(\sqrt{2\gamma_b}\right)$
QPSK,4QAM:	$P_s \approx 2\,Q\left(\sqrt{\gamma_s}\right)$	$P_b \approx Q\left(\sqrt{2\gamma_b}\right)$
MPAM:	$P_s \approx \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\bar{\gamma}_s}{M^2-1}}\right)$	$P_b \approx \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{6\bar{\gamma}_b \log_2 M}{(M^2-1)}}\right)$
MPSK:	$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin(\pi/M)\right)$	$P_b \approx \frac{2}{\log_2 M} Q\left(\sqrt{2\gamma_b \log_2 M}\sin(\pi/M)\right)$
Rectangular MQAM:	$P_s \approx \frac{4(\sqrt{M}-1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4(\sqrt{M}-1)}{\sqrt{M}\log_2 M} Q\left(\sqrt{\frac{3\bar{\gamma}_b \log_2 M}{(M-1)}}\right)$
Nonrectangular MQAM:	$P_s \approx 4Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\bar{\gamma}_b \log_2 M}{(M-1)}}\right)$

Table 6.1: Approximate Symbol and Bit Error Probabilities for Coherent Modulations

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Error Probability for Differential Modulation

Still for the AWGN-only channel !

For DPSK: $P_b = \frac{1}{2} e^{-\gamma_b}$ (6.40)

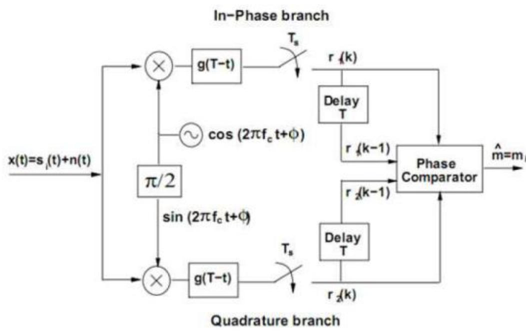

Figure 5.20: Differential PSK Demodulator.

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Alternative Q-Function Representation

- Recall that Q() is defined as the probability that a Gaussian random variable (W) with mean 0 and variance 1 is greater than z:

$$Q(z) = p(X > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \qquad (5.39)$$

- Not easy to work with (integrate infinite etc.). An alternative representation given to simplify the probability of error calculation for AWGN channels:

$$Q(z) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left[\frac{-z^2}{2 \sin^2 \varphi}\right] d\varphi \qquad (5.43)$$

- (5.43) is very useful for analysis with fading and diversity included !

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Adding Fading to the Channel – increasing the Challenge….

- For AWGN the BER depends on the SNR (or equivalently γ_s).
- With fading the received signal power varies randomly over distance or time. Therefore in this situation γ_s is a random variable with distribution : $p_{\gamma_s}(\gamma)$. The variable $P_s(\gamma_s)$ is therefore also random.
- Three different performance criteria can be used to describe the random variable P_s
  1. the outage probability, P_{out}
     defined as the probability that γ_s falls below a given value equivalent to the maximum allowable P_s
  2. the average error probability, $\overline{P}_s$
     averaged over the distribution of γ_s
  3. combined average error probability and outage
     defined as the average error probability that can be achieved some percentage of time or some percentage of spatial locations

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Calculating the **Outage Probability** - in Fading Channels

Outage probability: $$P_{out} = p(P_s > P_{target}) = p(\gamma < \gamma_{target})$$

…where γ_{target} is the minimum SNR required for acceptable performance

In Rayleigh fading channel:

- the average SNR is : $$\overline{\gamma}_s = \frac{\gamma_{target}}{-\ln(1-P_{out})} \qquad (6.48)$$

Example 6.4: Determine the required $\overline{\gamma}_b$ for BPSK modulation in slow Rayleigh fading such that 95% of the time (or in space), $P_b(\gamma_b) < 10^{-4}$.

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Calculating the **Average Error Probability** (1)

- in Fading Channels

- The average error probability is a performance indicator when the symbol time (T_s) is equal to the coherence time (T_c) of the channel ($T_s = T_c$) making the signal fading roughly constant over a symbol period.
- The average probability of error for BPSK (and BFSK) in a Rayleigh fading channel is (coherent):

$$\overline{P_b} = \frac{1}{4\overline{\gamma_b}} \qquad (6.58\ /\ 6.59)$$

- The average probability of error for DPSK in a Rayleigh fading channel is (non-coherent):

$$\overline{P_b} = \frac{1}{2\overline{\gamma_b}} \qquad (6.60)$$

- That is from BPSK to DPSK a 3-dB loss is introduced in the Rayleigh fading channel !

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Calculating the **Average Error Probability** (2)

- in Fading Channels

Some conclusions in Rayleigh fading channels !

- For binary PSK, FSK and DPSK the bit error probability in AWGN decreases exponentially with increasing SNR (γ_s)
- However, in fading the bit error probability decreases just linearly for all modulation types with increasing average SNR ($\bar{\gamma}_b$)
- The consequence is that the power necessary to maintain a given bit error probability is much higher in fading channels than in AWGN channels.

See following graphs for BPSK and MQAM

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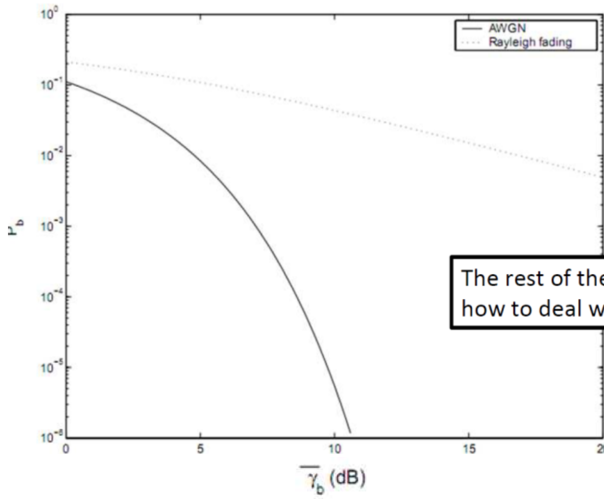
Some Graphical Examples - BPSK



The rest of the course is about how to deal with this !

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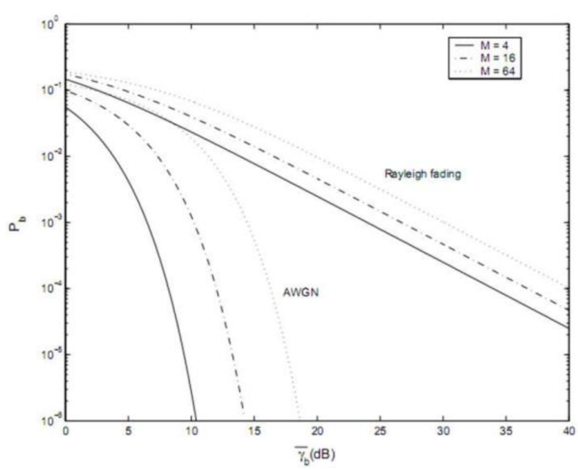
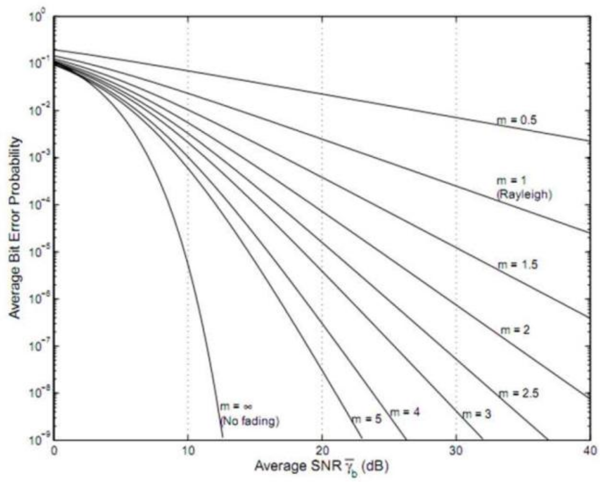
Some Graphical Examples - MQAM

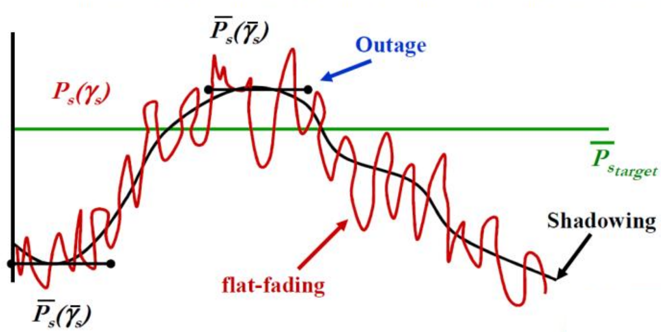


Some Graphical Examples – Nakagami fading



How to Combine Outage & Average Error Probability

- When the environment is a superposition of fast and slow fading (e.g. log-normal and Rayleigh fading) a common performance metric is **the combined outage and average error probability**.
  - Outage occur when the slow fading falls below the min. target value
  - The average performance in non-outage is obtained by averaging over the fast fading.



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The Irreducible Error-floor **from Doppler** (1)

- High Doppler causes channel phase to de-correlate between symbols
- Leads to an irreducible error floor **for systems with differential modulation/detection**, that is increasing the power does not reduce error
- For the uniform scattering model and Rayleigh fading DPSK has an irreducible error-floor of :

$$\bar{P}_{floor} = 0.5(\pi f_D T_b)^2 \qquad (6.93)$$

NOTE:
the error floor decreases as a function of the datarate (R=1/T_b), this a very rare phenomena in digital communications (performance increase with increased datarate) !

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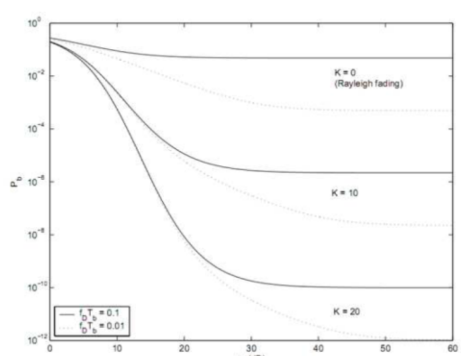
The Irreducible Error-floor **from Doppler** (2)

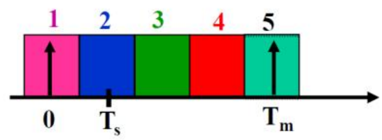
Example (Fig. 6.4) with error probability of DPSK in fast Rician fading, for uniform scattering and different values for $f_D T_b$.

Error-floor starts dominating at $\bar{\gamma}_b = 15\ dB$ for the Rayleigh fading case (K=0) and as K increases the value of $\bar{\gamma}_b$ where the error-floor starts dominating also increases



The Irreducible Error-floor from ISI (1)

- Delay spread exceeding a symbol time causes ISI (self interference).
- To avoid ISI it is required that $T_s >> T_m$ ($R_s << B_c$)

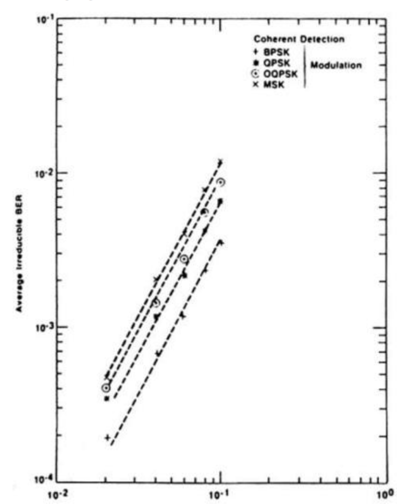


- ISI leads to irreducible error floor, since increasing the signal power increases the ISI power equivalently

The Irreducible Error-floor from ISI (2)

The error floor can be found for each modulation scheme from simulations. The absolute level is sensitive to the rms delay spread

d = normalised delay spread (σ_{T_m}/T_m) for BPSK, QPSK, MSK and OQPSK



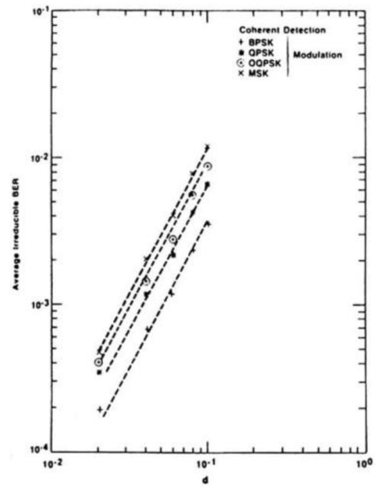
The Irreducible Error-floor from ISI (3)

From figure 6.5 we see:

With a typical urban rms delay spread of 2.5 micro-sec. If we have to keep $\sigma_{T_m} < 0.1T_s$ the datarate cannot exceed 40 kbaud !

In rural areas the rms delay spread is in the range of 25 micro-sec, meaning a max. datarate of 4 kbaud !

WE NEED TO DEAL WITH THAT !!!!



Main Points

- In fading P_s is a random variable, characterized by average value, outage, or combined outage/average
  - Outage probability based on target SNR in AWGN.
  - Fading greatly increases average P_s.
  - Alternate Q function approach simplifies P_s calculation
- Doppler spread only impacts differential modulation causing an irreducible error floor at low data rates
- Delay spread causes irreducible error floor or imposes rate limits from ISI
- To make wireless communication system offering both mobility and high data-rate we need to combat flat and frequency-selective fading

THAT WILL BE THE FOCUS OF THE REMAINDER OF COURSE