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DEPARTMENT OF ENGINEERING

1. SEPTEMBER 2011

LECTURE 6

EQUALIZATION

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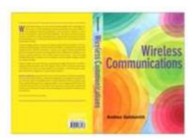


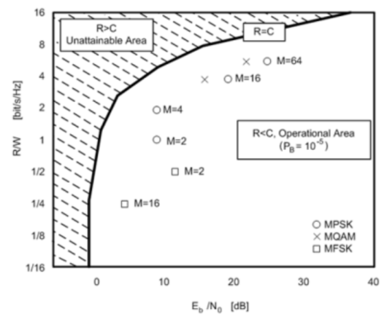
Review of Last Lecture

Performance of Digital Modulation over Wireless Channel

- AWGN by it self is "easy" !
- In fading P_s is a random variable, characterized by average value, outage, or combined outage/average
- Outage probability is based on a target SNR in AWGN.
- Fading greatly increases average P_s .
  - Alternate Q function approach simplifies P_s calculation
  - Doppler spread only impacts differential modulation causing an irreducible error floor at low data rates
  - Delay spread causes irreducible error floor or imposes rate limits from ISI

Theoretical Channel Capacity

- Shannon, 1948:

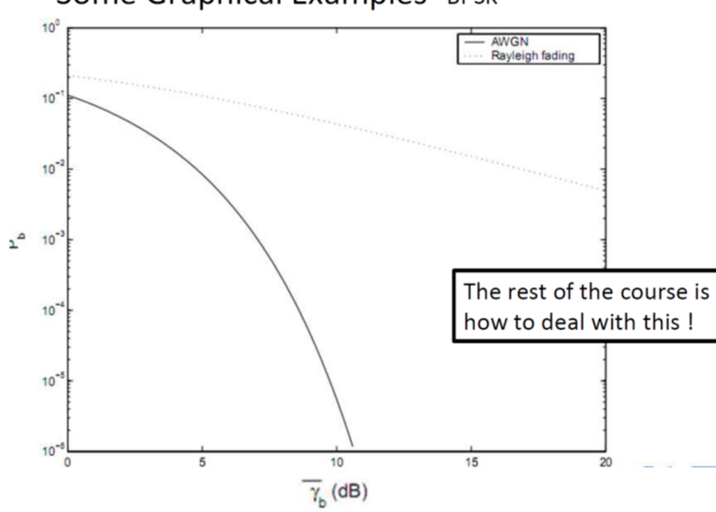


$$C = W \cdot \log_2\left(1 + \frac{S}{N}\right) \quad [\text{bits/s}]$$

$C =$ theoretical channel capacity
$W =$ bandwidth in Hz
$S =$ signal strangth
$N =$ noise

- Theoretically possible to transmit information at any rate R, where $R \leq C$, with an arbitrary small error probability by using a sufficiently complicated coding scheme.

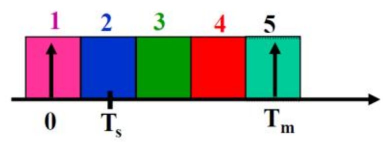
Some Graphical Examples - BPSK



The rest of the course is about how to deal with this !

The Irreducible Error-floor from ISI (1)

- Delay spread exceeding a symbol time causes ISI (self interference).
- To avoid ISI it is required that $T_s >> T_m$ ($R_s << B_c$)



- ISI leads to irreducible error floor, since increasing the signal power increases the ISI power equivalently

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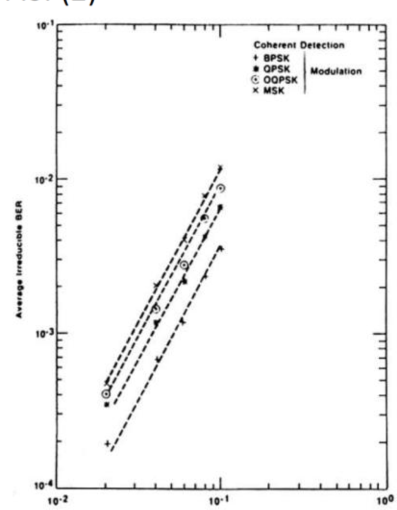
The Irreducible Error-floor from ISI (2)

The error floor can be found for each modulation scheme from simulations. The absolute level is sensitive to the rms delay spread

d = normalised delay spread (σ_{T_m}/T_m) for BPSK, QPSK, MSK and OQPSK



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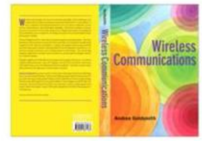
Topics of Todays Lecture - Equalizers



- Equalizer and the issue of minimizing Noise impact
- ISI-Free Transmission
- Linear Equalizers (Zero-forcing, Minimum Mean-Square Error)
- Maximum Likelihood Sequence Estimation
- Decision-Feedback Equalization
- Adaptive Equalizers: Training and Tracking

Your first tool to deal with some of the ISI in a challenging fading channel

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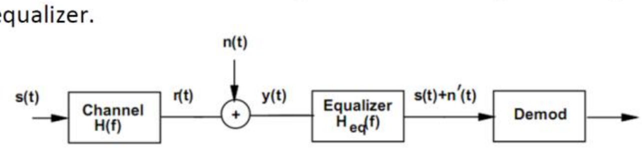
The Concept of Equalizers for Wireless Comm's (1)

- On the system level there are 2 major ways of dealing with a delay spread causing ISI, removing the influence
  1. Re-deign system to avoid it, we will see how that could be done in later lecture (OFDM)
  2. If we cannot do that we have to build a receiver that can "remove" the impact of a large delay spread in the received signal. An equalizer can do that using signal processing techniques – actually simply using advanced filtering techniques.
- Equalizer design must typically balance ISI mitigation with noise enhancements since both the signal and the noise pass through the equalizer.



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The Concept of Equalizers for Wireless Comm's (2)

- Equalizers can vary in complexity. For the equalizer to work properly is must use an estimate of the channel impulse or frequency response.
- Since the wireless channel vary over time, the equalizer must learn the response of the channel (training) and then update its estimate of the frequency response as the channel changes (tracking).
- This adaptive equalization can be extremely challenging in a rapidly changing channel.
- The equalizer can be implemented at baseband, the carrier RF or at an intermediate frequency. Most are implemented after the ADC because easier to physically implement.
- This course primarily treats digital equalizer implementations (except the next slide on noise – used for simplicity).

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$((c(t)+n(t))*H_{eq}(t)=1)$

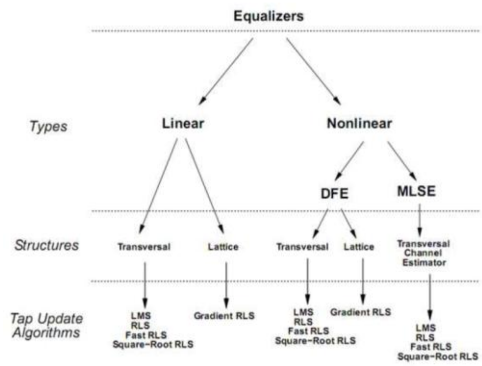
Equalizer Noise Enhancements – be careful

- Linear equalizers that simply inverts the filter of the channel (including the noise) can actually increase the impact of the noise in the equalized data. Non-linear equalizers cope better with this.
- With a signal H(f) with a spectral null (H(f)=0 for some frequency) make even a very low noise power level infinite !

Example 11.1: Consider a channel with impulse response $H(f) = 1/\sqrt{|f|}$ for $|f| < B$, where B is the channel bandwidth. Given noise PSD $N_0/2$, what is the noise power for channel bandwidth $B = 30$ KHz with and without a linear equalizer.

Different Types of Equalizers - hierarchy

We will examine these using the equivalent lowpass representation of bandpass systems as described in Appendix A.



Different Types of Equalizers – comments

- Normally categorized in two, linear and non-linear equalizers.
- Linear are simplest to implement and understand, however performs to well with respect to noise, i.e. not used in advanced wireless communication systems.
- Between non-linear, DFE are most common (not too complicated and performs well). Except with low SNR where we can have error propagation.
- Optimum technique is MLSE, problem is that complexity grows exponentially with the length of the delay spread making it impractical for most channels of our interest.
- Linear and non-linear equalizers are typically implemented using a traversal or lattice structure. We will focus on traversal structures with N-1 delay elements and N taps featuring tunable weights.
- Finally in addition to the structure the equalizers require algorithms for updating the filter coefficients during training and tracking. This is a trade-off between complexity, convergence rate and numerical stability.

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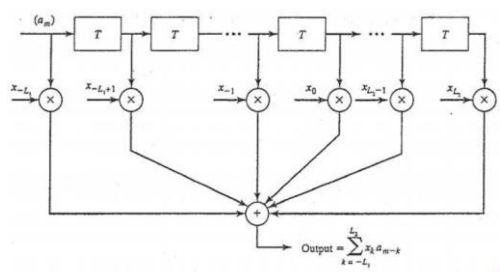
Traversal Structure - Equivalent Discrete-time Channel Model

- In practice ISI is assumed to span a finite number of symbols
- Consequently ISI can be viewed as passing the data sequence $\{a_m\}$ through a FIR filter with coefficients $\{x_n, -L_1 < n < L_2\}$
- This filter is the equivalent discrete-time channel filter



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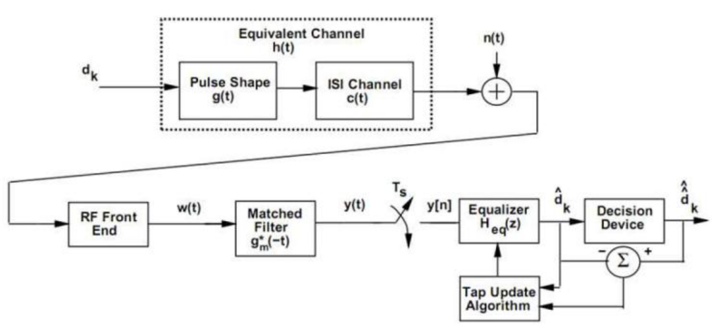
ISI-Free Transmission (when do we not need equalizer !)

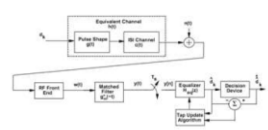
- End-to-end system view (based on equiv. lowpass representation):



Linear Equalizers – two examples

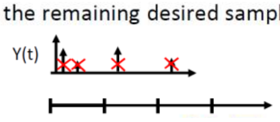
- Ideally we would like to optimize (the coefficients) towards minimization of error probability, however extremely difficult to do in real-life. Therefore indirect optimization. We look at two ways:
  1. Zero-forcing (ZF) equalizer as the algorithm and structure in $H_{eq}(z)$ in previous figure. ZF will cancel all ISI but leads to considerable noise enhancements.
  2. Minimum mean-square error (MMSE) equalizer as the algorithm and structure in $H_{eq}(z)$ in previous figure. MMSE minimizes the expected mean-squared error between the transmitted symbol (d_k) and the symbol detected at the equalizer output ($\hat{d}_k$).This gives better balance between ISI and noise handling

Linear Equalizers - Zero-forcing



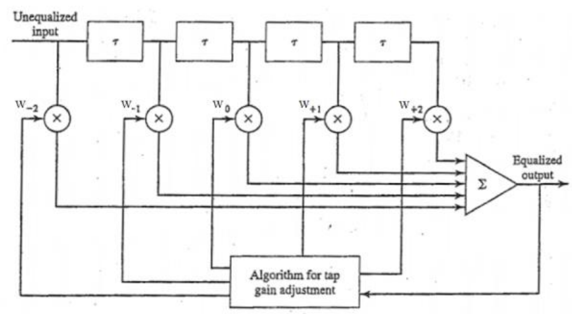
- Generally here with ZF we are looking for the **inverse channel filter** response for the channel response (c(t) + n(t)) * $H_{eq}(t) = 1$
- With zero-forcing we:
  - Achieve a solution by forcing the cross correlation between the error sequence and the desired information to be zero
  - …initially find H_{eq} according to the criteria above with a given filter size of 1+2L, where L is the number of samples that the ISI is limited to.
  - …then we apply the zero-forcing principle.
- When this filter is used the ISI is completely removed (if the length of the delay spread is no longer than L). The drawback is that the influence of the noise can be significantly increased on the remaining desired sample at t=0 if we have fading….
- …since the noise is given by:

$$\mathrm{N}(z) = \frac{N_0}{|H(z)|^2} \qquad (11.13)$$



Linear Equalizers – Equalizer Tap Spacing

- The time delay between taps can be as large as the symbol spacing
- Typically we use delay time between taps equal to half the symbol spacing, resulting in a doubling of the sampling compared to the symbol rate.



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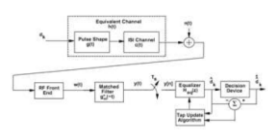
Linear Equalizers - MMSE (I)

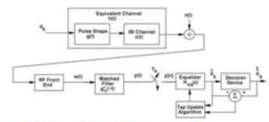


- With Minimum Mean-Square Error we go for a solution where we relax a bit on the ISI condition, but on the other hand select taps based on a minimization of the combination of ISI and noise.
- With MMSE the goal is to minimize the average mean-square error between the transmitted symbol (d_k) and the symbol detected at the equalizer output ($\hat{d}_k$). In other words, the weights w_i are chosen to minimize $\mathrm{E}\left[d_k - \hat{d}_k\right]^2$.
- Since this equalizer is linear its out ($\hat{d}_k$) is a linear combination of the input samples ($y[k]$):

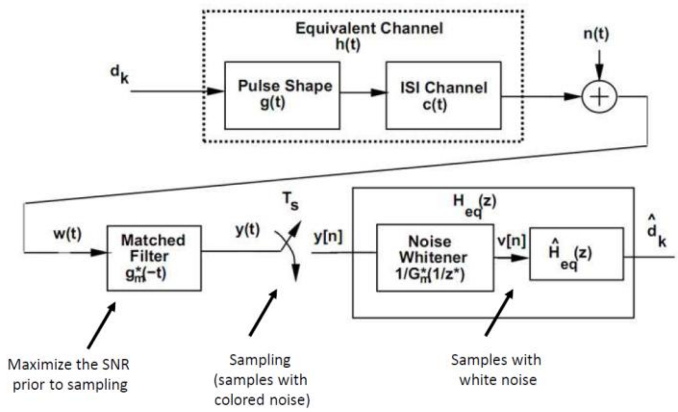
$$\hat{d}_k = \sum_{i=-L}^{L} w_i \, y[k-i] \qquad (11.15)$$

- This problem is basic linear estimation problem. If noise input at the equalizer is white it is a standard Wiener filtering problem !
- However due to matched filter in RF front-end the noise is NOT white.

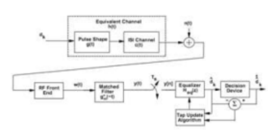
Linear Equalizers - MMSE (II)



- We apply a trick to make it white. We add another filter a whitening filter at the input of the equalizer



Linear Equalizers - MMSE (III)



- With the goal of finding the filter coefficients w_i to minimize $\mathrm{E}\left[d_k - \hat{d}_k\right]^2$ the full ideal MMSE equalizer can be simplified to:

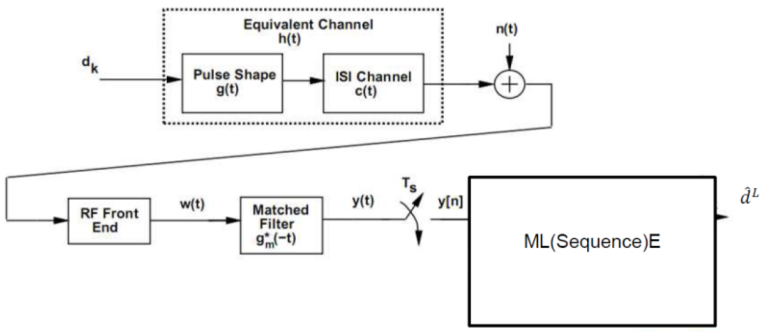
$$H_{eq}(z) = \frac{1}{F(z)+N_0} \qquad (11.26)$$

Fourier transform of combined equivalent lowpass impulse response f(t)

- We see a number of interesting things:
  - The ideal infinite-length MMSE equalizer cancels out the noise whitening filter
  - The ideal infinite-length MMSE equalizer is identical to the ZF equalizer except for the noise term N_0. With no noise they are equal.
  - There is a clear balance between inverting the channel and noise enhancements. When F(z) is highly attenuated at a specific frequency, then the noise in the denominator prevent the noise from being significantly enhanced by the equalizer.

Non-linear Equalizers - MLSE (I)

- Maximum-likelihood-sequence-estimation avoids the problem of noise enhancements since it does not use equalization filter.
- It estimates <u>the sequence</u> of transmitted symbols.



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Non-linear Equalizers - MLSE (II)

- Using Gram-Schmidt orthogonalization it is shown how the maximum-likelihood-sequence-estimation in our scenarios can be estimated to:

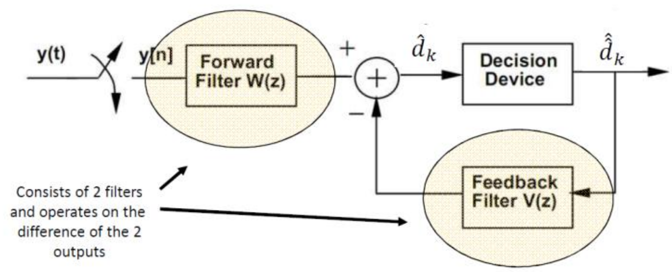
$$\hat{d}^L = \arg\max \left[2\,Re\left\{ \sum_k d_k^* y[k] \right\} - \sum_k \sum_m d_k d_m^* u[k-m] \right]$$

- The optimum sequence only depends on the sampler output y[k] and the channel parameters u[n-k] = u[nT$_s$ - kT$_s$], where u(t) = h(t) * h(-t).
- You will later see how the Viterbi algorithm can be used to reduce the complexity in finding the right sequence in real-life implentations

**Maximum-Likelihood decision criteria:
go back and read section 5.1.4.
in chapter 5, page 134**

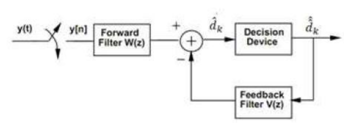
Non-linear Equalizers - DFE (I)

- The Decision-Feedback-Equalizer principle structure looks like this:



- The DFE determines the ISI contribution from the detected symbols $\{\hat{\hat{d}}\}$ by passing them through a feedback filter that approximated the composite signal F(z) convolved with the feedforward filter W(z). The resulting ISI is subtracted from the incoming symbols.

Non-linear Equalizers - DFE (II)



- Assuming that W(z) has N_1+1 taps and V(z) has N_2 taps the DFE output becomes:

$$\hat{d}_k = \sum_{i=-N_1}^{0} w_i\, y[k-i] - \sum_{i=1}^{N_2} v_i \hat{\hat{d}}_{k-i}$$

- Typically we select W(z) and V(z) to use ZF (remove all ISI) or MMSE (minimize expected MSE between the DFE output and the original symbol)

- Drawback: feedback equalizers exhibit feedback errors. Such errors propagate to the later bit decision stage. On top of that these errors cannot be corrected using channel coding since the feedback path operates on encoded symbols before decoding.
- Therefore error propagation seriously degrades performance for low SNRs.

General Comments on other Equalization Methods

- Although MLSE is optimum for equalization the very high level of complexity has caused some trouble, correspondingly alternative ways of simplifying MLSE.
- Most of the work is related to some of the coding techniques you will learn about in the following lectures, such as convolutional codes as Viterbi and interative codes like the Turbo code.

- Finally if the channel is **known at the transmitter** it can pre-equalize the transmitted signal by passing it through a filter that effectively inverts the channel frequency response. In that way there will be no noise enhancements !
- For a time-varying channel this is difficult, but do-able in more static channels.

More on Adaptive Equalizers – Training and Tracking (1)

- All equalizers so far based on the idea that the combined impulse response is known, h(t) = g(t) * c(t).
- Since the channel c(t) is generally not known the equalizer must be tunable and additionally periodically updated according to the channel dynamics.
- This process is called **equalizer training** or **adaptive equalization**.
- It can use the detected data themselves to adjust the filter coefficients, known as **equalizer tracking**.
- The ability of an equalizer is typically a question about compromising between complexity and speed of convergence.

Algorithm	# of multiply operations	Complexity	Convergence	Tracking
LMS	$2N+1$	Low	Slow ($> 10NT_s$)	Poor
MMSE	N^2 to N^3	Very High	Fast ($\approx NT_s$)	Good
RLS	$2.5N^2+4.5N$	High	Fast ($\approx NT_s$)	Good
Fast Kalman DFE	$20N+5$	Fairly Low	Fast ($\approx NT_s$)	Good
Square Root RLS DFE	$1.5N^2+6.5N$	High	Fast ($\approx NT_s$)	Good

More on Adaptive Equalizers – Training and Tracking (2)

Example 11.5: Consider a 5 tap equalizer that must retrain every $.5T_c$, where T_c is the coherence time of the channel. Assume the transmitted signal is BPSK with a rate of 1 Mbps for both data and training sequence transmission. Compare the length of training sequence required for the LMS equalizer versus the Fast Kalman DFE. For an 80 Hz Doppler, by how much is the data rate reduced in order to do periodic training for each of these equalizers. How many operations does each require for this training?

Main Points

- Equalization – a powerful tool to cope with ISI and noise. For situations with delay spread large causing self-interference.
- Linear equalizers
  - ZF: Removes all ISI (with long enough filter), but can introduce severe noise
  - MMSE: Compromise where both ISI and noise is equalized out.
    Most popular of linear types.
- Non-linear:
  - MLSE: The optimum, based on estimation on sequence of data, the complixity is very high – can be complicated in real life.
  - DFE: Based on feedback principles. Complexity much lower than MLSE, solve the issue of noise much better than the linear equalizers. The drawback it that the feedback error can propagate. This cannot be corrected through channel coding.
- Choice of equalization is a compromise between complexity and speed (convergence and tracking).

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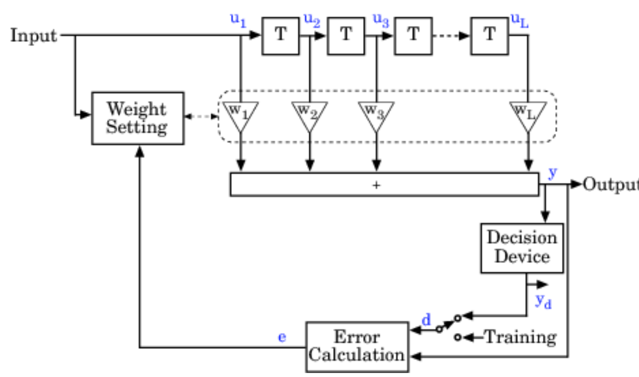
Symbol-spaced equalizer



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Decision feedback equalizer