AI intro - ov3

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AI - ov3

TDT4136 Introduction to Artificial Intelligence
Assignment 3 – Propositional and Predicate Logics
Deadline: 2016-10-11 23:59

Gruppe 13
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Assignment 3 - TDT4136 AI intro

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Problem 1

1) a) $A \wedge \neg B \models A \vee B$ **True**

A	B	$A \wedge \neg B$	$A \vee B$	
0	0	0	0	
0	1	0	1	
1	0	1	1	True
1	1	0	1	

b) $A \vee B \models A \wedge \neg B$ **False**

A	B	$A \vee B$	$A \wedge \neg B$	
0	0	0	0	
0	1	1	0	
1	0	1	1	True
1	1	1	0	False

c) $A \Leftrightarrow B \models A \Rightarrow B$ **True**

A	B	$A \Leftrightarrow B$	$A \Rightarrow B$	
0	0	1	1	True
0	1	0	1	
1	0	0	0	
1	1	1	1	True

$\models A \vee \neg B \vee \neg C$ True

d) $(A \Leftrightarrow B) \Leftrightarrow C \models A \lor \neg B \lor \neg C$ True

A	B	C	$(A \Leftrightarrow B) \Leftrightarrow C$	$A \lor \neg B \lor \neg C$	
0	0	0	1	1	True
0	0	1	0	1	
0	1	0	1	1	True
0	1	1	0	0	
1	0	0	1	1	True
1	0	1	0	1	
1	1	0	0	1	
1	1	1	1	1	True

e) $(\neg A \land B) \land (A \Rightarrow B)$ is satisfiable True

A	B	$(\neg A \land B) \land (A \Rightarrow B)$	
0	0	0	
0	1	1	True
1	0	0	
1	1	0	

f) $(\neg A \land B) \land (A \Leftrightarrow B)$ is satisfiable False

A	B	$(\neg A \land B) \land (A \Leftrightarrow B)$
0	0	0
0	1	0
1	0	0
1	1	0

2) $Q = 2^{100}$, A_1 will be satisfied by $\frac{1}{2} Q = \frac{1}{2} 2^{100} = 2^{99}$

a) $A_{31} \wedge \neg A_{76}$: $\frac{1}{2^2} Q = \underline{\underline{2^{98}}}$

b) $A_{44} \wedge A_{49} \wedge A_{78}$: $\frac{1}{2^3} Q = \underline{\underline{2^{97}}}$

c) $A_{44} \vee A_{49} \vee A_{78}$: $\frac{7}{2^3} Q = \underline{\underline{7 \cdot 2^{97}}}$

d) $A_{70} \Rightarrow \neg A_{92}$: $\frac{3}{2^2} Q = \underline{\underline{3 \cdot 2^{98}}}$

e) $(A_7 \Leftrightarrow A_{72}) \wedge (A_{83} \Leftrightarrow A_{84})$: $\frac{4}{2^4} Q = \frac{1}{2^2} Q = \underline{\underline{2^{98}}}$

f) $\neg A_9 \wedge \neg A_{19} \wedge A_{37} \wedge A_{68} \wedge A_{73} \wedge A_{79} \wedge A_{81} \wedge A_{50}$: $\frac{1}{2^8} Q = \underline{\underline{2^{92}}}$

Problem 2

1) a) $\neg A \vee (B \wedge C)$

$\underline{\underline{(\neg A \vee B) \wedge (\neg A \vee C)}}$

b) $\neg(A \Rightarrow B) \wedge \neg(C \Rightarrow D)$

$\neg(\neg A \vee B) \wedge \neg(\neg C \vee D)$

$\underline{\underline{A \wedge \neg B \wedge C \wedge \neg D}}$

c) $\neg(A \Rightarrow B) \vee \neg(C \Rightarrow D)$

$\neg(\neg A \vee B) \vee \neg(\neg C \vee B)$

$(A \wedge \neg B) \vee (C \wedge \neg D)$

$((A \wedge \neg B) \vee C) \wedge ((A \wedge \neg B) \vee \neg D))$

$\{C \vee (A \wedge \neg B)\} \wedge \{\neg D \vee (A \wedge \neg B)\}$

$(C \vee A) \wedge (C \vee \neg B) \wedge (\neg D \vee A) \wedge (\neg D \vee \neg B)$

d) $(A \Rightarrow B) \Leftrightarrow C$

$\{(A \Rightarrow B) \Rightarrow C\} \wedge \{C \Rightarrow (A \Rightarrow B)\}$

$\{(\neg A \vee B) \Rightarrow C\} \wedge \{C \Rightarrow (\neg A \vee B)\}$

$\{\neg(\neg A \vee B) \vee C\} \wedge \{\neg C \vee (\neg A \vee B)\}$

$\{\neg(\neg A \vee B) \vee C\} \wedge (\neg C \vee \neg A \vee B)$

$\{(A \wedge \neg B) \vee C\} \wedge (\neg C \vee \neg A \vee B)$

$(C \vee A) \wedge (C \vee \neg B) \wedge (\neg C \vee \neg A \vee B)$

2) KB:

$* (X \wedge \neg Y) \Rightarrow \neg B = \neg(X \wedge \neg Y) \vee \neg B = \neg X \vee Y \vee \neg B$

$* \neg X \Rightarrow C = X \vee C$

$* B \wedge \neg Y$

$* A \Rightarrow \neg C = \neg A \vee \neg C$

$$\frac{\neg A \vee \neg C, \quad X \vee C}{\neg A \vee X}, \qquad \frac{\neg A \vee X, \quad \neg X \vee Y \vee \neg B}{\neg A \vee Y \vee \neg B}$$

$Y \vee \neg B = \neg(\neg Y \wedge B) = \neg(B \wedge \neg Y)$

$$\frac{\neg A \vee \neg(B \wedge \neg Y), \quad B \wedge \neg Y}{\neg A}$$

$\Rightarrow \quad KB \models \neg A$

3) [7.18] F = Food, D = Drinks og P = Party

$$S: \; ((F \vee D) \Rightarrow P) \Rightarrow (\neg P \Rightarrow \neg F)$$

a)

F	D	P	S
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

S is valid

l.h.s. r.h.s.

b) $((F \vee D) \Rightarrow P) \Rightarrow (\neg P \Rightarrow \neg F)$

$$(\neg (F \vee D) \vee P) \Rightarrow (P \vee \neg F)$$

$$((\neg F \wedge \neg D) \vee P) \Rightarrow (P \vee \neg F)$$

$$((P \vee \neg F) \wedge (P \vee \neg D)) \Rightarrow (P \vee \neg F)$$

$$(P \vee \neg F) \wedge (P \vee \neg D) \Rightarrow (P \vee \neg F)$$

Er alltid true og bekrefter oppg a).

A	B	A ⇒ B
0	0	1
0	1	1
1	0	0
1	1	1

$P \vee \neg F$	$P \vee \neg D$	$((P \vee \neg F) \wedge (P \vee \neg D))$	S
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	1

c) Use proof by contradiction:

$\neg S$

$\neg\{((P \vee \neg F) \wedge (P \vee \neg D)) \Rightarrow (P \vee \neg F)\}$

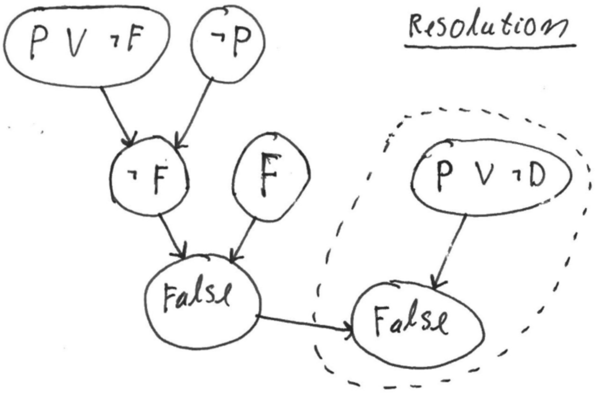
$\neg\{\neg((P \vee \neg F) \wedge (P \vee \neg D)) \vee (P \vee \neg F)\}$

$\neg\{\neg(P \vee \neg F) \vee \neg(P \vee \neg D) \vee (P \vee \neg F)\}$

$(P \vee \neg F) \wedge (P \vee \neg D) \wedge \neg(P \vee \neg F)$

$(P \vee \neg F) \wedge (P \vee \neg D) \wedge \neg P \wedge F$

Resolution



Hvis to ledd i uttrykket på CNF alltid blir false, så er hele uttrykket false.

$\neg S = false \iff S = true \Rightarrow$ Bekrefter oppg a).

Problem 3

1. a) PC(Batman, Christian Bale) $\wedge$ PC(Batman, George Clooney) $\wedge$ PC(Batman, Val Kilmer)

b) $\forall c$: $\neg$PC(c, Heath Ledger) $\vee$ $\neg$PC(c, Christian Bale)

c) $\forall m$: CIM(Batman, m) $\wedge$ D(Christoper Nolan, m) $\Rightarrow$ PIM(Christian Bale, m)

d) $\exists m$: CIM(Batman, m) $\wedge$ CIM(The Joker, m)

e) $\exists m: \; D(\text{Kevin Costner}, m) \land PIM(\text{Kevin Costner}, m)$

f) $\forall m: \; PIM(\text{George Clooney}, m) \Rightarrow \neg\{PIM(\text{Tarantino}, m) \lor D(\text{Tarantino}, m)\}$

g) $\exists m: \; PIM(\text{Uma Thurman}, m) \land D(\text{Tarantino}, m)$

2.

a) $\forall x, y: \; Divisible(x, y) \Longleftrightarrow \exists z: (z < x) \land (x = z \cdot y)$

b) $\forall x: \; Even(x) \Longleftrightarrow Divisible(x, 2)$

c) $\forall x: \; Odd(x) \Longleftrightarrow \neg Divisible(x, 2)$

d) $\forall x: \; Prime(x) \Longleftrightarrow \forall y: \neg(x = y) \Rightarrow \neg Divisible(x, y)$

Problem 4

1. a) $\theta = \{x / \text{Platon}\}$

b) $\theta = \{y / \text{The Republic}\}$

c) $\theta = \{x/\text{peter}, \; y/\text{Metaphysics}\}$

d) Not possible - Motsigende predicates

e) $\theta = \{y / \text{Critique Of Pure Reason}\}$

2. a) $Philosopher(A) \wedge StudentOf(B, A)$

A and B are skolem constants.

b) $\forall y, x: Philosopher(x) \wedge StudentOf(y, x)$
$\Rightarrow [Book(F(x,y)) \wedge Write(x, F(x,y)) \wedge Read(y, F(x,y))]$

$F(x,y)$ is a skolem function.

3. a) * $\forall x: \{ SA(x) \Leftrightarrow [\exists m: PIM(x,m) \wedge D(x,m)] \}$

$$\Leftrightarrow \quad \forall x: \{ SA(x) \Rightarrow [\exists m: PIM(x,m) \wedge D(x,m)] \wedge [\exists m: PIM(x,m) \wedge D(x,m)] \Rightarrow SA(x) \}$$

$$\Leftrightarrow \quad \forall x: \{ \neg SA(x) \vee [\exists m: PIM(x,m) \wedge D(x,m)] \wedge \neg[\exists m: PIM(x,m) \wedge D(x,m)] \vee SA(x) \}$$

$$\Leftrightarrow \quad \forall x: \{ \neg SA(x) \vee [\exists m: PIM(x,m) \wedge D(x,m)] \wedge [\forall m: \neg PIM(x,m) \vee \neg D(x,m)] \vee SA(x) \}$$

$$\Rightarrow \quad \forall x: \{ \neg SA(x) \vee [PIM(x, F(x)) \wedge D(x, F(x))] \wedge [\forall m: \neg PIM(x,m) \vee \neg D(x,m)] \vee SA(x) \}$$

$$\Rightarrow \quad \neg SA(x) \vee [PIM(x, F(x)) \wedge D(x, F(x))] \wedge [\neg PIM(x,m) \vee \neg D(x,m)] \vee SA(x)$$

$$\Rightarrow \quad [\neg SA(x) \vee PIM(x, F(x))] \wedge [\neg SA(x) \vee D(x, F(x))] \wedge [SA(x) \vee \neg PIM(x,m) \vee \neg D(x,m)] \qquad (1)$$

$$* \ \forall m: \{ D(T,m) \Leftrightarrow PIM(UT,m) \}$$

$$\Leftrightarrow \forall m: \{ D(T,m) \Rightarrow PIM(UT,m) \ \wedge$$
$$PIM(UT,m) \Rightarrow D(T,m) \}$$

$$\Leftrightarrow \forall m: \{ \neg D(T,m) \vee PIM(UT,m) \ \wedge$$
$$\neg PIM(UT,m) \vee D(T,m) \}$$

$$\Rightarrow \ [\neg D(T,m) \vee PIM(UT,m)] \wedge [\neg PIM(UT,m) \vee D(T,m)] \quad (2)$$

$$* \ \exists m: \{ PIM(UT,m) \wedge PIM(T,m) \} \qquad (M = \text{skolem const})$$

$$\Rightarrow \ PIM(UT,M) \wedge PIM(T,M) \quad (3)$$

P Proof by contradiction with resolution:

KB: (1), (2), (3) and $\alpha = SA(T)$

$[KB \models \alpha] \Leftrightarrow [KB \wedge \neg\alpha$ is not satisfiable$]$

KB: $PIM(UT,M)$ (4)

$PIM(T,M)$ (5)

$\neg D(T,m) \vee PIM(UT,m) \Rightarrow \neg D(T,M) \vee PIM(UT,M)$ (6)

$\neg PIM(UT,m) \vee D(T,m) \Rightarrow \neg PIM(UT,M) \vee D(T,M)$ (7)

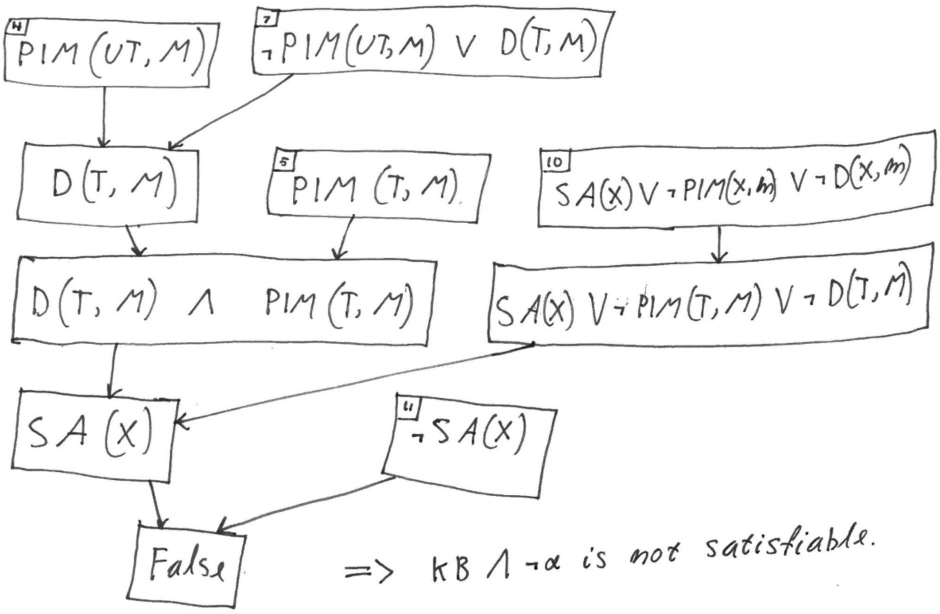
$\neg SA(x) \vee PIM(x, F(x))$ (8)

$\neg SA(x) \vee D(x, F(x))$ (9)

$SA(x) \vee \neg PIM(x, F(x)) \vee \neg D(x, F(x))$ (10)

(8)–(10): $F(x) = m$

$\neg\alpha = \neg SA(x)$ (11)



$\Rightarrow$ KB $\wedge \neg\alpha$ is not satisfiable.

b)

(1) En superActor er en person som både er regissør og skuespiller i samme film.

(2) Hvis Tarantino er regissør i en film, så vil alltid UmaThurman være skuespiller i timen (og omvendt).

(3) Det finnes en film der både UmaThurman og Tarantino er skuespillere.

Altså er Tarantino en superActor.