Automatic Stabilizers and the Volatility of the Business Cycle

ERASMUS UNIVERSITY ROTTERDAM
Erasmus School of Economics
Department of Economics

Supervisor: Julian Emami Namini

Name: Martin van Elp
Exam number: 271596
E-mail address: martin.vanelp@gmail.com

Automatic Stabilizers and the Volatility of the Business Cycle *

Martin van Elp

August 24, 2008

Abstract

This thesis exhibits several different models for showing the influence of automatic stabilizers on the effects of the business cycle. We achieve the intuition that they flatten the business cycle as commonly perceived. However we also theoretically show that automatic stabilizers worsen the shape of the economy. An empirical analysis establishes that countries with active automatic stabilizers indeed observe a lower volatility of their national income. The overall conclusion of this thesis is that automatic stabilizers indeed flatten the business cycle, but policymakers should always consider whether they are worth their price.

Contents

*Here I would like to especially thank my supervisor Julian Emami Namini for his useful comments and assistance during the course of writing my thesis. I always appreciated the positive input, and it kept coming even when I went my own way. A thank you should also go to Robert Dur for supervising on the static model in this thesis during one of my seminars, which effectively laid down the groundwork for the whole thesis. Others I will not name, but obviously my thanks also go to my family, friends, teachers of the past and the staff at the Erasmus School of Economics for making this piece of work possible. As always, remaining errors in the text are solely my responsibility.

List of Figures

List of Tables

1 Introduction

Our economy appears to be governed by business cycles. After some good years some bad years follow as if by definition. We aren't able to determine where the tilting point would lay in the future. Therefore policymakers are hardly capable of anticipating a change in the economic environment due to the business cycle. Automatic stabilizers are said to help these policymakers. A well-designed automatic stabilizer is supposed to react immediately to a change in the business cycle. If that is true policymakers don't necessarily need the capability of observing the future developments of the business cycle.

Automatic stabilizers are thus generally assumed to prevent the economy from suffering too much from business cycles. Automatic stabilizers are fiscal policy mechanisms that, by their nature, flatten the wave of the business cycle. That happens without policymakers taking any explicit action. One could think of unemployment benefits that offer an income (and consumption opportunity) to those that get unemployed because of worsening of the business cycle. When you assume that Ricardian Equivalence doesn't hold (perfectly; see for example Mankiw (2000)), one would expect consumption to remain at a higher level than without unemployment benefits.

The question underlying this thesis is whether fiscal mechanisms that supposedly are automatic stabilizers actually have that feature or that they maybe even amplify the wave of the business cycle. Policymakers should take the effects of automatic stabilizers into account when they use them. But then we should ascertain whether automatic stabilizers do what they should do in a business cycle environment and not for example the complete opposite.

The question we would like to answer with our research is whether automatic stabilizers have a distorting effect on the business cycle. Do automatic stabilizers really decrease the magnitude of the business cycle waves? Is it possible under circumstances that supposed automatic stabilizers aggravate the business cycle? Are there externalities to having automatic stabilizers in effect?

To answer these questions we will develop several theoretical models, all with their own characteristics. These models will then be analyzed to answer the questions we ask ourselves. Most of the automatic stabilization will revolve around the fiscal mechanism of unemployment benefits as we consider these the most obvious kind of automatic stabilization. These unemployment benefits can also be shown rather intuitively in a model environment.

When insight is achieved in what automatic stabilizers do in a theoretical model environment we can move on to check whether these intuitions are seen in reality. We will hypothesize that automatic stabilizers decrease the volatility of the GDP. We will use several variables that tell something about how generous a country is towards for example unemployed. Countries that spend more on these kind of fiscal instruments are supposed to have a less volatile GDP.

The structure of our thesis is as follows. After a short literature review we will start showing a static model in which we analyze the reaction of GDP to a productivity shock for a country with and without unemployment benefits. This static model also features a measure for the income distribution which will be interesting to investigate. As a static model only marginally resembles reality we will continue our analysis with a dynamic Keynesian model. This model again features productivity shocks and unemployment benefits but is also able to show the progress of the economy in time after a shock. This dynamic Keynesian model will also be tested against real United States data. For further research in the theoretical model domain we will show the implementation of an automatic stabilizer in a dynamic Ramseyian model. Though a thorough analysis of the model its behavior is beyond the scope of this thesis. After this we will show in an empirical analysis whether the obtained theoretical intuitions are mirrored by reality. To finalize we will give an overall conclusion on the subject of automatic stabilizers and the volatility of the business cycle.

2 Literature review

In this section we will attempt to give a good overview of the existing literature that concerns the questions that we pose. This section will have four different subjects. First we will show what automatic stabilization is perceived to be by the literature. After that we will spend some words on Ricardian equivalence and the potency of fiscal policy. Thirdly we will get into a number of empirical papers in the field of automatic stabilization. To finalize we slightly touch the subject of micro-founded decisions by households and automatic stabilization.

2.1 Characteristics of stabilization

In the introduction we said that automatic stabilizers automatically react to changes in the business cycle. If people get unemployed because of a negative shock, as part of the business cycle, progressive taxes and unemployment benefits soften the degree to which people lose disposable income. That effect is then assumed to stabilize the economy.

More technically put, an unemployment benefit program should withdraw money income from the economy in a boom and should pump money income into the economy during a recession in order to have a stabilizing effect on GDP, Palomba (1968). Auerbach and Feenberg (2000) put it slightly different and state that to stabilize output the 'cushioning' effect of taxes on changes in before-tax income results in lower volatility of household consumption with regard to goods and services. Another way of defining stabilization is by saying that the economy is Keynesian in the short run and thus an increase in aggregate demand, coming from automatic stabilizers, raises national income, Elmendorf and Mankiw (1998) and Cohen and Follette (1999).

These mechanisms thus imply that the government runs a deficit when in recession and a surplus in economic boom. Andersen and Dogonowski (2002) show that a constant tax rate, compared to a balanced budget, lowers the variance in after-tax income. They further show that a progressive tax is in fact the optimal fiscal policy from a stabilization point of view: higher taxes in boom, lower taxes in recession. The argument that budget restrictions would counter these effects, as they force a country to move back towards a balanced budget, doesn't seem to hold according to empirical research on several European Union countries, Leeftink (2003). Countries that have a worse record on their budget sustainability, which indicates stronger reactions to the business cycle in the past, appear to have the effect of their automatic stabilizers to be hampered. Neither disagree on the effectiveness of stabilization, but apparently they do differ in the margin within which automatic stabilizers should function.

Other than the positive effects of increased aggregate demand Elmendorf and Mankiw (1998) also point to negative effects. They state that as a consequence of a higher government debt level the capital stock gets reduced in the long run. They also note the deadweight loss attached to the taxes necessary to service that debt. According to them, eliminating the United States debt would add about two percent to national income. Monetary policy could also get influenced if the debt becomes substantial, at which point seignorage might become interesting. The possibility of running a deficit in general, and thus creating a (higher) debt, also influences political decisions and dependency and also the vulnerability to external effects as a drop of confidence in the international financial system. In this thesis we will mostly refrain from these monetary and more political discussions.

An interesting result is the following, as found in Cohen and Follette (1999): "in the context of a standard model of an optimizing representative consumer, Lucas argues that perfect stabilization, i.e., complete elimination of the variance of consumption in the United States, would yield virtually no utility gain to households both in absolute terms and relative to the huge utility gain associated with only a modest increase in the growth rate of consumption." Such means that if we can associate having no (or less) benefits with a higher GDP automatic stabilization might be a worse solution from a utility perspective. We will shed more light on this in our dynamic Keynesian model of section 4.

2.2 Ricardian equivalence and potency of fiscal policy

The extent to which Ricardian equivalence actually exists, and consequently the potency of fiscal policy, is of great importance to the findings and assumptions in this thesis. Ricardian equivalence means that government debt is completely neutral, Mankiw (2000). Under Ricardian equivalence households anticipate a future increase in tax by exactly saving the amount of the deficit that the government runs, their investment decision aside. If Ricardian equivalence holds the economy isn't Keynesian in the short run, but such is a requirement as we noted in the previous subsection.

Mankiw (2000) gives reasons why Ricardian equivalence doesn't generally hold. They have estimated in an earlier empirical paper that about half of all consumers are so-called rule-of-thumb consumers which means they simply spend their current income. That means that consumers do not smooth their consumption as Ricardian equivalence requires. That earlier paper was confirmed by Shapiro and Slemrod (1995) whom used survey data on the reaction of households to a change in tax withholding. They establish that the amount of rule-of-thumb consumers is about 43 percent of all consumers which can be considered similar to the Mankiw (2000) estimate.

Mankiw (2000) puts forward several propositions that are relevant in a world with about half of the consumers being rule-of-thumb consumers and where the general notion of Ricardian equivalence evidently doesn't hold. They state that "temporary tax changes have large effects on the demand for goods and services", and hence that automatic stabilization could work. "Government debt need not crowd out capital in the long run", which contradicts the position of Elmendorf and Mankiw (1998) that we mentioned in the previous subsection. This proposition is reached as the savers, the other half of the consumers, will restore the capital stock to its desirable level. However, if taxes are distortionary, they influence the before-tax interest rate, there indeed could be a case of crowding out of private investment. Because of that effect even the rule-of-thumb consumers would opt for a zero tax rate on capital income, as that is in their best interest. This result is collaborated by Chari et al. (1994) whom find that the optimal capital tax in a business cycle model is indeed zero. Interesting is that Mankiw (2000) also notes that "Government debt increases steady-state inequality" because interest payments on the debt go to savers. This conclusion lies in line with the previously mentioned notion of Lucas, Cohen and Follette (1999), that the gains from automatic stabilization could diminish because of the consequences of a future higher debt (though obviously Lucas reaches this result from a different angle).

A more thorough discussion of Ricardian equivalence and the literature on the subject can for example be found in Elmendorf and Mankiw (1998).

The fact that Ricardian equivalence doesn't hold, which is a common opinion among all cited authors in this literature review, makes the question whether fiscal policy is relevant maybe a bit strange. However there has been a discussion between monetarist and Keynesian economist which steps beyond the issue of Ricardian equivalence. In our opinion Blinder and Solow (1973) make a very strong case in favor of the potency of fiscal policy. Their theoretical model shows that if the economy is stable under bond finance that fiscal policy is "normally effective". Therefore we have confidence that the analysis in the following sections which involve fiscal policy make sense.

2.3 Statistics on automatic stabilizers their effectiveness

In section 6 we will show an empirical analysis that tries to establish a link between 'generosity', automatic stabilizers, and the volatility of the business cycle. Obviously we aren't the first to do empirical research in search of this link. Therefore we give an overview of some previous research in this field and we will then explain what our analysis should add to the previous research.

The most simple approach is to check whether a recession is paired by an almost immediate increase in the level of unemployment benefits being paid. Rejda (1966) takes this approach and shows that especially during downswing unemployment benefits change in a desirable counter-cyclical manner. Cohen and Follette (1999) collaborate this view as they state that unemployment benefits appear to be effective as an automatic stabilizer of income. The level of unemployment benefit payments increases significantly when the unemployment rate raises. Rejda (1966) however also has to conclude that when the business cycle moves from recession to boom that the level of unemployment benefits seems more sticky than it should for automatic stabilization. In subsection 4.3.3, our dynamic Keynesian model, this same observation seems to prevail because of how we designed the economy. Another reasoning for the slow reaction of unemployment benefits during upswing could lay in the lag in the response of employment to the business cycle. The turning points for the business cycle indeed seem to be followed by unemployment with a half year lag in the Netherlands, DNB (1998).

Another more sophisticated route is to consider the marginal propensity to consume an extra dollar. This tells us whether handing out money, through an automatic stabilizer, changes consumption behavior. Souleles (1999) shows that this marginal propensity to consume is 0.64 based on research of expenditure behavior following a tax refund in the United States. They establish that almost two-thirds of refund gets spend already during the same quarter. Along these lines we can also mention the result by Auerbach and Feenberg (2000) who use the TAXSIM model to "estimate that individual federal taxes perhaps offset as much as 8 percent of initial shocks to GDP".

A step further is to view how the variance of GDP gets affected. Thus one essentially creates a what-if world in which there were no automatic stabilizers for a specific shock and compare that to what did happen. Palomba (1968) concludes, based on their research, that different kinds of business cycles exist and that some of them aren't softened by automatic stabilizers. This seems to relate to the remark in Cohen and Follette (1999) that aggregate supply shocks, higher oil prices for example, are hardly stabilized by automatic stabilizers in their framework. They do find that income taxes offer modest stabilization against demand-side shocks.

There might be a welfare loss for households if their income gets more variable, less automatic stabilization, against which they can't (or don't) insure. Kniesner and Ziliak (2002) show that "an income tax provides implicit insurance by dampening the variability of disposable income and consumption". They then show based on data following a change in the United States tax regime that a decrease in the number of tax brackets not only decreases the automatic stabilization effect, but also creates a welfare loss because of a reduction in collective insurance (this insurance is implicit through the income tax). Oddly enough the decrease in automatic stabilization because of fewer tax brackets gets more or less contradicted by Auerbach and Feenberg (2000) whom state that according to their research the decline in marginal tax rates (fewer tax brackets) didn't decrease the role of the tax system in stabilizing aggregate demand.

The closest resemblance to our own empirical analysis is found in Leeftink (2003). Based on data of several European Union countries they show that government size, instrumental for 'generosity', is positively correlated with stability and that debt unsustainability is negatively correlated with stability. Especially the first observation is of importance as we would also like to establish, but then from a broader perspective, whether a more generous country actually observes lower variance in its business cycle. In fact all previous empirical papers we mentioned establish for a specific point in time that *if* there would have been more or less automatic stabilizers whether that would have made a difference. Both our dynamic Keynesian model, section 4 and our empirical work, section 6, would like to step beyond the ceteris paribus assumption but would like to review whether a more generous country, which applies more automatic stabilizers, actually has a more stable GDP in the long run and whether there are side-effects.

2.4 Micro-foundations and automatic stabilizers

It goes beyond the scope of this thesis to thoroughly discuss the literature on dynamic general equilibrium models. Such is as we will only expose a proposal for such a model in section 5. For an idea of the contents of such models we would like to for example refer to Lau et al. (2002), Ambler et al. (2002) and Emami Namini (2006). For the groundwork of these models we refer to Ramsey (1928).

Though we wouldn't want to skip the subject of micro-foundations as a whole. For example Agell and Lundborg (1992) reach a strikingly different intuition on what is optimal for a capital tax rate than Mankiw (2000) and Chari et al. (1994). While the latter conclude that in the end everyone is best off with a zero capital tax rate Agell and Lundborg (1992) find that "a general capital tax always raises the wage-rental rate and therefore unambiguously lowers the rate of unemployment". Because Agell and Lundborg (1992) 'create' unemployment through an effort function on the part of workers there apparently is the need for raising the wage-rental rate in such a fashion that more people are willing to work.

Kiley (2003) also reaches an interesting position. To strengthen the effect of automatic stabilization Kiley (2003) concludes that when in recession unemployment benefits should be higher than when in boom. Their micro-foundation lies in the fruitfulness of searching for a job. If it is less fruitful to look for a job it could be optimal to hand out a larger benefit from a welfare point of view. On the other hand when it is easier to find a job a lower benefit gives the right welfare optimizing incentives. Such a result is less contrasting than that of Agell and Lundborg (1992), but when introducing decisions of individuals at the micro level it appears that there is more than that meets the eye at the macro level.

3 Static model with automatic stabilization

In this section we will model a static environment for automatic stabilizers to work. Our environment will contain a value tax and unemployment benefits as its automatic stabilizers. In this environment we take a look at our hypothesis being that unemployment benefits and the value tax indeed flatten the business cycle.

In this model the workers are heterogenous in their productivity. Hiring one worker is more or less profitable to a firm than hiring another. Workers are rewarded according to their productivity, as they extract some of the rents they create for their employer. Wages have a fixed component and a variable component determined by a variable and the worker's productivity.

These heterogenous workers observe the wage they will get if they accept to work at a firm and they also observe the unemployment benefit level. They only care about getting the largest sum of money. Effort is of no consideration in this analysis, and wouldn't add to the intuitions developed in this section. We will explicitly state that at least the least productive worker is better off with an unemployment benefit. Without such we can't be sure that there are actually people unemployed.

The tax system itself is very simple. As said it only entails a value tax rate on all labor income. Unemployment benefits are not taxed and they are the other automatic stabilizer of our model. The level of these are set exogenously by the government. They must be set at a feasible level, so the least productive worker should always be unemployed as a minimum.

After stating the initial equations we can write down some macro-relations for consumption, taxes, unemployment benefits and a simple aggregate of the last two gives government net expenditures. We will then look for the indifferent worker, the worker that's indifferent between accepting the wage he is offered or staying unemployed and thus getting an unemployment benefit.

Knowing which worker is indifferent will make it possible to write down our macro-relations more explicit. These rewritten relations can then be differentiated to our exogenous variables: taxes, unemployment benefits, the fixed wage component and the variable wage component. Intuitively we would expect that our automatic stabilizers lower the effects of changes in these exogenous variables. The derivations will tell whether intuitive expectations of these variables their effects are confirmed by the model. In the two following subsections we will then compare our model with a situation without unemployment benefits and we will also look at what happens if government net expenditures are restricted.

3.1 Initial equations of the static model

We observe a population [0,1] with differing θ, productivity, and this population is uniformly distributed. The subscript i besides θ points to the θ of that individual i. θ runs from the lowest productivity, $\theta_0 = 0$, to the highest productivity, $\theta_1 = 1$. It does so linearly.

Wages are determined as follows:

$$w_i = \delta\theta_i + w_f \tag{3.1}$$

So there is a fixed component w_f and a component dependent on θ and δ. w_f has no specific meaning, essentially it is the reservation wage of the least productive worker. Other than that it just part of our ad hoc description of wage formation. δ determines the extra reward for an individual's productivity.

Taxes on these wages are determined through the tax rate:

$$t_i = t_f \tag{3.2}$$

So there is only a fixed component t_f, a value tax.

The level of the unemployment benefits, u^*, is not endogenously determined. Moreover the variable that determines the extra reward for an individual's productivity δ is also exogenously determined. However we do state that:

$$\delta > u^* > (1 - t_f)w_f \tag{3.3}$$

Else nobody will become unemployed and the income distribution would be too flat to be able to tell something about the derivations later on.

We introduce an indicator function which tells whether someone is unemployed (0) or employed (1):

$$I^*(\theta_i) = \begin{cases} 1 & \text{if } (1 - t_i)w_i > u^* \\ 0 & \text{if } (1 - t_i)w_i \leq u^* \end{cases} \tag{3.4}$$

Having stated so we can write down the relevant macro-relations.

People consume their whole net income (and there is no tax on u^*):

$$C = \int_{i=0}^{1} [I(\theta_i)(1 - t_i)w_i + (1 - I(\theta_i))u^*] \tag{3.5}$$

Total tax revenue:

$$T = \int_{i=0}^{1} [I(\theta_i)t_i w_i] \tag{3.6}$$

Total expenditures on unemployment benefits:

$$U = \int_{i=0}^{1} [(1 - I(\theta_i))u^*] \tag{3.7}$$

Government net expenditures (where G_0 is public consumption):

$$G = G_0 + U - T \tag{3.8}$$

3.2 Finding the indifferent worker in the static model

First we would like to know which (θ_i^*) worker is indifferent between working and staying unemployed (getting an unemployment benefit).

We find this worker by solving the following equation:

$$\begin{aligned} u^* &= (1 - t_i)w_i \\ &= w_i - t_f w_i \\ &= (\delta\theta_i + w_f) - t_f(\delta\theta_i + w_f) \\ (1 - t_f)\delta\theta_i &= u^* - (1 - t_f)w_f \\ \theta_i^* &= \frac{u^*}{(1 - t_f)\delta} - \frac{w_f}{\delta} \end{aligned} \tag{3.9}$$

From the inequality of equation (3.3) follows that $\theta_i^* > 0$. This θ_i^* tells us which workers are unemployed. Consequently $(1 - \theta_i^*)$ tells us which workers are employed.

3.3 Static model outcomes

Knowing θ_i^* offers the opportunity to rewrite the integral functions (3.5), (3.6) and (3.7), the model its macro-relations. Because of expositional reasons we do this in a reverse order like in Appendix A[1].

Total expenditures on unemployment benefits:

$$U = \theta_i^* u^* \tag{3.10}$$

Total tax revenue:

$$T = \frac{1}{2} t_f \delta (1 - (\theta_i^*)^2) + t_f w_f (1 - \theta_i^*) \tag{3.11}$$

People consume their whole net income (and there is no tax on u^*):

$$C = \frac{1}{2}(1 - t_f)\delta(1 - (\theta_i^*)^2) + (1 - t_f) w_f (1 - \theta_i^*) + \theta_i^* u^* \tag{3.12}$$

We will now have to differentiate to u^*, w_f, δ and t_f to improve our understanding of the effects of these exogenous variables on the macro-relations.

For unemployment benefits we get the following expressions.

$$\frac{\mathrm{d}U}{\mathrm{d}u^*} = \frac{2u^*}{(1-t_f)\delta} - \frac{w_f}{\delta} > 0 \tag{3.13}$$

$$\frac{\mathrm{d}U}{\mathrm{d}w_f} = -\frac{u^*}{\delta} < 0 \tag{3.14}$$

$$\frac{\mathrm{d}U}{\mathrm{d}\delta} = -\frac{(u^*)^2}{(1-t_f)\delta^2} + \frac{w_f u^*}{\delta^2} < 0 \tag{3.15}$$

$$\frac{\mathrm{d}U}{\mathrm{d}t_f} = \frac{(u^*)^2}{(1-t_f)^2\delta} > 0 \tag{3.16}$$

The signs of (3.13) and (3.15) stem from the inequality of (3.3).

A rise in the unemployment benefits level raises the total expenditures on these benefits. Of course because all unemployed get a higher benefit. But also because more people their net income becomes lower than this benefit, which in our model makes them unemployed. An increase in people their income because of an increase in the fixed wage or the productivity reward variable leads to a decrease of U. Interesting is the difference between these effects. The effect of a change of δ appears to be lower than that of w_f. So when an increase in the income of the population is more equally distributed one would expect more people to leave unemployment. To finish we see that an increase in the tax rate also increases the expenditures on unemployment benefits. Such is again due to more people their net income becoming lower than the unemployment benefit.

[1] Appendix A gives some more elaborate steps to get to the expressions of this section.

For tax revenues we get the following expressions.

$$\frac{\mathrm{d}T}{\mathrm{d}u^*} = -\left(\frac{t_f\theta_i^*}{1-t_f} + \frac{t_f w_f}{(1-t_f)\delta}\right) < 0 \tag{3.17}$$

$$\frac{\mathrm{d}T}{\mathrm{d}w_f} = t_f + t_f\frac{w_f}{\delta} > 0 \tag{3.18}$$

$$\frac{\mathrm{d}T}{\mathrm{d}\delta} = [1-(\theta_i^*)^2]\frac{t_f}{2} + t_f u^*\left(\frac{u^*}{(1-t_f)\delta^2} - \frac{w_f}{\delta^2}\right) > 0 \tag{3.19}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t_f} = [1-(\theta_i^*)^2]\frac{\delta}{2} + (1-\theta_i^*)w_f - t_f u^*\frac{u^*}{(1-t_f)^2\delta} > \text{or} < 0 \tag{3.20}$$

The sign of (3.19) partially stems from the inequality of (3.3).

A rise in the unemployment benefits level lowers the tax revenue. Such is intuitive as more people their net income becomes lower than this benefit and thus become unemployed workers which don't pay taxes. An increase in people their income because of an increase in the fixed wage or the productivity reward variable leads to an increase of T. The difference between these effects is less striking than in the unemployment benefit expenditures case. Nonetheless it is intuitive that tax revenues increase when income increases, and there obviously is an indirect effect through less people being unemployed. To finish we see that an increase in the tax rate has an ambiguous effect on the tax revenues. Such could happen because an increase in the tax rate crowds out taxes paid by people that turn unemployed which could be insufficiently compensated by the higher taxes received from those that remain working.

Consumption looks like a combination of both taxes and unemployment benefits:

$$\frac{\mathrm{d}C}{\mathrm{d}u^*} = \frac{u^*}{(1-t_f)\delta} - \frac{w_f}{\delta} > 0 \tag{3.21}$$

$$\frac{\mathrm{d}C}{\mathrm{d}w_f} = (1-t_f) + (1-t_f)\frac{w_f}{\delta} - \frac{u^*}{\delta} > 0 \tag{3.22}$$

$$\frac{\mathrm{d}C}{\mathrm{d}\delta} = [1-(\theta_i^*)^2]\frac{(1-t_f)}{2} > 0 \tag{3.23}$$

$$\frac{\mathrm{d}C}{\mathrm{d}t_f} = [(\theta_i^*)^2-1]\frac{\delta}{2} + (\theta_i^*-1)w_f < 0 \tag{3.24}$$

The signs of (3.21) and (3.22) stem from the inequality of (3.3).

A rise in the unemployment benefits level increases consumption. One would expect such obviously from this model as there are only individuals that observe an increase in their income (some get a higher unemployment benefit, others move from a low wage to an unemployment benefit). An increase in people their income because of an increase in the fixed wage or the productivity reward variable leads to an increase of C. Again, that is intuitive as those at work get a higher wage, and some unemployed are able to find work to get a better income. Comparing the two effects is difficult as they largely depend on the level of exogenous variables. However looking at the equations one would expect a more equally distributed increase in national income to increase consumption more than an increase in δ. And such is thus not due to differing propensities to consume, which are not part of this model, but due to the effect that more people can find a job with decent pay when the shock is more equally distributed. Finally we see that an increase in the tax rate decreases consumption. That is fairly logical as those that stay working have a lower net income and those that become unemployed don't get their prior net income as an unemployment benefit but a fixed amount lower than they got while being employed.

3.4 The static model without unemployment benefits

We now know how the model reacts to changes in its exogenous variables including the effect of the existence of unemployment benefits. All these equations in the end tell us nothing if we don't compare them for example with the extreme case of no unemployment benefits. We simply set u^* to zero and will find the outcomes without unemployment benefits. We add a prime to our new equations to denote the case without unemployment benefits.

Unemployment benefits become zero:

$$\begin{aligned} U &= \theta_i^* u^* \\ U' &= 0 \end{aligned} \tag{3.25}$$

Total tax revenue increases as θ_i^* disappears from the equation. Such happens because once unemployed now also will pay taxes:

$$\begin{aligned} T &= \frac{1}{2} t_f \delta (1 - (\theta_i^*)^2) + t_f w_f (1 - \theta_i^*) \\ T' &= \frac{1}{2} t_f \delta + t_f w_f \end{aligned} \tag{3.26}$$

Consumption seems somewhat ambiguous, however knowing that without unemployment benefits those at the low end have less disposable income (and thus consume less in this model) it is easy to see that private consumption decreases. We found such in fact already under equation (3.21).

$$\begin{aligned} C &= \frac{1}{2}(1 - t_f)\delta(1 - (\theta_i^*)^2) + (1 - t_f) w_f (1 - \theta_i^*) + \theta_i^* u^* \\ C' &= \frac{1}{2}(1 - t_f)\delta + (1 - t_f) w_f \end{aligned} \tag{3.27}$$

As we want to develop some intuition on how unemployment benefits might flatten the GDP curve we will look at how consumption in both cases changes if an external shock occurs (w_f or δ changes). Equations (3.22) and (3.23) are also repeated.

$$\begin{aligned} \frac{\mathrm{d}C}{\mathrm{d}w_f} &= (1 - t_f) + (1 - t_f)\frac{w_f}{\delta} - \frac{u^*}{\delta} > 0 \\ \frac{\mathrm{d}C'}{\mathrm{d}w_f} &= (1 - t_f) > 0 \end{aligned} \tag{3.28}$$

$$\begin{aligned} \frac{\mathrm{d}C}{\mathrm{d}\delta} &= [1 - (\theta_i^*)^2]\frac{(1 - t_f)}{2} > 0 \\ \frac{\mathrm{d}C'}{\mathrm{d}\delta} &= \frac{(1 - t_f)}{2} > 0 \end{aligned} \tag{3.29}$$

It is immediately obvious that without unemployment benefits the same exogenous shocks result in larger changes in consumption. It is also clear that a higher u^*, and thus θ_i^*, increases the counter-cyclical effect in these equations. Unemployment benefits indeed appear to flatten an exogenous shock in our model and if they are higher they do so more effectively. This feature resembles the findings of Rejda (1966) and Kiley (2003) as we discussed in section 2, the literature review.

Another observation that now becomes clearer is that our simple tax system indeed flattens the business cycle as well. An increase in w_f or δ isn't fully translated into an increase in C (or C') because t_f lowers the effect somewhat. The government absorbs some of the improvement of the workers earning potential which will flatten the business cycle if G_0, u^* and t_f don't get changed by the government in response.

3.5 Restrictions on the government budget in the static model

At the end of our initial equations we defined government net expenditures. We haven't done anything with the equation yet as we implicitly assumed that the government is always able to fulfill its obligations. So if more people become unemployed the government will pay these people their unemployment benefit ceteris paribus. The government will not change the tax rate or the level of the unemployment benefit. It will just run a debt when necessary without any repercussions.

But what if for example EMU regulations restrict the government in its capabilities to indebt the country? We will therefore take a closer look at what happens to equation (3.8) now. We will repeat that equation first.

$$G = G_0 + U - T$$

Again, we look at what happens if an external shock occurs (change in w_f or δ). This is given by the following repeated equations (3.14), (3.15), (3.18) and (3.19).

$$\begin{aligned}
\frac{\mathrm{d}U}{\mathrm{d}w_f} &= -\frac{u^*}{\delta} < 0 \\
\frac{\mathrm{d}U}{\mathrm{d}\delta} &= -\frac{(u^*)^2}{(1-t_f)\delta^2} + \frac{w_f u^*}{\delta^2} < 0 \\
\frac{\mathrm{d}T}{\mathrm{d}w_f} &= t_f + t_f \frac{w_f}{\delta} > 0 \\
\frac{\mathrm{d}T}{\mathrm{d}\delta} &= [1-(\theta_i^*)^2]\frac{t_f}{2} + t_f u^* \left(\frac{u^*}{(1-t_f)\delta^2} - \frac{w_f}{\delta^2} \right) > 0
\end{aligned}$$

So aggregating these for G we get that in the case of a negative shock the government net expenditures increase because T decreases and also increase because U increases. If we take the case without unemployment benefits only T would decrease, and although more strongly still not as strong as the combined effect of U and T with unemployment benefits. This means, as expected in contrast to the reaction of consumption, that the government budget reacts stronger to an external shock.

So countries with (higher) unemployment benefits react (more) stronger to external shocks. Countries that would try to smooth the business cycle through such a mechanism might get more easily into trouble with regulations like the Stability and Growth Pact of the EMU. What happens if they have to bring their budget in line with such regulations?

We assume that after an exogenous shock the net expenditures of the government exceed regulations. Again equation (3.8):

$$G = G_0 + U - T$$

First, lets say that the government decides to lower its public consumption G_0. It easily follows that aggregate consumption $(C+G_0)$ drops, so (part of) the counter-cyclical behavior of unemployment benefits is undone because government will have to drop public consumption.

It is a lot harder to touch the tax rate. First of all equation (3.20) tells us that depending on the present situation an increase in the tax rate might even lead to lower tax revenues. Moreover equation (3.16) already warns that a higher tax rate comes at the cost of higher unemployment benefit expenditures. So in the odd case that a country could benefit from lower taxes, a case with very many unemployed probably, it will be possible to lower taxes, increase tax revenue and decrease spending on unemployment benefits (a sort of Laffer curve situation if you wish). In general, it is more feasible to assume that a tax increase makes people unemployed which means the government loses tax revenue from these people and has to pay them an unemployment benefit. Although tax revenue probably will improve when the tax rate increases, its improvement is partly offset by people getting unemployed.

Getting more people to work, lowering the level of the unemployment benefit, appears to work a lot better. Equation (3.13) obviously states that a lower u^* leads to lower expenditures on unemployment benefits. All unemployed get less. Moreover equation (3.17) adds that tax revenue increases as well as some people decide to find a job that pays better than sticking to their, now lower, unemployment benefit. Following equation (3.21) we know that private consumption drops as a consequence of lowering u^*. This effect isn't as strong on aggregate consumption $(C+G_0)$ as directly lowering G_0 as some people now start working. However there is a downside as well to lowering unemployment benefits, which isn't immediately expressed in the aggregate consumption $(C+G_0)$ though. As we noted at the end of section 3.4 higher unemployment benefits flatten the business cycle better. If one would attach a future cost to more volatile aggregate consumption lowering unemployment benefits might not be the best option either. Another critique could come from Kiley (2003) whom state that in recession unemployment benefits should not be lowered because it is harder to find a job. Such means that it might be better from a stabilization point of view to keep or increase the level of unemployment benefits.

However if there is the risk that an unemployment benefit schedule creates an unsustainable debt the outcome might go the other way, Leeftink (2003). Their research shows that an unsustainable debt appears to lower the effectiveness of automatic stabilizers.

Concluding this section it is clear that restricting a government its net expenditures might lead to (partially) obsoleting automatic stabilization of the business curve through unemployment benefits and/or taxes. Though, there is some empirical literature that states that an unsustainable debt might also lower automatic stabilization effectiveness. Either way, a budget restriction might lead a government to making difficult decisions on which it needs very good information to choose the right magnitude of its policy changes as there are ambiguous effects at play.

3.6 Static model summary and conclusion

We have developed a static model environment where a value tax and unemployment benefits are supposed to stabilize the reaction to exogenous shocks. These shocks are introduced through changes in the earning potential of the population. We find that taxes and unemployment benefits indeed stabilize the reaction of consumption to exogenous shocks when compared to a situation where these fiscal mechanisms don't exist. However we do note that if we restrict the government budget in our model, EMU regulations for example, that this effect might become obsolete again.

The literature mostly supports our assumptions, most dominantly our assumptions on people just spending their income without taking Ricardian equivalence or alike into account.

One of the other interesting outcomes is that if an increase in the income of the population is more equally distributed one would expect more people to leave unemployment. This stretches to the other findings, which leads to the impression that a more equal distribution of income increases the effects of government policy in the fields we modeled.

4 Dynamic Keynesian model with automatic stabilization

The static model of the previous section provides intuition why unemployment benefits are thought to function as automatic stabilizers. It clearly shows the mechanics underlying automatic stabilization. Unemployment benefits automatically react to a negative shock and that makes that consumption stays at a higher level. It also clearly shows the negative correlation between a shock and government expenditures and debt.

However, as the previous model is static it doesn't account for future effects of having taxes and unemployment benefits. The static model doesn't account for supply effects either. In this section we will present a dynamic Keynesian model with taxes and unemployment benefits that observes more than one period and accounts for the supply side of the economy. Given the complex structure of this dynamic Keynesian model we will provide a numerical analysis as the main insights can't be derived analytically. However, it does give the opportunity to look beyond the direct effect of unemployment benefits, which was the mere subject of the previous section. The dynamic model is capable of showing how the economy evolves after a productivity shock and that is what we need to make a stronger case in favor (or against) automatic stabilizers. The chosen mechanics of the economy will become apparent alongside the description of the model equations in the first subsection.

Inspiration for the model was taken from the Economics courses at Erasmus University Rotterdam in general, and more specifically from the short-run model presented by Sørensen and Whitta-Jacobsen (2005), the savers-spenders theory model by Mankiw (2000) and the exposition by Cooley and Prescott (1995).

4.1 The dynamic Keynesian model equations

In this section we will describe the equations of the dynamic Keynesian model. After reading this section the reader is supposed to understand how our Keynesian economy functions. The subscript t in all beneath equations stands for the present time period. This addition compared to the static model makes it dynamic. Shocks, and automatic stabilization, in one period can now affect subsequent periods.

When necessary we also mention the boundaries of the equations. For some variables we can't allow them to become positive or negative. When one chooses wrong exogenous and/or starting values for these equations the model could cross these bounds and will then deteriorate. For example, we restrict the government debt from becoming negative. A negative government debt would mean the government is investing excess tax revenues, but our model does not account for that possibility. Therefore government debt is restricted from becoming negative.

National income definitions

$$\begin{aligned} Y_t^D &= C_t + I_t + G_t, & (4.1)\\ Y_t^S &= A_t {K_t}^\alpha {L_t^D}^{1-\alpha} & (4.2)\\ \text{and } Y_t^D = Y_t^S &\geq 0 \end{aligned}$$

These two equations are pretty standard economic descriptions of demand Y^D and supply Y^S. Demand consists of consumption C, investments I and government consumption G. Supply is created by a Cobb-Douglas production function with two production factors, capital K and labor L. A stands for overall productivity and will later on be our entry point for introducing impulse and business cycle shocks. α establishes in the optimum how much capital earns from total income. It determines the extent in which capital contributes to income.

Obviously national income should be equal on the demand and supply side. The number also has to be greater than zero else there is no economy to analyze.

Supply side

$$\begin{aligned} K_t &= \delta K_{t-1} + I_{t-1} & (4.3)\\ L_t^D &= \left(\frac{\lambda}{(1-\tau)A_t(1-\alpha){K_t}^\alpha}\right)^{\frac{1}{-\alpha}} & (4.4)\\ \text{and } K_t > 0 \quad &, \quad 0 < L^D \leq L^S \end{aligned}$$

The production factors on the supply side, capital K and labor L, are established as follows. Capital simply consists of the depreciated capital stock of last period, δ determines the depreciation rate, and last period's investments I.

Labor demand L^D is determined slightly more sophisticated in our model[2]. The equation uses a reservation/minimum wage λ, corrects for taxes τ, and then determines the labor demand through the derivative of the production function to labor. With negative shocks this will lead to temporary unemployment as the remaining wage, given the existing capital stock, can't be high enough to employ all workers.

Both variables are constrained to be larger than zero, else there is no production left. Obviously labor demand is restricted not to go beyond labor supply, L^S.

[2]A more thorough derivation of this labor demand equation is found in Appendix B.

Demand side

$$
\begin{aligned}
C_t &= (1-\tau)Y_t^S - I_t - B_t && (4.5)\\
I_t &= (1-\delta)K_t^F && (4.6)\\
G_t &= G^C + r_t D_{t-1} + \theta\lambda U_t && (4.7)\\
\text{and } C_t, I_t, G_t &> 0
\end{aligned}
$$

The demand side is given by consumption C, investments I and government consumption G. Consumption consists of after-tax income, where τ stands for the tax rate, minus private investments I and government borrowing B. Households lend the whole government deficit to the government as their rate of time preference is always met by the government. We will get to that in more detail when discussing the desired capital stock. Clear from this statement is that government lending can't crowd out private investment in our model during the first round. In subsequent periods this could change though.

Investments are determined in such a way that households invest the amount that would make up for the depreciation of the desired capital stock K^F, δ determines the depreciation rate. We will show the determination of the desired capital stock beneath.

Government spending consists of a fixed component G^C, interest payments over the debt of the previous period, $r_t \cdot D_{t-1}$, and possibly spending on unemployment benefits. The θ determines the height of the unemployment benefit in correspondence with the height of the reservation/minimum wage λ. That is obviously multiplied by the number of unemployed U.

We do not allow negative consumption, investment or government spending. From both a theoretical and practical point of view this is infeasible and allowing it will merely lead to deterioration of the model output.

Supply side prices

$$
\begin{aligned}
r_t &= \frac{\partial Y_t^S}{\partial K_t} = A_t \alpha {K_t}^{\alpha-1} {L_t^D}^{1-\alpha} && (4.8)\\
w_t &= \frac{\partial Y_t^S}{\partial L_t^D} = A_t(1-\alpha){K_t}^{\alpha} {L_t^D}^{-\alpha} && (4.9)
\end{aligned}
$$

The interest rate r and wage rate w are determined in a very similar fashion. The above derivatives mean that capital K and labor L^D earn their respective marginal products. This equation is kept simple because we assume that capital and labor income is equally taxed[3]. The wage rate is of low importance as labor demand immediately adjusts so to satisfy equation (4.4). Workers are essentially hired as long as $w \geq \frac{\lambda}{1-\tau}$. However, if labor demand exceeds supply the wage rate is higher than the reservation/minimum wage.

[3]Equal taxes for both income types means that taxes don't lead to some kind of arbitrage between them by households. For that reason we can simply take the derivatives of the production function Y^S. If there would have been a difference in the tax level we would have to take a closer look at price setting behavior. Maybe such is interesting for future research.

Government

$$\begin{aligned} D_t &= D_{t-1} + B_{t-1} &(4.10)\\ B_t &= G_t - \tau Y_t^S &(4.11)\\ \text{and } D_t &\geq 0 \end{aligned}$$

The above equations tell something about the government debt D and the government deficit B. Both are pretty straightforward. Present government debt simply consists of last period's debt and the deficit of last period.

The deficit could be positive occasionally (when introducing shocks), however we restrain the debt from becoming negative (an investing government). Not restraining the debt would create an infeasible capital income for the government, see equation (4.7). Moreover, if we would allow the deficit to become negative it would have to enter the production function, equation (4.2), as it then supposedly creates income. That doesn't not add any insight and would make the analysis more difficult than necessary, hence the restraint.

Optimal capital stock

$$K_t^F = \left(\frac{\rho}{(1-\tau)A_t \alpha L_t^{D\,1-\alpha}} \right)^{\frac{1}{\alpha-1}} \tag{4.12}$$

This equation states that households 'know' how high the capital stock should be to optimize for their rate of time preference ρ[4]. What the equation essentially does is determining how high the capital stock should be to get that the interest rate $r \geq \frac{\rho}{1-\tau}$. Restraints on this variable stem from the restraints on equations (4.3) and (4.6).

Unemployment level and tax rate

$$\begin{aligned} U_t &= L_t^S - L_t^D &(4.13)\\ \tau_t &= \frac{G^C + r_{t-1}D_{t-1}}{Y_{t-1}^S} &(4.14)\\ \text{and } U_t &\geq 0, \text{ essentially the same restraint as on (4.4)} \end{aligned}$$

Then these two equations complete the dynamic Keynesian model. Unemployment U is the remainder of labor supply L^S when labor demand L^D is subtracted. Taxes τ are endogenously determined with a delay of one period. Such means that the government makes an educated guess based on last period's government spending and the income of households to determine the tax rate in the present period. The government does recognize that its unemployment benefits payments are only temporary. Determining the optimal tax in the same period is impossible as labor demand, and thus government spending, is dependent on the tax rate. Production thus depends on the tax rate, so the tax rate can't depend on production which is necessary if one would like to determine it so to balance the budget.

[4] A more thorough derivation of this optimal or desired capital stock equation is found in Appendix B.

4.2 Input of static and starting values in the dynamic Keynesian model

The model has to take off with setting some exogenous figures and giving some starting values. We will minimize the latter, and compute as much endogenously as possible during the starting period. The figures beneath create a near steady-state situation, honestly we should add extra decimals for an immediate perfect steady state. However adding these doesn't change the intuitions derived.

Exogenous static variables

$$\begin{aligned} A_t &= 1 \\ \alpha &= 0.36 \\ \delta &= 0.95 \\ \lambda &= 0.4 \\ \theta &= 0 \text{ or } 0.5 \\ \rho &= 0.2 \\ L^S &= 300 \\ G^C &= 90 \end{aligned}$$

When applicable we state our sources for these exogenous values. Productivity is set at unity for now, but is exogenously amendable to introduce shocks. According to Mankiw (2000) capital earns about one-third of income and Cooley and Prescott (1995) refer to more precise measures which lead to $\alpha = 0.36$. The depreciation rate is set at 0.05 ($\delta = 0.95$) which is a reasonable figure for yearly depreciation in our model, again relating to Cooley and Prescott (1995). The other variables are arbitrarily set to establish a steady-state, while trying to mimic characteristics comparable to the U.S. economy, again due to Mankiw (2000).

Exogenous starting variables

$$\begin{aligned} K &= 402 \\ \tau &= 0.33 \\ D &= \frac{1}{6}K = 67 \end{aligned}$$

Mankiw (2000) states that for the U.S. economy taxes are about one-third of income and that government debt equals about one-sixth of the capital stock. Knowing that and trying to have the model in a steady state the above values follow. Obviously one could choose other values, but the general idea stays the same as long as you let the model start in a steady state.

4.3 Analysis of the dynamic Keynesian model output

With the model set up for starting in steady state we can now look at how the model output changes when we introduce productivity shocks through changing A. We will sketch three different theoretical and thus artificial scenarios. The first is a one-period negative shock to productivity. The second is the same shock but then fading away over consecutive periods. The last is the application of a sinusoid function to productivity. The sinusoid function gives both positive and negative shocks for a longer period of time.

We will not discuss steady state. The above explanation of the chosen static and starting values discusses the state of that static situation sufficiently. We will show for each scenario the situation with and without unemployment benefits. Upfront we can mention that the reader should keep an eye on how the debt service stemming from unemployment benefit payments changes the state of the economy. The productivity shocks were chosen arbitrarily to show a noticeable change in the output and not to let the model economy deteriorate.

For each scenario we will show four different figures showing all variables for the situation with and without unemployment benefits, respectively $\theta = 0.5$ and $\theta = 0$. The output with unemployment benefits is recognized by the "(w)" suffix. All variables except the deficit B and total unemployment U are normalized by dividing the output of each scenario by the steady state output as given by the starting and static values of the previous section. Doing so sheds more light on which variables react stronger or weaker to the shock.

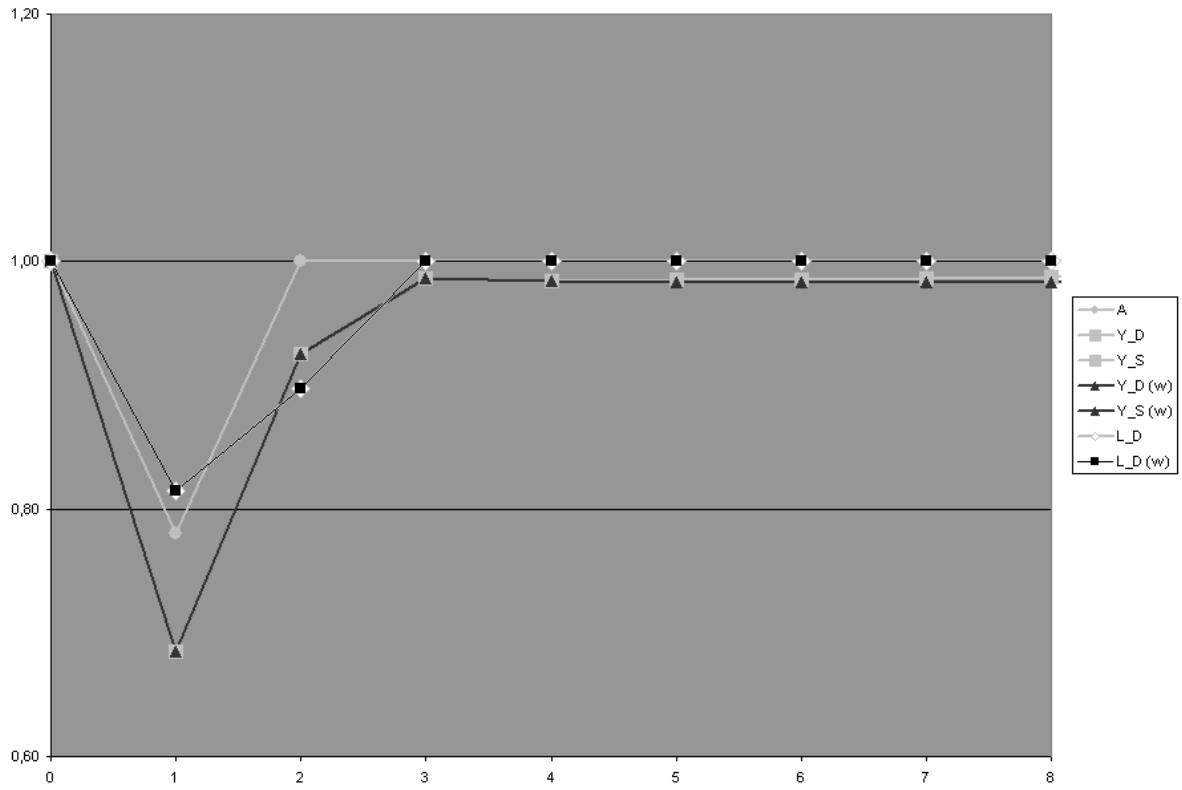
4.3.1 Single impulse shock analysis

We start with a very simple scenario, being that of a single impulse shock to productivity A that lasts only one period. The model starts in steady state in period 0 and the shock is given by a single impulse in period 1 of twenty-two percent, $A_1 = 0.78$.

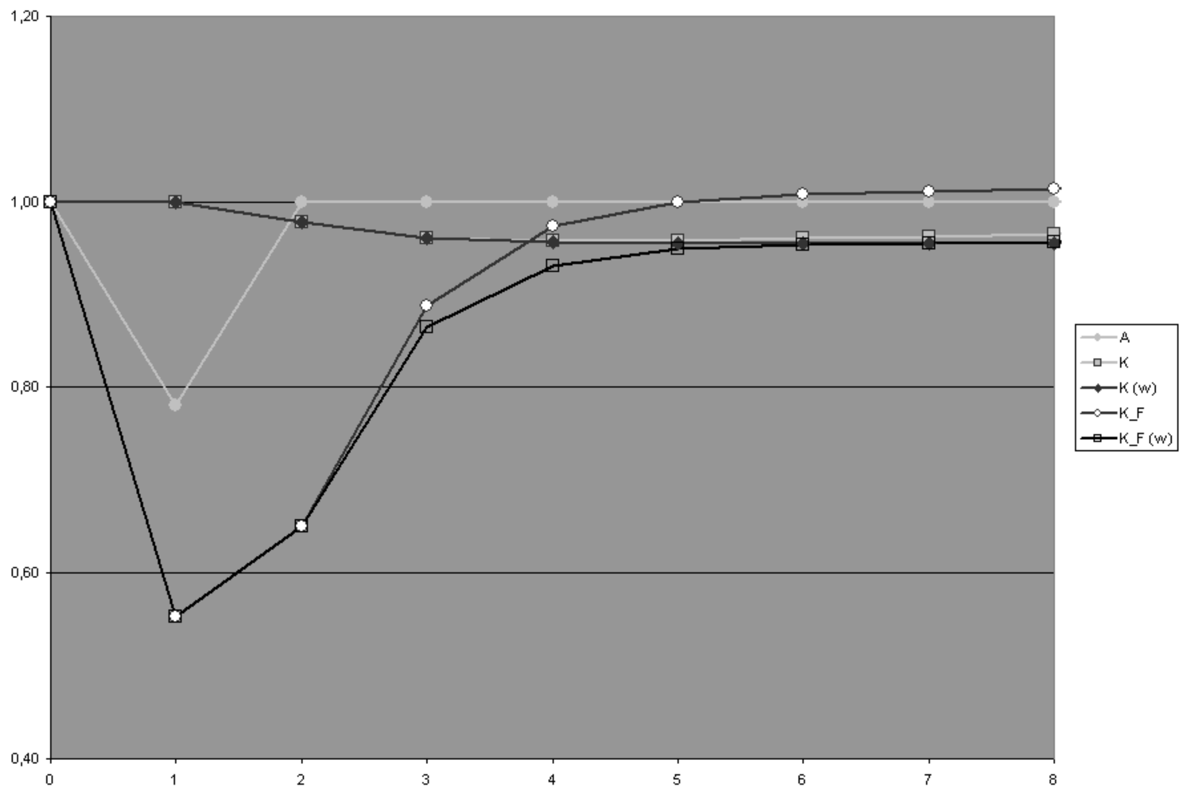
Figure 1 shows the evolution of A, Y^D, Y^S, K, L^D and K^F after the shock. There is no direct clear difference between the output. A first conclusion should indeed be that a single shock doesn't give very different outcomes with and without unemployment benefits. However a closer look shows that the capital stock isn't returning to the steady state of period 0 in both situations. Because of the non-anticipation of the government the deficit becomes pretty significant in period 1 when having unemployment benefits. Given the nature of our model the government eventually sticks to its new debt level which lowers the optimal level of capital through the higher tax level. The economy without unemployment benefits doesn't suffer from this. Even on the contrary, because of the lag in the government's response, leading to a lower debt level, the optimal level of capital becomes higher than in period 0.

Figure 2 shows the evolution of A, C, I, G and D after the shock. Here the differences are more obvious. It is indeed clear that the higher deficit in period 1 leads to a higher debt for the economy with unemployment benefits. Private consumption, in our model consumption from unemployed is excluded from C, also gets a stronger hit in that economy as the government guarantees the desired return on capital for that period. A strange observation would be that government expenditures in the non-benefit economy drops in period 1. That is explained by the lower interest rate of that period, which lowers the government its interest payments. The interest rate of that period is lower as it is an expression of the income capital produces. As overall productivity drops the interest rate can only follow the shock. Through the investment function, equation (4.6), the interest rate then slowly adjusts back to its desired level.

Overall it is clear that the higher debt service, the consequential higher government expenditures

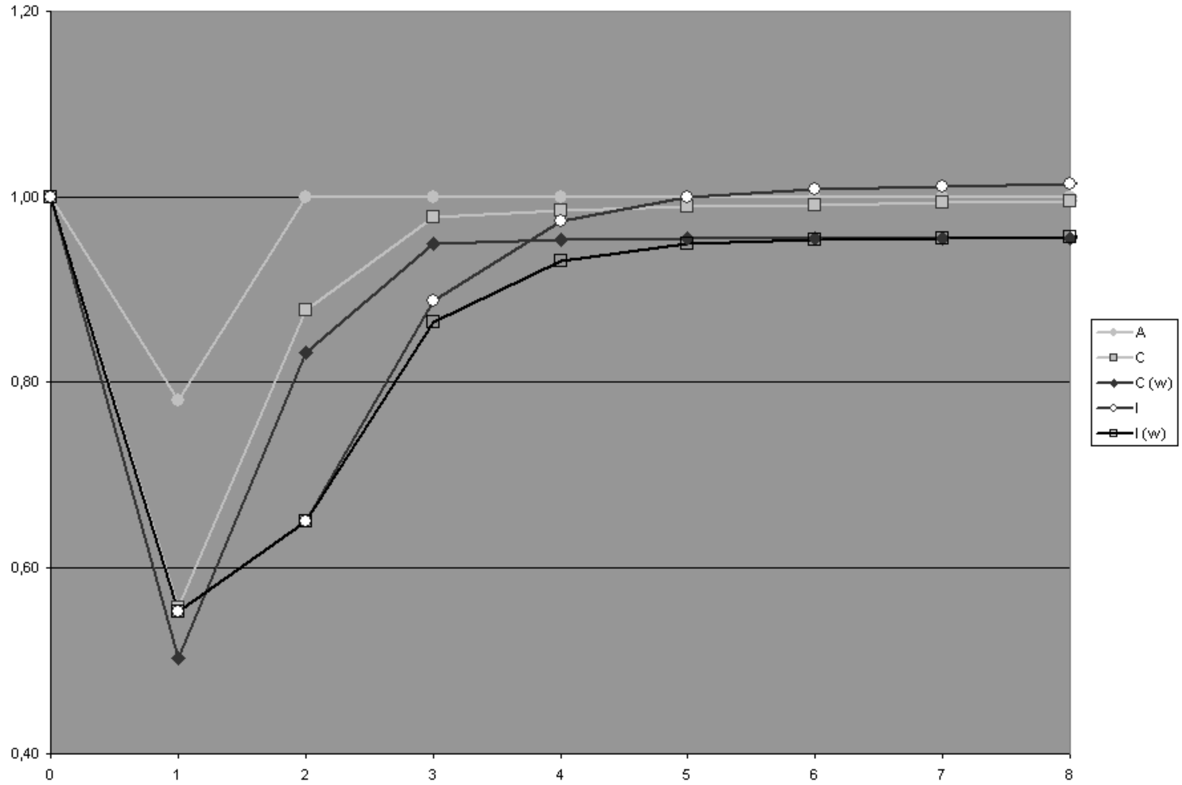


(a) National income and labor demand

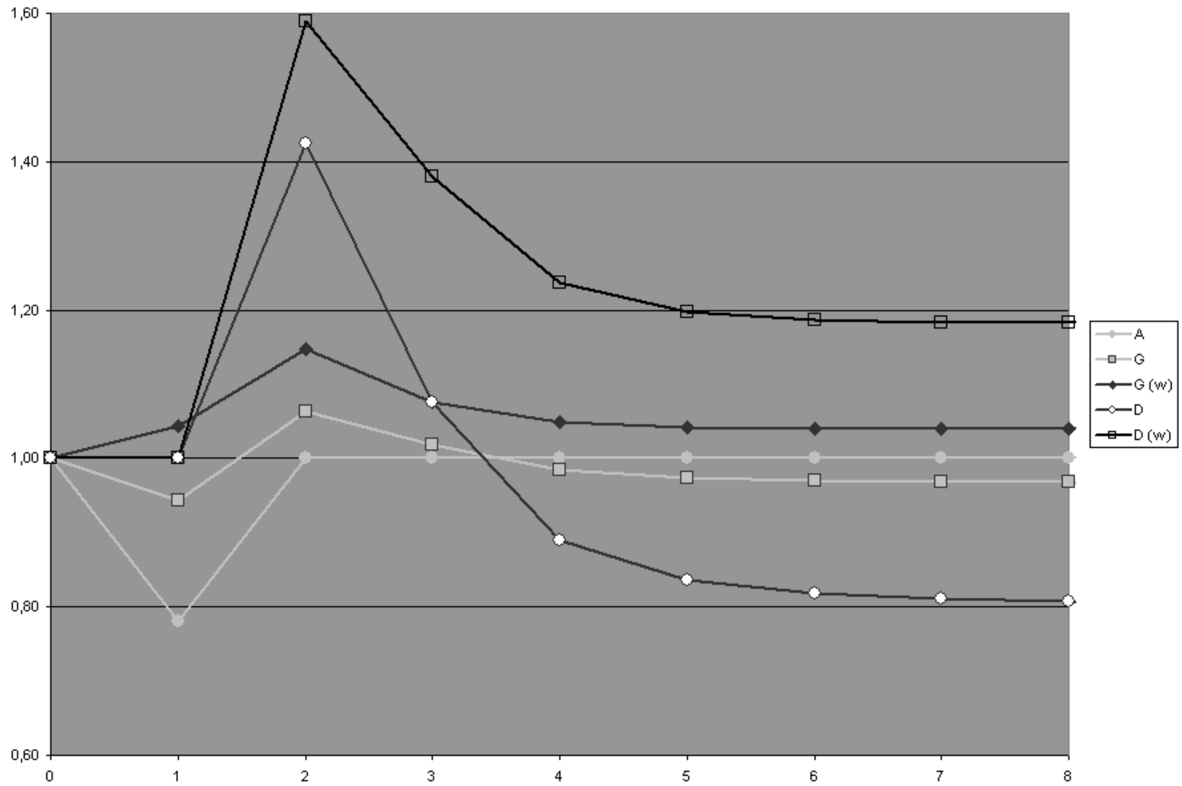


(b) (Optimal) capital stock

Figure 1: Graphical representation of A, Y^D, Y^S, K, L^D, K^F in the single impulse shock scenario.



(a) Consumption and investment



(b) Government consumption and debt

Figure 2: Graphical representation of A, C, I, G, D in the single impulse shock scenario.

leading to a higher tax rate, lead to lower aggregate consumption and investments post-shock in the unemployment benefit economy.

Figure 3 shows the evolution of A, t, r and w after the shock. We already discussed the higher level of government expenditures after the shock for the unemployment benefit economy, the income tax indeed remains higher. Not really evident from the figures, but as a consequence the non-benefit economy in the end observes a lower nominal interest rate. Such is obvious as the income tax rate is also lower in that economy. That then leads to a higher optimal capital stock which eventually leads to a higher wage rate, as a consequence of full employment in equilibrium. All these long-run effects will become more obvious when we look at the fading impulse shock analysis.

Figure 4 shows the evolution of U and B after the shock. Unemployment is the same in both situations with a single impulse shock. The structural characteristics of the model economy are apparently such that after a single shock everything is the same, with or without unemployment benefits. The deficit does react differently. We notice two effects. The unemployment benefit economy exhibits a higher deficit when the shock strikes and shows a lower surplus in period 2 as unemployment persists. That leads to a higher debt which we already discussed. Remains the subsequent surpluses of the government under both scenarios. That stems from the effect that the government sets its tax rate based on the previous period. After a negative shock the government thus (decreasingly) underestimates tax revenues.

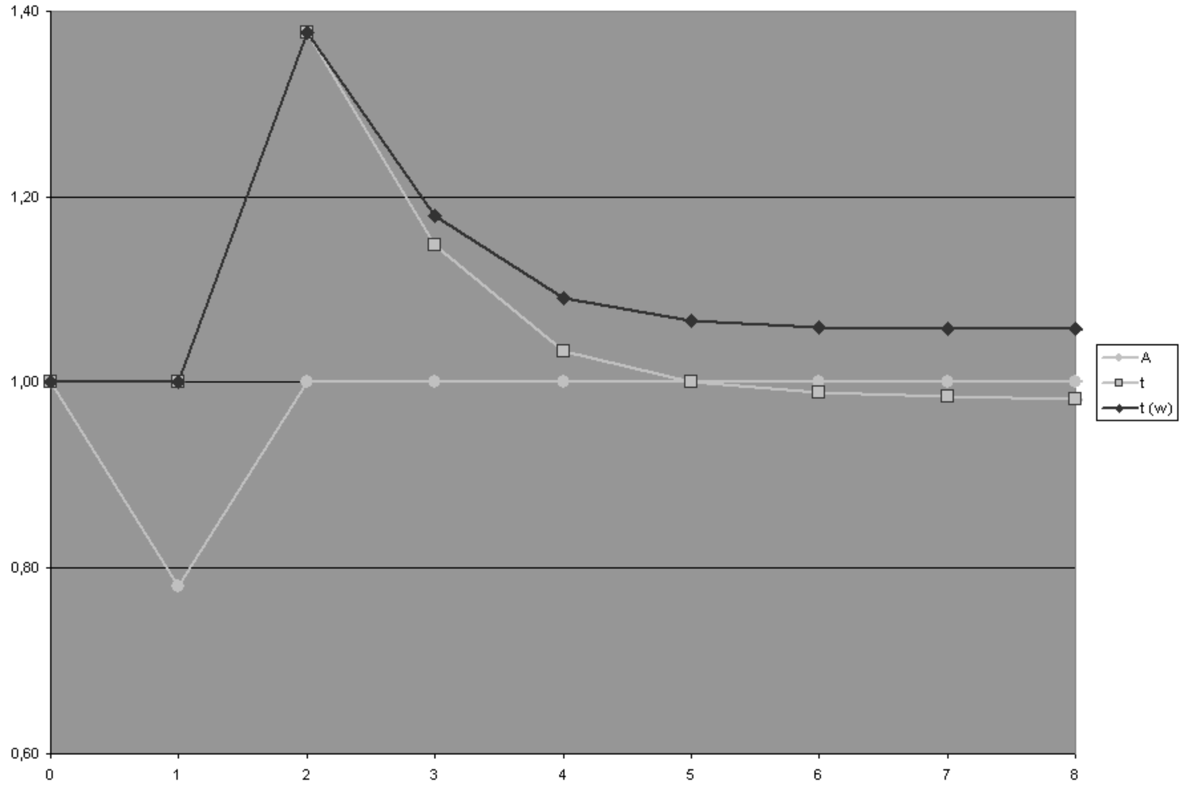
Spread of the output after the single impulse shock

Other than looking at the model output in figures we could also look at the variance of the output. As we in the end would like to say something about the stabilizing effects of a measure like unemployment benefits we will do so now. For this scenario we calculated the standard deviation[5] of all model variables over the starting period (period 0) and the following fifteen periods. These fifteen periods in our opinion exhibit the strongest results from the single impulse shock. The long-run results have already been discussed sufficiently above.

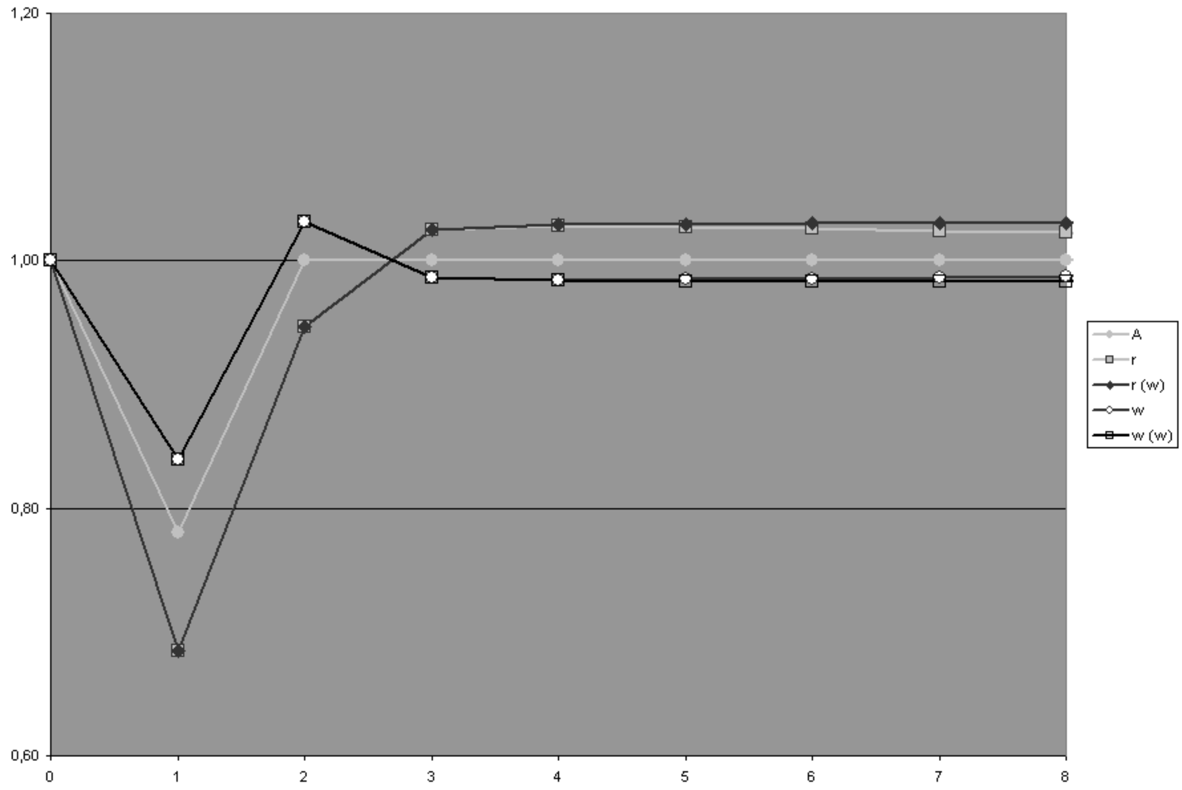
Table 1 shows the standard deviations for all variables. We also derived one extra variable, total private consumption including unemployment benefits C^*. Leaving that measure out would give an incorrect impression of how consumers would experience the volatility of their consumption. We here make the implicit assumption that unemployed spend their whole benefit, which is not unreasonable given the mediocre level of it as defined by us. The volatility of A is merely shown to get a feel for how high the standard deviation is when a variable only drops twenty-two percent for one period.

Within the boundaries of our calculation national income Y is indeed less volatile when having unemployment benefits. The same holds for the optimal capital stock K^F. However that stems from the fact that the post-shock non-benefit economy has a higher optimal capital stock than in the initial steady state. Total consumption including unemployment benefits C^* has a lower volatility in the unemployment benefit economy as expected, consumers are hit less by the negative shock. We then don't even account for the fact that unemployed in the non-benefit economy wouldn't get any money. From an utility perspective, decreasing marginal utility of money, consumption volatility would have been even

[5]Standard deviation is a measure of the spread of a set of values. It reveals how volatile a variable has been and that makes it very useful for our analysis.



(a) Tax rate



(b) Wage and interest rate

Figure 3: Graphical representation of A, t, r, w in the single impulse shock scenario.

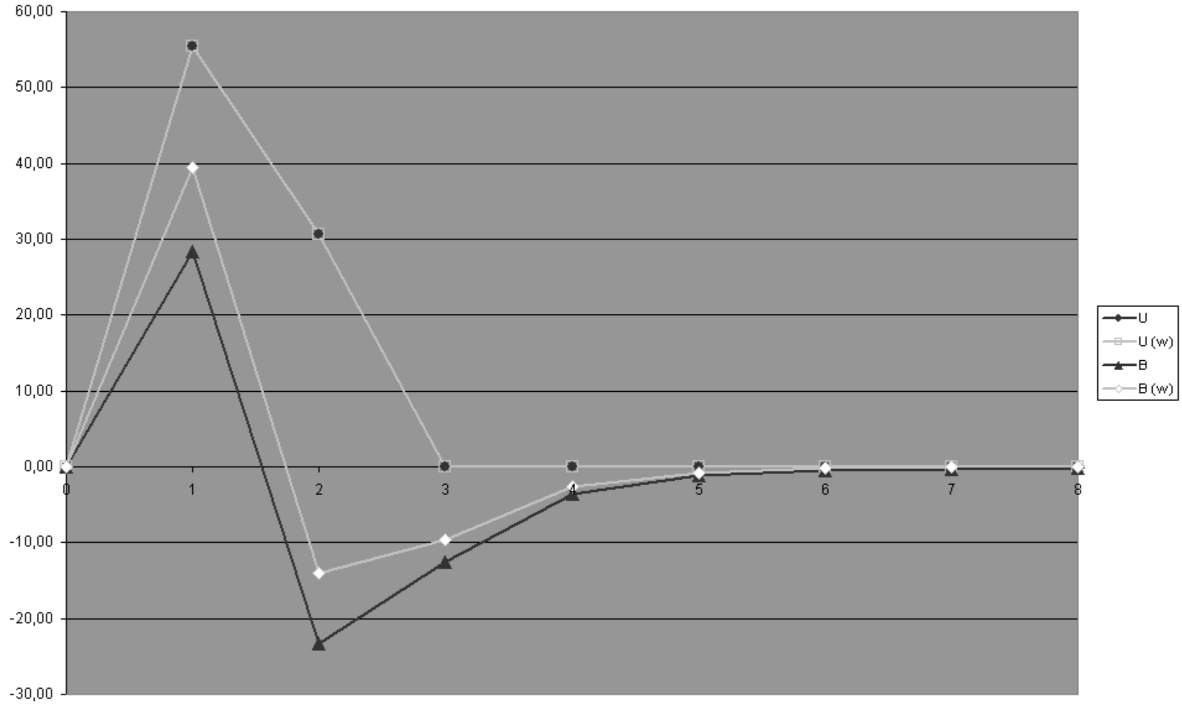


Figure 4: Graphical representation of U, B in the single impulse shock scenario.

Table 1: Standard deviations in the single impulse shock scenario (t = 0 to 15)

Variable	With benefits	Without benefits
productivity (A)	0.055	0.055
national income (Y)	25.245	25.643
capital stock (K)	6.291	5.223
labor demand (L^D)	15.392	15.392
optimal capital stock (K^F)	49.557	56.906
private consumption (C)	23.702	22.725
C + unemployment benefits (C^*)	20.747	22.725
investments (I)	2.478	2.845
government consumption (G)	3.329	3.120
government debt (D)	8.966	11.286
tax rate (τ)	0.029	0.034
interest rate (r)	0.026	0.025
wage rate (w)	0.028	0.028
unemployment (U)	15.392	15.392
government deficit (B)	11.095	10.025

worse in the non-benefit economy if we would have added a measure for that. The stronger reaction of the government budget to unemployment is seen in the higher volatility of government expenditures G and the deficit B. Together with figures 2 and 4 it is clear that the unemployment benefit economy reacts more instantaneous to a negative shock.

We are cautious towards the other variables to infer intuitions from. When looking at the actual output much of the differences between those seem to come from the differing steady states the unemployment benefit and non-benefit economy diverge to. Some also appear to be inherent to the model. The unemployment benefit economy shifts to a higher debt level which gives a higher income tax. This higher income tax for example smooths the volatility of investment as changes to the optimal capital stock are also smoothed by that higher income tax.

4.3.2 Fading impulse shock analysis

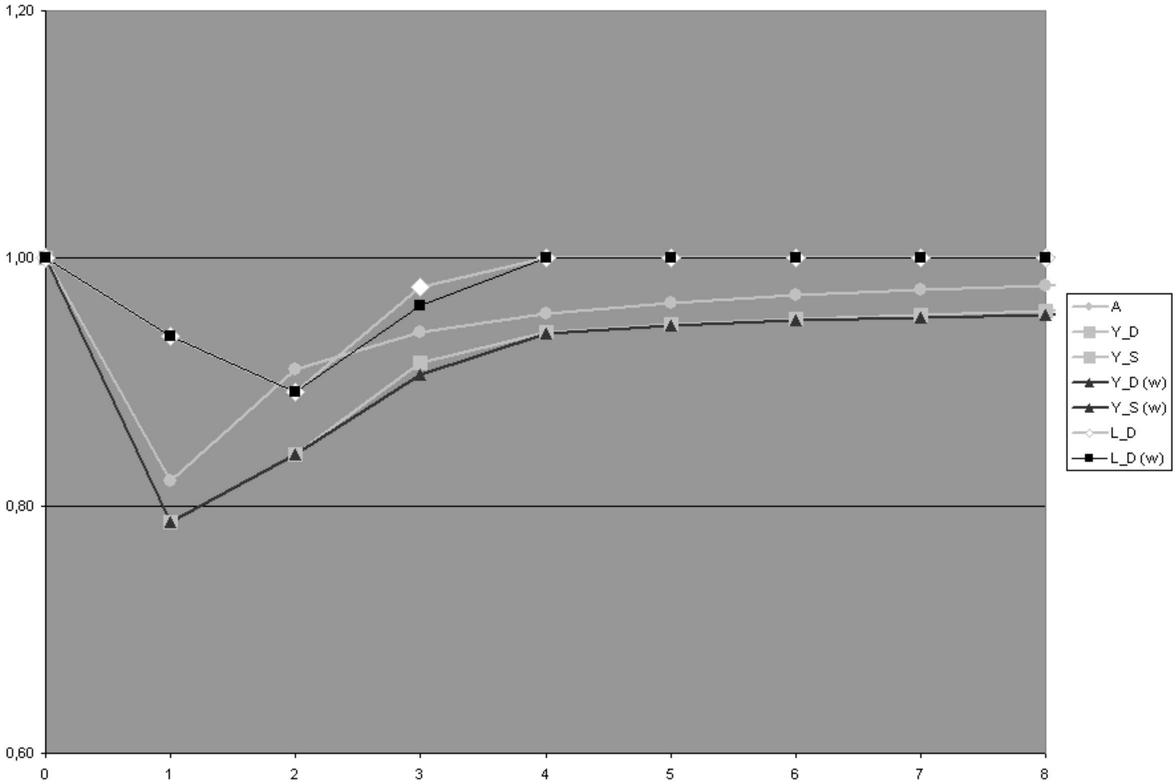
Now we will analyze the situation of a fading impulse shock. The shock will theoretically last indefinitely but will tend to zero. The model starts in steady state in period 0 and the shock is given by a fading impulse shock of the following form:

$$A_{1\to\infty} = 1 - 0,18 \cdot t^{-1}$$

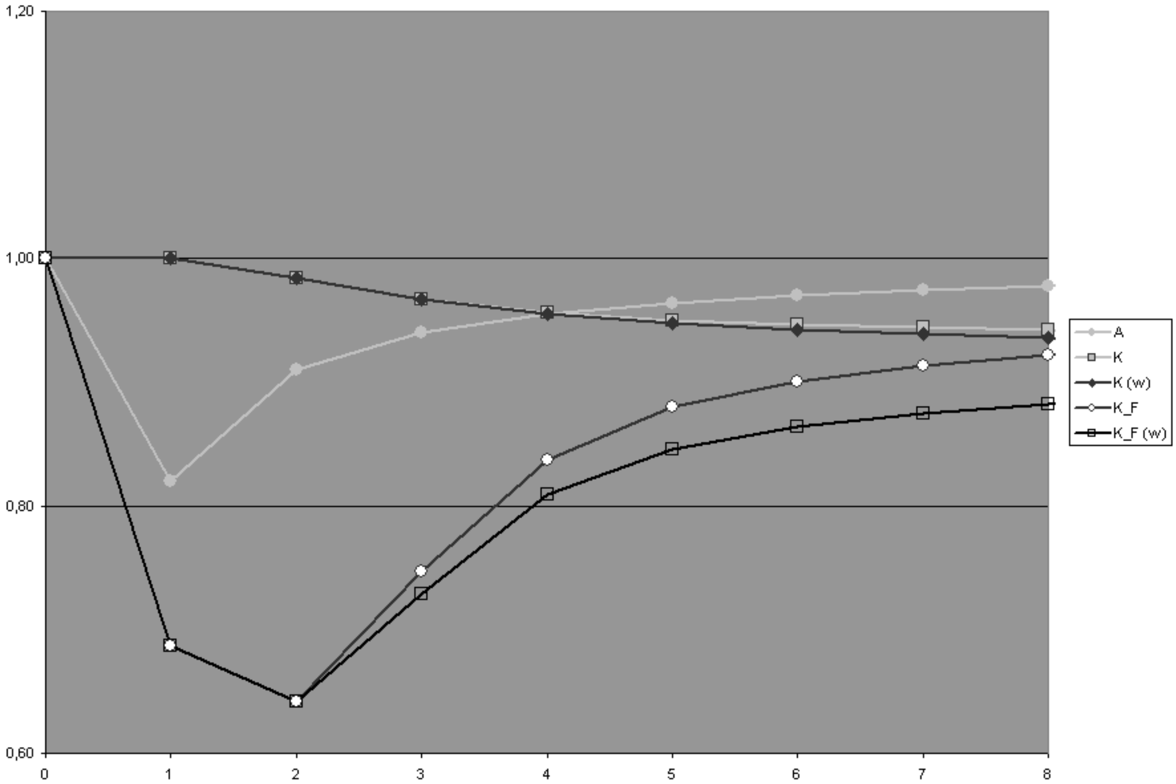
We have chosen for a different initial shock than the single shock under the previous scenario. We did this because a compromise to have them both at the same level led to less discernable effects on the other variables under both scenarios. In favor of our numerical analysis we therefore decided not to take the same starting level. Hence we advise not to compare the results of this section and the previous in detail, but to evaluate them in their own right. We don't regard this a problem as our research question doesn't concern the comparison of business cycle modalities and thus we are less concerned with this difference in starting level.

Figure 5 shows the evolution of A, Y^D, Y^S, K, L^D and K^F after the shock. Probably hard to observe, but evident from the given definition, productivity A does not return to 1. As with the single shock there isn't that much to see here. We again observe that the non-benefit economy returns to a higher steady-state capital stock level. The significant borrowing by the government of the unemployment benefit economy keeps that economy at a higher debt in the following periods. Most dominantly because of the persistence of the shock, although fading, the non-benefit economy does not end up at a higher optimal capital stock level under the fading impulse shock contrary to what we saw with the single impulse shock. Unless you are a hawk-eye you probably won't immediately notice the difference in labor demand for both economies in period 3. That observation is more evident in figure 8 and will be discussed in the corresponding paragraph.

Figure 6 shows the evolution of A, C, I, G and D after the shock. Very clear is that the unemployment benefit economy is running a very high debt and is, by definition, destined to stay at that higher debt level. While the non-benefit economy is already paying off some of its debt in period 3 the other economy is still increasing its debt. The reason for that is the persisting unemployment in period 3, and the difference in labor demand aggravates that. As promised more on that in a following paragraph. The

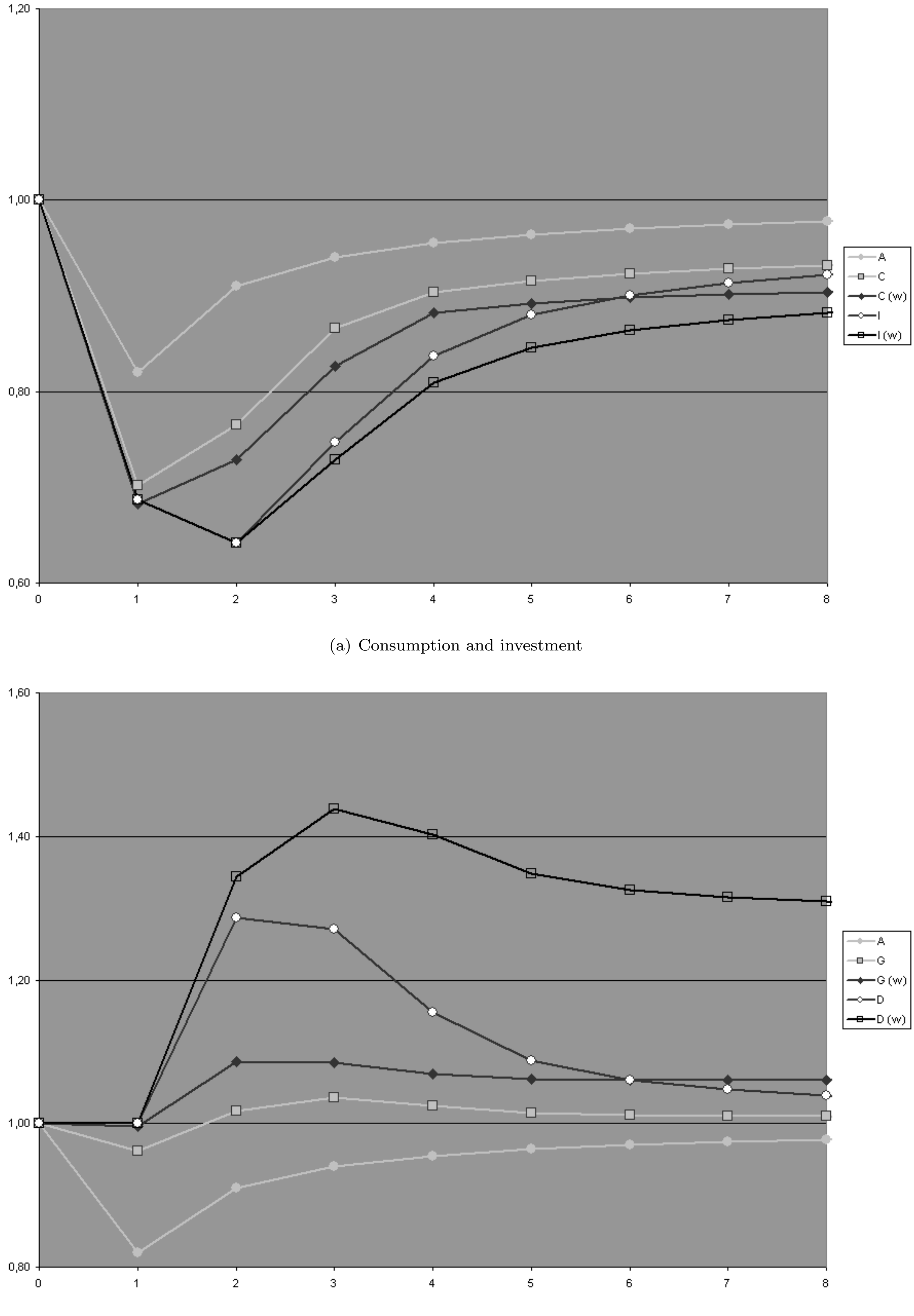


(a) National income and labor demand



(b) (Optimal) capital stock

Figure 5: Graphical representation of A, Y^D, Y^S, K, L^D, K^F in the fading impulse shock scenario.



(a) Consumption and investment

(b) Government consumption and debt

Figure 6: Graphical representation of A, C, I, G, D in the fading impulse shock scenario.

other variables work as under the previous scenario and as one would expect. Higher debt D leads to higher government expenditures G which leads to a higher tax rate τ (see figure 7). The higher tax rate leads to lower consumption C, less disposable income. It also leads to lower investment I as the optimal capital stock becomes lower because of lower net return at the same capital stock level (an effect of higher taxes in our model).

An interesting observation is that government expenditures in the unemployment benefit economy has its top in period 2 and in the non-benefit in period 3. One reason is that the gravest unemployment occurs in period 2 and hence the higher expenditures of the unemployment benefit economy in period 2. However the top in period 3 of the non-benefit economy is also explained by looking at how the interest rate evolves (see figure 7). As that economy appears to restore more swiftly the government also faces a higher interest rate earlier on. Once the unemployment benefit economy gets to that point of restoration the effects of the impulse shock have already sufficiently faded.

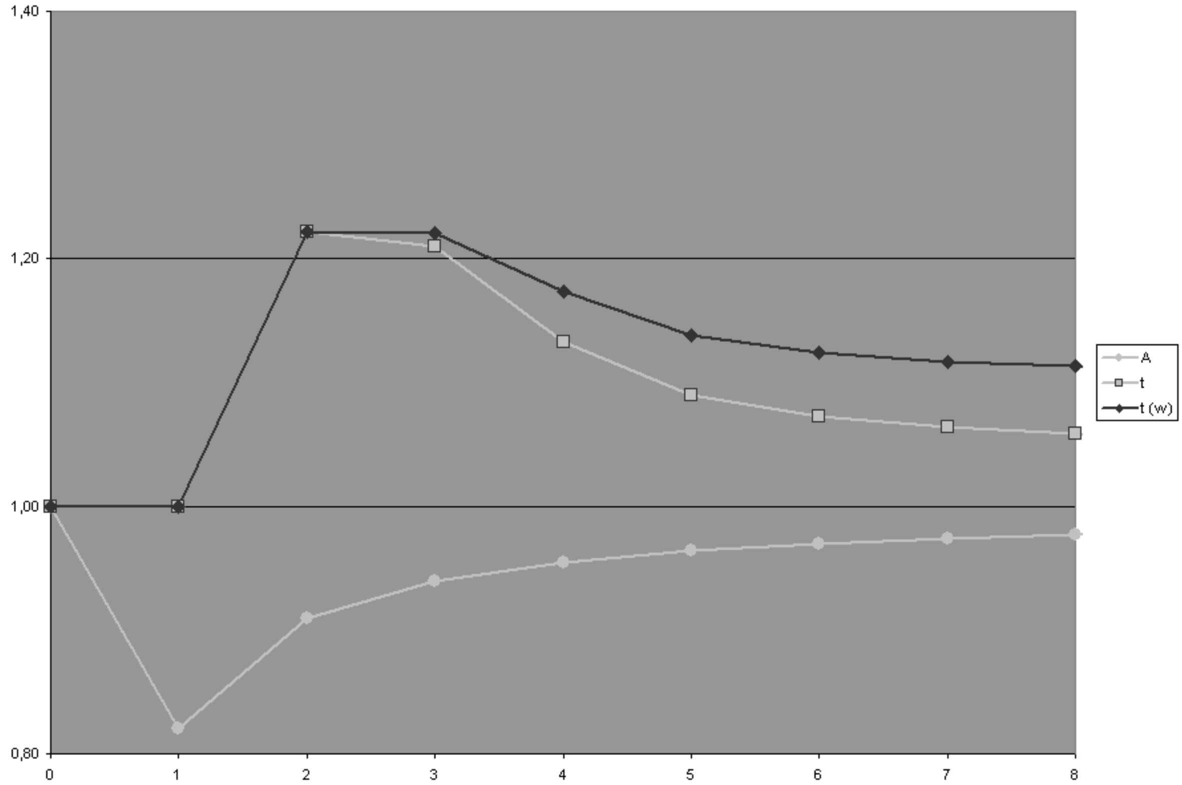
Figure 7 shows the evolution of A, t, r and w after the shock. The following is in fact just a repetition of what we said in our single impulse shock analysis. As you can see higher government expenditures lead to higher taxes in the following periods of the unemployment benefit economy. These higher taxes lower the optimal capital stock, from which the wage rate suffers under full employment, and the before taxes interest rate also becomes higher. These differences become more evident when we analyze the sinusoid business cycle scenario.

Figure 8 shows the evolution of U and B after the shock. As we already mentioned, unemployment now shows a slight difference in period 3. Unemployment remains higher when the government uses unemployment benefits to support consumption. The extra accumulated debt puts such a pressure on the economy, a lower optimal capital stock, through which labor demand restores slightly slower. On the one hand one could argue that the immediate response of the government is in itself good, but it does seem to harm future periods. We again also observe that the non-benefit economy, obviously, has a lower deficit in the worst periods and turns to higher surpluses in subsequent periods. The lag in adjusting the tax rate combined with the persistence of unemployment also makes us see that the unemployment benefit economy has its highest surplus in a later period than the non-benefit economy. Apparently the unemployment benefit economy has more difficulty to return to its new equilibrium, or to put it more neutral, takes more time to return to that new equilibrium.

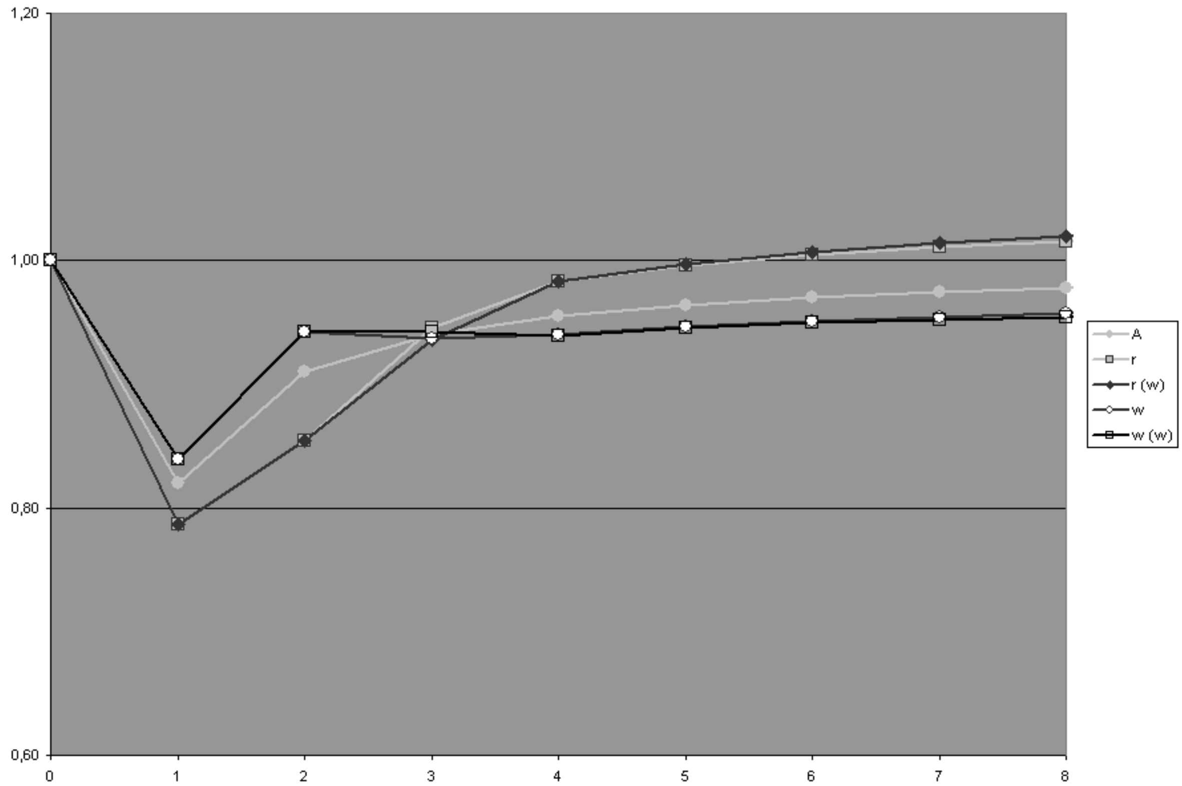
Spread of the output after the fading impulse shock

As we did with the single impulse shock scenario we will also show the standard deviations of the output in the fading impulse shock scenario. For this scenario we calculated the standard deviation of all model variables over the starting period (period 0) and the following fifteen periods. These fifteen periods in our opinion exhibit the strongest results from the fading impulse shock. The long-run results have already been discussed sufficiently above.

Table 2 shows the standard deviations for all variables. We again also derived the extra variable, total private consumption including unemployment benefits C^*. The volatility of A is again shown to get a sense of the standard deviation of a function as we used.



(a) Tax rate



(b) Wage and interest rate

Figure 7: Graphical representation of A, t, r, w in the fading impulse shock scenario.

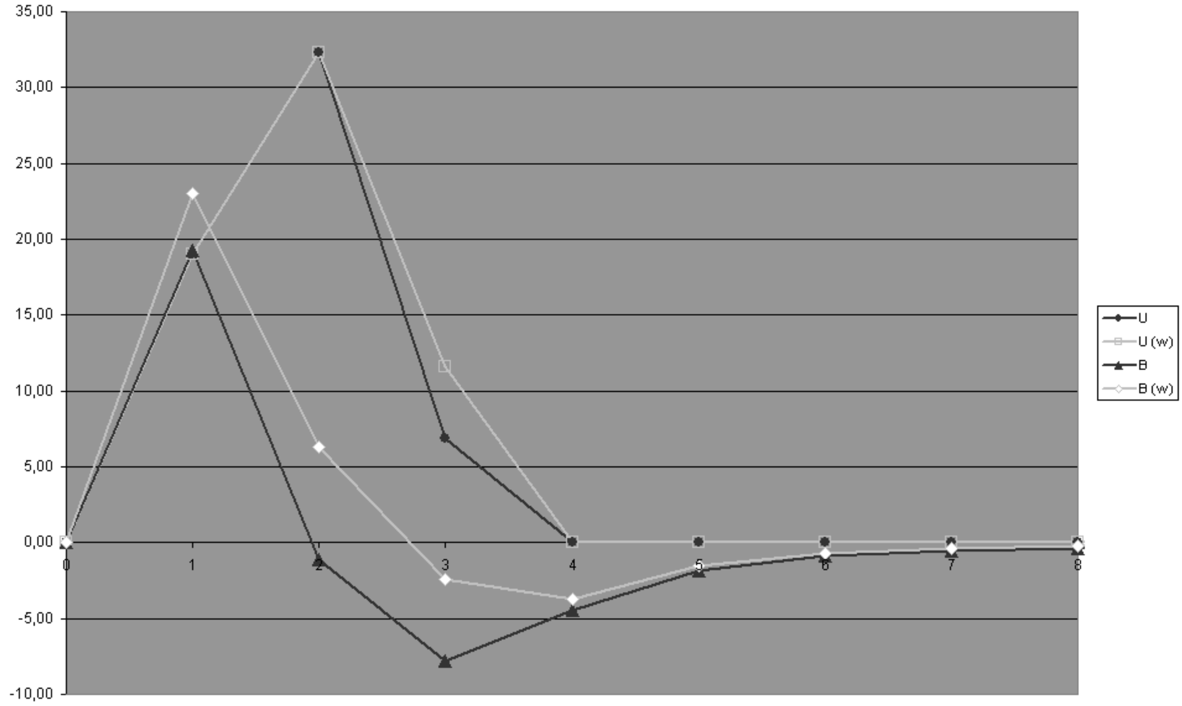


Figure 8: Graphical representation of U, B in the fading impulse shock scenario.

Table 2: Standard deviations in the fading impulse shock scenario (t = 0 to 15)

Variable	With benefits	Without benefits
productivity (A)	0.044	0.044
national income (Y)	17.378	17.654
capital stock (K)	10.611	8.557
labor demand (L^D)	9.286	9.101
optimal capital stock (K^F)	36.627	40.962
private consumption (C)	15.386	15.050
C + unemployment benefits (C^*)	13.791	15.050
investments (I)	1.831	2.048
government consumption (G)	2.701	1.683
government debt (D)	7.977	6.020
tax rate (τ)	0.020	0.021
interest rate (r)	0.022	0.021
wage rate (w)	0.023	0.023
unemployment (U)	9.286	9.101
government deficit (B)	6.149	5.514

The results in this table correspond pretty much with those in the previous table. Again both national income Y and our adjusted consumption measure C^* are indeed less volatile when having unemployment benefits. The latter indicates a more positive situation for the general populace. We can also see that the government (automatically) reacts stronger to the impulse shock, government expenditures, the debt and the deficit are more volatile.

Again, we should be cautioned not to infer too much information from the above statistics. We commented on that under table 1. However the higher standard deviation of the interest rate in the unemployment benefit economy might correspond with the earlier remark that the unemployment benefit economy has a longer/tougher trajectory back to a (near) stable state. The difference in the volatility of labor demand (unemployment is merely a mirror statistic of labor demand) stems from the difference in period 3 between both figures. That is therefore not that interesting, other than a confirmation that labor demand is indeed worse in the unemployment benefit economy. Moreover, if one would observe only the volatility of labor demand of the first 5 periods, including period 0, the outcome would be the other way around. During the worst of the shock, unemployment would be regarded less volatile. But that is only because it remains higher in period 3 and one could easily argue that that isn't a positive feature.

4.3.3 Sinusoid business cycle analysis

Now we will analyze the situation of a sinusoid business cycle. We designed this sinusoid function to be quite volatile; we didn't necessarily intend to mimic a empirical business cycle. The idea is to see how both economies react to negative and positive shocks following each other. The model starts in steady state in period 0 and the shock is given by a sinusoid function of the following form:

$$A_{0\to 30} = 1 + |0.25 \cdot \sin(t \cdot \frac{\pi}{6})| - |0.22 \cdot \sin(t \cdot \frac{\pi}{10})|$$

There is no special reason for this form. However we do need to comment on some characteristics. First of all the sinusoid lasts for only 30 periods. This is necessary as continuing it for infinity will make the output get out of bound. That happens because of the lag in government tax setting behavior, equation (4.14). The unemployment benefit economy becomes too indebted after a while, and the non-benefit economy will get into a negative debt which is by definition illegal in our model.

Theoretically one could argue that unemployment benefit payments in recession should perfectly alternate with budget surpluses in boom periods so that we shouldn't have to restrict the time span to 30 periods. If that would be the case the model wouldn't deteriorate indeed. However, our government or model in general isn't designed to perfectly alternate as the government lags in its response to changes in the economy. So the model could move out of bounds if continuing shocks keep the economy of a balanced path. Each subsequent shock pushes the government further away from the steady state, the direction depends on the direction of the shock and whether the government hands out unemployment benefits. And each step further away from steady state means a step closer to deterioration. Hence the limitation on the duration of the sinusoid function.

Now we will discuss the components of the sinusoid business cycle function. The base level of productivity A is unity. Then putting the period as a variable, modified by multiplying by a division with π as the dividend, into a sinus function creates a positive shock of six periods with a maximum magnitude of $+0.25$ and a negative shock of ten periods with a maximum magnitude of -0.22. Both repeat themselves again after one cycle. The combination of both creates a business cycle of differing magnitudes both up and down. Figures 9, 10 and 11 all show how the function progresses over time.

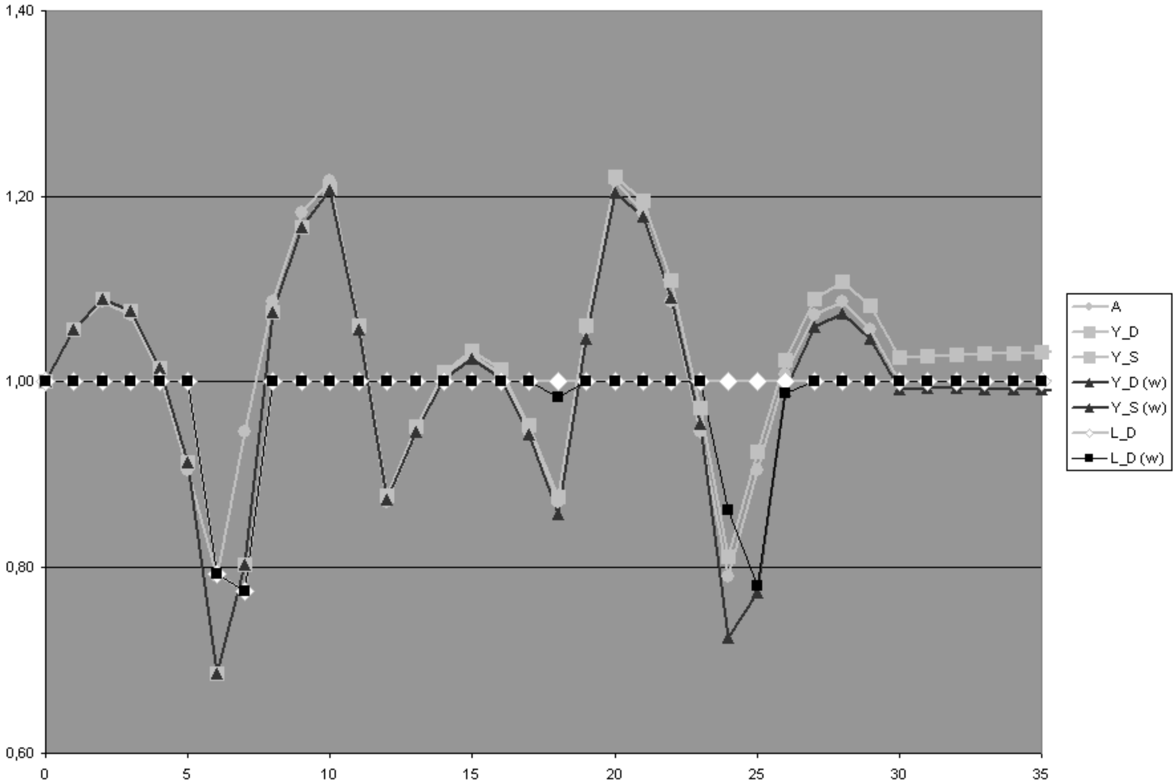
Figure 9 shows the evolution of A, Y^D, Y^S, K, L^D and K^F after the shock. The period for the graphs has been lengthened and the volatility of the productivity function might make the figures somewhat overwhelming at first. However, as in the previous sections, we keep focusing on differences. The most evident difference is that the non-benefit economy over the periods is able to attain a higher level of the optimal and actual capital stock, K^F and K. We already extensively discussed that the unemployment benefit government its borrowing behavior is the reason for that. The consequential higher taxes 'crowd out' private investments.

Another striking event is that labor demand L^D in period 18, 24, 25 and 26 is not at its maximum (1.00 in the graphs) for the unemployment benefit economy, contrary to the non-benefit economy that only suffers unemployment like the unemployment benefit economy during period 6 and 7. What is happening here? The extra debt burden created when handing out unemployment benefits puts the unemployment benefit economy in a worse shape which makes it more vulnerable during future negative shocks. It is even that bad that period 26 has a slightly higher than 1.00 productivity A but still has some unemployment. Period 26 is also the third consecutive period of unemployment and under the single impulse shock scenario we saw that the economy only took two periods to adjust after a severe negative shock. This effect combined with the lower (optimal) capital stock overall leads to a lower national income, Y^D and Y^S. That is easily seen when comparing with productivity A, the non-benefit economy performs better than A and the unemployment benefit economy performs worse than A.

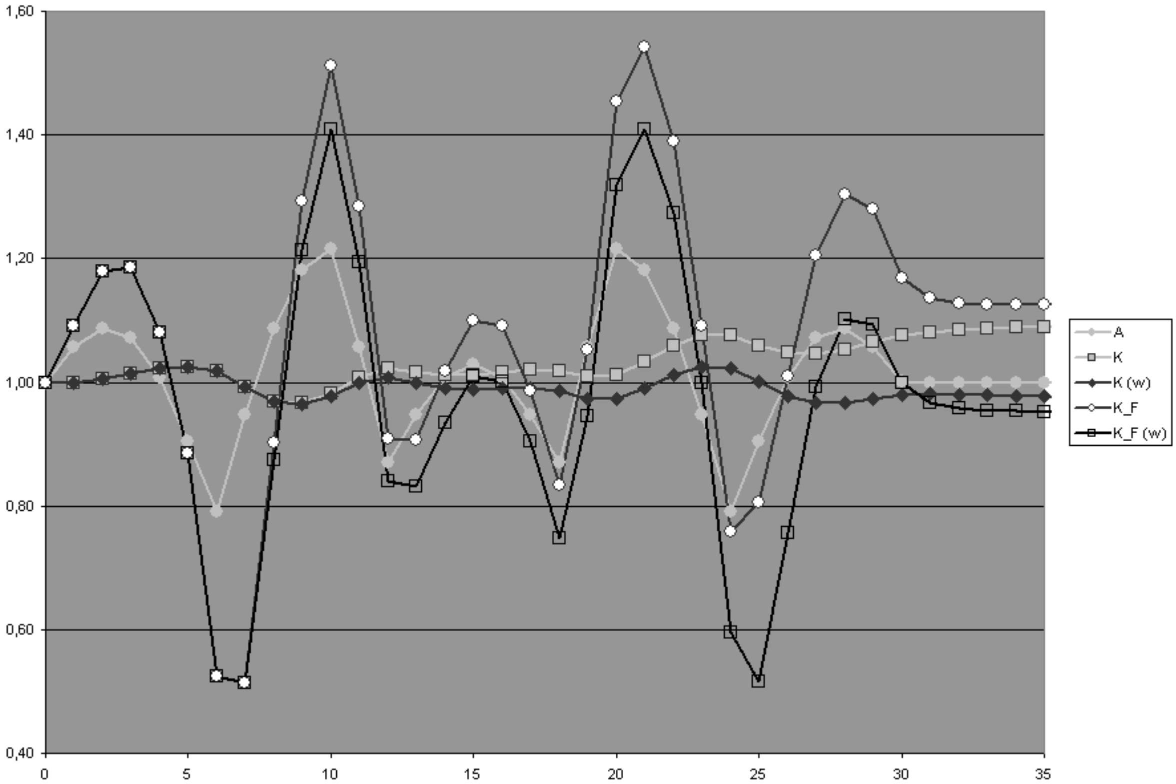
Figure 10 shows the evolution of A, C, I, G and D after the shock. These figures first of all show that the government debt D of the non-benefit economy indeed gets in a better shape every time a negative shock moves over to a positive state. The unemployment benefit economy however ends in a worse state than it started in, and each unemployment period worsens that state. The general shape the economy is in influences the other depicted variables. Consumption C and investments I become higher (lower) in the non-benefit (unemployment benefit) economy and because of the lower (higher) debt the government spends less (more), G.

More specifically we notice that again G reacts instantaneous in the unemployment benefit economy and is more volatile, thus having a more responsive nature. In the fading shock impulse section that different behavior was already discussed. The non-benefit economy returns to a stable state faster but because of that it also earlier faces a higher interest rate on its debt. Already seen in figure 9 is that the latest two unemployment periods, exclusively seen under the unemployment benefit government, creates a lower need for investments I. When looking at the graphs, period 25 shows sign of improvement in the non-benefit economy after the shock, but it worsens for the unemployment benefit economy.

Figure 11 shows the evolution of A, t, r and w after the shock. At the risk of getting repetitive we summarize our previous findings that are still applicable. The higher debt of the unemployment benefit economy leads to a higher tax rate τ. That in turn increases the return on investment, interest r, that

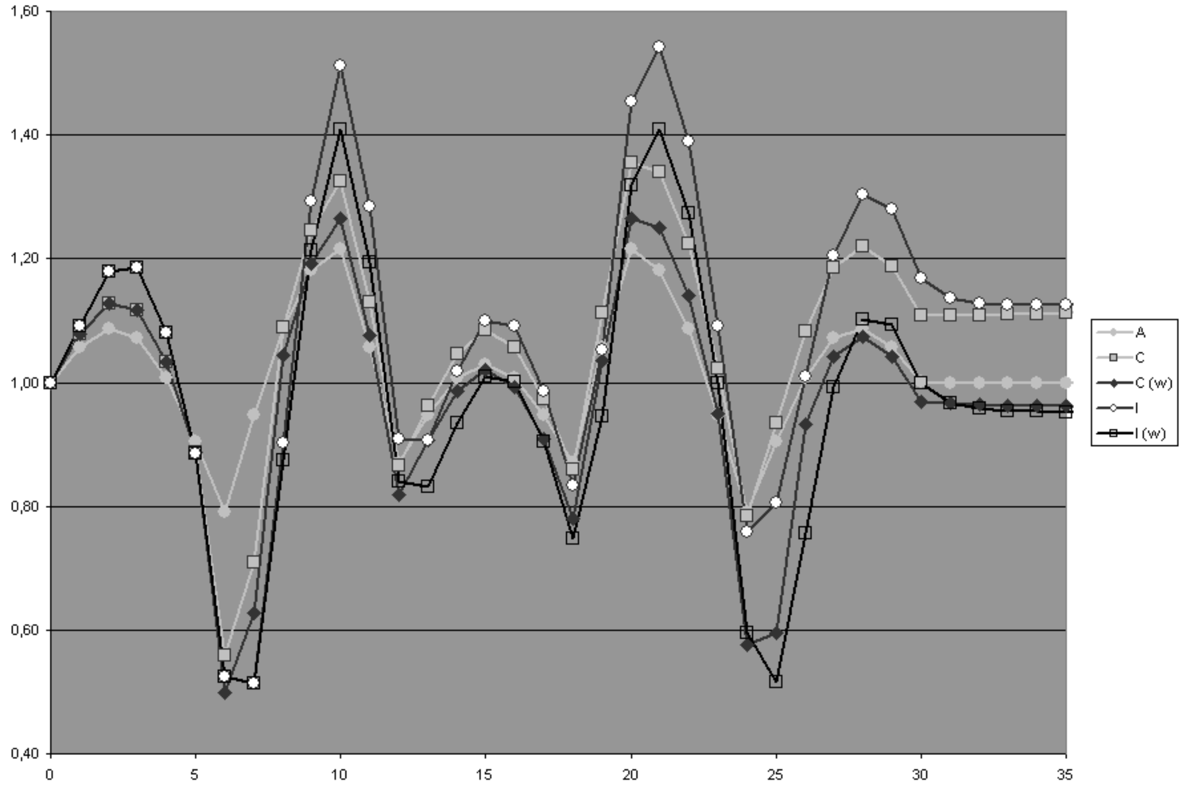


(a) National income and labor demand

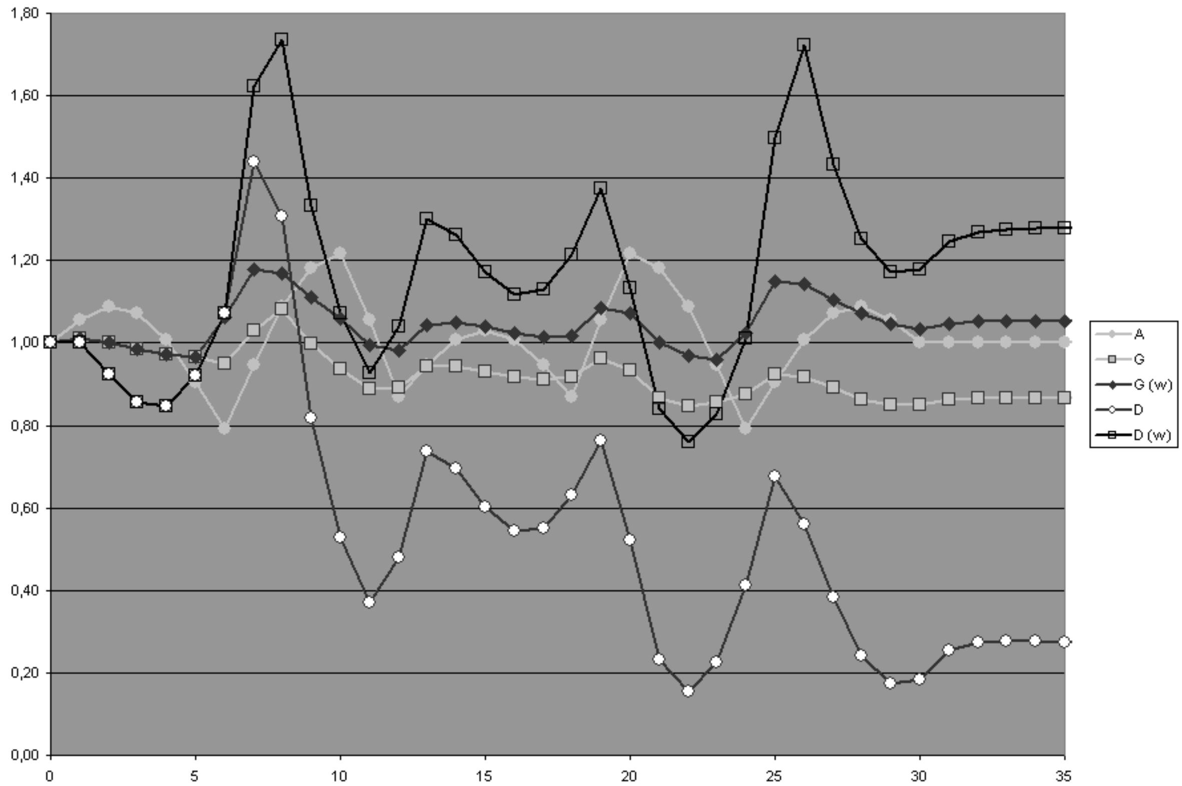


(b) (Optimal) capital stock

Figure 9: Graphical representation of A, Y^D, Y^S, K, L^D, K^F in the sinusoid business cycle scenario.

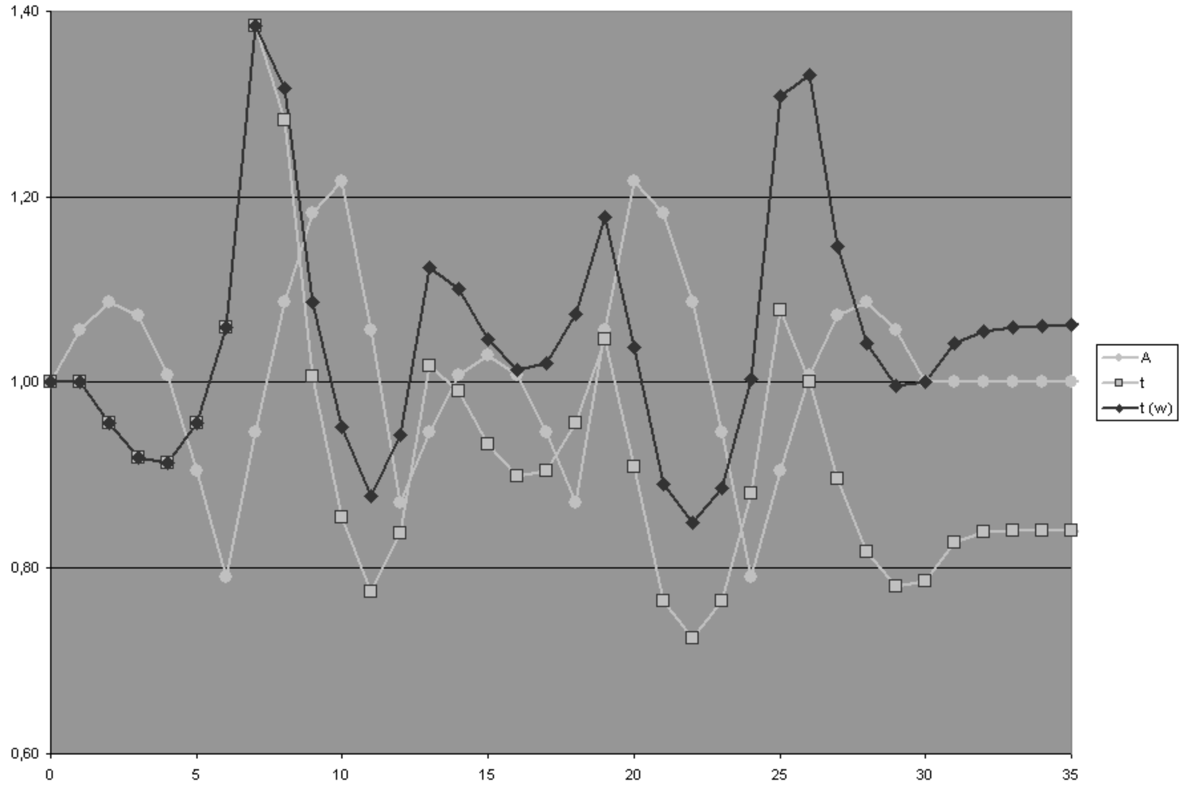


(a) Consumption and investment

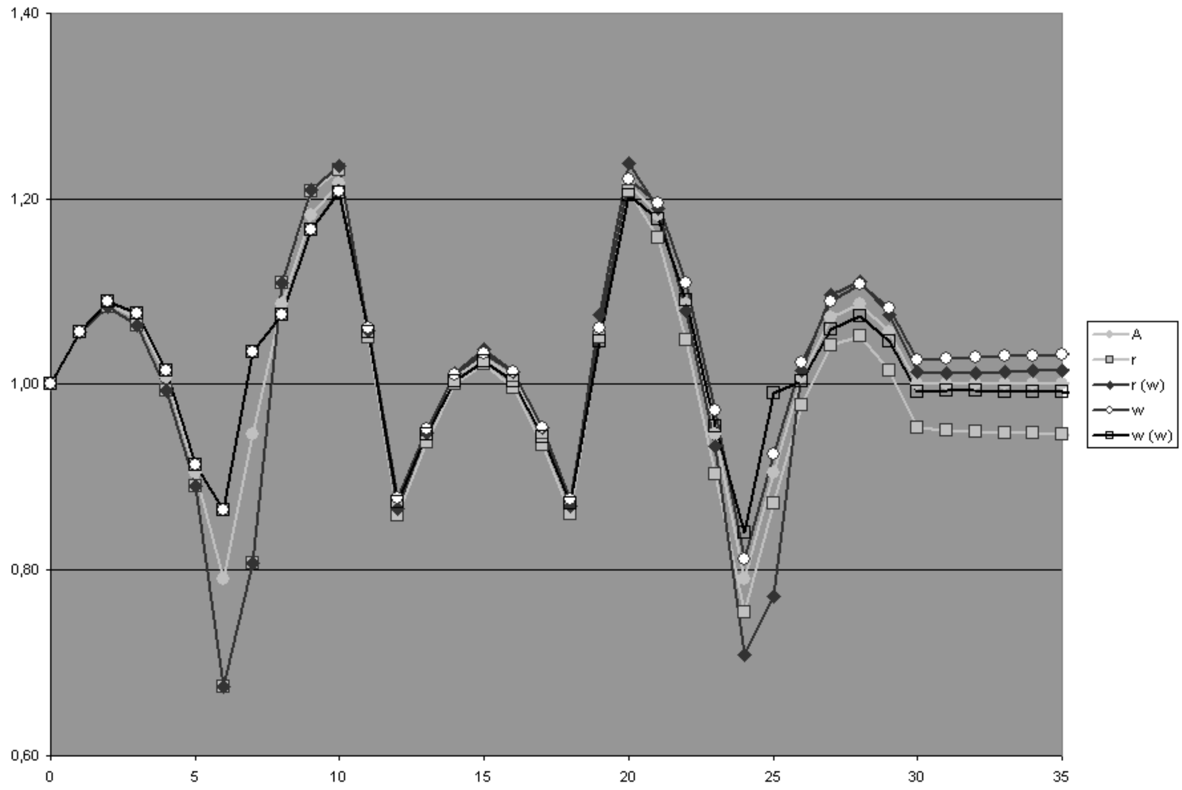


(b) Government consumption and debt

Figure 10: Graphical representation of A, C, I, G, D in the sinusoid business cycle scenario.



(a) Tax rate



(b) Wage and interest rate

Figure 11: Graphical representation of A, t, r, w in the sinusoid business cycle scenario.

households desire. That desire is depicted by their rate of time preference, ρ, equation (4.12). Overall the wage rate w is affected from another direction, namely the total capital stock. The higher (optimal) capital stock in the non-benefit economy also creates higher income for those employed in that economy, their labor earns more money as there is more capital per worker. Moreover some interesting things happen with the wage rate in the third unemployment period ($t = 24, 25, 26$). The severe relative drop in the non-benefit economy seems to be mitigated in the unemployment benefit economy, especially period 25 shows a strong improvement for the latter economy. On the contrary the interest rate is still struggling in that period for the unemployment benefit economy. The reason for that is essentially the tax rate and the reservation wage λ. Equation (4.4) makes that workers demand a relatively high wage in period 25 as they are faced by a high tax rate as a consequence of the unemployment situation of the previous period. Their demands are even that high that not everyone can get employed at a satisfying wage rate. Another consequence of that is that essentially a large portion of the available capital stock isn't working at its full capacity. The capital stock in those periods, we specifically discuss period 25, is far too high to get a reasonable return on the total stock. One can imagine a situation in which roughly half the available capital stock gives its normal return and the other half is redundant in those periods. Averaging leads to an overall lower return on capital, the interest rate.

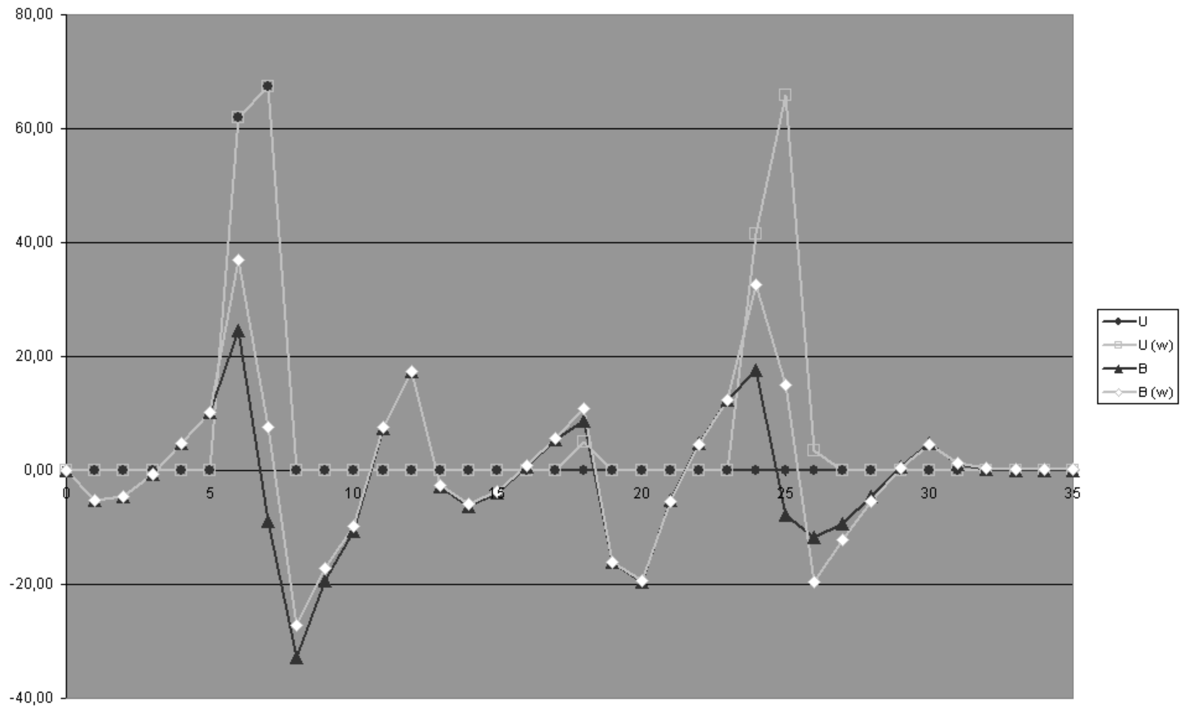


Figure 12: Graphical representation of U, B in the sinusoid business cycle scenario.

Figure 12 shows the evolution of U and B after the shock. Most that could be discussed here has been discussed in the preceding paragraphs. The two extra periods of unemployment for the unemployment benefit economy are now clearly discernable. The automatic stabilization effect is also pretty clear, the non-benefit economy has a deficit that reacts less strongly to productivity (in figure 12 implicitly shown by unemployment U).

Spread of the output after the sinusoid business cycle

As we did with the single and fading shock scenario we will finally show the standard deviations of the output in the sinusoid business cycle scenario. For this scenario we calculated the standard deviation of all model variables over the starting period (period 0) and the following thirty periods. These thirty periods are directly affected by the sinusoid business cycle function and that has under this scenario our primary interest.

Table 3: Standard deviations in the sinusoid business cycle scenario (t = 0 to 30)

Variable	With benefits	Without benefits
productivity (A)	0,109	0,109
national income (Y)	43,426	40,539
capital stock (K)	7,455	11,890
labor demand (L^D)	20,450	16,167
optimal capital stock (K^F)	97,551	100,730
private consumption (C)	39,799	36,575
C + unemployment benefits (C^*)	36,623	36,575
investments (I)	4,878	5,037
government consumption (G)	6,659	6,458
government debt (D)	17,323	22,096
tax rate (τ)	0,046	0,047
interest rate (r)	0,042	0,038
wage rate (w)	0,067	0,072
unemployment (U)	20,450	16,167
government deficit (B)	14,131	12,032

Table 3 shows the standard deviations for all variables. We again also derived the extra variable, total private consumption including unemployment benefits C^*. The volatility of A is again shown to get a sense of the standard deviation of a function as we used.

This table has some differences with the previous ones. First of all that has something to do with not taking a phasing out period into account contrary to the other tables. However under this scenario there is a lot more happening while the function is active and the considered period is already longer than in the previous tables. We should not dismiss this influence, but we can evaluate the above figures in their own right.

Maybe disturbing but national income Y and even our adjusted consumption measure C^* are now more volatile in the unemployment benefit economy. While the business cycle lasts, our model suggest that having an automatic stabilizer like unemployment benefits creates overall higher volatility. In the severest periods people might be better off with unemployment benefits. But, as we already showed in the figures of this section, that instrument might weaken the economy in such a fashion that it will make it less dependable in the future. The stronger reaction of the unemployment benefit economy is again clear, both the expenditures and, consequently, the deficit are more volatile. However, one of the weaknesses of our model, specifically equation (4.14), becomes apparent. The non-benefit economy is running such high surpluses that the debt appears to be more volatile, which should be read in our case as that it merely diverges very strongly from the defined steady state. The difference in tax rate τ volatility is probably related to this as well. The eventually lower tax rate of the non-benefit economy pulls it further away from the base value and thus appears more volatile.

The other statistics should be considered with caution, which was commented on under table 1. The difference in the volatility of the interest rate indicates that the unemployment benefit economy is apparently struggling more with returning to steady state. We discussed this also based on figure 11, the capital stock is far too high when unemployment occurs, which appears to happen more often when having unemployment benefits. That puts pressure on the interest rate, and hence the higher standard deviation in our table. The lower tax rate of the non-benefit economy also creates more room for a lower pre-tax wage rate, which we regard the main reason for the difference seen in the table for that variable.

4.3.4 General theoretical analysis conclusions

We have shown the specific results of simulating different kind of fictive shocks in our dynamic Keynesian model. Step by step we evolved an understanding of how the economy in our model responds to shocks when having unemployment benefits or not. Now we will draw some general summarizing conclusions of the above analysis.

Initially both economies seem to respond in a similar fashion to a shock. Contrary to the non-benefit economy the unemployment benefit economy however extracts money from the money market and gives this to the unemployed, and hence the total (private) consumption under that regime gets hurt less. In the following periods the unemployment benefit government is however faced by a higher debt which increases the tax rate and thus harms private investment and consumption. In the non-benefit economy we observed a decrease in the debt and noticed that the lower tax gave leave to a lower desired interest rate. That consequently led to a higher capital stock and hence a higher wage rate, as labor earns a higher income because of a higher capital per worker ratio.

Especially the analysis of a sinusoid business cycle suggested some disturbing features of our model, and thus possibly revealed negative features of automatic stabilization. Under the fading impulse shock we already demonstrated that the unemployment benefit economy has more difficulty with restoring full employment. The longer sinusoid business cycle analysis after that showed that the higher debt situation also leads to more difficulties in future periods to maintain full employment. The unemployment benefit on the one hand appears to take away the tough edges of the business cycle, by providing alternative income, but on the other hand seems to weaken the economy in such a way that it is less able to brace itself against future negative shocks. It also appears that the non-benefit economy is faster in returning to a steady state.

The standard deviation statistics mostly collaborated the intuitions gathered based on the figures. They underlined that the unemployment benefit government indeed reacts more instantaneous to, especially negative, shocks because of the unemployment benefit. That perfectly mirrors the situation we saw under our static model. Intuitively that is obvious of course and considering the lag in the tax setting equation, equation (4.14), that characteristic is already true by definition.

4.4 Comparison of the dynamic Keynesian model with empirical data

In this subsection we will attempt to show how well our model mirrors reality. We gathered data for the US for 1980 through 2004 from the World Development Indicators (WDI) database of The World Bank. The data series we have used are the following:

- GDP (constant 2000 US$)
- Capital stock (constant 2000 US$, computed)
- Gross capital formation (constant 2000 US$)
- Labor force, total
- Labor force growth rate (computed)
- Final consumption expenditure (constant 2000 US$)
- General government final consumption expenditure (constant 2000 US$)
- Household final consumption expenditure (constant 2000 US$)
- Productivity (computed)

We computed the growth rate of the labor force in an obvious and simple fashion. It is just the difference between the level of the labor force between each period and the previous period.

To calculate the capital stock we did the following. We use the depreciation rate stated earlier on, $\delta = 0.95$, to calculate the capital stock for the first year (1980). If we assume the same kind of depreciation as in our model we can use the formula $s = \frac{a}{1-r}$ for the sum of a geometric series. This gives the expected/desired sum for the next period, as we use the investment figure of 1980, so we modify that number with one cycle of depreciation to get the computed capital stock for 1980, $K_{1980} = \delta \cdot \frac{I_{1980}}{1-\delta}$. For the subsequent periods we depreciate the capital stock with δ and add the investments of the previous period. Applying that summation formula to later periods learns that our computed guestimate might be an overestimation, as the subsequent periods based on depreciation and investments fall short compared to a summation solely based on that periods investments. However, we can explain this partially by saying that in a growing economy the actual capital stock, K, is always under the desired capital stock, K^F. This is indeed the case in our model when setting the starting values reasonably and adding economic growth (increase in productivity and population). K then is consistently clearly below K^F. Thus, our computed capital stock is not perfect, but in our view still a feasible estimation.

We calculated productivity for each period as if the US economy its production function resembles our chosen Cobb-Douglas function, equation (4.2). Productivity is thus computed as follows: $A_t = \frac{Y}{K^\alpha \cdot (L/10)^{1-\alpha}}$. The reason for dividing labor by ten is to get a more useful unit of measurement. This way it reasonably matches with the capital stock and resembles the kind of magnitude as we used in the previous sections.

The next step we take is to put these statistics in our model environment. We will choose two different procedures in which we use more and less of the empirical data. The first series of data, the “fictive” series, is mainly computed by using the same values as under subsection 4.2. The second series, the "realistic" series, will use much more of the available US data.

The fictive series is enhanced by applying the labor force growth rate to the labor supply, L^S. Productivity in each period as computed replaces the value of unity for A. Moreover the value of the reservation wage, λ, is made that low that it becomes redundant. This is because our model isn't suitable to mimic empirical unemployment. Obviously the level of the unemployment benefit, θ, doesn't play a role either. The value of the rate of time preference, ρ, is adjusted so that the model output approaches the empirical values we gathered for the US. We had to rewrite the fixed government consumption parameter, G^C, as we now face both productivity and labor force growth. G^C is now a function of L^S and t, more specifically $G_t^C = L_t^S \cdot \frac{3}{10}$, and this is also reflected in the tax setting function, τ.

The realistic series takes the empirical values for labor supply, again divided by ten. It also uses the capital stock starting value for 1980, computed as explained above. Again, productivity in each period as computed replaces the value of unity for A. Both λ and θ have been made redundant as our model can't mimic empirical unemployment. As above ρ is again adjusted to approach the empirically observed investment behavior. Fixed government consumption is defined slightly different in this case: $G_t^C = L_t^S \cdot \frac{2}{30}$. The tax rate, τ, is adjusted accordingly, and moreover we had to drop the approximation of section 4.2. τ moves between 0.215 and 0.120 in this generated realistic series.

Both generated series have specific characteristics which might make their feasibility somewhat doubtful. As we mentioned our model appeared incapable of simulating empirical unemployment and we thus essentially assume full employment. This is because empirical shocks to productivity as calculated by us don't have the magnitude to move our model economy in an unemployment state. Our model wasn't designed to simulate structural unemployment either. Moreover we had to observe that our model economy is decreasing its debt while it should more or less increase proportional with national income. Probably the lack of unemployment partially explains why our model government runs a surplus. We also had to adjust the level of ρ to get our model output approximating the empirical data. The academical value of this trial-and-error procedure is obviously debatable.

We would now like to show how these two generated series and the existing empirical series compare to each other. There are three measures for comparison. First we calculated the coefficient of variation[6] (CV) of Y, K, C, I, G and $C+G$ to compare those for all three series. Second we calculated the correlation, R^2, between the generated series and the empirical series. Table 4 shows our findings. Third we show the result of a F-test with which we test whether the null hypothesis that the variances in all subgroups are equal holds. Such is therefore a measure of whether the output is really as similar as it may seem.

Quite clear from table 4 is that when one does some rudimentary calibration our model is already capable of generating rather similar behavior. The correlation coefficients, R^2, are exceedingly high. That is of course explained by the fact that we applied manual calibration for the static and starting values. We defined these to have the output to be in line with reality. The Durbin-Watson test for autocorrelation shows that there is serial correlation. That is obvious as we test whether figures that are, hopefully, exactly the same actually are. That combined with our manual calibration gives disturbing

[6]The coefficient of variation is calculated by dividing the standard deviation figures by the mean of the sample. That makes them, contrary to standard deviation, comparable across data sets that differ in absolute levels.

Table 4: Comparison of empirical data and generated series (US: 1980-2004)

	US Data	Fictive			Realistic		
	CV	CV	R^2	F-test (prob.)	CV	R^2	F-test (prob.)
Y	0,2299	0,2377	0,9995	0,7324	0,2247	0,9996	0,9485
K	0,1648	0,2083	0,9895	0,2257	0,1712	0,9935	0,8228
C	0,2482	0,2609	0,9962	0,6845	0,2453	0,9961	0,9158
I	0,3307	0,3159	0,9561	0,9675	0,3056	0,9573	0,8595
G	0,1363	0,1442	0,9021	0,5975	0,0502	0,8634	0,0000
C + G	0,2251	0,2183	0,9920	0,9743	0,2077	0,9925	0,8238
A (SD)	0,10954						
α	-	0,36			0,36		
δ	0,95	0,95			0,95		
ρ	-	0,075			0,100		

test figures at face value. However, in this kind of regression analysis these statistics are not relevant. There is no reason to assume a normal distribution of the error terms as part of the data is generated by our model. When observing the residuals one sees increasing error terms. This is probably largely explained by the fact that it is easier for our model to generate an early period than some future period as errors accumulate in the generation process.

We are more interested in the way our model is able to mimic the volatility of the different output figures. There are several ways of testing whether the different series are as similar as they seem. To test for this we first normalized all series output by dividing by the mean of the respective series. Otherwise we can't compare them as the absolute values differ for the fictive series case. We don't show the results of our mean and median tests. We regard them less important though they did show that the real data and the generated series seem comparable if the assumptions underlying these tests hold. But given these assumptions our data doesn't really lend itself for these tests. The data set is small ($N = 25 < 30$). The series aren't normally distributed, which we discovered by applying the Jarque-Bera test. They aren't independent either as they all have their origin in the same input. Therefore the more common mean and median tests aren't applicable. Given the uncommon structure of our data we don't feel presenting our attempt at a mean/median test is very informative.

Though, we were able to apply the common F-test which can show whether the variance in one group is equal to that in another group. The null hypothesis for this test is that variance is similar between the two groups and is rejected if the probability figure reported is close to zero. Both the fictive as the realistic series were tested against the real US data. We only have to reject the null hypothesis once for the variance in government expenditures in the realistic series. That raises some doubts about whether the behavior of the government as defined by us is realistic. That aside the other F-tests convincingly prove that our model is indeed capable to closely mimic the stream of real data.

Finally we take a look at the coefficient of variation for the different variables. When we keep most of the basic values of subsection 4.2 intact for our fictive series we can clearly see that for example investments are roughly twice as volatile as government consumption for both the fictive output and the real US data. These same kind of similarities are also seen between the real US data and the realistic output. Overall, across the board the volatility of our model output variables thus mimic those of the real data pretty well. Although we haven't been able to get unemployment involved, as our model isn't suited for simulating long lasting unemployment, we have shown that our model is a reasonable simulation. This is especially important for our findings under subsection 4.3.3 as that analysis also shows the situation with a continuing fluctuating productivity figure.

4.5 Weaknesses of our dynamic Keynesian model

As with the static model this dynamic Keynesian model has weaknesses of its own. We will discuss these in short.

The income tax is set quite arbitrarily. When simulating we observed that certain expressions gave better results than others. One can imagine that the government is somewhat oblivious towards the optimal income tax. However there is no theoretical background for the chosen expression that makes it better than other possibilities. As seen in our analysis this lag in setting the income tax gives some possibly dubious results. Would the government really leave the situation as it is once it increased the debt to a higher level because of unemployment benefits?

The answer to that question would depend on how the government is assembled. When one believes that the government is an intelligent forward-looking institution that maximizes welfare into infinity one should apply well-known social planning functions to government behavior. However, we could also consider for example a democracy where the government would have a strong bias towards maximizing present welfare, which would maximize their vote at the next election. Such a government wouldn't ask from itself to repay the debt as it hardly has an incentive to exchange present welfare (expressed in utility) against future welfare. Then we don't even argue against the capacity of the government to measure and exchange present and future welfare in terms of utility units. If that democratically chosen government seems more feasible then our chosen government spending and income tax setting functions might not be even that far off reality than social planning literature might suggest. Moreover, in reality it is highly doubtful whether the government is capable of measuring welfare in utility and then perfectly balancing present welfare against future welfare.

In the proceeding part of this thesis we will shortly exhibit a proposal for a, from a micro-foundation point of view, more sophisticated model. Such a proposal certainly makes sense. Our model is purely macro-economical and does not consider the individual preferences of households. It does not consider differences in the way goods get produced. Everything in our dynamic Keynesian model is pretty straightforward and simple. However in defense of our model we would like to point out that about half of the income in an economy goes to rule-of-thumb consumers according to Mankiw (2000). These rule-of-thumb consumers just spend their present income without smoothing their consumption of their lifetime. And that kind of behavior approaches the simple macro-economical definition we chose in our model. In our view there is nothing principally wrong about trying to introduce micro-foundations to the model environment, but we should be realistic about whether these micro-foundations really exhibit a better reproduction of reality. For the intentions of this thesis, given rule-of-thumb behavior of certain consumers, we think our model is in this respect a decent attempt at predicting the effect of automatic stabilization.

We already pointed to the total lack of social planning by the government. The government in our model is almost completely oblivious towards the future and appears to maximize present consumption, essentially even in favor of consumption in the next period. Social planning in our model would intuitively mean that the government simply sets out a path towards zero debt. That would significantly favor future households. In our model that would implicitly mean that we consider infinitely living households. These infinitely living households allow for infinitely smoothing governments. Dropping that assumption will by definition make the government react more oblivious, like in our model. If one would like to do something about consumption smoothing and optimizing investment behavior an overlapping generations model should be applied. Just making the government somewhat more intelligent wouldn't suffice.

We extensively showed the results of the tax setting function. The unemployment benefit economy ran a higher debt and the non-benefit economy ended up in a better state than it started in. We are the first to acknowledge that these new (long-run) equilibria after the imposition of a shock might be infeasible. That also raises some doubts about the intermediate effects of the shocks and for example makes the re-occurrence of unemployment later on in the sinusoid business cycle scenario highly debatable. On the one hand we indeed have our own doubts about using our results in the long-run as the debts significantly differ. However we also think that differing debts between an unemployment benefit and non-benefit economy makes some sense. The unemployment benefit economy occasionally runs higher deficits because of its immediate response (automatic stabilization). Knowing that makes our prediction that the non-benefit economy will have a lower debt maybe not perfect, but perfectly reasonable.

Lastly, the money market is more or less left out of the model. The lending and borrowing behavior in the economy is marginally defined and the unlimited borrowing of the government probably meets some theoretical opposition from the monetary field. The effects of the interest rate in our model should therefore be reviewed with caution. It intuitively does what it should do, but the exact magnitudes of its influence might be infeasible, especially in the extremes of very low and high productivity.

4.6 Dynamic Keynesian model summary and conclusion

In this section we have exhibited a Keynesian model that contrary to the static model featured more than one period. That way we have been able infer some intuitions on how the economy evolves, with and without unemployment benefits, after and during productivity shocks. After our description of the model equations we showed an extensive analysis of the model output by applying three different fictive impulse shock scenarios.

We showed that the unemployment benefit economy indeed reacts as expected and initially alleviates the negative effects of the impulse shock. However that automatic stabilization effect leads to a higher debt service which puts the unemployment benefit economy in a worse shape. Both the wage and interest rate are negatively affected. The wage rate becomes lower, lower wages for everyone, and the interest rate higher, lower investment and thus lower future income. At face value this leads to a lower national income and higher unemployment when new shocks occur in subsequent periods.

After that we shortly showed how our model matches with reality. We compared the volatility of the core variables of our model with those of the US economy. Table 4 showed that our model is reasonably capable of mimicking a real world economy with some rudimentary calibration.

To conclude we discussed some points that seem weaknesses of the model. Overall we have shown that if one takes the way the real world works (e.g. public choice theory and rule-of-thumb consumers) our model is doing a fine job. The difference between the debt levels for a non-benefit and unemployment benefit economy is merely an intuitive reflection of active automatic stabilization.

Our conclusion based on this section therefore is that our theoretical model has convincingly shown that policymakers should take automatic stabilization into account when setting their (fiscal) policies. However they should consider that present automatic stabilization comes at a future cost and they should always at least try to weigh these future costs against present benefits.

5 Dynamic Ramseyian model with automatic stabilization

We already discussed the downside of a static model. The dynamic Keynesian model of the previous section amended this by introducing time. However it is still macro-founded, decisions by individual households are arbitrarily given. Consumer behavior is simply defined by one equation that reserves a certain portion of income for consumption. For investment behavior holds the same, households observe the interest rate and make their optimal investment.

A further step in research would be to develop a dynamic model that considers decision-making at the micro level. It oversteps the scope of this thesis to give an elaborate analysis, but we will exhibit a proposal for a theoretical dynamic Ramseyian model with micro-foundations[7]. The model will have different households endowed with different resources (labor) and preferences. Investment consists of a combination of these goods. Therefore the optimal consumption and investment choice is influenced by the different price levels. That means that contrary to the previous model unemployment benefits have a future influence through these price levels. More interesting intuitions could surface in this automatic stabilization model. For example, in the beneath model we introduce taxes through an investment tax and that would evidently have an impact on goods prices. Especially those that dominantly require capital as an input factor. That in turn will influence consumption and investment decisions.

After the exhibition of the equations, which of course could be subject to change in actual future research, we will shortly discuss what we should expect from this model.

5.1 The dynamic Ramseyian model equations

Beneath we will explain the equations of our dynamic growth model. The model consists of one country (H), 2 goods (X and Y), 2 households (A and B) and 3 factor inputs being unqualified labor (U), qualified labor (Q) and capital (K). Household A is endowed with qualified labor, and household B with unqualified labor. The tax on investments (I) are given by the tax rate τ. Unemployment U occurs because of unqualified labor wages are constrained by the level of qualified labor wages and a salary differential d. Therefore only unqualified labor workers might get unemployed. E modifies the unqualified labor that household B gets endowed with.

We will first describe the used subscripts. i stands for good X or good Y. j stands for the other good. h stands for household A or B. t determines the present period. T is the terminal period.

[7] This Ramseyian model is founded on the basis of Emami Namini (2006), Ramsey (1928) and Lau et al. (2002).

5.1.1 Zero profit

All equations in this subsection assure that there are zero profits. There is no possible deviation for agents in this model from which they could make a profit. We thus assume perfect markets.

$$\begin{aligned} \text{good } i\text{: } p_{it}/A_{it} \leq {w_{Ut}}^{\alpha_i^U} \cdot {w_{Qt}}^{\alpha_i^Q} \cdot r_t^{\alpha_i^K}, \\ \alpha_X^U = 0.3, \alpha_X^Q = 0.3, \alpha_X^K = 0.4, \\ \alpha_Y^U = 0.2, \alpha_Y^Q = 0.2, \alpha_Y^K = 0.6, \perp Q_{it} \geq 0 \end{aligned} \tag{5.1}$$

Thus, the price p_{it} of good i in period t is lower or equal to the cost of producing it. A_{kit} is the inefficiency parameter (technology, productivity shock) that corrects for productivity of good i in period t. The production cost is determined by a Cobb-Douglas function with wages w_{Ut} & w_{Qt} and award on capital r_t in period t. It mirrors the production function. The α_i's determine the productivity of labor and capital for good i. $Q_{it} \geq 0$ assures there is no negative quantity produced.

$$\begin{aligned} \text{intertemporal utility } h\text{: } p_h^U \quad \leq \quad & \prod_{t=0}^{T=100} \left(p_{ht}^W \cdot (1+\rho)^t\right)^{\phi_t}, \perp U_h \geq 0, \\ & \text{with } p_{ht}^W = p_{Xt}^{\gamma_{hX}} \cdot p_{Yt}^{\gamma_{hY}}, \\ & \gamma_{AX} = 0.4, \gamma_{AY} = 0.6, \gamma_{BX} = 0.6, \gamma_{BY} = 0.4, \\ & \phi_t = \left(\frac{1+g}{1+\rho}\right)^t \Bigg/ \sum_{t=0}^{T=100} \left(\frac{1+g}{1+\rho}\right)^t, \\ & g = 0.02, \rho = 0.05 \end{aligned} \tag{5.2}$$

Households A and B have different preferences, hence the h subscript in the above equations. Given the above the price of one unit of lifetime utility U (p^U) is equal or lower to the product of the weighted prices of utility in each period. The price of utility p^W consists of a package of good X and Y as defined on the second line. $(1+\rho)^t$ is a discounting mechanism, that makes future utility relatively more expensive (not expressed through the price in that period). ϕ_t is the weighing factor in this product function. With the chosen growth factor g and discounting factor ρ future periods get a lower weight. The meaning of all this is that agents in this model should optimally 'determine' the prices of the produced goods. They would try to get the prices of both goods over the lifetime as low as possible, to in the end get the price of one unit of lifetime utility as low as possible.

$$\begin{aligned} \text{capital rental activity: } p_t^K \geq \begin{cases} r_t \cdot (\delta+\rho) + (1-\delta) \cdot p_{(t+1)}^K, & \text{if } t < T \\ r_T \cdot (\delta+\rho) + (1-\delta) \cdot p_{(T+1)}^K, & \text{if } t = T \end{cases}, \\ \delta = 0.05, \perp K_t \geq 0 \end{aligned} \tag{5.3}$$

To buy one unit of capital K one has to pay p_t^K in period t. This price is determined by the rental rate of that period (the income on capital) and the price of capital in the next period. $p_{(t+1)}^K$ thus in fact stands for the future stream of rental income on capital. As stated here the price of capital is thus higher or equal to the value of (future) benefits. $(1-\delta)$ simply stands for depreciation. $K_t \geq 0$ assures the capital stock can't become negative.

$$\text{investment activity: } p_{Xt}^{\beta_X} \cdot p_{Yt}^{\beta_Y} \cdot (1+\tau) \geq \begin{cases} p_{(t+1)}^K, & \text{if } t < T \\ p_{(T+1)}^K, & \text{if } t = T \end{cases}, \qquad (5.4)$$
$$\tau = 0.2, \beta_X = \beta_Y = 0.5, \perp I_t \geq 0$$

Here we see how the price of capital p^K is composed by the prices of the separate goods X and Y. β defines the fixed composition. As we assume zero profit the price of investing this composition $p_{Xt}^{\beta_X} \cdot p_{Yt}^{\beta_Y}$ is higher or equal to the price of capital in the next period $p_{(t+1)}^K$. There is no 'profit' in investing another unit of this composition, so households will invest up till this point. The $(1+\tau)$ part introduces an investment tax, which increases the cost of doing an investment.[8] Negative investment isn't allowed ($I_{kt} \geq 0$).

5.1.2 Market clearing

The following equations assure that the market indeed clears, which is another feature of a perfect market. This means that all goods get consumed or invested.

$$\text{good } i\text{: } Q_{it} \geq \sum_h \phi_t \cdot \left(p_h^U \cdot U_h\right) / p_{ht}^W \cdot \gamma_{hi} \cdot \left(\frac{p_{jt}}{p_{it}}\right)^{\gamma_{hi}} + \beta_i \cdot \left(\frac{p_{jt}}{p_{it}}\right)^{\beta_i} \cdot I_t, \qquad (5.5)$$
$$i, j = X, Y, i \neq j, \perp p_{it} \geq 0$$

The quantity of good i produced in period t (Q_{it}) is greater than or equal to all demand. Demand in our model consists of two parts. The first part starts with a summation to get the total demand of both households. The first half of this part sets the price in the present period of the consumption package W relative to lifetime utility U and corrects for the weight that lifetime utility has in this period, ϕ_t. This part de facto establishes the consumption demand following Shephard's lemma: given the prices over the lifetime the consumer will decide on present demand, by using cost minimization[9]. The second half of the first part corrects for the price difference between itself, good i, and the other good j and places this in the context of the utility function as stated under equation (5.2). This therefore determines the actual composition of the consumption package in this period. The second part depicts the amount of good i being demanded because of investment I_t. This amount is determined by correcting for the price difference between itself, good i, and the other good j and having it placed in the context of the investment function as stated under equation (5.4). Again, this thus establishes demand through determining the optimal cost minimizing composition of investment in this period.

$$\text{intertemporal utility: } p_h^U \cdot U_h \geq \Omega_h, \perp p_h^U \geq 0 \qquad (5.6)$$

This simply means that total income of both households over the lifetime has to be spent on utility. It is not possible to not spend income. The income definition (5.13 & 5.14) corrects for investments/divestures in the capital stock.

[8] In equation (5.14) the investment tax revenues are given back as an 'unemployment benefit' to household B.
[9] A more extensive explanation of Shephard's lemma can be found in Appendix C.

$$\text{intertemporal capital: } K_t \leq \begin{cases} (1-\delta)\cdot K_{t-1} + I_{t-1}, & \text{if } t \geq 1 \\ K_0, & \text{if } t = 0 \end{cases}, \perp p_t^K \geq 0 \tag{5.7}$$

The capital stock K in period t consists of the capital stock in the previous period after depreciation ($(1-\delta)\cdot K_{t-1}$) and having added investments (I_{t-1}) of that same previous period. It is obviously impossible to have a higher capital stock than that there could physically be.

$$\text{intratemporal capital: } K_t \cdot (\delta + \rho) \geq \sum_i \alpha_i^K \cdot w_{Ut}^{\alpha_i^U} \cdot w_{Qt}^{\alpha_i^Q} \cdot r_t^{\alpha_i^K - 1} \cdot A_{it} \cdot Q_{it}, \quad \perp r_t \geq 0 \tag{5.8}$$

This equation makes sure that there can't be more production of good i in period i (Q_{it}) than possible with the given capital stock. Everything behind the greater than or equal sign is a translation of production Q_{it} to capital K_t.

$$\text{terminal capital: } K_{(T+1)} \leq (1-\delta)\cdot K_T + I_T, \perp p_{(T+1)}^K \geq 0 \tag{5.9}$$

Through this it is assured that terminal capital isn't higher than that could be possible. It is in a way a specific version of equation (5.7).

$$\text{unqualified labor: } (L_{U0} \cdot E - U)\cdot(1+g)^t \geq \sum_i \alpha_i^U \cdot w_{Ut}^{\alpha_i^U - 1} \cdot w_{Qt}^{\alpha_i^Q} \cdot r_t^{\alpha_i^K} \cdot A_{it} \cdot Q_{it} = L_{Ut}, \quad E = 1, \perp w_{Ut} \geq 0 \tag{5.10}$$

$$\text{qualified labor: } L_{Q0} \cdot (1+g)^t \geq \sum_i \alpha_i^Q \cdot w_{Ut}^{\alpha_i^U} \cdot w_{Qt}^{\alpha_i^Q - 1} \cdot r_t^{\alpha_i^K} \cdot A_{it} \cdot Q_{it} = L_{Qt}, \quad \perp w_{Qt} \geq 0 \tag{5.11}$$

The above two equations are both quite comparable to (5.8). Production Q_{it} can't be higher than possible given the labor capacity L_t. Such holds for both kinds of labor. Labor grows with a constant factor $(1+g)^t$. E determines the actual amount of unqualified labor with which household B gets endowed with, for now this is set at unity.[10] U is an unemployment figure and 'clears' the unqualified labor market to make sure (5.12) holds.

$$\text{unemployment: } w_{Ut} \geq w_{Qt} \cdot d, d = 1 \tag{5.12}$$

[10] In equation (5.14) we endow household B with a fixed amount of labor. For modeling purposes we can then later on change the labor endowment by changing E instead of changing the fixed figure. If we increase E from 1 to 2 that simply means we double the amount of labor with which household B gets endowed.

This equation simply says that unqualified labor workers demand a certain wage dependent on the wage of qualified labor workers. If the wage of unqualified labor workers would like to drop below this fixed figure unemployment U increases according to equation (5.10). At present we set the salary differential d at unity.[11]

5.1.3 Income definition

$$\Omega_A = \sum_{t=0}^{T=100} w_{Qt} \cdot L_{Qt} + \frac{1}{2} \cdot (p_0^K \cdot K_0 - p_{(T+1)}^K \cdot K_{(T+1)}), \qquad L_{Q0} = 50, K_0 = 1000 \tag{5.13}$$

$$\Omega_B = \sum_{t=0}^{T=100} w_{Ut} \cdot (L_{U0} \cdot E - U) + \frac{1}{2} \cdot (p_0^K \cdot K_0 - p_{(T+1)}^K \cdot K_{(T+1)}) + p_{Xt}^{\beta_X} \cdot p_{Yt}^{\beta_Y} \cdot \tau \cdot I_t, L_{U0} = 50, K_0 = 1000 \tag{5.14}$$

Both equations determine the disposable total income over the lifetime from $t = 0$ to $T = 100$ for each separate household. It adds all labor income $w_t \cdot L_t$ and the income from the capital stock (see equation (5.3)) minus the remaining capital stock ($p_{k(T+1)}^K \cdot K_{k(T+1)}$). Both households have an equal share in the capital stock, hence the $\frac{1}{2}$ in both equations. For household B there is a third part which means that they also get an 'unemployment benefit' which is paid for by the investment tax τ. The level of the benefit is determined by investment activity, equation (5.4).

5.1.4 Investment constraint

$$I_T / I_{(T-1)} \geq Q_{iT} / Q_{i(T-1)}, i = X \text{ or } Y \tag{5.15}$$

This equation is necessary to make sure that there is still a capital stock beyond the terminal period T. These kind of models are often simulated for about 100 periods and then halted. By having this equation we virtually extent the lifetime of the economy to infinity. It says that in the terminal period the investment growth is equal or larger than the growth in production. Households are thus not capable of completely divesting the capital stock.

5.2 Discussion of the dynamic Ramseyian model

As said it overextends the scope of this thesis to give a thorough analysis as we did with the dynamic Keynesian model. Moreover as this is only a proposal separate parts could be subject to change. For example, it requires further investigation to introduce a labor tax or overall income tax. Moreover this model features infinitely living households which should be questioned as well. In this section we will just

[11] When the salary differential d is set at 1 it simply means that the unqualified labor wage level must be higher or equal to that of qualified labor. If that doesn't clear the unqualified labor market it means that all the remaining unqualified labor workers become unemployed U. If d would be set at 0.5 it means that unqualified labor workers are still willing to work for half of what qualified laborers receive.

superficially discuss what we should expect from this model. For this overview we predominantly used the intuitions gathered by Emami Namini (2006).

The introduction of a negative shock to the labor intensive good X will lead to a price increase of that good, equation (5.1). This will in the end among other things influence the demand for good X, equation (5.5). Through the zero profit equation for good X (5.1) this will lead to a lower wage rate. This decrease could lead to unemployment for unqualified labor, however such is dependent on unqualified labor endowment ($L_{U0} \cdot E$, equation (5.10)) and the chosen salary differential (d, equation (5.12)). If there is a benefit system ($\tau > 0$) a smaller portion of disposable income is lost as can be seen in equation (5.14). It is expected that as the shock at face value influences a smaller portion of disposable income that demand stays more steady. Here one can cast another critical view, contrary to our previous models the level of benefits doesn't react to the occurrence of unemployment but to the level of investments. Preliminary testing on our part showed that non-automatic stabilization through an investment tax was the best we could achieve. Hence, again here is definitely an area for further research and effort.

What one would observe when the proposed model would be executed through GAMS/MPSGE is that if there are taxes and benefits that the economy will be slightly more stable right after a shock as the disposable income of household B is partially fixed, equation (5.14). However, the big question is how the equilibria before the shock differ between no benefits and benefits to household B and how they differ after the shock. Under the dynamic Keynesian model we showed that (unemployment) benefits could be detrimental to future periods as they put a burden on the economy. The burden of investment taxes in this model could be detrimental as well and maybe not worth the stabilization effect they have. As we said such is however up to future research and efforts.

6 Empirical analysis

In the foregoing sections we have shown several ways of introducing automatic stabilizers in a theoretical model. By showing these models and analyzing them we gathered the general intuition that automatic stabilizers, like unemployment benefits, could very well stabilize the volatility of the national income. In this section we would like to show whether these theoretically established intuitions are mirrored by reality.

Our hypothesis is roughly as follows:

> *The volatility (variance) of the GDP of a country is lower if a country has a higher level of social security.* – This then depicts automatic stabilization.

First we will have to discuss the variables we have chosen and how we had to construct them. After that we will get to the actual statistical analysis, for which we will use ordinary least squares. Then we finalize with a conclusion.

6.1 Discussion of the empirical variables

The data set that we use consists of two groups of variables. We have volatility measures, all based on some sort of GDP statistic. To explain these volatility measures we gathered explanatory variables from several kinds and sources. We will discuss these groups subsequently.

6.1.1 Volatility measures

For these measures we have used three different data sources. We will first sum up the volatility measures and will then discuss their meaning.

World Bank, World Development Indicators (annual data)

- Coefficient of variation (GDP)
- Standard deviation (GDP growth)

OECD (quarterly data)

- Coefficient of variation (GDP)
- Standard deviation (GDP growth)

European Union (quarterly data)

- Coefficient of variation (GDP)
- Standard deviation (GDP growth)

Obviously the World Bank database (WDI) offers the vastest amount of data in the number of available countries. As that database lacks in direct explanatory measures we have chosen to observe some counterparts as well. We did consider per capita GDP figures in our statistical analysis but these gave the same or worse results than total GDP volatility measures. As can be seen we have modified several GDP measures to our liking. These modifications are now explained.

Presumably 'standard deviation' is a known measure. It is a measure that shows the volatility (variance) of a variable in absolute terms. Because it gives an absolute figure it is sometimes not possible to use it for comparison between countries. That is why we have also calculated the coefficient of variation for some variables.

The 'coefficient of variation' is calculated as follows. The standard deviation of a variable is simply divided by the mean of that variable. The result is a single number that tells us how high the standard deviation is compared to the mean. This statistic is in a format that makes it possible to compare it across countries with differing absolute levels of, for example, GDP.

Our calculations for the volatility measures obviously depend on availability of data. The World Bank figures are based on the available figures for 1961 through 2006; the OECD figures are based on the available figures for 1959 through 2006; the European Union figures are based on the available figures for 1990 through the third quarter of 2007. For all these measures it holds that for specific countries they don't cover the total time span of the database. We haven't corrected for this. First, a shorter time span doesn't necessarily give a very different picture of the volatility of a country its GDP. Moreover, these differences in time span are mostly mirrored by those in the time span of the explanatory variables. To us that is an implicit correction as within a country there is at least a match in time span.

Especially for the WDI data we attempted to test separate groups based on the time span of the data. The downside to that is that one loses a big portion of the data and that harms the regression analysis. Also, overall high-income countries have a longer available time span for their data. So grouping on availability of data also implicitly means grouping for very similar countries. As there are many influences on GDP and we only test one aspect it is very hard to distinguish for differences when a group only holds very similar countries. In earlier research[12] we already found that finding enough distinguishing elements to separate countries is a tough undertaking and leaving out data because of, mostly slight, time span differences will for certain lead to insignificant outcomes.

6.1.2 Explanatory variables

We would like to test whether we can explain the variation in the measures of the previous section by variables that intuitively match our hypothesis of automatic stabilization. The following set of explanatory variables have been chosen by us, again grouped by source.

World Bank, World Development Indicators (annual data)

- GDP (in US 2000 dollars, year 2000)
- Unemployment as % of the labor force (average)
- GINI index (average)
- Rigidity of employment index (average, years 2003-2006)

OECD (annual data)

- Aggregated benefits and transfers (average; % of GDP)
- Social expenditures on unemployment (average; % of GDP)
- Replacement rates (bi-annual; average)

European Union (annual data)

- Social benefits per capita (average)
- Social benefits per capita (average; adjusted through dividing by GDP per capita)
- Benefits unemployment function (average; % of GDP)

[12] This research is unpublished.

International Labour Organization (single figure; national surveys)

- Social benefits as % of household income

Overall it is clear that we again reduced strings of data to a single measure. That is the only way to match our explanatory variables with the volatility measures. We shouldn't get into specifics that are more obvious to comment on when we show our statistical analysis. Here we just explain some of the above variables that aren't that commonly known or deserve some explanation on why we chose them. The discussion on availability of data of the previous section holds here as well. The World Bank figures are based on the available figures for 1961 through 2006; the OECD figures are based on the available figures for 1959 through 2006; the European Union figures are based on the available figures for 1990 through the third quarter of 2007; the ILO measure comes from national surveys which were held during different years for different countries ranging from 1995 up to 2004. If some years lack we only have the average for that variable for that country for the period available. Again, we use these variables as instrumental for automatic stabilization and we are therefore less concerned with getting perfect measures but are more concerned with getting significant outcomes. We don't attach much meaning to the level of the coefficients we will find but we will primarily look at the sign of these coefficients. Therefore it is more important to at least get some idea of the 'generosity' of a country, which should be depicted by the variables chosen by us.

We let the GDP of each country enter the equation because one can imagine that a larger economy is affected differently by the business cycle. On the one hand one might argue that a larger economy might balance within its own borders. On the other hand a smaller economy probably isn't affected as much by a negative shock as most smaller economies are more exposed to the world economy through imports and exports. An internal shock doesn't change so much then. All in all, we don't a priori define an expectation of the influence of the size of an economy. The chosen year 2000 is an arbitrary one, and we think that doesn't meet theoretical opposition as we consider the GDP level to be merely an instrumental variable for the size of the economy.

The GINI index[13] of the World Bank is used by us because it measures the distribution of income. It could thus show whether our findings under the static model on the effectiveness of automatic stabilization under differing income distributions is also apparent in reality. Moreover it is probably instrumental for how generous a country is, and thus how strong supposed automatic stabilizers might be in effect. The rigidity of employment index[14] is added as a supplement variable that doesn't directly say something about our subject but might improve the overall regression. More rigid regulations in the field of employment might influence the volatility of the business cycle.

[13] Definition taken from the World Bank: "GINI index measures the extent to which the distribution of income (or, in some cases, consumption expenditure) among individuals or households within an economy deviates from a perfectly equal distribution. A Lorenz curve plots the cumulative percentages of total income received against the cumulative number of recipients, starting with the poorest individual or household. The GINI index measures the area between the Lorenz curve and a hypothetical line of absolute equality, expressed as a percentage of the maximum area under the line. Thus a GINI index of 0 represents perfect equality, while an index of 100 implies perfect inequality. World Bank staff estimates based on primary household survey data obtained from government statistical agencies and World Bank country departments. Data for high-income economies are from the Luxembourg Income Study database."

[14] Definition taken from the World Bank: "The rigidity of employment index measures the regulation of employment, specifically the hiring and firing of workers and the rigidity of working hours. This index is the average of three subindexes: a difficulty of hiring index, a rigidity of hours index, and a difficulty of firing index. The index ranges from 0 to 100, with higher values indicating more rigid regulations."

The replacement rate[15] of the OECD gives the percentage that an unemployment benefit represents of someone's income when employed. Under the European Union variables we added an adjusted variable. We suppose that a correction for GDP per capita might increase the viability of our test. At face value it seems that the worth of an unemployment benefit in absolute value is less important than the level it represents compared to the income per person in a country. The single measure coming from the International Labour Organization is interesting as well. Based on survey data (from different years) one can calculate how much of household income stems from benefits. We reasonably assume that a higher percentage indicates a more generous country with more supposed automatic stabilizers.

6.2 Test of our empirical hypothesis

Now that we have extensively discussed the data available to us and how we modified it we can get to the actual analysis. We divided this section in three parts. First, we will discuss our findings based on the coefficient of variation and standard deviation measures from the World Bank. The second part shortly discusses our findings based on the OECD data. Then we get to our analysis of the European Union data set.

As we use different data sets we use different equations, but they all boil down to the following common standard equation. For the beneath description we borrowed from Souleles (1999). However our actual regression analysis principally differs from theirs with regard to the chosen subject and variables.

$$\begin{gathered} V_i = \alpha + \beta_1 \cdot E_{1i} + \beta_2 \cdot E_{2i} + \beta_3 \cdot E_{3i} + \\ + \beta_4 \cdot \ln GDP_i + \beta_5 \cdot REI_i + \varepsilon_i \end{gathered} \tag{6.1}$$

We will estimate this equation (6.1) by ordinary least squares (OLS). As is common under OLS we assume a random variable with mean zero, ε_i, for the error terms. V_i is the volatility measure in question for country i, which can be found in the headings of the tables. E_i is an explanatory variable which definition can be found in the first column of each table. GDP_i refers to the "GDP (in US 2000 dollars, year 2000)"-variable from the WDI database. We take the natural log as during our tests we found that this modification gave better results. We figure that there are 'decreasing' returns to size, regardless of whether big is better per se. REI_i refers to the rigidity of employment index which we discussed above. The top row of each table will tell you which explanatory variables actually entered the regression. α is the constant for the regression outcome, which we will not report, as it doesn't offer extra information. Each β connects with an explanatory variable and is the coefficient which is found through the OLS regression.

Upfront we do need to spend a few words on our R^2 results[16]. We can be frank that even the more convincing regressions have rather low R^2 figures. However, one reason is that we only use a small set of explanatory variables that doesn't even intend to explain all volatility. We do not think that

[15] Definition taken from the OECD: "The OECD summary measure [of benefit entitlements] is defined as the average of the gross unemployment benefit replacement rates for two earnings levels, three family situations and three durations of unemployment."

[16] R^2 stands for the portion of variation explained by the regression. An R^2 of 0.20 means that our regression only explains 20% of all volatility in a measure.

automatic stabilizers can explain all GDP volatility. The area we venture into with our hypothesis is also pretty unknown to our knowledge. We try to link volatility to overall 'generosity' levels which are both indirect variables, and we had to calculate them ourselves. The closest approximation to our chosen OLS regression is that of Leeftink (2003) who has very comparable levels of R^2 figures. Other papers with some resemblance to our regression mostly (possibly conveniently) left out any R^2 figures.

To ameliorate for these relatively low R^2 results we have tried to add more variables that might also have some influence on GDP volatility. From the WDI database we took several so-called CPIA ratings[17] but neither of these improved our OLS regressions. Sometimes they were insignificant themselves and at other times they made the regression insignificant. Attempts to let a variable on long term unemployment enter didn't succeed and a variable on profit tax didn't give significant outcomes either (both variables from the WDI database). Though our attempts did succeed sometimes and that is why the natural log of GDP and the rigidity of employment index do enter some of the equations, as stated in equation 6.1. These latter modifications did improve our results to a, in our opinion, satisfying level.

6.2.1 Empirical test: World Bank data

The first test has been conducted with the World Bank data and we have also let the ILO measure, that gives the social benefits as % of household income, enter the equation. In this subsection we will show whether there is correlation between (instrumental) variables for presumed automatic stabilizers and the volatility of the GDP as found in the World Bank database. The natural log of GDP and the rigidity of employment index are added to improve the explanatory value of our regressions. Tables 5 and 6 show the outcomes of applying our hypothesis to different GDP volatility measures.

In Table 5, all three measures individually are significant in explaining a portion of the volatility (here: coefficient of variation) in GDP. Even better, unemployment and the GINI index combined still give a significant outcome. In the fourth column we see that a higher percentage of social benefits as part of the household income can apparently explain more than 50% of variation in the coefficient of variation of GDP. Moreover the direction of the correlation is in all cases as expected.

Higher unemployment might very well be instrumental for more automatic stabilization (higher unemployment benefits). Such is the case because we showed under the dynamic Keynesian model that higher unemployment benefits probably lead to higher unemployment in the long run. In Table 5 higher unemployment is indeed said to give lower volatility[18].

The GINI index could be considered instrumental for the generosity of a country. The lower the GINI index the higher the level of social security (higher unemployment benefits and other automatic stabilizers). The GINI index also tells us something about how effective automatic stabilization can be. Under the static model we showed that a more equally distributed national income leads to a stronger effect of an automatic stabilizer on GDP. That direct effect works in the same direction as the instrumental effect. So, we expect a lower GINI index to give a lower GDP volatility and that is the case according to

[17] CPIA ratings are reported by the International Development Association, which is a part of the World Bank. We selected as possibly most relevant the following: business regulatory environment rating, debt policy rating, fiscal policy rating, macroeconomic management rating, policies for social inclusion/equity cluster average, property rights and rule-based governance rating, quality of public administration rating, social protection rating.

[18] The negative sign in front of the coefficient indicates that unemployment moves in the opposite direction of the coefficient of variation of GDP.

Table 5: WDI Coefficient of Variation (GDP)

	= 1) + 4) + 5)	= 2) + 4) + 5)	= 3) + 4) + 5)	= 1) + 2) + 4) + 5)
1) Unemp.	-0.004672			-0.005890
p-value	*0.0242***			*0.0137***
2) GINI		0.005400		0.006677
p-value		*0.0005**		*0.0001**
3) ILO			-0.011975	
p-value			*0.0001**	
4) ln GDP	0.017187	0.026897	0.028270	0.024298
p-value	*0.0125***	*0.0002**	*0.0020**	*0.0026**
5) REI	-0.001546	-0.001855	0.003945	-0.001696
p-value	*0.0773****	*0.0318***	*0.0080**	*0.0766****
R^2	0.126379	0.200657	0.529970	0.249796
(obs.)	137	127	30	112

* = significant at 1%
** = significant at 5%
*** = significant at 10%
1) = World Bank: Unemployment as % of the labor force (average)
2) = World Bank: GINI index (average)
3) = ILO: Social benefits as % of household income
4) = World Bank: Natural log of the GDP of the country
5) = World Bank: Rigidity of employment index (average)
p-value = probability of getting a value as extreme as observed if the null hypothesis (no correlation) is true. As for all other p-values in this section it holds that a p-value near zero gives a certain level of confirmation of our posed hypothesis.
R^2 = portion of variation explained by the regression
obs. = number of observations

Table 6: WDI Standard Deviation (GDP growth)

	= 1) + 4)	= 2) + 4)	= 3) + 4)	= 1) + 2) + 4)	= 1) + 2) + 3) + 4)
1) Unemp.	0.114715			0.097891	0.150712
p-value	*0.0018**			*0.0012**	*0.0027**
2) GINI		-0.068626		-0.035988	0.086005
p-value		*0.0087**		*0.0801****	*0.0482***
3) ILO			0.005468		-0.050486
p-value			*0.9130*		*0.1249*
4) ln GDP	-0.268221	-0.653139	-0.789436	-0.432746	-1.077332
p-value	*0.0269***	*0.0000**	*0.0002**	*0.0000**	*0.0000**
R^2	0.124575	0.203286	0.374541	0.266551	0.865907
(obs.)	147	129	33	113	27

Explanation of abbreviations and signs under Table 5.

Table 5.

The ILO measure that gives the percentage of household income stemming from social benefits is also significant. A higher percentage means in our view that an economy has more automatic stabilizing measures in effect. That means that we expect a higher volatility if only a small portion of household income consists of social benefits. According to Table 5 that is indeed the case. We would also like to point to the R^2 of this regression. The differences in the percentage of household income stemming from social benefits can explain about 53% of total volatility in the data set. That is impressive, though we must note that the data set for that particular regression is small so this high R^2 should be viewed with caution.

We didn't show a full regression for all variables in Table 5 as the results were too insignificant. That is probably the case because the number of observations becomes pretty low in that regression. Also every explanatory variable adds another restriction on the empirical model, which lowers the chance of a successful regression. Another effect that negatively impacts significance is that the different variables might be instrumental for each other or some 'background variable'. A higher portion of household income coming from social benefits also indicates a lower GINI index. Combining them in one regression might 'overestimate' their impact as probably some background variable (call it 'generosity') enters the equation twice, both through the ILO measure and the GINI index. And that overestimation might in turn explain the low significance of the individual variables when combined in one regression. A similar explanation will also hold for the following subsections.

The two extra variables, ln GDP and REI, are both significant in all the regressions we show in Table 5. Maybe slightly against intuition we see that a higher GDP correlates positively with higher volatility. Maybe that has something to do with what we already mentioned: larger economies their volatility is less shaped by the world economy and that might give a bigger effect of internal shocks. The rigidity of employment index only has the expected sign in column 4. More rigid regulations lead to a higher volatility of GDP. Such is intuitive as rigid regulations makes the economy less capable of smoothly adjusting the allocation of labor and thus shocks have a higher impact. As that column also exhibits the best explanatory value we assume that rigidity of employment indeed has the expected effect. We can't explain the counter-intuitive results of the other regressions, though it could be that these stem from the combination with the other explanatory variable.

We left the rigidity of employment index out of Table 6 because of insignificance. Table 6 shows the same regressions of Table 5 but now on the standard deviation of GDP growth. Oddly enough columns 2, 3, 4 and 5 all contradict our previous findings. Higher unemployment appears to give higher volatility. We could say that unemployment comes at an economic cost which could weaken the economy as we have shown in the dynamic Keynesian model. From that perspective the direction of the correlation between unemployment and GDP volatility could go both ways intuitively. It all depends on the strength of the positive and negative effects of automatic stabilization on unemployment.

A less balanced distribution of income lowers volatility according to column 3, though we do get that a higher GDP leads to lower volatility which is more along the lines of our intuition. Column 6 ameliorates these contradictory results somewhat as the GINI index gets the right sign and the ILO measure approaches significance with the expected sign. Though we still put question marks besides the influence of unemployment as an instrumental variable. Lastly, this last column of Table 6 does show a very high R^2, so we do think we are on to something.

Summarizing this WDI analysis we can state that we have convincingly shown correlation between unemployment and GDP volatility. The direction however is ambiguous and could also intuitively go both directions. Distribution of income as an instrumental variable seems to support our hypothesis that automatic stabilizers lower GDP volatility. Social benefits as a percentage of household income confirms this finding and does so convincingly, especially in Table 5.

6.2.2 Empirical test: OECD data

Secondly we would like to show our test on OECD data. The OECD offers more direct measures of the benefit system of a country. Therefore it would be interesting to see whether we can find correlation between the benefit system and GDP volatility. In advance we must stress that because of our calculations with a database like that of the OECD, with only a small number of countries included, we are left with a very small data set which is detrimental for applying a regression. We have also let the natural log of GDP and the rigidity of employment index enter the regressions when they improved the results. Tables 7 and 8 show our tests with the OECD data set.

Table 7: OECD Coefficient of Variation (GDP)

	= 1) + 4)	= 1) + 4) + 5)	= 2) + 4)	= 3) + 4)
1) Agg. ben. & trans.	-0.536785	-0.825247		
p-value	*0.1879*	*0.0266***		
2) Soc. exp. on unemp.			-0.010926	
p-value			*0.6747*	
3) Repl. rates				0.001208
p-value				*0.5632*
4) ln GDP	0.034738	0.050404	0.033178	0.034694
p-value	*0.0238***	*0.0006**	*0.0341***	*0.0280***
5) REI		0.001851		
p-value		*0.1658*		
$\boldsymbol{R^2}$	0.234537	0.470776	0.170491	0.173498
(obs.)	28	27	28	29

* = significant at 1%
** = significant at 5%
*** = significant at 10%
1) = OECD: Aggregated benefits and transfers (average; % of GDP)
2) = OECD: Social expenditures on unemployment (average; % of GDP)
3) = OECD: Replacement rates (bi-annual; average)
4) = World Bank: Natural log of the GDP of the country
5) = World Bank: Rigidity of employment index (average)
p-value = see Table 5.
$\boldsymbol{R^2}$ = portion of variation explained by the regression
obs. = number of observations

As we warned this regression doesn't offer much new insight. Two of the three variables are unambiguously insignificant. We take out two of the columns, namely column 3 of Table 7 and column 2 of Table 8, which seem to make some sense. Those columns show that a country with a higher aggregated

Table 8: OECD Standard Deviation (GDP growth)

	= 1) + 5)	= 2) + 5)	= 3) + 5)	= 4) + 5
1) Agg. ben. & trans.	-2.605745			
p-value	*0.0305***			
2) Soc. exp. on unemp.		-0.057399		
p-value		*0.5428*		
3) Repl. rates			-0.001168	
p-value			*0.8946*	
4) ln GDP				-0.045854
p-value				*0.4825*
5) REI	0.007887	0.009222	0.011873	0.011246
p-value	*0.0712****	*0.0875****	*0.0613****	*0.0697****
R^2	0.218126	0.130223	0.134514	0.149975
(obs.)	27	27	28	29

Explanation of abbreviations and signs under Table 7.

benefits and transfers as percentage of GDP level has a lower GDP volatility. Such is a clear confirmation that automatic stabilization through benefits and transfers lower the volatility of GDP. Nonetheless we think the results from the previous subsection are more convincing.

On a side note we need to comment on the ln GDP and REI variables. The correlation between GDP level and volatility is as in Table 5: a higher GDP level gives a higher volatility. Table 8 which regresses on the standard deviation of GDP gives the expected sign for the rigidity of employment index. If regressing against standard deviation of GDP we get that a higher rigidity of employment leads to a higher volatility. The intuition behind that has been explained in the previous subsection. More rigid regulations nerve the ability of the economy to adjust to a shock, which intuitively leads to higher volatility.

6.2.3 Empirical test: European Union data

We now get to testing our hypothesis with European Union data. Though we have only two explanatory variables that are instrumental variables for automatic stabilization. As with the OECD, this database has its pros and cons. Beneficial is that it offers more direct measures of presumed automatic stabilizers, but it is of a small size which might harm the significance of our regression. Tables 9 and 10 show our tests with the European Union data set.

Table 9 seems to confirm our hypothesis. As our well-intended correction of the “Social benefits per capita (average)” variable for GDP per capita made no sense, insignificant outcomes, we don’t report our regressions with that variable. It is not intuitive to see that the relative value of the benefits handed out probably is of no importance. However, we do see that higher social benefits per capita leads to lower GDP volatility. Such is as expected as we showed under the static model that higher benefits lead to lower volatility. The dynamic Keynesian model didn’t offer a strong confirmation of that intuition, but within reasonable boundaries this statement seems to hold in that model as well. The other regressions are shown for expositional reasons. They show why we focused on column 2 of Table 9. It also apparent that with concern to the coefficient of variation that the rigidity of employment index variable has the expected sign: higher regulations rigidity leads to a higher GDP volatility.

Table 10 confirms our findings with regard to the social benefits per capita. However, the third column

Table 9: EU Coefficient of Variation (GDP)

	= 1) + 5)	= 2) + 5)	= 1) + 2) + 5)	= 1) + 4) + 5)	= 2) + 4) + 5)
1) Soc. ben.	-0.000100		-0.000292	-6.13E-06	
p-value	*0.0307*****		*0.0333*****	*0.8763*	
2) Unem. fun.		-1.621354	4.529311		0.599975
p-value		*0.1254*	*0.1278*		*0.4842*
4) ln GDP				-0.022651	-0.045854
p-value				*0.0003**	*0.0001**
5) REI	0.001665	0.001952	0.001299	0.001486	0.001527
p-value	*0.0343*****	*0.0170*****	*0.0980******	*0.0136*****	*0.0095**
R^2	0.401253	0.333143	0.465259	0.682481	0.689603
(obs.)	25	25	25	25	25

* = significant at 1%
** = significant at 5%
*** = significant at 10%
1) = EU: Social benefits per capita (average)
2) = EU: Benefits unemployment function (average; % of GDP)
4) = World Bank: Natural log of the GDP of the country
5) = World Bank: Rigidity of employment index (average)
p-value = see Table 5.
R^2 = portion of variation explained by the regression
obs. = number of observations

Table 10: EU Standard Deviation (GDP growth)

	= 1)	= 2)	= 1) + 2)	= 4)	= 5)
1) Soc. ben.	-0.007603		-0.024173		
p-value	*0.0597******		*0.0134*****		
2) Unem. fun.		-91.81225	413.0370		
p-value		*0.3312*	*0.0573******		
4) ln GDP				-0.646060	
p-value				*0.2519*	
5) REI					0.148031
p-value					*0.0832******
R^2	0.129744	0.036342	0.249012	0.045012	0.110972
(obs.)	28	28	28	31	28

Explanation of abbreviations and signs under Table 9.

raises some doubts about the influence of benefits with an unemployment function. Table 6 also showed standard deviation as a volatility measure and returned unexpected results. Though, the explanatory value of the regressions of this table seem low either way. The ln GDP and REI variables weren't helpful either as they made the actual explanatory variables insignificant in the regression. We do again find some confirmation in column 6 that rigid regulations increases the volatility of GDP.

6.3 Overall empirical conclusion

Summarizing we can conclude that there certainly is some empirical evidence of automatic stabilizers stabilizing the GDP of countries. We found that a more equally distributed national income overall convincingly signals a country that also has a lower GDP volatility. Countries with households that have a larger share of their income coming from social benefits also observe a more stable GDP. OECD and European Union data suggest that actual social benefit levels correlate with GDP volatility and in a fashion that confirms automatic stabilization.

We also tested the correlation between unemployment, instrumental for a generous social security system, and GDP volatility. These results were only ambiguous. Some tests seemed to prove that economies with higher unemployment, presumably a higher incentive to stay unemployed from unemployment benefit programs, have a more stable GDP. However other tests showed the opposite and might suggest that a stable GDP is that valuable that it eventually leads to lower unemployment.

Concluding we find that there is evidence of automatic stabilization functioning as suggested by our theoretical models. Some of our tables show room for improvement and we think that there is enough material for future research to increase the explanatory value of regressions on GDP volatility. The most important outcome of this empirical analysis is that a more equal distribution of national income, instrumental for active automatic stabilizers, clearly and unambiguously lowers GDP volatility and thus flattens the business cycle.

7 Summary and conclusion

We have shown several different theoretical models for observing the influence of automatic stabilizers on the stability of national income. Each instance improved the previous and increased the intuitions connected to automatic stabilizers. The differences weren't too striking although we have shown that adding the element time can change some intuitions from the static case. In future research there could be done more with micro-foundations as consumer preferences, effort functions and job search effectiveness. In that light the remark of Cohen and Follette (1999) that the effectiveness of automatic stabilizers is invariant to the choice of framework might be a bit too bold.

After a review of the literature on automatic stabilizers and the volatility of the business cycle we exhibited a static model with a value tax and unemployment benefits, section 3. We establish theoretically that these taxes and unemployment benefits indeed stabilize the national income. A restriction on the budget hampers the effectiveness of these automatic stabilizers. Finally we show that under a more equally distributed national income automatic stabilizers are more effective.

To extend our analysis over more than one period we then showed a dynamic Keynesian model with more than one period, section 4. The economy has an endogenous tax setting function and we can compare the case with and without unemployment benefits. We then 'simulated' three different kinds of shocks through the productivity parameter of the Cobb-Douglas production function. The theoretical results are that the economy with unemployment benefits indeed at first alleviates the negative effects but gets in a worse shape because of increased debt service. To support our theoretical model we exhibited how it could simulate 25 periods (years) of the United States economy with some modest calibration. The message that policymakers should read in this section is that automatic stabilizers function as they should but that the benefits of that should be weighed against the future costs (higher debt).

Future research should probably go in the direction of a micro-founded model like our dynamic Ramseyian model proposal, section 5. These models could increase the understanding on how automatic stabilizers like unemployment benefits and (progressive) income taxes influence the present and future periods.

To offer some insight on the question how automatic stabilizers affect the economy we also showed an empirical analysis in which we determined the relation between 'generosity', instrumental for automatic stabilization, and GDP volatility. Our OLS regressions convincingly proved the expected negative relation between the level of automatic stabilization and GDP volatility. Most important finding is that a more equal distribution of national income clearly and unambiguously lowers GDP volatility and thus flattens the business cycle.

To answer the questions of the introduction we can say that automatic stabilizers are effective to lower the variance in national income, at least in the short run. Because of that they are definitely beneficial, but these benefits should always be weighed against the costs of a worsened shape of the economy in the long run. This worsened shape could be more costly than the gains stemming from a more stable national income. Therefore we conclude that automatic stabilizers indeed flatten the business cycle, but policymakers should always consider whether they are worth their price.

A Static model outcomes - More elaborate steps

In this appendix we will give some more elaborate steps to get to the expressions of section 3.3.

We start with rewriting the integral functions (3.5), (3.6) and (3.7). We rewrite them in a reverse order as C logically follows from U and T.

$$
\begin{aligned}
U &= \int_{i=0}^{1} [(1 - I(\theta_i))u^*] \\
U &= \theta_i^* u^*
\end{aligned}
$$

$$
\begin{aligned}
T &= \int_{i=0}^{1} [I(\theta_i)t_i w_i] \\
&= FT(1) - FT(\theta_i^*) \\
&= \left(\frac{1}{2}t_f\delta + t_f w_f\right) - \left(\frac{1}{2}t_f\delta(\theta_i^*)^2 + t_f w_f \theta_i^*\right) \\
T &= \frac{1}{2}t_f\delta(1-(\theta_i^*)^2) + t_f w_f(1-\theta_i^*)
\end{aligned}
$$

$$
\begin{aligned}
C &= \int_{i=0}^{1} [I(\theta_i)(1-t_i)w_i + (1-I(\theta_i))u^*] \\
C &= \frac{1}{2}(1-t_f)\delta(1-(\theta_i^*)^2) + (1-t_f)w_f(1-\theta_i^*) + \theta_i^* u^*
\end{aligned}
$$

The derivations in more steps are as follows (on unemployment benefits we don't show more elaborate steps). We start with U, then T and end with C as this serves understanding the steps that are taken.

$$
\begin{aligned}
\frac{\mathrm{d}U}{\mathrm{d}u^*} &= \frac{2u^*}{(1-t_f)\delta} - \frac{w_f}{\delta} > 0 \\
\frac{\mathrm{d}U}{\mathrm{d}w_f} &= -\frac{u^*}{\delta} < 0 \\
\frac{\mathrm{d}U}{\mathrm{d}\delta} &= -\frac{(u^*)^2}{(1-t_f)\delta^2} + \frac{w_f u^*}{\delta^2} < 0 \\
\frac{\mathrm{d}U}{\mathrm{d}t_f} &= \frac{(u^*)^2}{(1-t_f)^2\delta} > 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}T}{\mathrm{d}u^*} &= 0 - \frac{1}{2}t_f\delta\frac{2\theta_i^*}{(1-t_f)\delta} - \frac{t_f w_f}{(1-t_f)\delta} \\
&= -\left(\frac{t_f\theta_i^*}{1-t_f} + \frac{t_f w_f}{(1-t_f)\delta}\right) < 0 \\
\frac{\mathrm{d}T}{\mathrm{d}w_f} &= t_f - \left(\frac{1}{2}t_f\delta\frac{-2\theta_i^*}{\delta} + t_f\frac{-w_f}{\delta} + t_f\theta_i^*\right) \\
&= t_f + t_f\frac{w_f}{\delta} > 0 \\
\frac{\mathrm{d}T}{\mathrm{d}\delta} &= \frac{t_f}{2} - \left[\frac{t_f}{2}(\theta_i^*)^2 + \frac{t_f\delta}{2}2\theta_i^*\left(\frac{-(1-t_f)u^*}{[(1-t_f)\delta]^2} - \frac{-w_f}{\delta^2}\right) + t_f w_f\left(\frac{-(1-t_f)u^*}{[(1-t_f)\delta]^2} - \frac{-w_f}{\delta^2}\right)\right] \\
&= [1-(\theta_i^*)^2]\frac{t_f}{2} + t_f(\delta\theta_i^* + w_f)\left(\frac{(1-t_f)u^*}{[(1-t_f)\delta]^2} - \frac{w_f}{\delta^2}\right) \\
&= [1-(\theta_i^*)^2]\frac{t_f}{2} + t_f u^*\left(\frac{u^*}{(1-t_f)\delta^2} - \frac{w_f}{\delta^2}\right) > 0 \\
\frac{\mathrm{d}T}{\mathrm{d}t_f} &= \frac{\delta}{2} + w_f - \left(\frac{\delta}{2}(\theta_i^*)^2 + \frac{t_f\delta}{2}2\theta_i^*\frac{\delta u^*}{[(1-t_f)\delta]^2} + w_f\theta_i^* + t_f w_f\frac{\delta u^*}{[(1-t_f)\delta]^2}\right) \\
&= [1-(\theta_i^*)^2]\frac{\delta}{2} + (1-\theta_i^*)w_f - t_f(\delta\theta_i^* + w_f)\frac{\delta u^*}{[(1-t_f)\delta]^2} \\
&= [1-(\theta_i^*)^2]\frac{\delta}{2} + (1-\theta_i^*)w_f - t_f u^*\frac{u^*}{(1-t_f)^2\delta} > \text{or} < 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}C}{\mathrm{d}u^*} &= -\frac{1}{2}(1-t_f)\delta\left(\frac{2\theta_i^*}{(1-t_f)\delta}\right) - \frac{(1-t_f)w_f}{(1-t_f)\delta} + \theta_i^* + \frac{u^*}{(1-t_f)\delta} \\
&= \frac{u^*}{(1-t_f)\delta} - \frac{w_f}{\delta} > 0 \\
\frac{\mathrm{d}C}{\mathrm{d}w_f} &= -\frac{1}{2}(1-t_f)\delta\left(\frac{-2\theta_i^*}{\delta}\right) + (1-t_f) - (1-t_f)\theta_i^* - (1-t_f)w_f\left(\frac{-1}{\delta}\right) - \frac{u^*}{\delta} \\
&= (1-t_f) + (1-t_f)\frac{w_f}{\delta} - \frac{u^*}{\delta} > 0 \\
\frac{\mathrm{d}C}{\mathrm{d}\delta} &= \frac{(1-t_f)}{2} - \frac{(1-t_f)}{2}(\theta_i^*)^2 - \frac{(1-t_f)\delta}{2}2\theta_i^*\left(\frac{-(1-t_f)u^*}{[(1-t_f)\delta]^2} - \frac{-w_f}{\delta^2}\right) + \\
&\quad -(1-t_f)w_f\left(\frac{-(1-t_f)u^*}{[(1-t_f)\delta]^2} - \frac{-w_f}{\delta^2}\right) + u^*\left(\frac{-(1-t_f)u^*}{[(1-t_f)\delta]^2} - \frac{-w_f}{\delta^2}\right) \\
&= [1-(\theta_i^*)^2]\frac{(1-t_f)}{2} + \\
&\quad + [(1-t_f)\delta\theta_i^* + (1-t_f)w_f - u^*]\left(\frac{(1-t_f)u^*}{[(1-t_f)\delta]^2} - \frac{w_f}{\delta^2}\right) \\
&\quad \text{and } [(1-t_f)\delta\theta_i^* + (1-t_f)w_f - u^*] = 0, \text{ thus} \\
&= [1-(\theta_i^*)^2]\frac{(1-t_f)}{2} > 0 \\
\frac{\mathrm{d}C}{\mathrm{d}t_f} &= \frac{-\delta}{2} - \left(\frac{-\delta}{2}(\theta_i^*)^2 + \frac{(1-t_f)\delta}{2}2\theta_i^*\frac{\delta u^*}{[(1-t_f)\delta]^2}\right) - w_f + \\
&\quad - \left(-w_f\theta_i^* + (1-t_f)w_f\frac{\delta u^*}{[(1-t_f)\delta]^2}\right) + u^*\frac{\delta u^*}{[(1-t_f)\delta]^2} \\
&= [(\theta_i^*)^2 - 1]\frac{\delta}{2} + (\theta_i^* - 1)w_f + \\
&\quad + [u^* - (1-t_f)\delta\theta_i^* - (1-t_f)w_f]\frac{\delta u^*}{[(1-t_f)\delta]^2} \\
&\quad \text{and } [u^* - (1-t_f)\delta\theta_i^* - (1-t_f)w_f] = 0, \text{ thus} \\
&= [(\theta_i^*)^2 - 1]\frac{\delta}{2} + (\theta_i^* - 1)w_f < 0
\end{aligned}
$$

B Dynamic Keynesian model - Derivation of the level of the production factors

In this appendix we will give a more thorough derivation of equations (4.4), labor demand, and (4.12), optimal (or desired) capital stock. Equation (4.4) determines the optimal level of labor input given all other factors being productivity, the reservation wage, the tax rate and the present capital stock. Equation (4.12) determines the optimal, desirable, level of the capital stock given all other factors being productivity, the rate of time preference, the tax rate and the present input of labor.

For a better understanding we first repeat equations (4.8), the interest rate, and (4.9), the wage rate.

$$
\begin{aligned}
r_t &= \frac{\partial Y_t^S}{\partial K_t} = A_t \alpha {K_t}^{\alpha-1} {L_t^D}^{1-\alpha} \\
w_t &= \frac{\partial Y_t^S}{\partial L_t^D} = A_t(1-\alpha){K_t}^{\alpha} {L_t^D}^{-\alpha}
\end{aligned}
$$

As explained in the main text the production factors both earn their marginal product, hence the derivatives. Equations (4.4) and (4.12) then determine the corresponding levels of labor demand and desired capital stock. We talk about desired capital stock as in our model the investment function takes time to move from the present capital stock to a different optimal capital stock.

We will now discuss how we arrived at the labor demand equation in (4.4). First a repetition of this equation for convenience.

$$L_t^D = \left(\frac{\lambda}{(1-\tau)A_t(1-\alpha){K_t}^{\alpha}}\right)^{\frac{1}{-\alpha}}$$

The labor demand function has to determine how many people are hired given the wage rate. People are assumed to have a reservation wage and if their wage doesn't exceed this level they prefer to stay unemployed, possibly getting an unemployment benefit. That means that the net wage rate, determined through the derivative of the production function as shown above, should be equal to the reservation wage. We will now show this in a few more elaborate steps.

$$\frac{\lambda}{(1-\tau)} = A_t(1-\alpha){K_t}^{\alpha}{L_t^D}^{-\alpha} \tag{B.1}$$

$${L_t^D}^{-\alpha} = \left(\frac{\lambda}{(1-\tau)A_t(1-\alpha){K_t}^{\alpha}}\right) \tag{B.2}$$

$$L_t^D = \left(\frac{\lambda}{(1-\tau)A_t(1-\alpha){K_t}^{\alpha}}\right)^{\frac{1}{-\alpha}} \tag{B.3}$$

The left hand side of step (B.1) corrects the reservation wage for the tax level to get the net wage level that workers demand to accept a job. The right hand side is merely a repetition of equation (4.9), the wage rate as determined by the production function. After understanding that it is all pretty straightforward. In step (B.2) the $[A_t(1-\alpha){K_t}^{\alpha}]$ part is taken to the other side after which we simply reverse sides. Then only remains the negative power on the left hand side which is moved to the other side as well in step (B.3), which is the same equation as equation (4.4) in the main text.

The same explanation holds for equation (4.12), optimal (or desirable) capital stock. We will show the same three steps as for labor demand.

$$\frac{\rho}{(1-\tau)} = A_t\alpha{K_t^F}^{\alpha-1}{L_t^D}^{1-\alpha} \tag{B.4}$$

$${K_t^F}^{\alpha-1} = \left(\frac{\rho}{(1-\tau)A_t\alpha{L_t^D}^{1-\alpha}}\right) \tag{B.5}$$

$$K_t^F = \left(\frac{\rho}{(1-\tau)A_t\alpha{L_t^D}^{1-\alpha}}\right)^{\frac{1}{\alpha-1}} \tag{B.6}$$

The description of the rewriting steps of the previous paragraph hold here as well. On the left hand side we now start with the rate of time preference corrected for taxes. The only difference is that the right hand side in equation (B.4) doesn't take the present interest rate, equation (4.8), but takes the interest that would prevail if the capital stock would be at its optimal level (K_t is replaced by K_t^F).

C Dynamic Ramseyian model - Shephard's lemma

Shephard's lemma was proven by Shephard (1953). It states how to chose ones demand for good i given the expenditure function over all goods and the price of good i. In our case that means that a consumer maximizes its lifetime utility when he chooses its demand for a good in a specific period.

C.1 Definition of Shephard's lemma

The lemma is defined as follows. The demand for good i, x_i, is given by the derivative of the cost function, in our case that refers to 'cost' in terms of lifetime utility, Q which is divided by the derivative of the price function of good i, p_i:

$$x_i = \frac{\partial Q}{\partial p_i} \tag{C.1}$$

Shephard proved his lemma by using the distance formula, today's proofs are given by the envelope theorem. A more thorough explanation could for example be found in Mas-Colell et al. (1995).

C.2 Shephard's lemma in our dynamic model

Shephard's lemma is used in the following equations, (5.5), (5.8), (5.10) and (5.11). We will show the application of Shephard's lemma in equation (5.5). We first repeat the equation for convenience:

$$\text{good } i\text{: } Q_{it} \geq \sum_h \phi_t \cdot \left(p_h^U \cdot U_h\right)/p_{ht}^W \cdot \gamma_{hi} \cdot \left(\frac{p_{jt}}{p_{it}}\right)^{\gamma_{hi}} + \beta_i \cdot \left(\frac{p_{jt}}{p_{it}}\right)^{\beta_i} \cdot I_t$$

Our interest has this particular part which stems from Shephard's lemma. We left out the household subscript, as it doesn't matter for the explanation:

$$\phi_t \cdot \left(p^U \cdot U\right)/p_t^W \tag{C.2}$$

Applying Shephard's lemma in our case would read:

$$\text{demand for good } W \text{ in } t = \frac{\partial p^U}{\partial p_t^W} \cdot U \tag{C.3}$$

The expenditure function is $\partial p^U \cdot U$ in this case. We now give steps to get from Shephard's lemma to the expression in our model. From (5.2) we take the function for p^U, leaving aside the discounting factor as it does not change the final outcome of this explanation:

$$p^U = \left(p_1^W\right)^{\phi_1} \cdot \left(p_2^W\right)^{\phi_2} \cdot \ldots \cdot \left(p_T^W\right)^{\phi_T} \tag{C.4}$$

$$\begin{aligned} \frac{\partial p^U}{\partial p_1^W} &= \phi_1 \cdot \left(p_1^W\right)^{\phi_1 - 1} \cdot \left(p_2^W\right)^{\phi_2} \cdot \ldots \cdot \left(p_T^W\right)^{\phi_T} \\ &= \phi_1 \cdot \frac{\left(p_1^W\right)^{\phi_1} \cdot \left(p_2^W\right)^{\phi_2} \cdot \ldots \cdot \left(p_T^W\right)^{\phi_T}}{p_1^W} \\ &= \phi_1 \cdot \frac{p^U}{p_1^W} \end{aligned} \tag{C.5}$$

Put this this back into (C.3) and compare it with (C.2) and one sees how Shephard's lemma has entered the equations of our dynamic model. For the other applicable equations the same explanation holds.

References

Agell, Jonas and Per Lundborg. Fair wages, involuntary unemployment and tax policies in the simple general equilibrium model. *Journal of Public Economics*, (47):299–320, 1992.

Ambler, Steve, Emanuela Cardia, and Christian Zimmerman. International transmission of the business cycle in a multi-sector model. *European Economic Review*, 46(2):273–300, 2002.

Andersen, Torben M. and Robert R. Dogonowski. Social insurance and the public budget. *Economica*, 69(275):415–431, 2002.

Auerbach, Alan J. and Daniel Feenberg. The significance of federal taxes as automatic stabilizers. *The Journal of Economic Perspectives*, 14(3):37–56, 2000.

Blinder, Alan S. and Robert M. Solow. Does fiscal policy matter? *Journal of Public Economics*, 2(4):319–337, 1973.

Chari, V. V., Lawrence J. Christiano, and Patrick J. Kehoe. Optimal fiscal policy in a business cycle model. *The Journal of Political Economy*, 102(4):617–652, 1994.

Cohen, Darrel and Glenn Follette. The automatic fiscal stabilizers: quietly doing their thing. Finance and Economics Discussion Series 1999-64, Board of Governors of the Federal Reserve System (U.S.), 1999. URL `http://ideas.repec.org/p/fip/fedgfe/1999-64.html`.

Cooley, Thomas F. and Edward C. Prescott. *Frontiers of business cycle research*, chapter Economic Growth and Business Cycles, pages 1–38. Princeton University Press, 1995. ISBN 0-691-04323-X.

DNB. Conjunctuur en werkloosheid. *Economische Statistische Berichten*, (4138):114, 1998.

Elmendorf, Douglas W. and N. Gregory Mankiw. Government debt. Working Paper 6470, National Bureau of Economic Research, 1998. URL `http://www.nber.org/papers/w6470`.

Emami Namini, Julian. *International Trade and Multinational Activity.* Number 573 in Lecture Notes in Economics and Mathematical Systems. Springer, 2006. ISBN 3-540-32718-5, 978-3-540-32718-9.

Kiley, Michael T. How should unemployment benefits respond to the business cycle? *Topics in Economic Analysis & Policy*, 3(1), 2003.

Kniesner, Thomas J. and James P. Ziliak. Tax reform and automatic stabilization. *The American Economic Review*, 92(3):590–612, 2002.

Lau, Morten I., Andreas Pahlke, and Thomas F. Rutherford. Approximating infinite-horizon models in a complementarity format: A primer in dynamic general equilibrium analysis. *Journal of Economic Dynamics & Control*, 26(4):577–609, 2002.

Leeftink, Bertholt. Rules vs. flexibility - is there a trade-off between budgetary sustainability and budgetary stabilisation? Research Memorandum 0001, Dutch Ministry of Finance, OCFEB, 2003. URL `http://hdl.handle.net/1765/835`.

Mankiw, N. Gregory. The savers-spenders theory of fiscal policy. *The American Economic Review*, 90(2):120–125, 2000.

Mas-Colell, Andreu, Michael Dennis Whinston, and Jerry R. Green. *Microeconomic Theory*. Oxford University Press, 1995. ISBN 0195073401.

Palomba, Neil A. Unemployment compensation program: Stabilizing or destabilizing. *The Journal of Political Economy*, 76(1):91–100, 1968.

Ramsey, Frank P. A mathematical theory of saving. *The Economic Journal*, 38(152):543–559, 1928.

Rejda, George E. Unemployment insurance as an automatic stabilizer. *The Journal of Risk and Insurance*, 33(2):195–208, 1966.

Shapiro, Matthew D. and Joel Slemrod. Consumer response to the timing of income: Evidence from a change in tax witholding. *The American Economic Review*, 85(1):274–283, 1995.

Shephard, Ronald W. *Cost and Production Functions*. Princeton University Press, 1953.

Sørensen, Peter Birch and Hans Jørgen Whitta-Jacobsen. *Introducing advanced macroeconomics : growth and business cycles*. McGraw-Hill/Maidenhead, 2005. ISBN 0-07-710425-0.

Souleles, Nicholas S. The response of househould consumption to income tax refunds. *The American Economic Review*, 89(4):947–958, 1999.