Aircraft Design

Project 1: Business Jet

The complete step-by-step medium business jet project

Exercise

2021

-This is a practical exercise designed specifically for the Aircraft Design course-

Contents

Project 1:

Design a civil transport aircraft for business needs. The type of the aircraft is airplane also called Business Jet. The airplane shall be allowed increased cabin space for its passengers. The only class available shall be business class standard.

The aircraft should meet the following requirements:

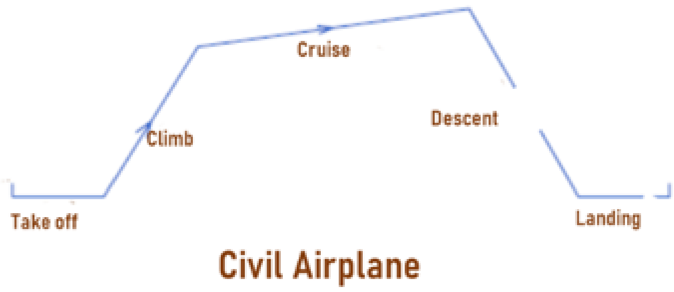
- Be able to transport at least 12 passengers excluding the cabin crew.
- Hs maximum wing span of 20 meters.
- Be transonic aircraft to meet the noise standards. Its cruise speed should not exceed 980 km/h.
- Maximum flight level should not exceed F500 or 15000 meters.
- The aircraft shall allow overflying a range of at least 3000 kilometres with a single fuel tank.
- The engines shall allow take-off distance of no more than 2000 meters.

1. Preparation for statistical analysis

№	Parameters and Characteristics	Dassault Falcon 900 (France)	Learjet 45 (USA & Canada)	Embraer Legacy 500 (Brasil)	Business Fly
	Mass				
1	Take Off Mass, kg	20640	9750	17400	13500
2	Number of Passengers	19	9	12	12
3	Mass of the Empty Aircraft, kg (Approximate mass*)	10255	5830	12750	7900
4	Fuel Mass, kg (Approximate mass*)	9482	2610	5963	4000
5	Wing loading, daN/m2	-	-	430	380
	Geometry				
6	Wingspan, m	20	14.58	20.25	17
7	Wing Area, m2	50	28.95	40.1	34
8	Aspect Ratio	8	7.34	10.22	8.5
9	Aircraft Length, m	20	17.68	20.47	17
10	Fuselage Diameter, m	2.5	1.8	1.8	2.15
	Flying Capabilities				
11	Minimum Speed, km/h	158		193	170
12	Maximum Speed, km/h	950	804	863	800
13	Range, km	7400	3519	5788	3000
14	Max Flight Height, km	-	13	13.7	13
15	Take Off Distance, m	1560	1536	1245	1510
16	Landing Distance, m	715	811	647	750
	Powerplant				
17	Engine Make	Honeywell TFE731-60	Honeywell TFE731-20	Honeywell HTF7500E	Honeywell TFE731-60
18	Number of Engines	3	2	2	2
19	Thrust, daN	6672	3114	6260	4448
20	Take Off Relative Thrust, kW/daN	0.32	0.32	0.36	0.34
21	Specific Fuel Consumption, kg/kNh	397			

2. Types of flight mission

The aircraft will be designed for civil peruses only. The flight segments are: Take-off, climb, cruise flight, descent, land.



3. Determination of the main aircraft parameters

3.1. Initial estimations of the aircraft mass

The take-off mass is the mass with which the aircraft starts its mission. This is not necessary the maximum possible pass for the aircraft. Some aircrafts include military equipment and could be overloaded, this reduces their manoeuvrability. At this stage you shouldn't aim for accuracy, but to include the main components of the mass:

$$m0 = m_{cw} + m_{ld} + m_{fl} + m_{ey} \quad (1)$$

- Crew mass- m_{cw}
- Loading mass (passengers, luggage, armoury, ect) - m_{ld}
- Fuel mass - m_{fl}
- Empty mass - m_{ey}
- Take off mass – m_0

No	Variable	Paraeter	Value
1	Crew mass	m_{cw}	270
2	Loading mass	mld	1320
3	Fuel mass	m_{fl}	4246.004
3	Empty mass	m_{ey}	7663.996
4	Take off mass	m_0	13500
5	Relative ey	m^*_{ey}	0.567703

3.1.1. Estimating the mass of the empty aircraft.

At that stage the empty mass can be calculated based on the statistical analysis. It is usually from 0.3 to 0.7 of the whole mass, depending on the aircraft size and mission. The Figure 2 shows the relative empty mass depending on the type and the take off mass of the aircraft. Table 3 shows summarized statistical analysis to determine m^*_{ey}.

$$m_{ey}^* = A(2.2m_0)^C K_p \;=7664\ \text{kg} \tag{4}$$

- A, C – Coefficients;
- m_0 – Take-off mass (from the statistics), kg;
- K_p – Coefficient of the propeller type.

3.1.2. Estimating the fuel mass

The fuel needed and basically its mass can be estimated from all segments of the flight that require significant fuel consumption: take-off, climbing, cruise flight, descending, landing, manoeuvring. Once you determine the segments of your flight by choosing its mission, the sum of the fuel used in each segment will give you the final fuel volume and mass for the mission.

If you the number of segments is denoted with i then the fuel in the end of this segment will be m_i. So the difference between the starting point and the final point is due to the fuel burn. To determine the change in the mass due to fuel burn depending on the number of segments, you can use the following equation:

$$m_{fl}^* = m_{fl}/m_0 = 1.06(1-(m_2/m_1)(m_3/m_2)\ \ldots m_n/m_{n-1}) \tag{5}$$

No	Regio, m_i	Take-off	Climb	Cruise	Reserve
1	m_i	13095	12477.9581	7902.92519	7665.837
2	m_i-1	13500	13095	12477.9581	7902.925
3	m_i/m_i-1	0.97	0.95287958	0.63335083	0.97

dH	10
L	3000
c	1.9
V_{cr}	780
K_{cr}	16
n_{cr}	3

3.1.3. Estimating the mass of the cabin crew

The mass of the cabin crew is usually approximated taking for each bomber of the cabin crew a weight of 90kg.

$$m_{cr} = 90n_{cr} \;= 270\ \text{kg} \tag{9}$$

, where n_{cr} is the number of the cabin crew members.

3.1.4. Estimating the mass of the aircraft loading

The mass of the aircraft load includes the passengers and their luggage.

$$m_{ld}=90n_{pas}+180V_l=1320 \text{ kg} \quad (10)$$

, where n_{pas} – number of passengers

V_l – volume of the baggage aera, m^3.

3.2. Determination of the wing loading

The wing loading p = G/S is determined from one of three estimation cases: from the landing conditions, from the conditions for maintaining the cruise flight and from the maneuvering conditions. From the three cases you have to choose the minimum wing loading.

- Landing conditions

$$P_0^I = (c^{land}_{ymax}(V_{min}*1.3)^2)/(30,2(1-m_{fl.hf})) = 588, \text{ daN/m}^2 \quad (11)$$

where: c^{land}_{max} = 2.5…3.2 –max lift coefficient when landing for effective wing mechanization

V_{min} – landing velocity

$m*_{fl.hf}$ – relative horizontal flight fuel mass ($m*_{fl.hf} = 1 - m_i/m_{i-1}$)

- Cruise flight conditions

$$P_0^{II} = (c_{ycr}\rho_{cr}V^2_{cr})/(20(1-0.6m*_{fl.hf}) = 491.8 \text{ , daN/m}^2 \quad (12)$$

where: c_{ycr} - lift coefficient for cruise flight

ρ_{cr} - air density at flight altitude

V_{cr} – cruise flight velocity

- Manoeuvring conditions

$$P^{III}_0 = 0.1(c_{yman}/n_{ymax})q_{max}= 965, \text{ daN/m}^2 \quad (13)$$

where: c_{yman} - lift coefficient for manoeuvring

ρ_{max} – maximal structure loading coefficient

q_{max} – dynamic pressure, Pa

Once the estimation of the above coefficients is completed, you can take the once having the minimum value.

No	Varible	Coefficient	Value
1	Max landing lift coefficient	$c_{y\ max\ land}$	3
2	Cruise lift coefficient	$c_{y\ cr}$	0.53
3	Maneuvering lift coefficient	$c_{y\ man}$	2
4	Cruise velocity	V_{cr}	216.1
5	Minimum Velocity	V_{min}	47.1
6	Relative fuel mass	m^*_{fl}	0.366649
7	Air density at 10km	ρ	0.31
8	Overloading coefficient	n_{max}	1.5
9	Dynamic pressure	q_{max}	7238.378
10	Landing Overload	P1	588.0286
10	Cruise Overload	P2	491.8319
11	Maneuvering Overload	P3	965.117

3.3. Aircraft traction and qualities

There are certain factors that ensure the aircraft has enough thrust and lift to overtake climb, descend, flying with one engine out of order and so on. Here you will consider again three cases: from the conditions for climbing with failed engine, from the condition for a cruise flight and from the condition for take off distance condition.

- climbing with one failed engine

$$P^{I}_0 = k_v(n_{eg}/(n_{eg}-1)(1/K_{cb}+tg\theta) = 0.351, \quad (14)$$

where: k_v = 1.5- coefficient

K_{cb} – aerodynamic quality when climbing

n_{eg} – number of engines

tgθ =– climbing gradient

- horizontal flight

$$P^{II}_0 = 1/(K_{cr}\,\delta^{0.85}\phi) = 0.211, \quad (15)$$

where: φ = 0.8 – throttling coefficient

δ – relative density at cruise altitude

- take-off condition

$$P^{III}_0 = 1.05[520/(C_{ymaxto}l_{to}) + 0.5(3f_{to} + 1/K_{to}) = 0.232, \quad (16)$$

where: C_{ymaxto} =1,7…2.3 – Lift coefficient for take-off

f_{to} – friction coefficient of the landing gear (*Table 4*)

No	Airport Surface	f_{to}
1	Snow and ice	0.02
2	Dry concrete	0.02
3	Wet concrete	0.03
4	Solid primer	0.07
5	Wet grass	0.06
6	Grass	0.08

Table 1: Friction coefficient values

Once the estimation of the above coefficients is completed, you can take the once having the maximum value.

No	Varible	Coefficient	Value
1	Climb coefficient	k_v	1.5
2	Aerodynamic quality for climbing	K_{cb}	10
3	Number of engines	n_{eg}	2
4	Climbing gradient	tgθ	0.017
5	Cruise flight density	ρ_{cr}	0.31
6	Take off lift	c_{ymaxto}	2.3
7	Rolling friction coefficient	f_{to}	0.02
8	Take-off distance	l_{to}	1510
9	Aerodynamic quality for take-off	K_{to}	12
10	Engine Failure	P*1	0.351
11	Cruise Condition	P*2	0.021141
12	Take off Condition	P*3	0.232463

4. Aerodynamic scheme

The term “Aerodynamic Scheme” describes some scheme that includes all surfaces on the aircraft, regardless whether they are control surfaces or static. This scheme determines the shape, position and dimensions of those surfaces. A typical aircraft scheme consists of main surfaces, such as wing, that provide most of the total lift and supporting surfaces such as horizontal and vertical surfaces that help for the stabilization and control of the aircraft.

Depending the position of the supporting surfaces to the wing, we can divide the aerodynamic schemes to the following:

- Standard Scheme: Horizontal surfaces are positioned behind the wing
- Foreplane: Horizontal surfaces are positioned in from of the wing

- Flying wing: There is only a single surface

4.1. Choosing the aerodynamic scheme

When designing a new aircraft there has always been the problem for designing its general scheme. The general scheme is something more than the typical aerodynamic scheme and depends on the applications for which the machine will be used for. Once we know the tasks that the aircraft will perform, we can determine the components of the general aircraft scheme as:

- Positioning the crew and the load
- Choosing the take off and landing mechanization and the aerodynamic scheme
- Choosing the powerplant scheme, such as number, type of the engines and their position on the aircraft.
- Aircraft sections and armoury

We can organize all elements from the general scheme in a table for simplicity. We can use the values in that table later on, when determining the dimensions of each component in the aerodynamic scheme. The table is called features matrix and is provided on *Table 5*.

Aerodynamic scheme	standard
Wing position	low wing
Wing shape	trapezoidal
Wing sweep	standard
Control surfaces	T-shaped
Cabin shape	circular
Lending gear	front wheel
Engine type	turbofan
Number of engines	2
Engines position	tail

Table 2: Features Matrix

4.2. Wing airfoil and geometry

Based on the loading and traction coefficients, we can estimate the wing area (S) and re-estimate the required thrust produced by the engines:

$$S = \frac{0.1 m_0^l g}{P_0}$$

$$T_0 = 0.1 P_0^* m_o^l g$$

With the date from the previous section, we can find the wing span (l):

- S – wing area, m^2

- λ - aspect ratio (from the statistics and the scheme)

$$l = \sqrt{S\lambda} \tag{17}$$

Then based on the statistical analysis we can now choose the inner and outer chords of the whing and find the taper ratio.

$$\eta = \frac{b_0}{b_r}, \tag{18}$$

where η - the taper ration, b_0 – inner chord, b_r – outer chord.

We can now estimate the geometric and the aerodynamic chords of the wing:

$$b_m = \frac{b_0 b_r}{2} = \frac{S}{l}, \text{ where } b_m \text{ is the median geometric chord} \tag{19}$$

$$b_A = \frac{2}{3} b_0 \frac{1+\eta+\eta^2}{\eta(1+\eta)}, \text{ where } b_A \text{ is the median aerodynamic chord} \tag{20}$$

Once we choose the front wing sweep X_{fe}, we can now estimate:

$$X_{re} = arctg\left[\frac{2}{l}(b_r - b_0 + \frac{l}{2}tgX_{fe}\right], \text{ rear sweep} \tag{21}$$

$$X_{0.25} = arctg\left[\frac{2}{l}(\frac{b_k - b_0}{4} + \frac{l}{2}tgX_{fe}\right], \text{ crank sweep at 0.25 from the inner chord} \tag{22}$$

We can finally choose dihedral angle (Ψ)

No	Variable	Coefficient	Value
1	Wing area	S, m^2	29.9187617
2	Take off thrust	T_{0}, daN	5164.965
3	Aspect ratio	λ	8.5
4	Wing span	L, m	15.9470836
5	Taper Ratio	η	3
6	Inner chord	b_{0}, m	2.81419122
7	Outer chord	B_{r}, m	0.93806374
8	Median geometric chord	B_{m}, m	1.87612748
9	Median aerodynamic chord	b_{A}, m	2.03247143

10	Sweep angle	X_{fe}, deg	10
11	Rear sweep	X_{re},deg	21
12	Crank sweep	$X_{0.25}$, deg	28
13	Dihedral angle	Ψ, deg	5
	New Engine Make Thrust/ Relative Thrust	Honeywell F124-GA-100 5220daN / 0.36 kg/daN	

4.3. Horizontal surface geometry

The following parameters related to the geometry of the horizontal pane have to be chosen:

- A_{hs} = [0.8…1.1] – static moment
- L_{hs}/b_A = [2.0…3.0], L_{hs} – distance from the center of gravity to the aerodynamic center of the horizontal surface
- λ_{hs} = [3.4…4.5] – HS aspect ratio
- η_{hs} = [2.0…3.5] – HS taper ratio
- X_{hs} – HS sweep angle
- Ψ_{hs} – HS dihedral angle

We can find A_{hs} with the following equation:

$$A_{hs} = \frac{S_{hs}L_{hs}}{Sb_A} \tag{23}$$

The rest of the parameters for the horizontal surface (L_{hs}, S_{hs}, l_{hs}, b_{0hs}, b_{rhs}, $b_{Ahs,}$ X_{rehs}) are estimated with the equations from the wing geometry section (17) – (22).

No	Variable	Coefficient	Value
1	Static moment	A_{hs}	0.91
2	Distance from center of gravity to the aerodynamic center	L_{hs}/b_{Ahs}	2.2
3	HS span	l_{hs}	7.21
4	HS aspect ratio	λ_{hs}	4
5	HS taper ratio	η_{hs}	3
6	HS sweep angle	X_{hs}	35.1
7	HS dihedral angle	ψ_{hs}	0
8	HS area	S_{hs}	13
9	Inner chord	b_{0hs}	2.70
10	Outer chord	b_{rhs}	0.90
11	Median aerodynamic chord	b_{Ahs}	1.95
12	Rear sweep	X_{rehs}	6

4.4. Vertical surface geometry

The following parameters related to the geometry of the vertical pane have to be chosen:

- A_{vs} = [0.05…0.08] – static moment
- L_{vs}/b_A = [2.0…3.0], L_{vs} – distance from the center of gravity to the aerodynamic center of the vertical surface
- λ_{vs} = [0.8…1.2] – VS aspect ratio
- η_{vs} = [2.0…3.5] – VS taper ratio
- X_{vs} – VS sweep angle

We can find A_{vs} with the following equation:

$$A_{vs} = \frac{S_{vs}L_{vs}}{Sl} \tag{24}$$

The rest of the parameters for the vertical surface (L_{vs}, S_{vs}, l_{vs}, b_{0vs}, b_{kvs}, b_{avs}, X_{revs}) are estimated with the equations from the wing geometry section (17) – (22).

No	Variable	Coefficient	Value
1	Static moment	A_{vs}	0.075
2	Distance from center of gravity to the aerodynamic center	L_{vs}/b_A	2
3	VS span	l_{vs}	2.549
4	VS aspect ratio	λ_{vs}	1
5	VS taper ratio	η_{vs}	3
6	VS sweep angle	X_{vs}	39
7	VS dihedral angle	ψ	0
8	VS area	S_{vs}	6.5
9	Inner chord	b_{0vs}	3.824
10	Outer chord	b_{rvs}	1.274
11	Median aerodynamyc chord	b_{Avs}	2.761
12	Rear sweep	X_{revs}	45

4.5. Elevator and rudder design

The parameters typically used for the design of the control surfaces are:

- Elevator area S_{el}, m^2 – S_{el}/S_{hs} = [0.3…0.4]
- Rudder area S_{rd}, m^2 – S_{rd}/S_{vs} = [0.35..0.45]
- Aileron area S_{ai}, m^2 – S_{ai}/S = [0.05…0.07]
- Aileron span l_{ai}, m – l_{ai}/l = [0.3…0.4]
- Aileron chord b_{ai}, m b_{ai}/b = [0.20..0.25]

No	Variable	Coefficient	Value
1	Elevator Area	S_{el}	4.55
2	Rudder Area	S_{rd}	2.6
3	Aileron area	S_{ai}	1.79
4	Aileron span	l_{ai}	5.58
5	Aileron chord	b_{ai}	0.467

5. Cabin design

5.1. Estimating the fuselage width

The width of the fuselage can be estimated based on the design of the passenger cabin. In this case the width of the body shall be about 635..650 mm above the cabin floor and can be estimated using the following equation:

$$B_m = B_2 n_2 + B_3 n_3 + c_n n_n + 2\delta_1 + 2\delta_2, \tag{25}$$

where:

B_2 and n_2, B_3 and n_3 – width and quantity of the double and triple blocks of seats respectively. They are shown on *Figure 3.*

c_n and n_n – width of the cabin corridors (*c_n >509mm*)

δ_1 = 30…50mm – distance between the seats and the inner wall of the cabin

δ_2 = 120…130mm – cabin wall thickness

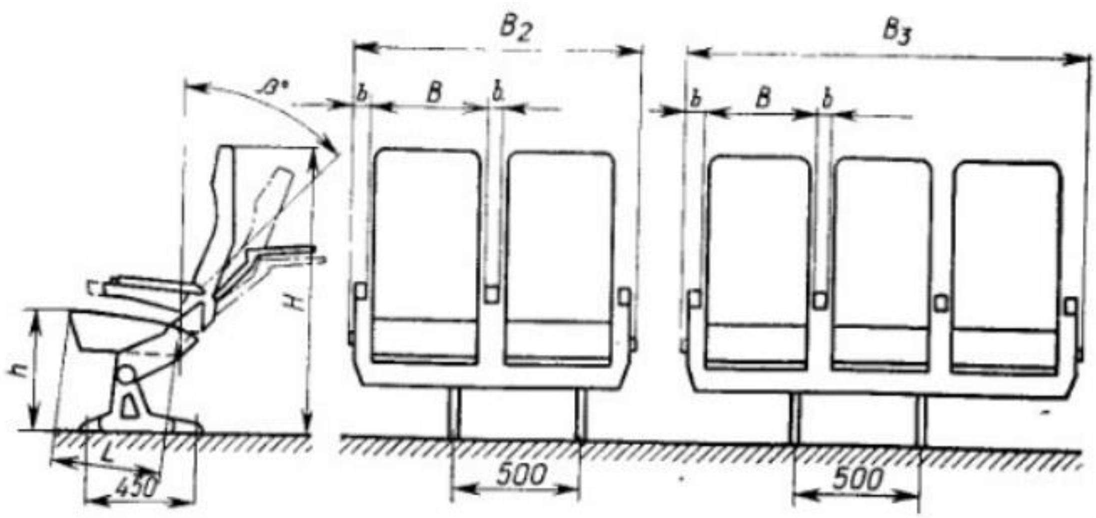


Figure 1: Transport aircraft seat dimensions

We can use *Table 6* to determine each of the cabin dimensions.

Class	Forearm distance B, mm	Forearm size b, mm	Seat length L, mm	Seat height, h	Seat and backrest height H, mm	Backrest shift angle, deg	Seat block width	
							B2	B3
I	500	70	500	445	1140	45	1260	-
II	440	50	470	445	1120	36	1030	1520

Figure 2: Transport aircraft cabin dimensions

5.2. Estimating cabin length

The cabin length can be determined with the following equation:

$$L_c = l_1 + (i_r - 1)t + l_2, \tag{26}$$

where:

l_1 – minimum distance between the front compartment to the front row *(Figure 4).* Ist class = 630mm, IInd class = 615mm, IIIrd class = 585mm;

l_2 – minimum distance between the rear compartment to the back row *(Figure 4).* Ist class = 1000mm, IInd class = 800mm, IIIrd class = 750mm;

i_r – number of rows;

t – distance between two seat blocks. Ist class = 980…1080mm, IInd class = 840…870mm, IIIrd class = 780…810mm;

5.3. Estimating cabin height

The height of the cabin H_{cab} shall be between 1900mm and 2500mm. The ratio between the cabin width and cabin length is represented by the coefficient k_φ = 0.2…0.5.

The relative volume in the cabin for each passenger shall be 0.84...1.2 m_3/passenger

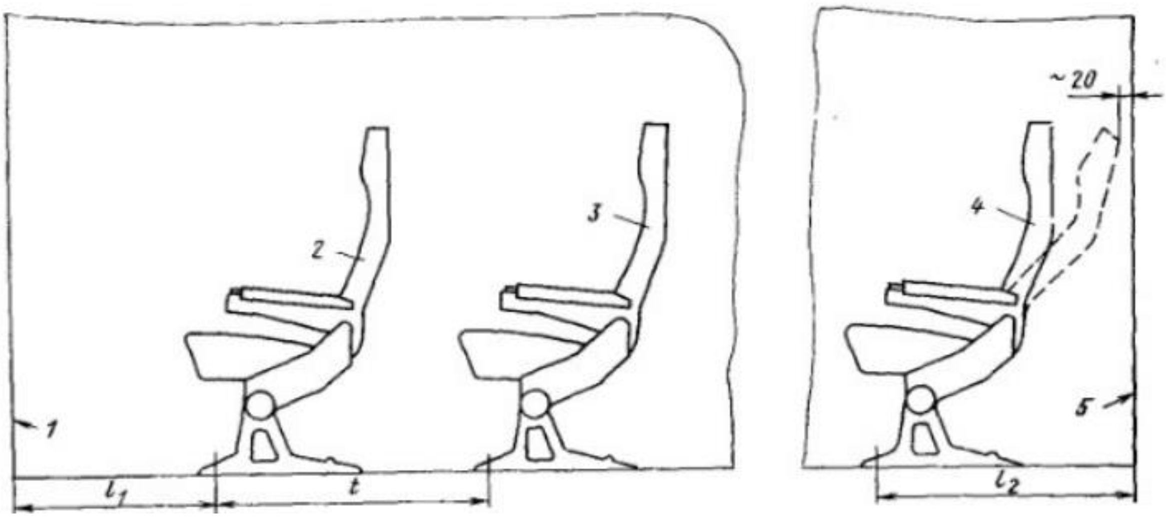


Figure 3: Scheme for mounting the passenger seats in the aircraft

5.4. Estimating the body length

The length of the aircraft body is estimated by choosing the extensions of the front and rear compartments of the body with the equation:

$$L_m = L_c + \lambda_f D_m + \lambda_b D_m, \quad (27)$$

where:

$\lambda_f = 1.2\ldots2$ and $\lambda_b = 2\ldots3$ – extensions of the front and rear compartment respectively.

D_μ – diameter of the median section estimated with $D_\mu = \sqrt{4S_\mu/\pi}$, where S_μ is section area.

No	Variable	Coefficient	Value, mm
	Fuselage Width	**B_m**	**2070**
1	Width of the double seat blocks	B_2	1260
2	No of double seat blocks per row	n_2	1
3	Width of the cabin corridors	c_n	510
4	Number of corridors	n_n	1
5	Seat to inner wall distance	δ_1	30
6	Cabin wall thickness	δ_2	120
	Cabin Length	**L_c**	**6580**
1	Front compartment to row distance	l_1	630
2	Back compartment to row distance	l_2	1000
3	number of rows	i_r	6
4	Seat blocks distance	t	990
	Cabin Height	**H_{cab}**	**2100**
	Body Length	**L_m**	**14980**
1	Front extension	λ_f	1.5

2	Rear extension	λ_b	2.5
3	Median section diameter	D_μ	2100

6. Re-estimation of aircraft masses

6.1. Estimating the masses of aircraft construction

6.1.1. Estimating final relative wing mass

$$m_w^* = 1{,}15.10^{-4}.K_{mec}K_{con}K_{mat}\varphi\frac{\lambda n_e\sqrt{S}}{\sqrt{\theta c_0}cos^{1,5}X}\frac{\eta+4}{\eta+1}\left(1-\frac{\mu-1}{\eta+3}\right), \qquad (29)$$

where:

- K_{con} – Coefficient related to the type of construction. K_{con} = 0.9 for wings with honey comb structure. K_{con} = 0.95 for riveted + glued wing. K_{con} = 1 for riveted wing.
- K_{mec} – Mechanization coefficient. K_{mec} = 0.9 for wing without mechanization. K_{mec} = 1 for wing with flaps. K_{mec} = 1.15 for full mechanization.
- K_{mat} – Wing material coefficient. K_{mat} = 1 for material D16, K_{mat} = 1.2 for material AMG6.
- Θ = 0.9 – coefficient determining the construction strength
- λ – Aspect ration
- η – Taper ration
- c_0 – Core wing thickness
- c_t – Tip wing thickness
- $\mu = c_0/c_k$ – thickness ratio
- φ = 0.68 – Unloading coefficient
- n_e = 5.5…6 – Overloading

6.1.2. Estimating final relative fuselage mass

$$m_m^* = 1{,}14K_{ep}(1-0{,}4p_{cab}^e)l_m^{1,5}m_{0,}^{-0.75}, \qquad (30)$$

where:

- K_{ep} =1.14 for engines mounted inside the fuselage. K_{ep} = 1 for engines mounted outside the fuselage.
- l_m - body length, m
- m_0 – take off mass, kg
- p_{cab} cabin pressure, atm

6.1.3. Estimating the final control surfaces mass

$$m^*_{tail} = \frac{K_v K_m}{m_0}(4.4 + 0.8 * 10^{-3} m_0) S_{tail}, \quad (31)$$

where:

- K_v - velocity coefficient, $Kv = 0.643 + 1{,}02.10^{-3}V_{cr}$
- K_m – 1 for low manoeuvrability aircrafts, K_m = 1.5 for hight manoeuvrability aircrafts
- V_{cr} – cruse speed, km/h
- $S_{tail} = S_{hs} + S_{vs}$, m^2

6.1.4. Estimating the final landing gear mass

$$m^*_{ld} = K_{mt} K_{fl} (6.H_{hl} + 11.3) 10^{-3} + 0{,}0625 K_{tr} \frac{\sqrt{p_{tr}}}{1 + p_{tr}} + 0.005\,, \quad (32)$$

- K_{mt} = 0.65..0.7 – landing gear material coefficient
- K_{fl} – aerodynamic coefficient of the landing gear. K_{fl} = 1.2 aerodynamic shape, K_{fl} = 1 for unshaped landing gears
- K_{tr} = 1 for cambered tires, K_{tr} = 0.93 for uncambered tires
- H_{lh} – landing gear height, m
- P_{tr} – pressure in the tires, atm

6.1.5. Estimating the final landing gear mass

$$m^*_c = m^*_w m^*_{fs} m^*_{tl} m^*_{ld}, \quad (33)$$

6.2. Powerplant, fuel and equipment final relative mass

The powerplant mass denoted by $m^*_{pp} = 0.05\ldots0.1m_0$, or between 0.5 and 1 % of the aircraft take off mass.

The equipment mass includes also the aircraft loading mass and can be found with the following equation:

$$m_c^* = \frac{200}{m_0} + 0.02m_{lm}^*(1 + 0.1\frac{L}{V_{cr}}) + 0.08\,, \tag{34}$$

where:

- $m^*_{ld} = m_{ld}/m_0$ – the relative loading mass
- L – range, km
- V_{cr} – cruise velocity, km/h

Finally the fuel relative mass is simply the ratio between the actual fuel mass and the take-off mass:

$$m_{fl}^* = \frac{m_{fl}}{m_0}, \tag{35}$$

No	Variable	Coefficient	Value, mm
	Relative wing mass	**m^*_w**	**0.139**
1	Construction type coefficient	K_{con}	0.950
2	Mechanization coefficient	K_{mec}	1.150
3	Material coefficient	K_{mat}	1.000
4	Unloading coefficient	ϕ	0.684
5	Overloading	n_e	5.500
6	Wing aspect ratio	λ	8.500
7	Wing aera	S	29.919
8	Wing structure coefficient	θ	0.900
9	Core wing thickness	c_0	0.120
10	Crank sweep	$X_{0.25}$	0.489
11	Taper ratio	η	3.000
12	Core to tip thickness ratio	μ	1.091
	Fuselage relative mass	**m^*_m**	**0.041**
1	Powerplant position coefficient	K_{ep}	1.140
2	Cabin pressure coefficient	p_{cab}	0.800
3	Body length	l_m	15.000
4	Initial aircraft mass	m_0	13500.000
	Control surfaces relative mass	**m^*_{tail}**	**0.032**
1	Velocity coefficient	K_v	1.439
2	Maneuvering coefficient	K_m	1.000
3	Tail area	S_{tail}	19.500
	Landing gear relative mass	**m^*_{ld}**	**0.039**
1	Landing gear material coefficient	K_{mt}	0.700
2	Landing gear aerodynamic coefficient	K_{fl}	1.200
3	Landing gear height	H_{lh}	1.500
4	Tire clatch	K_{tr}	1.000
5	Tire pressure	p_{tr}	12.000
	Construction relative mass	**m^*_c**	**0.250**

	Powerplant relative mass	**m*pp**	**0.077**
	Aircraft Equipment relative mass	**m*eq**	0.098
1	Aircraft loading relative mass	m*lm	0.118
2	Range	L	3000.000
3	Cruise velocity	Vcr	780.000
	Relative fuel mass	**m*fl**	**0.315**

7. Aircraft mass balance characteristics

The balancing characteristics are important for the relative positioning of the wing, tail, fuselage, passengers, engines etc.

One of the most important tasks of the engineers in determining the center of masses of the aircraft. The relative position of the center of masses and the median aerodynamic chord is of significant importance for its dynamic and balancing characteristics. The aircraft changes its mass during flight is this relative distance has to ensure that when the center of masses is moving back (tailwise), there is enough longitudinal stability. Also, in case the center of masses moves forward, there is enough lever displacement to balance the aircraft for take-off and landing.

Here we are going to center the horizontal axis of the aircraft as since it is symmetric, the vertical axis moments can be neglected. The origin of the coordinate system can be either taken at the nose of the aircraft or at the tip of the coot wing chord.

We first need to estimate the sum of static horizontal moments with the equation:

$$x_m = \frac{\sum(mgx)i}{\sum(mg)i}, \tag{36}$$

where:

- x is the distance of the element from the chosen coordinate system origin.
- m is the mass of the element

Then we can estimate the balancing characteristics with the equation:

$$x_m^* = \frac{x_m - x_A}{b_A}, \tag{37}$$

where:

- x_m – coordinates of the center of masses
- x^*_m longitudinal balancing
- x_A – coordinate of the tip of the median aerodynamic chord.

The coordinates of the elements are taken from a simplified aircraft blueprint, showing their relative positions.

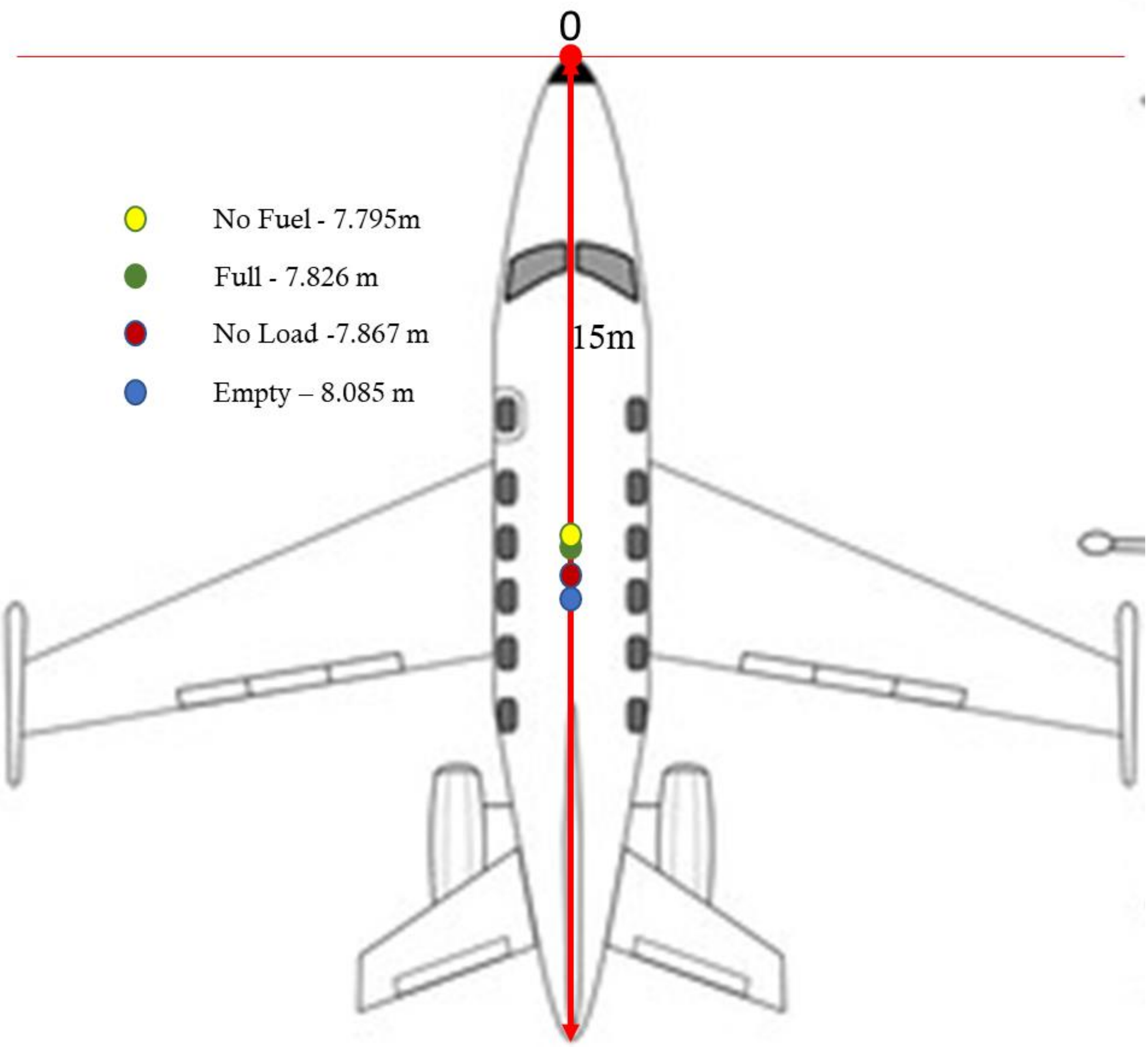
The balancing characteristics of the aircraft have to be estimated in all of those cases:

- Maximum take-off mass
- Fully equipped aircraft without fuel.
- Full fuel tank without any load.
- Empty aircraft without fuel and load.
-

No	Component	Relative mass		Actual mass, kg	
		m*	Value	m	Value
1	Wing	m^*_w	0.139	m_w	1875.56
2	Fuselage	m^*_m	0.041	m_m	553.40
3	Landing gear	m^*_{ld}	0.039	m_{ld}	522.54
4	Construction	m^*_c	0.250	m_c	3377.90
5	Powerplant	m^*_{pp}	0.077	m_{pp}	1039.50
6	Equipment	m^*_{eq}	0.098	m_{eq}	1324.03
7	Aircraft loading	m^*_{lm}	0.118	m_{lm}	1590.00
8	Fuel	m^*_{fl}	0.315	m_{fl}	4246.00
9	Control surfaces	m_{tail}	0.032	m_{tail}	426.40

The moving masses such a passengers and cabin crew have to be positioned near the center of masses of the aircraft.

Aircraft Balancing			
Component	**Moment**	**Distance**	**Force**
	mgx, daNm	**x, m**	**mg, daN**
1.1 Wing	13799.4621	7.5	1839.928281
1.2 Fuselage	4071.63854	7.5	542.8851391
1.3 Horizontal surface	2342.47675	14	167.3197681
1.4 Vertical surface	3388.2253	13.5	250.9796521
1.5 Front landing gear	322.942626	2.1	153.782203
1.6 Main landing gear	2583.54101	7.2	358.8251404
2. Powerplant	9942.55763	9.75	1019.7495
3. Equipment	9871.4438	7.6	1298.874185
3.1 Front compartment	1818.42386	2.8	649.4370923
3.2 Back compartment	7923.13253	12.2	649.4370923
4. Cabin crew	768.123	2.9	264.87
4.1 Pilots		2.4	
4.2 Flight attendants		7.5	
4.3 Crew luggage		2.5	
5. Fuel	32906.1093	7.9	4165.330297
6. Load	11698.425	7.5	1559.79
7. Passengers	7063.2	7.5	941.76
Σ(mgi x)	**108499.701**	Σ(mg)i	**13862.96835**
Σ(mgi x) No Load	96801.2765	Σ(mg)i	12303.17835
Σ(mgi x) No Fuel	75593.5922	Σ(mg)i	9697.638053
Σ(mgi x) Empty	56063.8442	Σ(mg)i	6931.218053



Standard positions of the aircraft components are:

- Wing center of mass position – 40…42% b_A
- Tail center of mass position – 45…50% b_{Ahot}
- Fuselage center of mass position – 50% of fuselage length
- Re-estimating passenger and cabin crew mass: 80kg for a pilot, 70kg for cabin crew, 75kg for passenger and 15kg for luggage per passenger.

The balance characteristic of the aircraft in each case shall be between x^*_m = 0.2…0.3.

		Full	No Load	No Fuel	Empty
Coordinates of the mass center	$Xm=\sum_{k=0}^{n}\frac{(mgx)i}{(mg)i}$	7.99919287	7.93570082	7.955894181	8.088599108
Final aircraft centering coefficient		**0.24590782**	**0.21463095**	**0.224578414**	**0.289950299**
Median aerodynamic chord		2.03			
Median aerodynamic chord coordinate		7.5			