Homework Assignment 5

Due Date: March 25, 2022, 23:59

EXERCISE 1. For the KMP algorithm design an algorithm to compute the sequence *NEXT* in time in $O(m)$, where m is the length of the pattern sequence.

total points: 10

EXERCISE 2. Modify the KMP algorithm such that it will find all occurrences of a pattern sequence P in target sequence S in time in $O(n)$, where n is the length of S.

total points: 8

EXERCISE 3.

(i) Represent a polynomial $P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_dx^d$ of degree d by an array P of length $d+1$ containing the coefficient $a_0, \ldots, a_d$. If $a_d = 1$, such a polynomial is called *monic*.

Assume you are given already an algorithm to multiply a polynomial of degree i with a polynomial of degree 1 in time in $O(i)$. Assume further to be given another algorithm to multiply two polynomials of degree i in time in $O(i \log i)$.

Let $n_1, \ldots, n_d$ be integers. Design an efficient divide-and-conquer algorithm to find the unique monic polynomial $P(x)$ of degree d such that $P(n_1) = P(n_2) = \cdots = P(n_d) = 0$ holds, and analyse the complexity of your algorithm.

(ii) Let $x_1, \ldots, x_n$ be pairwise distinct values. Design an efficient algorithm to compute the coefficients of the unique monic polynomial $P(x)$ of degree n such that $P(x_1) = P(x_2) = \cdots = P(x_n) = 0$ holds. The algorithm is to require time in $O(n \log^2 n)$, provided that all necessary operations are elementary.

total points: 12

EXERCISE 4. Implement the KMP algorithm on arbitrary lists.

Programming instructions. Extend the unbounded array data structure for lists by a member function computing the function *next*. Then add a member function *kmp*, which takes a pattern list as input and decides whether this pattern is a sublist of the given list.

Testing instructions. Test your implementation with several pattern strings P, represented as lists with elements of type **char**. For each pattern string explore several target strings S. You must cover cases, where P is a substring of S and where this is not the case.

total points: 20