CS 225: Data Structures
Homework 5

Group D1

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Please turn over for our answers.

Ex. 1 **OUR ANSWER.** (This part is written by Zhong Tiantian)

We assume that the list `next[]`, the so-called "prefix table", satisfy the following properties:

(a) `next[0]` $= 0$, since the first element has no prefixes.

(b) `next[`i`]` $= t$ indicates that the i-th element in `pList[]` is equal to the t-nd one.

Assuming that we have known `next[0]`, `next[1]`, ..., `next[`$x-1$`]`, and denote `next[`$x-1$`]` as ptr:

(a) If `pList[`x`]` $=$ `pList[`ptr`]`, which indicates the ptr-th element in `pList[]` is equal to the x-th, then extend the current sublist: `next[`x`]:=next[`$x-1$`]`.

(b) If `pList[`x`]` $\neq$ `pList[`ptr`]`, then go backward by reducing ptr to `next[`$ptr-1$`]`.

The algorithm is defined below.

Algorithm 1: Build `Next[]`

```
1  next.append(0)
2  x ← 1
3  ptr ← 0
4  while x < getLength(pList) do
5      if pList[ptr] = pList[x] then
6          ptr ← ptr + 1
7          x ← x + 1
8          call next.append(ptr)
9      end
10     else if ptr ≠ 0 then
11         ptr ← next(ptr − 1)
12     end
13     else
14         call next.append(0)
15         x ← x + 1
16     end
17 end
```

Time Complexity Analysis In most cases, the algorithm iterate one by one with $x \leftarrow x+1$, so it will perform `Length(pList)`$= m$ times; in relatively rare cases, where ptr goes backward,

the extra time spent can be amortised to the general cases. Thus the amortised time complexity is $O(\texttt{length(pList)}) = O(m)$.

Ex. 2 **OUR ANSWER.** (This part is written by Jiang Wenhan)

Algorithm Description Assume the length of pattern sequence is m.

- Calculate the next sequence of the pattern sequence, which takes time $O(m)$, $m < n$ and align $P[0]$ and $S[0]$.
- Begin comparison between $P[0]$ and $S[0]$, if $P[0] \neq S[0]$.
- Turn back to $S[1]$, and keep doing things above until we found $P[0] = S[k], k \leq n$,

  Once we found such k,

  (a) Align $P[0]$ with $S[k]$ and keep comparing them until we found $P[a] \neq S[k+a]$.

  (b) Right shift the pattern sequence for $m - next[a]$ where front maximal common sublist overlap with the original position of back maximal common sublist.

  (c) Repeat the comparison at this new position (begin with $P[next[a]]$ and $S[k+a]$).

  Whenever we found that $a = m$,

  (a) Record the position k , where the pattern sequence has occur.

  (b) Right shift the pattern sequence by $m - next[m]$ and start new comparison at $P[next[m]]$ and $S[k+m]$.

  (c) Repeat doing this until we reach the end, where the $P[m-1]$ is aligned with $S[n-1]$.

Time Complexity Analysis To calculates its time complexity, we make an assumption of the worst case, where the pattern sequence and target sequence both consists of pairwise different elements. In such case, all elements in *next* sequence are all 0 . The times of shift is $\frac{n}{m}$, and the comparisons for each shift is m , so the time complexity for comparison is $m \times \frac{n}{m}$, which is n. The total time complexity is $O(m)+n$, since m is less equal than n, so the total time complexity is $O(n)$.

Ex. 3 **OUR ANSWER.** (This part is written by Zhou Ruidi)

(i) **Method 1** (Might not be correct; see **Method 2** for a better solution)

Here is the algorithm.

Algorithm 2: Multiplying two polynomials.

```
  /* From n_1 to n_d we choose one by one. */
  /* Denote the length of the list as d. */
1 forall n_i ∈ list and 1 ≤ i ≤ d do calculate each number from n_1 to n_d
2   for j = 2,3,··· ,d do
        /* calculate P(n_i) */
3       for m = 2,3,··· , j do
            /* figure out each element in the polynomial */
4           n_i ← n_i · n_i
            /* record this number, k iterates from 2 to d, and we know that
               a_d = 1 */
5           P_k ← a_k · n_i
6       end
7   end
8 end
```

Calculate $P(n_i)$ by adding up the $P_k + a_0 + a_1 \cdot n_i$, and finally each $P(n_i) = 0$. After that we can calculate the unknown coefficients $a_0, a_1, \cdots, a_{d-1}$. And the time complexity is $O(d \cdot d \cdot (d+1)/2) \approx O(d^3)$.

Method 2

For monic polynomial, it has the form like: $P(x) = (x-n_1) \cdot (x-n_2) \cdots (x-n_d)$ (because $a_d = 1$) .

We pair two elements to make one pair and we will get $d/2$ pairs: $(x-n_1) \cdot (x-n_2), (x-n_3) \cdot (x-n_4)$... and the time complexity for calculate each pair is $O(1)$ from the function before we know (Multiplying polynomials of degree i and degree 1 has time complexity $O(i)$).

Then we pair these $d/2$ polynomials to get $d/4$ polynomials and for each polynomial we spend $2\log 2$ using the function above (Degree-i polynomial multiplying the degree-i need

$O(i\log i)$. So for this step we need $(d/4)\cdot(2\log 2)$

Repeat the above steps until getting the answer by multiplying two polynomials or three polynomials. The total time we need is

$$\begin{aligned}\sum_i \frac{d}{2^i}\cdot 2^{i-1}\cdot\log 2^{i-1} &= \frac{d}{2}(1+\log 2+\log 4+...)\\ &= \frac{d}{2}(\log d)^2\\ &\approx O(d\log^2 d)\end{aligned}$$

(ii) The algorithm we designed in the previous part has satisfied the requirement of time complexity $O(n\log^2 n)$.

Now let's prove the functions mentioned above are elementary.

Obviously, the time complexity of multiplying a degree-1 polynomial with a degree-i polynomial is $O(i)$.

Let's consider multiplying two degree-i polynomials:

(1) Suppose $P(x)=[x_0,x_1,x_2,\cdots,x_i]$, $P'(x)=\left[x'_0,x'_1,x'_2,\cdots,x'_i\right]$ (assuming i is the power of 2; yet things will remain similar if i is not).

(2) Perform $FFT(P(f_x)), FFT(P(f_y))$, where

$$P(f_x)=[x_0,x_1,x_2,\cdots,x_i,0,0,0,\cdots]$$

and

$$P'(f_x)=[x'_0,x'_1,x'_2,\cdots,x'_i,0,0,0,\cdots]$$

(NOTE: The numbers of ending 0-s in the above lists are i, respectively).

(3) Denote

$$W=e^{2\cdot\pi m/2n}=e^{\pi m/n}$$

and thus we get

$$L_1\equiv\left[P(W^0),\cdots,P(W^{2i-1})\right]$$

and

$$L_2\equiv\left[P'(W^0)\cdots P'(W^{2i-1})\right]$$

.

(4) Let $L_3\equiv L_1\cdot_2\left[P(W^0)\cdot P'(W^0)\cdots P(W^{2i-1})P'(W^{2i-1})\right]$.

(5) Perform $IFFT(L_3)$, which takes $O(i\log i)$.

(6) The time complexity: $O(2\cdot 2i\cdot\log 2i+2i+2i\cdot\log 2i)=O(i\log i)$.

Therefore, the operations are all elementary.