

# Lab #2 Report

## DIGITAL SIMULATION

Tiantian Zhong  
3200110643  
Section LA1

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**Total:** \_\_\_\_/50

### Prelab Exercises

We have

$$H(s) = \frac{Y(s)}{U(s)} = \frac{25}{s^2 + 6s + 25} = \frac{25}{(s + 3)^2 + 16}.$$

Thus

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 6s + 25$$

and we yields

$$\begin{cases} \zeta = \frac{3}{5} = 0.6 \\ \omega_n = 5. \end{cases}$$

Solve  $s^2 + 6s + 25 = 0$  we have the poles

$$s_{\text{poles}} = \frac{-6 \pm \sqrt{36 - 100}}{2} = -3 \pm 4j.$$

Figure 1 shows the block diagram for this system.

### 1 STATE SPACE MODEL OF $H_1(s)$ \_\_\_\_/8

(Compare the plots of  $\dot{y}$  and  $y$  obtained in Part 1 of the lab with the plots previously made for the prelab. Why are they identical? Attach plots—if your prelab plot was wrong, fix it and attach the corrected plot.)

The plots of  $y$  and  $\dot{y}$  from the prelab and lab are shown in Figures 2 and 3.

### 2 EFFECTS OF AN EXTRA ZERO \_\_\_\_/22

The step responses after adding an extra zero are shown in Figure ??.

#### 2.1 Effects of a Zero on $M_p$ , $t_r$ , and $t_s$ \_\_\_\_/2

Fill the table of specs of time domain responses.

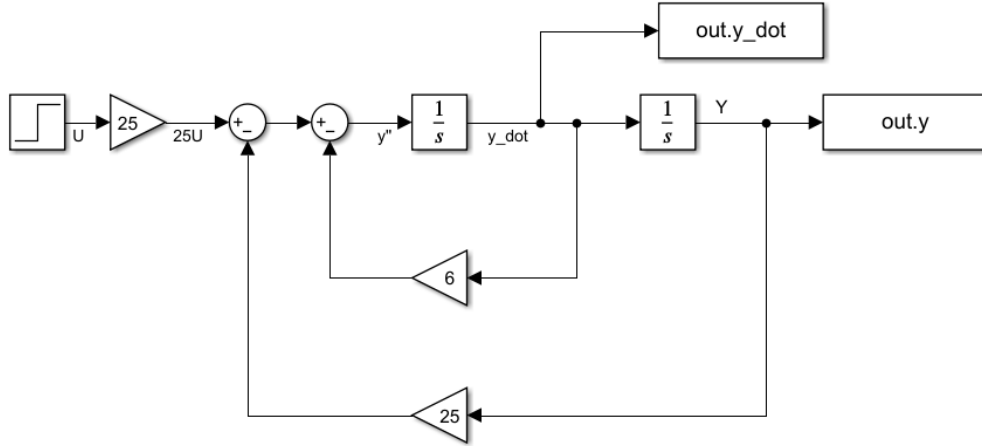


Figure 1: Block diagram of  $H(s) = \frac{25}{s^2 + 6s + 25}$

Table 1: Effects of Zero

Specs	No zero $H_1(s)$	$H_2(s)$ zero at $s = -30$	$H_2(s)$ zero at $s = -3$	$H_2(s)$ zero at $s = -1.5$	$H_2(s)$ zero at $s = 1.8$	$H_2(s)$ zero at $s = 18$
$M_p$ [%]						
$t_r$ [s]						
$t_s$ [s]						

## 2.2 Discuss the Effects of a LHP Zero

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(Explain how  $M_p$ ,  $t_r$ , and  $t_s$  are affected by the zero location. When can the zero be ignored?)

## 2.3 Effects of a Non-minimum Phase Zero

\_\_\_/2

(What is unique in this situation?)

## 2.4 Decomposition of $H_2(s)$

\_\_\_/14

(Take  $H_2(s)$ , set  $\zeta$  to the value found in the prelab, separate the numerator into two terms so that  $H_2(s)$  is a sum of 2 fractions. Discuss how this decomposition helps to explain the effect of the zero location. In particular, discuss what each term represents. Also discuss  $\alpha$ 's effect. Which term dominates as  $\alpha$  approaches 0? As  $\alpha$  approaches  $\infty$ ? What happens when  $\alpha$  is negative?)

$$H_2(s) = \frac{25 \left( 1 + \frac{s}{\alpha\zeta} \right)}{s^2 + 10\zeta s + 25}$$

= INSERT FRACTION 1 + INSERT FRACTION 2

## 3 EFFECTS OF AN EXTRA POLE

\_\_\_/20

The step responses after adding an extra pole are shown in Figure ??.

### 3.1 Effects of a Pole on $M_p$ , $t_r$ , and $t_s$

\_\_\_/2

(Fill out the table of specs of time domain responses.)

Table 2: Effects of Pole

Specs	No pole $H_1(s)$	$H_2(s)$ pole at $s = -30$	$H_2(s)$ pole at $s = -3$	$H_2(s)$ pole at $s = -1.5$
$M_p$ [%]				
$t_r$ [s]				
$t_s$ [s]				

### 3.2 Discuss the Effects of an Extra Pole

\_\_\_\_/4

(Explain how  $M_p$ ,  $t_r$ , and  $t_s$  are affected by the location of the additional pole. When can the extra pole be ignored?)

### 3.3 Decomposition of $H_3(s)$

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$$\begin{aligned}
 H_3(s) &= \frac{25}{\left(1 + \frac{s}{\alpha\zeta}\right)(s^2 + 10\zeta s + 25)} \quad (\zeta = 0.6) \\
 &= \frac{k_1}{1 + \frac{5s}{3\alpha}} + \frac{k_2 s}{s^2 + 6s + 25} + \frac{k_3}{s^2 + 6s + 25}
 \end{aligned}$$

Using a partial fraction expansion,

$$\begin{aligned}
 k_1 &= -\frac{25}{\alpha^2\zeta^2 - 10\alpha\zeta + 25} \\
 k_2 &= -\frac{25\alpha\zeta}{\alpha^2\zeta^2 - 10\alpha\zeta + 25} \\
 k_3 &= 25 + \frac{625}{\alpha^2\zeta^2 - 10\alpha\zeta + 25}
 \end{aligned}$$

(Discuss how this decomposition helps to explain the effect of the location of an additional pole. In particular, discuss what each term represents. Also discuss  $\alpha$ 's effect. Which term dominates as  $\alpha$  approaches 0? As  $\alpha$  approaches  $\infty$ ?)

## Attachments

- Plot from the prelab
- Plots of overlaid step responses after adding a zero; and after adding a pole
- Sample figures for calculating  $M_p$ ,  $t_s$  and  $t_r$  in Table 1 and Table 2
- MATLAB code

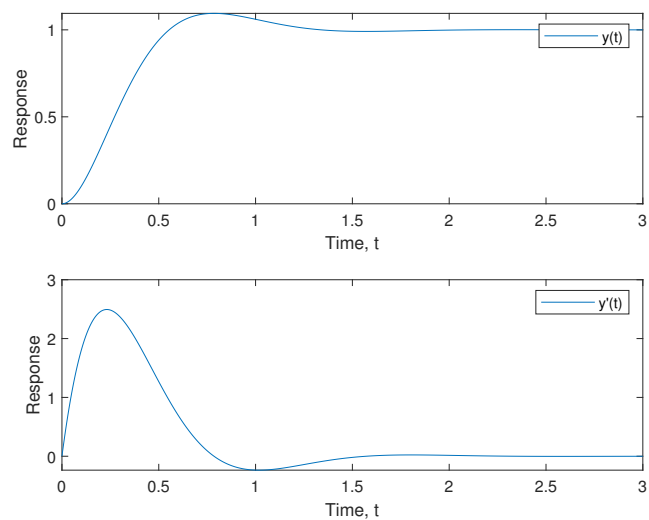


Figure 2: Plot from the prelab

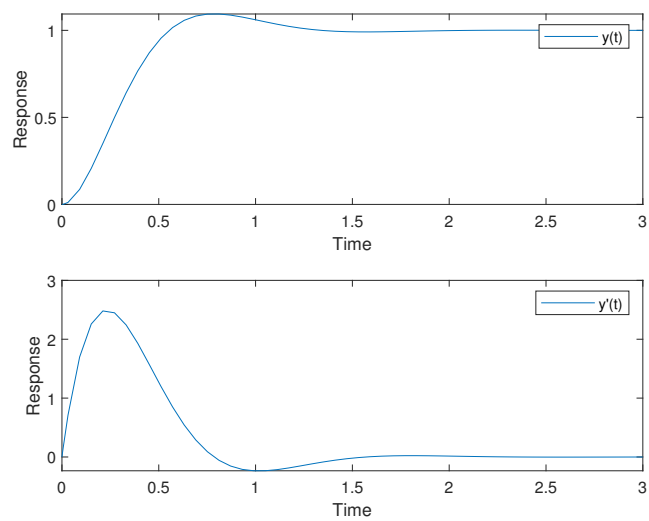


Figure 3: Plot from part 1