

# Lab Report

## Lab #1: ANALOG SIMULATION

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### 1 Prelab Exercises

#### Ex a.

From Newton's 3rd law, I derive the following equation:

$$\begin{aligned}f &= ma_{tot} + bv + kx \\a_{tot} &= \frac{d^2}{dt^2}x \\v &= \frac{d}{dt}x\end{aligned}$$

thus the ODE should look like this:

$$f(t) = m\ddot{x}(t) + b\dot{x}(t) + kx(t)$$

Substituting the values for  $m$ ,  $b$ ,  $k$ , and  $f$  I obtain

$$f(t) = 2\ddot{x}(t) + 0.7\dot{x}(t) + x(t). \quad (1)$$

#### Ex b.

Laplace Transform of 2nd-order ODE (with zero initial condition) in Equation 1:

$$f(t) = 2\ddot{x}(t) + 0.7\dot{x}(t) + x(t) \leftrightarrow F(s) = 2s^2X(s) + 0.7sX(s) + X(s)$$

and move  $s^2X$  to LHS and  $F$  to RHS:

$$s^2X = \frac{0.7}{2}sX + \frac{1}{2}X - \frac{1}{2}F \quad (2)$$

which gives the diagram in Figure 1.

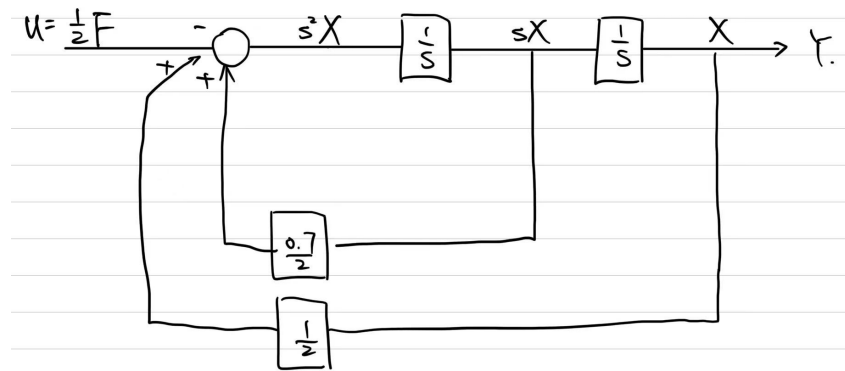


Figure 1: Block diagram for Equation 2

**Ex c.**

For Figure 2(b) in the manual I can derive a group of equations based on KVL,

$$q_C = C e_o(t) \quad (3)$$

$$e_i(t) - e_o(t) = i(t)R \quad (4)$$

$$dq_C = i(t)dt \quad (5)$$

which gives

$$e_i(t) - \frac{q_C}{C} = i(t)R$$

$$\Rightarrow e_i(t) - \frac{1}{C} \int i(t)dt - i(t)R = 0.$$

## 2 Lab Exercises

### Ex 1: Solving Differential Equations using Analog Computer

**Damp Coefficient**  $b = 0.7$

Prelab exercise provides a 2nd-order ODE

$$m\ddot{x} + b\dot{x} + kx = F$$

which, using Laplace Transform, can be represented as

$$s^2X(s) + \frac{b}{m}sX(s) + \frac{k}{m}X(s) = \frac{F}{m}.$$

For step 2, I found the I/O relationship at **adder1** can be represented as

$$Y = -\frac{R_f}{F_1}x_1 - \frac{R_f}{F_2}x_2$$

and substituting  $x_1, x_2$  with its corresponding signals  $-\dot{x}, F(t) = 0.5$  I obtain

$$\frac{F(t) - b\dot{x}}{m} = \frac{0.5 - 0.7\dot{x}}{2} = -\dot{x}\frac{10k}{28.6k} + 0.5\frac{10k}{20k}$$

which matches the original parameter in the SIMUNLINK model.

This step produces a plot of  $x(t)$  and  $x'(t)$  over time, which is shown in Figure 2 and 3.

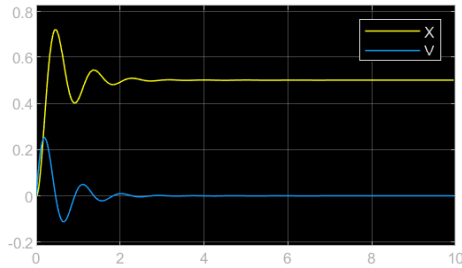


Figure 2: Function  $x(t)$  and  $v(t)$  with the change of  $t$ ; generated by SIMULINK model

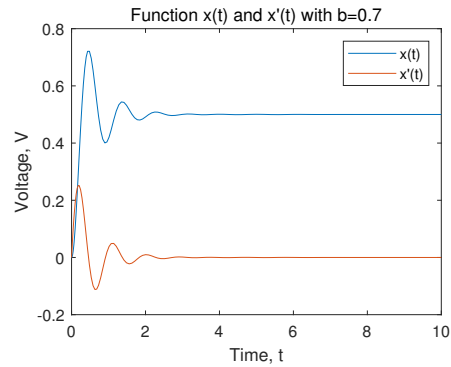


Figure 3: Function  $x(t)$  and  $v(t)$  with the change of  $t$ ; plotted by MATLAB

**Damp Coefficient**  $b = 1$

Let's focus on  $R_1$  in Figure 4.

As is discussed above, the I/O relationship of **adder1** is

$$\frac{F(t) - b\dot{x}}{m} = \frac{0.5 - \dot{x}}{2} = -\dot{x}\frac{10k}{R_1} + 0.5\frac{10k}{20k}$$

which yields

$$\frac{1}{2} = \frac{10k}{R_1}.$$

Thus here I take  $R_1 = 20k\Omega$ . The plots for function  $x_{b=1}(t)$  and  $x'_{b=1}(t)$  is shown in Figure 5 and 6.

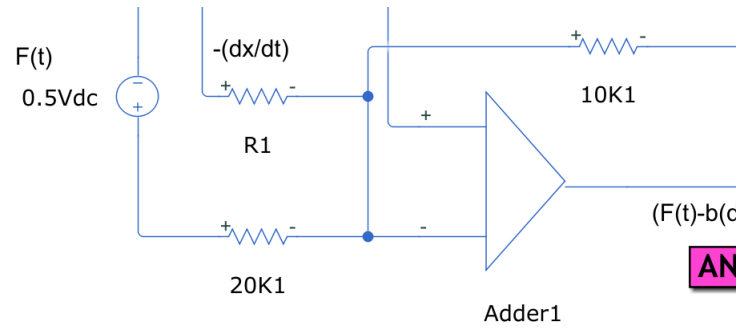
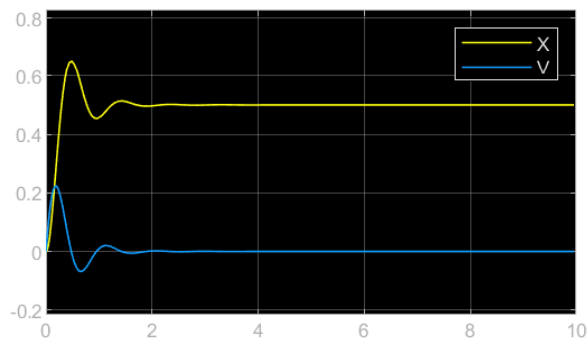
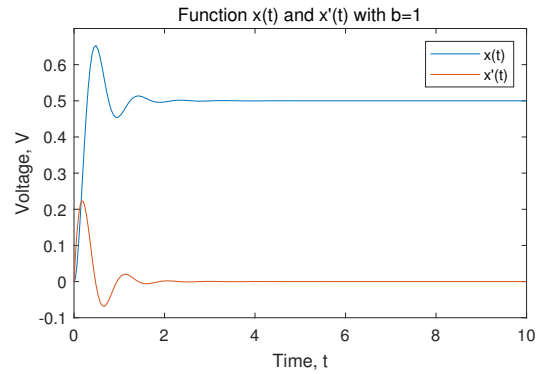


Figure 4: Block diagram near adder1

Figure 5: Function  $x_{b=1}(t)$  and  $x'_{b=1}(t)$  with the change of  $t$ ; generated by SIMULINK modelFigure 6: Function  $x(t)$  and  $v(t)$  with the change of  $t$ ; plotted by MATLAB

### Discussion about the Changes in Response

**For the changes with  $b$ ,** I derived an expression for  $R_1$  in Figure 4:

$$R_1 = 10k \frac{m}{b}$$

which indicates that I can explore the change of functions with different  $R_1$ . It is concluded that with an increasing  $R_1$ , i.e. decreasing  $b$ ,

1. the system will reach its steady state with longer time.
2. there will be a higher maximum value of system response.
3. frequency remains unchanged.

I plotted function  $x(t)$  for different  $R_1$ , which is shown in Figure 7.

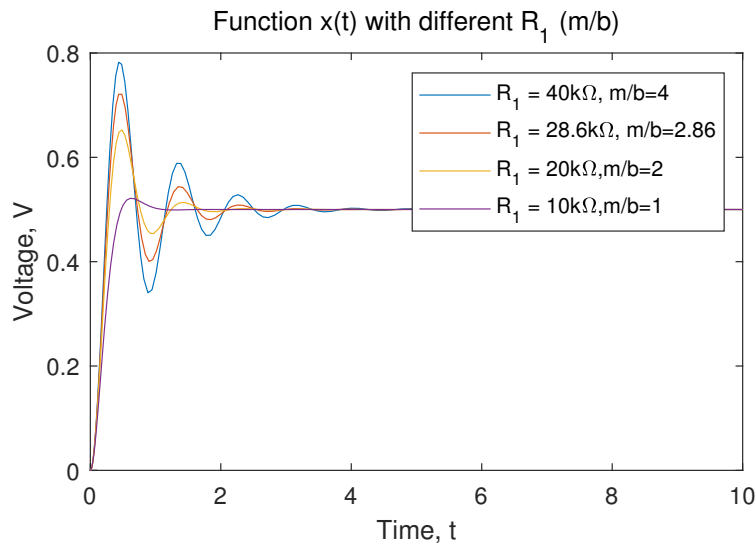
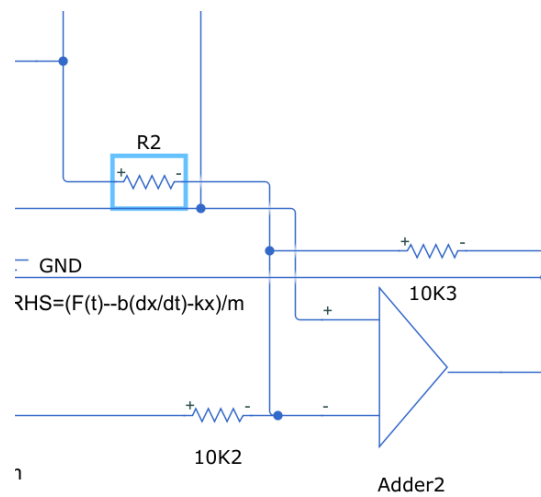
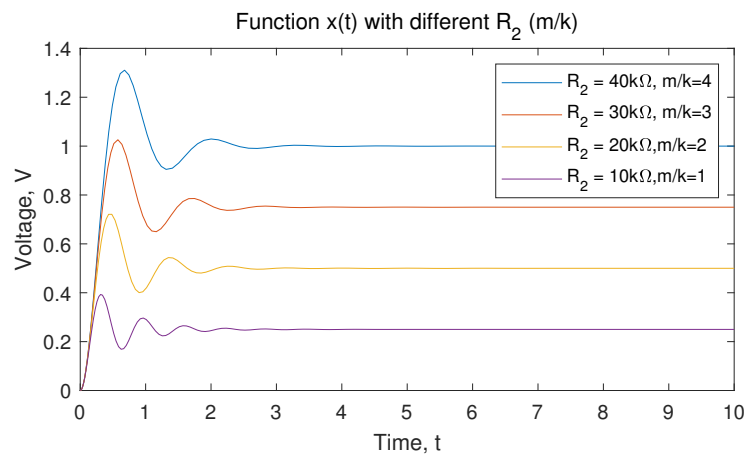
**For the changes with  $F(t)$ ,** the steady state response is exactly the value of  $F(t)$ . As  $F(t)$  changes, the response changes accordingly.

**For the changes with  $k$ ,** similar as before I derived an expression for  $R_2$ , which is

$$R_2 = 10k \frac{m}{k}.$$

I plotted  $x(t)$  with different value of  $R_2$  as below in Figure 9.

With increasing value of  $R_2$ , i.e. decreasing  $k$ ,

Figure 7: Function  $x(t)$  with different values of  $\frac{m}{b}$ Figure 8: Block diagram near **adder2**Figure 9: Function  $x(t)$  with different values of  $\frac{m}{k}$

1. The system will reach its steady state with longer time.
2. The frequency will decrease.
3. The steady state value will increase, as  $k$  changes the conditions at equilibrium.

## Ex 2: System Identification Experiment

### Hardware & Software Setup

After setting up both hardware and software I ran the simulation and obtained the two plots (shown as Figure 10 and 11), following the instructions on lab manual.

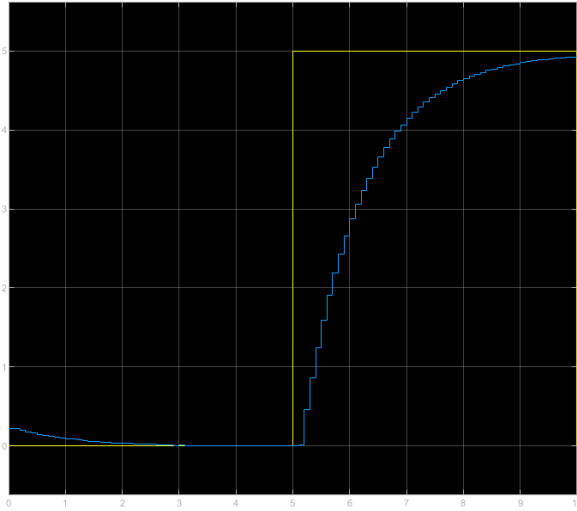


Figure 10: Voltage output of Arduino

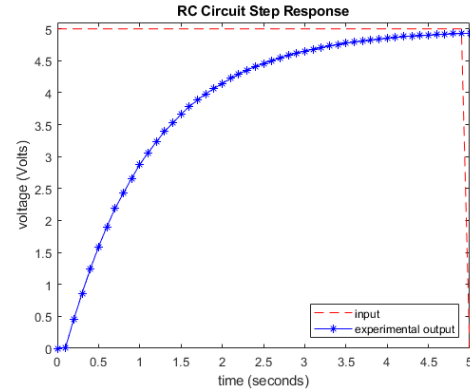


Figure 11: Voltage change within 5 seconds, generated by MATLAB

### Parameter Identification

To get the value of  $\tau$ , we have to find the time when the response reaches 63.2% of its total changes, i.e. around 3.16V.

From data gathered we find the response at 1.1s is 3.07V, and at 1.2s is 3.19V. Using linear approximation we have

$$V(t) = V(1.1) + \frac{V(1.2) - V(1.1)}{1.2 - 1.1}t = 3.07 + 1.2(t - 1.1), 1.1 \leq t \leq 1.2$$

Solving  $V(\tau) = 3.16$  we have  $\tau \approx 1.175$ s.