Lab Report
Lab #3: Digital Simulation of a Closed-Loop System

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Prelab Exercises

Ex *a*.

The block diagram is shown as Figure 1, 2, and 3.

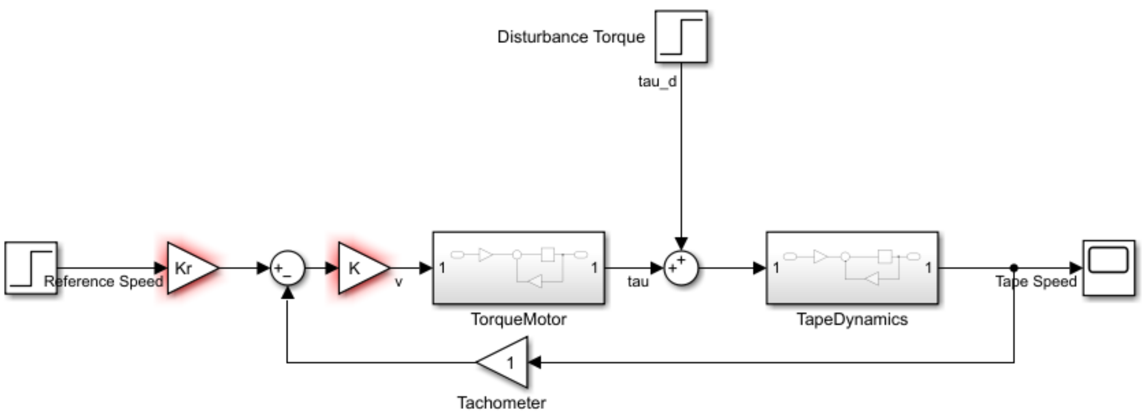

Figure 1: Block Diagram for the System

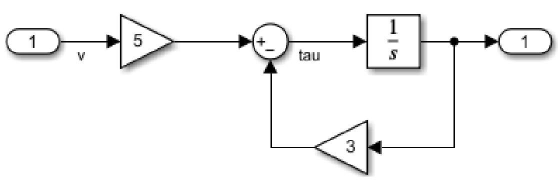

Figure 2: Block Diagram for Torque Motor Subsystem

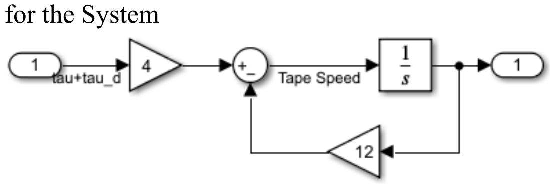

Figure 3: Block Diagram for Tape Dynamics Subsystem

Ex *b*.

The system can be represented with, in s-domain,

$$\Omega = [(\Omega_r K_r - \Omega) K H_1 + T_d] H_2 \tag{1}$$

where H_1 and H_2 are the transfer function for Torque Motor and Tape Dynamics, respectively.

Using the conditions given, we derive the transfer functions H_1 and H_2 as

$$H_1(s) = \frac{T(s)}{V(s)} = 5 \cdot \frac{3}{s(s+3)} \cdot \frac{1}{\frac{1}{s}} = \frac{15}{s+3} \tag{2}$$

$$H_2(s) = \frac{\Omega(s)}{T(s)+T_d(s)} = \frac{1}{\frac{1}{4}s+3} = \frac{4}{s+12} \tag{3}$$

As we take $\tau_d = 0$, whose Laplace transform should be $T_d = \mathscr{L}\{\tau_d\} = 0$, we will find

$$\frac{\Omega(s)}{\Omega_r(s)} = \frac{K_r\Omega_r K H_1 H_2}{KH_1H_2+1} = \boxed{\frac{60K \cdot K_r}{s^2+15s+60K+36}}(\tau_d = 0) \tag{4}$$

With $\omega_r = 0 \leftrightarrow \Omega_r = 0$, we derive the ratio of tape speed and disturbance torque as

$$\frac{\Omega(s)}{T_d(s)} = \frac{H_2}{KH_1H_2+1} = \boxed{\frac{4s+12}{s^2+15s+60K+36}}(\omega_r = 0) \tag{5}$$

Ex *c*.

Solve for poles of the system and we have

$$s_{\text{pole}} = \frac{-15 \pm \sqrt{15^2 - 4(60K+36)}}{2} = \frac{-15 \pm \sqrt{81-240K}}{2}, \tag{6}$$

all of which have to be located on OLHP. This indicates that

$$\begin{aligned} 81 - 240K &< 0 && (7)\\ \mathrm{Re}(s_{\text{poles}}) &< 0 && (8)\\ 60K+36 &> 0\\ K \cdot K_r &> 0 \end{aligned}$$

Thus $\boxed{K > \frac{81}{240} = \frac{27}{80}}$.

Ex *d*.

With Equation 5, we pull in $T_d = \frac{1}{s}$ and $\omega \le 0.01 \leftrightarrow \Omega \le \frac{0.01}{s}$ and the FVT gives

$$\begin{aligned} \Omega_{\text{ss}}(s) = \lim_{s\to 0}\Omega(s) &= \lim_{s\to 0}\frac{60K \cdot K_r}{s^2+15s+60K+36} \cdot \frac{1}{s}\\ &= \frac{60K \cdot K_r}{60K+36} \cdot \frac{1}{s} \le \frac{0.01}{s} \end{aligned} \tag{9}$$

which yields $\boxed{K \le \frac{0.03}{5K_r - 0.05}}$.

Ex *e*.

The solution for $\Omega_r = \frac{1}{s}$ gives