Lab Report
Lab #1: Analog Simulation

Tiantian Zhong
3200110643

October 7, 2022

1 Prelab Exercises

Ex *a*.

From Newton's 3rd law, I derive the following equation:

$$ma_{tot} = f + b\dot{x} + kx$$
$$a_{tot} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}x$$

thus I have the second-order ODE for the system,

$$m\ddot{x} = b\dot{x} + kx + f$$

and by simplifying the equation, I obtain

$$\ddot{x} = \frac{b}{m}\dot{x} + \frac{k}{m}x + \frac{f}{m}.$$

Substituting the values for m, b, k, and f I obtain

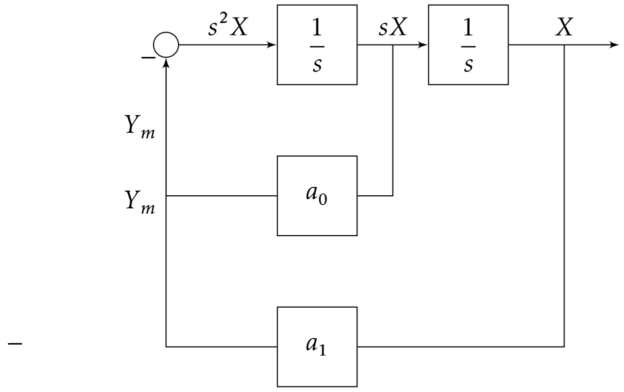
$$\begin{cases} \ddot{x} &= \frac{0.7}{2}\dot{x} + \frac{1}{2}x + \frac{0.5}{2} \\ x_{t=0} &= 0 \\ \dot{x}_{t=0} &= 0. \end{cases} \tag{1}$$

Ex *b*.

Laplace Transform of 2nd-order ODE (with zero initial condition) in Equation 1:

$$\mathcal{L}\{\ddot{x}\} = s^2X(s) = \frac{0.7}{2}sX(s) + \frac{1}{2}X(s) + \frac{0.5}{2s}$$

which gives the diagram below:



Ex *c*.

For Figure 2(b) in the manual I can derive a group of equations based on KVL,

$$q_C = Ce_o(t) \tag{2}$$
$$e_i(t) - e_o(t) = i(t)R \tag{3}$$
$$\mathrm{d}q_C = i(t)\,\mathrm{d}t \tag{4}$$

which gives

$$\begin{aligned} & e_i(t) - \frac{q_C}{C} = i(t)R \\ \Rightarrow & e_i(t) - \frac{1}{C}\int i(t)\,\mathrm{d}t - i(t)R = 0. \end{aligned}$$

2 Lab Exercises

Ex 1: *Solving Differential Equations using Analog Computer*

Damp Coefficient $b = 0.7$

Prelab exercise provides a 2nd-order ODE

$$m\ddot{x} + b\dot{x} + kx = F$$

which, using Laplace Transform, can be represented as

$$s^2X(s) + \frac{b}{m}sX(s) + \frac{k}{m}X(s) = \frac{F}{m}.$$

For step 2, I found the I/O relationship at `adder1` can be represented as

$$Y = -\frac{R_f}{F_1}x_1 - \frac{R_f}{F_2}x_2$$

and substituting x_1, x_2 with its corresponding signals $-\dot{x}, F(t) = 0.5$ I obtain

$$\frac{F(t) - b\dot{x}}{m} = \frac{0.5 - 0.7\dot{x}}{2} = -\dot{x}\frac{10\text{k}}{28.6\text{k}} + 0.5\frac{10\text{k}}{20\text{k}}$$

which matches the original parameter in the SIMUNLINK model.

This step produces a plot of $x(t)$ and $x'(t)$ over time, which is shown in Figure 1 and 2.

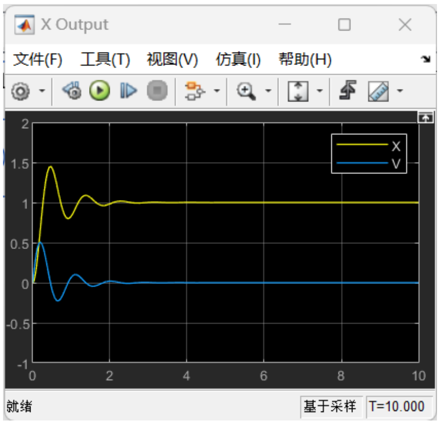


Figure 1: Function $x(t)$ and $v(t)$ with the change of t; generated by SIMULINK model

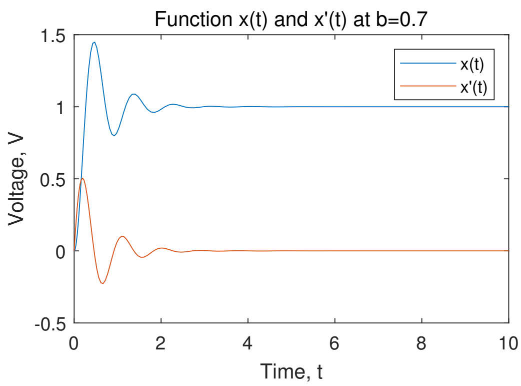


Figure 2: Function $x(t)$ and $v(t)$ with the change of t; plotted by MATLAB

Damp Coefficient $b = 1$

Let's focus on R_1 in Figure 3.

As is discussed above, the I/O relationship of `adder1` is

$$\frac{F(t) - b\dot{x}}{m} = \frac{0.5 - \dot{x}}{2} = -\dot{x}\frac{10\text{k}}{R_1} + 0.5\frac{10\text{k}}{20\text{k}}$$

which yields

$$\frac{1}{2} = \frac{10\text{k}}{R_1}.$$

Thus here I take $R_1 = 20\text{k}\Omega$. The plots for function $x_{b=1}(t)$ and $x'_{b=1}(t)$ is shown in Figure 4 and 5.

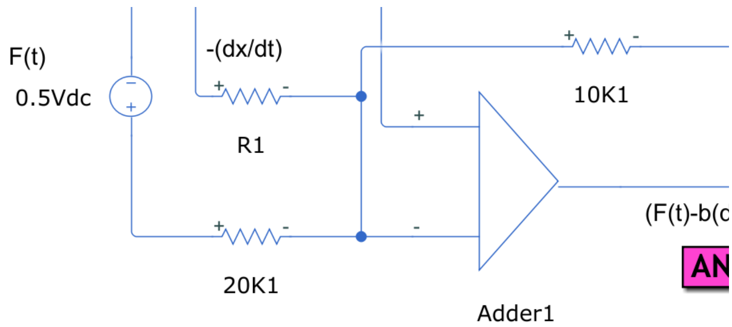


Figure 3: Block diagram near adder1

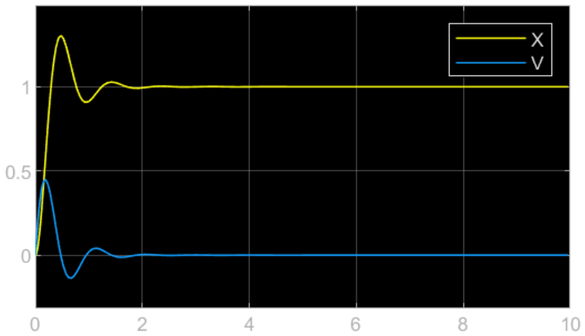


Figure 4: Function $x_{b=1}(t)$ and $x'_{b=1}(t)$ with the change of t; generated by SIMULINK model

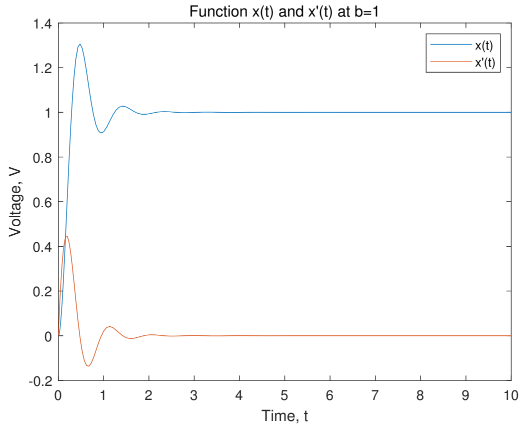


Figure 5: Function $x(t)$ and $v(t)$ with the change of t; plotted by MATLAB

Discussion about the Changes in Response

For the changes with b, I derived an expression for R_1 in Figure 3:

$$R_1 = 10\mathrm{k}\frac{m}{b}$$

which indicates that I can explore the change of functions with different R_1. It is concluded that with an increasing R_1, i.e. decreasing b,

1. the system will reach its steady state with longer time.
2. there will be a higher maximum value of system response.
3. frequency remains unchanged.

I plotted function $x(t)$ for different R_1, which is shown in Figure 6.

For the changes with $F(t)$, the steady state response is exactly the value of $F(t)$. As $F(t)$ changes, the response changes accordingly.

For the changes with k, similar as before I derived an expression for R_2, which is

$$R_2 = 10\mathrm{k}\frac{m}{k}.$$

I plotted $x(t)$ with different value of R_2 as belo in Figure 8.

With increasing value of R_2, i.e. decreasing k,

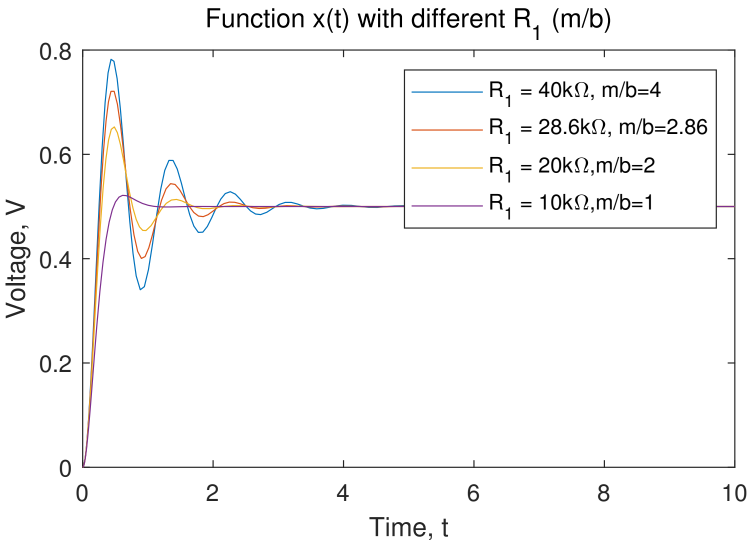


Figure 6: Function $x(t)$ with different values of $\frac{m}{b}$

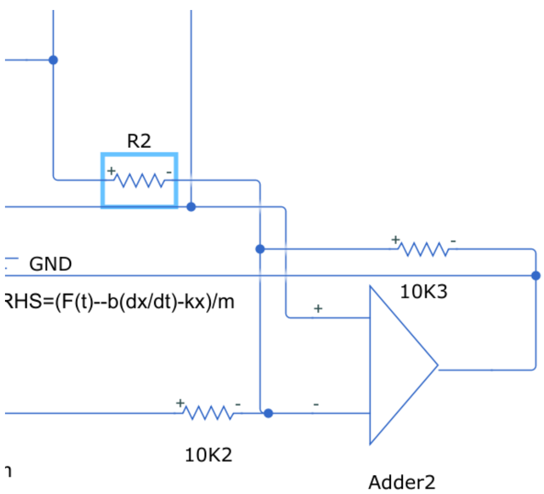


Figure 7: Block diagram near adder2

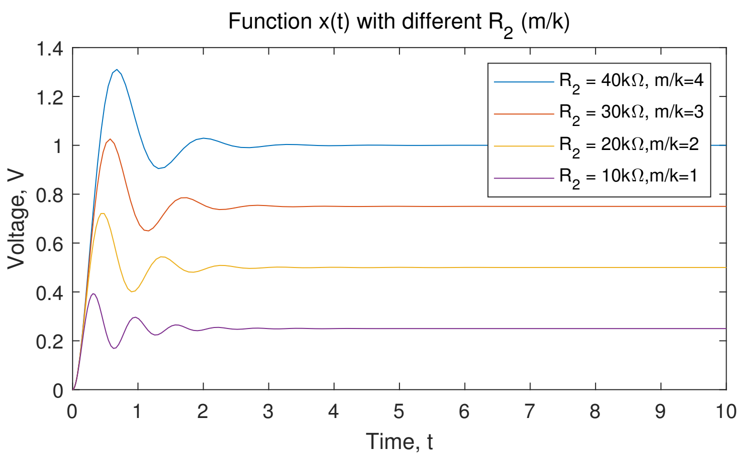


Figure 8: Function $x(t)$ with different values of $\frac{m}{k}$

1. The system will reach its steady state with longer time.
2. The frequency will decrease.
3. The steady state value will increase, as k changes the conditions at equilibrium.

Ex 2: *System Identification Experiment*

Hardware & Software Setup

After setting up both hardware and software I ran the simulation and obtained the two plots (shown as Figure 9 and 10), following the instructions on lab manual.

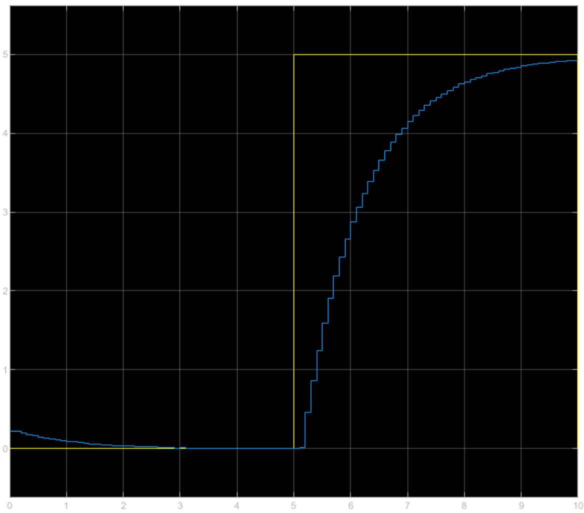


Figure 9: Voltage output of Arduino

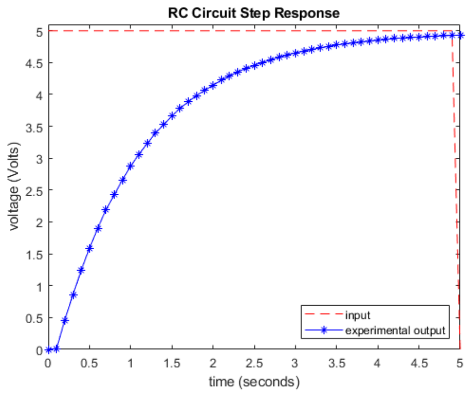


Figure 10: Voltage change within 5 seconds, generated by MATLAB

Parameter Identification

To get the value of τ, we have to find the time when the response reaches 63.2% of its total changes, i.e. 3.16V.

From data gathered we find the response at 1.1s is 3.07V, and at 1.2s is 3.19V. Using linear approximation we have

$$V(t) = V(1.1) + \frac{V(1.2) - V(1.1)}{1.2 - 1.1}t = 3.07 + 1.2(t - 1.1), 1.1 \leq t \leq 1.2$$

Solving $V(t) = 3.16$ we have $t \approx 1.175$s.