Let's consider an n-pendulum, with n rods of length ℓ_i and n bobs of mass m_i linked together. Let's also take a set of Cartesian coordinate axes centered at the anchor point of rod 1, with an upward–directed y–axis.

Given that our system satisfies the Principle of Virtual Work, we may apply the Lagrangian formalism. If we take our generalised coordinates θ_i to be the counterclockwise angle between each rod and the negative y direction, the (x_i, y_i) coordinates of each bob will be:

$$\begin{aligned}
x_1 &= \ell_1 \sin\theta_1 & y_1 &= -\ell_1 \cos\theta_1 \\
x_2 &= \ell_1 \sin\theta_1 + \ell_2 \sin\theta_2 & y_2 &= -\ell_1 \cos\theta_1 - \ell_2 \cos\theta_2 \\
&\vdots & &\vdots \\
x_i &= \sum_{j=1}^{i} \ell_j \sin\theta_j & y_i &= -\sum_{j=1}^{i} \ell_j \cos\theta_j
\end{aligned}$$

The velocities of the i^{th} bob are then

$$\dot{x}_i = \sum_{j=1}^{i} \ell_j \dot{\theta}_j \cos\theta_j \qquad \dot{y}_i = \sum_{j=1}^{i} \ell_j \dot{\theta}_j \sin\theta_j$$

The kinetic energy of the system is thus

$$T = \frac{1}{2} \sum_{i=1}^{n} m_i \left[\left(\sum_{j=1}^{i} \ell_j \dot{\theta}_j \cos\theta_j \right)^2 + \left(\sum_{j=1}^{i} \ell_j \dot{\theta}_j \sin\theta_j \right)^2 \right]$$

And the potential energy is

$$V = -g \sum_{i=1}^{n} \sum_{j=1}^{i} m_i \ell_j \cos\theta_j$$

The system's Lagrangian is $\mathcal{L} = T - V$. Let's derive its Euler–Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \theta_k} = \sum_{i=k}^{n} m_i \left[-\ell_k \dot{\theta}_k \sin\theta_k \sum_{j=1}^{i} \ell_j \dot{\theta}_j \cos\theta_j + \ell_k \dot{\theta}_k \cos\theta_k \sum_{j=1}^{i} \ell_j \dot{\theta}_j \sin\theta_j \right] - g \sum_{i=k}^{n} m_i \ell_k \sin\theta_k$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_k} = \sum_{i=k}^{n} m_i \left[\ell_k \cos\theta_k \sum_{j=1}^{i} \ell_j \dot{\theta}_j \cos\theta_j + \ell_k \sin\theta_k \sum_{j=1}^{i} \ell_j \dot{\theta}_j \sin\theta_j \right]$$

$$\begin{aligned}
\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}_k} &= \sum_{i=k}^{n} m_i \left[-\ell_k \dot{\theta}_k \sin\theta_k \sum_{j=1}^{i} \ell_j \dot{\theta}_j \cos\theta_j + \ell_k \dot{\theta}_k \cos\theta_k \sum_{j=1}^{i} \ell_j \dot{\theta}_j \sin\theta_j \right] + \\
&+ \sum_{i=k}^{n} m_i \left[\ell_k \cos\theta_k \sum_{j=1}^{i} \ell_j (\ddot{\theta}_j \cos\theta_j - \dot{\theta}_j^2 \sin\theta_j) + \ell_k \sin\theta_k \sum_{j=1}^{i} \ell_j (\ddot{\theta}_j \sin\theta_j + \dot{\theta}_j^2 \cos\theta_j) \right]
\end{aligned}$$

Thankfully a lot of terms will cancel.

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_k}-\frac{\partial \mathcal{L}}{\partial \theta_k}=\sum_{i=k}^{n} m_i \ell_k \left[g\sin\theta_k+\cos\theta_k\sum_{j=1}^{i}\ell_j(\ddot{\theta}_j\cos\theta_j-\dot{\theta}_j^2\sin\theta_j)+\sin\theta_k\sum_{j=1}^{i}\ell_j(\ddot{\theta}_j\sin\theta_j+\dot{\theta}_j^2\cos\theta_j)\right]=$$

$$=\ell_k\sum_{i=k}^{n} m_i \left[g\sin\theta_k+\sum_{j=1}^{i}\ell_j(\ddot{\theta}_j\cos(\theta_k-\theta_j)+\dot{\theta}_j^2\sin(\theta_k-\theta_j))\right]=0$$

Let's now split up the sums.

$$\sum_{i=k}^{n}\sum_{j=1}^{i} m_i\ell_j\ddot{\theta}_j\cos(\theta_k-\theta_j)+\sum_{i=k}^{n}\sum_{j=1}^{i} m_i\ell_j\dot{\theta}_j^2\sin(\theta_k-\theta_j)+g\sin\theta_k\sum_{i=k}^{n} m_i=0$$

Our next trick will be swapping the order of summation. This leaves us with

$$\sum_{j=1}^{n}\ell_j\ddot{\theta}_j\cos(\theta_k-\theta_j)\sum_{i=\max(j,k)}^{n} m_i+\sum_{i=k}^{n}\ell_j\dot{\theta}_j^2\sin(\theta_k-\theta_j)\sum_{i=\max(j,k)}^{n} m_i+g\sin\theta_k\sum_{i=k}^{n} m_i=0$$

Finally! Notice this expression sums over $\ddot{\theta}_j$ and $\dot{\theta}_j^2$. Let's see what it looks like in matrix form:

$$\begin{pmatrix}
\ell_1\sum\limits_{i=1}^{n} m_i & \ell_2\cos(\theta_1-\theta_2)\sum\limits_{i=2}^{n} m_i & \cdots & \ell_n\cos(\theta_1-\theta_n)m_n\\
\ell_1\cos(\theta_2-\theta_1)\sum\limits_{i=2}^{n} m_i & \ell_2\sum\limits_{i=2}^{n} m_i & \cdots & \ell_n\cos(\theta_2-\theta_n)m_n\\
\vdots & \vdots & \ddots & \vdots\\
\ell_1\cos(\theta_n-\theta_1)m_n & \ell_1\cos(\theta_n-\theta_2)m_n & \cdots & \ell_n m_n
\end{pmatrix}
\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \vdots\\ \ddot{\theta}_n\end{pmatrix}+$$

$$+\begin{pmatrix}
0 & \ell_2\sin(\theta_1-\theta_2)\sum\limits_{i=2}^{n} m_i & \cdots & \ell_n\sin(\theta_1-\theta_n)m_n\\
\ell_1\sin(\theta_2-\theta_1)\sum\limits_{i=2}^{n} m_i & 0 & \cdots & \ell_n\sin(\theta_2-\theta_n)m_n\\
\vdots & \vdots & \ddots & \vdots\\
\ell_1\sin(\theta_n-\theta_1)m_n & \ell_1\sin(\theta_n-\theta_2)m_n & \cdots & 0
\end{pmatrix}
\begin{pmatrix}\dot{\theta}_1^2\\ \dot{\theta}_2^2\\ \vdots\\ \dot{\theta}_n^2\end{pmatrix}+g\begin{pmatrix}\sin\theta_1\sum\limits_{i=1}^{n} m_i\\ \sin\theta_2\sum\limits_{i=2}^{n} m_i\\ \vdots\\ \sin\theta_n m_n\end{pmatrix}=0$$

If one stares at this expression for long enough they may notice something. Let's call $\sum\limits_{i=k}^{n} m_i=\mu_k$:

$$\sum_{j,\ell=1}^{n}\Re(\mathbf{Z})_{k\ell}\mathbf{L}_{\ell j}\ddot{\theta}_j+\sum_{j,\ell=1}^{n}\Im(\mathbf{Z})_{k\ell}\mathbf{L}_{\ell j}\dot{\theta}_j^2+g\,\mu_k\sin\theta_k=0$$

where $\Re(\cdot)$ denotes "the real part of" and $\Im(\cdot)$ denotes "the imaginary part of". Also $\mathbf{L}=\mathrm{diag}(\ell_1,...,\ell_n)$, $\mathbf{Z}=e^{i\boldsymbol{\Theta}}\mathbf{U}\mathbf{M}\mathbf{U}^{\mathbf{T}}e^{-i\boldsymbol{\Theta}}$, and the matrices $\mathbf{Z}$ factors into are $\boldsymbol{\Theta}=\mathrm{diag}(\theta_1,...,\theta_n)$, $\mathbf{M}=\mathrm{diag}(m_1,...,m_n)$, and $\mathbf{U}$ is an upper triangular matrix of ones.

$$\mathbf{U}=\begin{pmatrix}1 & 1 & \cdots & 1\\ 0 & 1 & \cdots & 1\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1\end{pmatrix}$$

It is now clear that if we want to write the ODE for the system in explicit form we may do the following:

$$\ddot{\theta}_k = -\sum_{i,j,\ell,m=1}^{n} \mathbf{L}_{ki}^{-1}\Re(\mathbf{Z})_{i\ell}^{-1}\Im(\mathbf{Z})_{\ell m}\mathbf{L}_{mj}\dot{\theta}_j^2 - g\sum_{j,\ell=1}^{n}\mathbf{L}_{k\ell}^{-1}\Re(\mathbf{Z})_{\ell j}^{-1}\mu_j\sin\theta_j$$

And we therefore need to calculate $\Re(\mathbf{Z})^{-1} = 2(\mathbf{Z}+\mathbf{Z}^*)^{-1}$. Applying the Woodbury matrix identity

$$(\mathbf{Z}+\mathbf{Z}^*)^{-1} = \mathbf{Z}^{-1} - \mathbf{Z}^{-1}\left(\mathbf{Z}^{-1} + (\mathbf{Z}^*)^{-1}\right)^{-1}\mathbf{Z}^{-1} = \mathbf{Z}^{-1} - \frac{1}{2}\mathbf{Z}^{-1}\Re(\mathbf{Z}^{-1})^{-1}\mathbf{Z}^{-1}$$

Furthermore, since $\mathbf{Z}+\mathbf{Z}^*$ is real

$$\begin{gathered}\Re(\mathbf{Z})^{-1} = 2(\mathbf{Z}+\mathbf{Z}^*)^{-1} = 2\Re\left((\mathbf{Z}+\mathbf{Z}^*)^{-1}\right) = \\ = 2\Re(\mathbf{Z}^{-1}) - \left[\Re(\mathbf{Z}^{-1})\Re(\mathbf{Z}^{-1})^{-1}\Re(\mathbf{Z}^{-1}) - \Im(\mathbf{Z}^{-1})\Re(\mathbf{Z}^{-1})^{-1}\Im(\mathbf{Z}^{-1})\right] = \\ = \Re(\mathbf{Z}^{-1}) + \Im(\mathbf{Z}^{-1})\Re(\mathbf{Z}^{-1})^{-1}\Im(\mathbf{Z}^{-1})\end{gathered}$$

And we can find further simplification for the matrix $\Re(\mathbf{Z})^{-1}\Im(\mathbf{Z})$ which multiplies $\dot{\theta}_j^2$:

$$\begin{gathered}\Re(\mathbf{Z})^{-1}\Im(\mathbf{Z}) = -i\Re(\mathbf{Z})^{-1}\left[\mathbf{Z} - \Re(\mathbf{Z})\right] = -i\left[\Re(\mathbf{Z})^{-1}\mathbf{Z} - \mathbf{I}\right] = \\ = -i\left[\mathbf{I} - \mathbf{Z}^{-1}\Re(\mathbf{Z}^{-1})^{-1}\right] = \Re\left(-i\left[\mathbf{I} - \mathbf{Z}^{-1}\Re(\mathbf{Z}^{-1})^{-1}\right]\right) = \\ = -\Im(\mathbf{Z}^{-1})\Re(\mathbf{Z}^{-1})^{-1}\end{gathered}$$

Note that our only task now is to invert a series of diagonal matrices to find $\mathbf{Z}^{-1}$, and also by inspection we can tell

$$\mathbf{U}^{-1} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

We can luckily obtain $\mathbf{Z}^{-1}$ via direct computation without too much effort.

$$\mathbf{Z}^{-1} = \begin{pmatrix} \frac{1}{m_1} & -\frac{\exp[i(\theta_1-\theta_2)]}{m_1} & 0 & \cdots & 0 \\ -\frac{\exp[i(\theta_2-\theta_1)]}{m_1} & \frac{1}{m_1}+\frac{1}{m_2} & -\frac{\exp[i(\theta_2-\theta_3)]}{m_2} & \cdots & 0 \\ 0 & -\frac{\exp[i(\theta_3-\theta_2)]}{m_2} & \frac{1}{m_2}+\frac{1}{m_3} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\frac{\exp[i(\theta_{n-1}-\theta_{n-2})]}{m_{n-2}} & \frac{1}{m_{n-2}}+\frac{1}{m_{n-1}} & -\frac{\exp[i(\theta_{n-1}-\theta_n)]}{m_{n-1}} \\ 0 & 0 & \cdots & -\frac{\exp[i(\theta_n-\theta_{n-1})]}{m_{n-1}} & \frac{1}{m_{n-1}}+\frac{1}{m_n} \end{pmatrix}$$

The only remaining hard matrix to invert in this problem is $\Re(\mathbf{Z}^{-1})$, but we have found it is tridiagonal (sparse) and symmetric. We'll leave its inversion to SciPy.