Estudio de Función

sábado, 11 de mayo de 2024

I) ESTUDIO DE FUNCIÓN

Realizar el estudio completo de las siguientes funciones, indicando:

- ✓ **Dominio**
- ✓ **Asíntotas**
- ✓ Cortes con los ejes
- ✓ **Intervalos de crecimiento**
- ✓ **Máximos y mínimos**
- ✓ **Intervalos de Concavidad**
- ✓ **Puntos de Inflexión**

Luego realice un gráfico aproximado **a partir de los datos obtenidos**.

19) $f(x) = \frac{x^2-2x+2}{x-1}$

- Dom = $\mathbb{R} - \{1\}$ — cand. a A.V
- ASINTOTAS

- AV

$$\lim_{x\to 1^+} \frac{\overbrace{x^2-2x+2}^{\to 1}}{\underbrace{x-1}_{\to 0^+}} = +\infty \qquad \lim_{x\to 1^-} \frac{\overbrace{x^2-2x+2}^{\to 1}}{\underbrace{x-1}_{\to 0^-}} = -\infty$$

Posee AV en $x=1$

- AH

$$\lim_{x\to\infty} \frac{x^2-2x+2}{x-1} \text{ "}\frac{\infty}{\infty}\text{" ind.} \qquad \lim_{x\to\infty} \frac{2x-2}{1} = \infty$$

no posee AH

Puede tener A.O

- AO $\quad y = mx+b$

$$m = \lim_{x\to\infty} \frac{f(x)}{x} \quad \circ \quad \lim_{x\to\infty} f(x) \cdot \frac{1}{x}$$

$$m = \lim_{x\to\infty} \frac{x^2-2x+2}{x-1} \cdot \frac{1}{x}$$

$$m = \lim_{x\to\infty} \frac{x^2-2x+2}{x^2-x} \text{ "}\frac{\infty}{\infty}\text{"}$$

$$m \overset{L'H}{=} \lim_{x\to\infty} \frac{2x-2}{2x-1}$$

$$m \overset{L'H}{=} \lim_{x\to\infty} \frac{2}{2}$$

$$b = \lim_{x\to\infty} [f(x) - mx]$$

$$b = \lim_{x\to\infty} \underbrace{\frac{x^2-2x+2}{x-1}}_{\infty} - \underbrace{1x}_{\infty} \qquad \text{"}\infty - \infty\text{" ind.}$$

$$b = \lim_{x\to\infty} \left(\frac{x^2-2x+2-x(x-1)}{x-1} \right)$$

$$b = \lim_{x\to\infty} \frac{\cancel{x^2}-2x+2\cancel{-x^2}+x}{x-1}$$

$$b = \lim_{x\to\infty} \frac{-x+2}{x-1} \text{ "}\frac{\infty}{\infty}\text{" ind.}$$

$m = \lim_{x\to\infty} \frac{2}{2}$

$m = 1$

$b = \lim_{x\to\infty} \frac{-x+2}{x-1}$ " $\frac{\infty}{\infty}$ " ind.

$b \overset{L'H}{=} \lim_{x\to\infty} \frac{-1}{1}$

$b = -1$

Posee AO en $y = x - 1$

- CORTE CON LOS EJES.

CORTE CON EL EJE X (RAÍCES)

$f(x) = 0 \qquad y = 0$

$\frac{x^2-2x+2}{x-1} = 0$

$x^2-2x+2 = 0$ Ec. CUADRÁTICA

No posee raíces reales.

$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2-4.1.2}}{2.1}$ discriminante Δ

$x_1, x_2 = \frac{2 \pm \sqrt{4-8}}{2}$

$\Delta > 0$ 2 raíces.

$\Delta = 0$ 1 raíz

$\Delta < 0$ no posee.

$x_1 x_2 = \frac{2 \pm \sqrt{-4}}{2}$

$\sqrt{-1} = i$

$\sqrt{-4} = \sqrt{-1 . 4}$

$\underbrace{\sqrt{-1}}_{i} . \underbrace{\sqrt{4}}_{\pm 2} =$

$x_1, x_2 = \frac{2 \pm 2i}{2}$

$x_1, x_2 = 1 \pm i$

CORTE CON EL EJE Y (ORDENADA)

$f(0) \qquad x = 0$

$f(0) = \frac{0^2 - 2.0 + 2}{0 - 1}$

$f(0) = -2$

$\cap y = (0; -2)$

INTERVALO DE CRECIMIENTO / EXTREMOS

$f(x) = \frac{x^2-2x+2}{x-1}$

$f'(x) = \frac{(2x-2)(x-1) - (x^2-2x+2) . 1}{(x-1)^2}$

$f'(x) = \frac{2x^2 \cancel{-2x} - 2x \cancel{+2} - x^2 \cancel{+2x} \cancel{-2}}{(x-1)^2}$

$f'(x) = \frac{x^2 - 2x}{(x-1)^2}$

$0 = \frac{x^2-2x}{\underbrace{(x-1)^2}_{\neq 0}}$

$f'(x) > 0 \quad f$ crece

$f'(x) < 0 \quad f$ decrece.

$f'(x) = 0 \quad$ cand-a M o m

Para que lo sea debe haber cambio de signo a la derecha e izquierda del candidato.

$0 = x^2 - 2x$

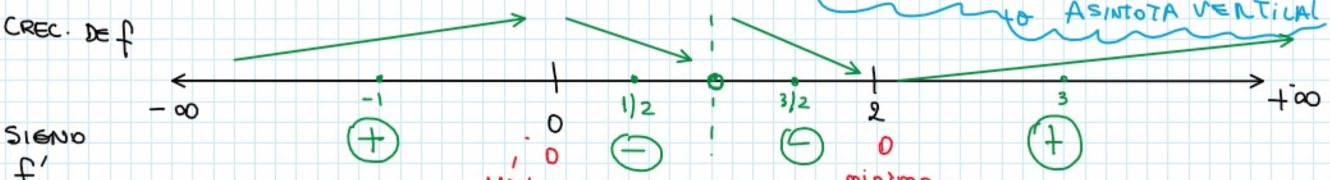
$0 = x.(x-2)$

$\boxed{x=0}$ ó $x - 2 = 0$

$\boxed{x = 2}$

candidatos a Máximo (M) o mínimo (m)

Recordar que hay valores excluidos del Dom. → ASINTOTA VERTICAL.



MAXIMO $(0; f(0)) = (0; -2)$

(coincide con OR-OR)

mínimo $(2; f(2)) = (2; 2)$

En la función en la derivada dio CERO.

$f'(x) = \dfrac{x.(x-2)}{(x-1)^2}$ siempre (+)

SIGNO $f'(-1) = \dfrac{-.-}{+} = (+)$

SIGNO $f'(1/2) = \dfrac{+.-}{+} = (-)$

SIGNO $f'\left(\frac{3}{2}\right) = \dfrac{+.-}{+} = (-)$

SIGNO $f'(3) = \dfrac{+.+}{+} = (+)$

CRECE $= (-\infty; 0) \cup (2; +\infty)$

DECRECE $= (0; 1) \cup (1; 2)$

MAXIMO $= (0; -2)$

mínimo $= (2; 2)$

$f(x) = \dfrac{x^2 - 2x + 2}{x - 1}$

$f(2) = \dfrac{2^2 - 2.2 + 2}{2 - 1}$

$f(2) = 2.$

CONCAVIDAD / PUNTO DE INFLEXIÓN

$f''(x) > 0 \quad f \cup$

$f''(x) < 0 \quad f \cap$

$f''(x) = 0$ cand. a PI. Para que lo sea debe haber cambio de concavidad a la derecha e izquierda.

$f'(x) = \dfrac{x.(x-2)}{(x-1)^2} = \dfrac{x^2 - 2x}{(x-1)^2}$

$f''(x) = \dfrac{(2x-2)(x-1)^2 - (x^2-2x).2(x-1).1}{(x-1)^4}$

$f''(x) = \dfrac{2\cancel{(x-1)}\left[(x-1)^2 - (x^2-2x)\right]}{(x-1)^{\cancel{4}\,3}}$

$f''(x) = \dfrac{2.\left[\cancel{x^2}\cancel{-2x}+1\cancel{-x^2}\cancel{+2x}\right]}{(x-1)^3}$

$f''(x) = 2$

$(x-1)^3$

$$f''(x) = \frac{2}{(x-1)^3}$$

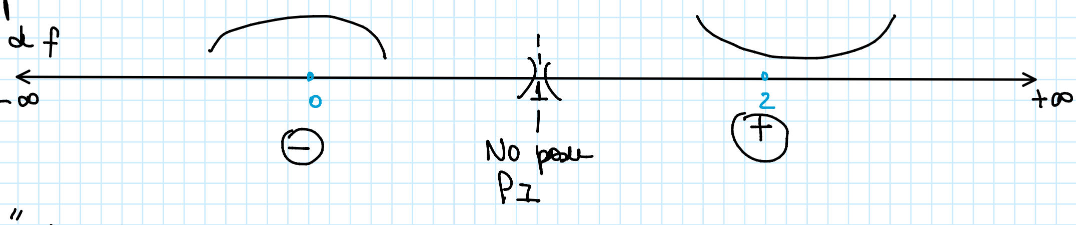
$$f''(x) = 0$$

$$\frac{2}{(x-1)^3} = 0$$

$f''(x)$ nunca es cero.

no posee cand. a PI ✓

Concavidad de f



Signo f''

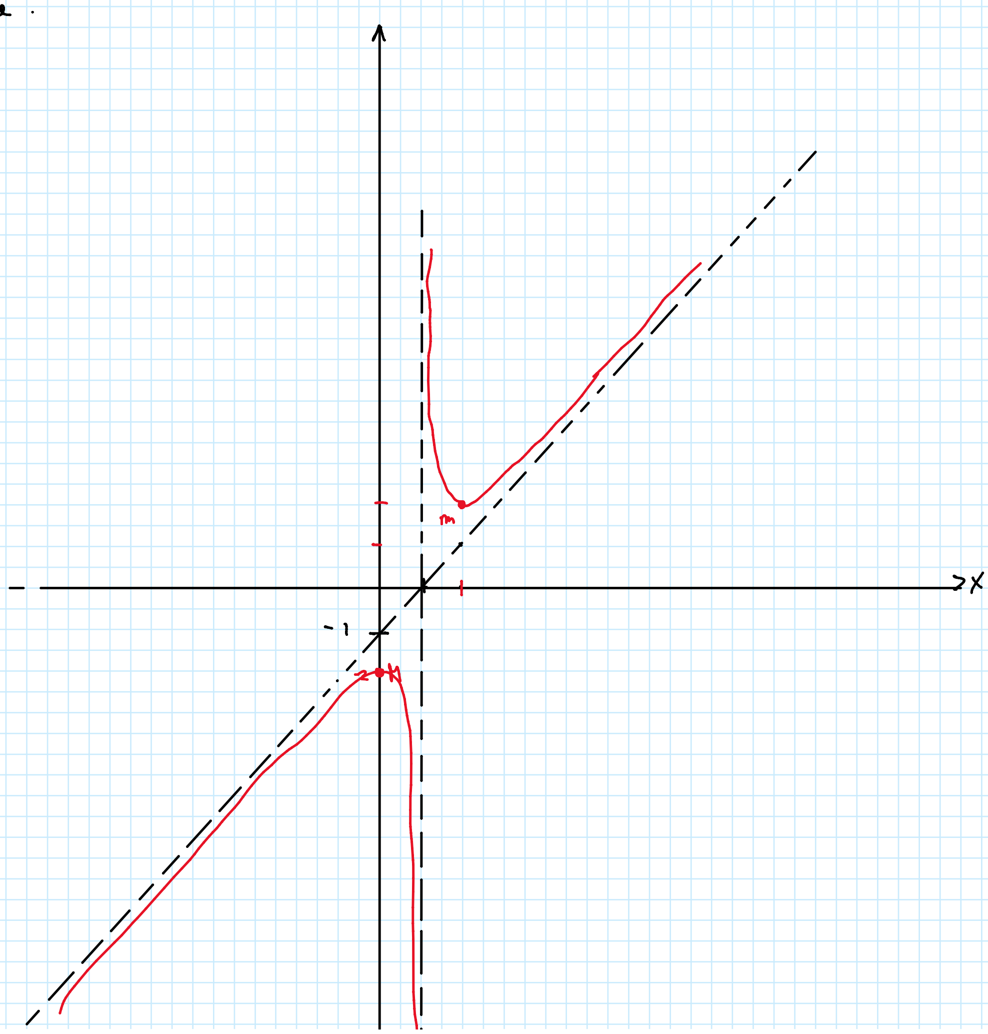
$$\text{Signo } f''(0) = \frac{+}{-} = -$$

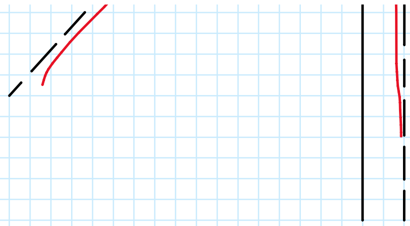
$$\text{Signo } f''(2) = \frac{+}{+} = +$$

CONCAVIDAD NEGATIVA = $(-\infty; 1)$

CONCAVIDAD POSITIVA = $(1; +\infty)$

PI: No posee.





9) $f(x) = -\frac{1}{4}x^4 + 2x^2 + 1$

$\left.\begin{array}{l} \text{Dom} = \mathbb{R} \\ \text{ASINTOTAS : no posee} \end{array}\right\}$ por ser función polinómica

- CORTE CON LOS EJES.

$\cap x \quad y=0$

$0 = \frac{-1}{4} x^4 + 2x^2 + 1$ Ec. Bicuadrática.

Cambio de variable $T = x^2$

$0 = -\frac{1}{4}(x^2)^2 + 2x^2 + 1$

$0 = \frac{-1}{4} T^2 + 2T + 1$ Ec. cuadrática

$T_1, T_2 = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot \left(\frac{-1}{4}\right) \cdot 1}}{2 \cdot \left(\frac{-1}{4}\right)}$

$T_1, T_2 = \frac{-2 \pm \sqrt{4+1}}{-\frac{1}{2}}$

$T_1, T_2 = \frac{-2 \pm \sqrt{5}}{\frac{-1}{2}}$.

$T_1 = \frac{-2+\sqrt{5}}{-1/2}$ $\qquad$ $T_2 = \frac{-2-\sqrt{5}}{-1/2}$

$T_1 \cong -0{,}47$ $\qquad$ $T_2 \cong 8{,}47$

Vuelvo a mi cambio de variable.

$T = x^2$

$\frac{-2+\sqrt{5}}{-1/2} = x^2$

$\sqrt{\frac{-2+\sqrt{5}}{-1/2}} = |x|$ $\quad -0{,}47$

$\sqrt{\frac{-2+\sqrt{5}}{-1/2}} = x_1 \qquad -\sqrt{\frac{-2+\sqrt{5}}{-1/2}} = x_2$

$T = x^2$

$\frac{-2-\sqrt{5}}{-1/2} = x^2$

$\sqrt{\frac{-2-\sqrt{5}}{-1/2}} = |x|$

$\sqrt{\frac{-2-\sqrt{5}}{-1/2}} = x_3 \qquad -\sqrt{\frac{-2-\sqrt{5}}{-1/2}} = x_4$

$$\underbrace{\sqrt{\frac{-2+\sqrt{5}}{-1/2}} = x_1 \qquad -\sqrt{\frac{-2+\sqrt{5}}{-1/2}} = x_2}_{\text{no posee raíces reales.}}$$

(raíz cuadrada de $-0,47$)

$$\sqrt{\frac{-2-\sqrt{5}}{-1/2}} = x_3 \qquad -\sqrt{\frac{-2-\sqrt{5}}{-1/2}} = x_4$$

$x_3 = 2,91$ $\qquad$ $x_4 = -2,91$

$\cap x = (2,91 ; 0) \quad (-2,91 ; 0)$

$\cap y = (0 ; 1)$ ⌊ $f(0)$

9) $f(x) = -\frac{1}{4}x^4 + 2x^2 + 1$

INTERV. CRECIMIENTO / EXTREMOS

$f'(x) = -x^3 + 4x$

$0 = -x^3 + 4x$

$0 = x(-x^2+4)$

$x = 0 \quad \vee \quad -x^2+4 = 0$

$-x^2 = -4$

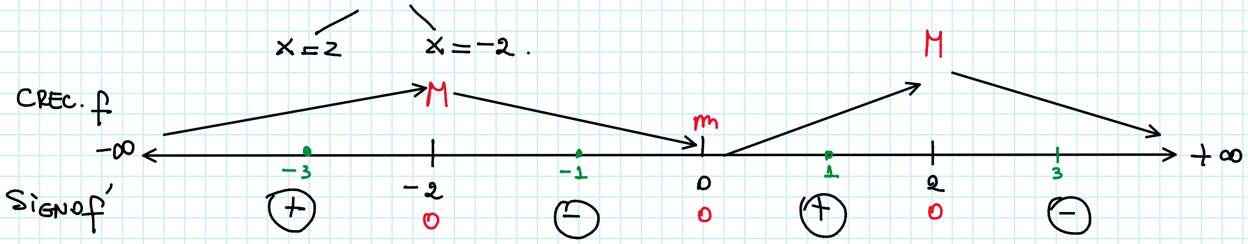
$x^2 = 4$

$|x| = \sqrt{4}$

$x = 2 \qquad x = -2.$

CANDID. $\{-2 ; 0 ; 2\}$

cand. a M ó m.



Signo $f'(-3) = - \cdot - = (+)$ $\qquad$ Signo $f'(1) = + \cdot + = (+)$

Signo $f'(-1) = - \cdot (+) = (-)$ $\qquad$ Signo $f'(3) = + \cdot - = (-)$

MÁXIMO $= (-2 ; f(-2)) = (-2 ; 5)$ $\qquad$ CRECE $= (-\infty ; -2) \cup (0 ; 2)$

MÁXIMO $= (2 ; f(2)) = (2 ; 5)$ $\qquad$ DECRECE $= (-2 ; 0) \cup (2 ; +\infty)$

mínimo $= (0 ; f(0))$ COINCIDE CON LA OR-OR $= (0 ; 1)$

$f(-2) = -\frac{1}{4}(-2)^4 + 2(-2)^2 + 1 = 5$

$f(2) = -\frac{1}{4}(2)^4 + 2(2)^2 + 1 = 5$

CONCAVIDAD / PI $\qquad f'(x) = -x^3 + 4x$

$f''(x) = -3x^2 + 4.$

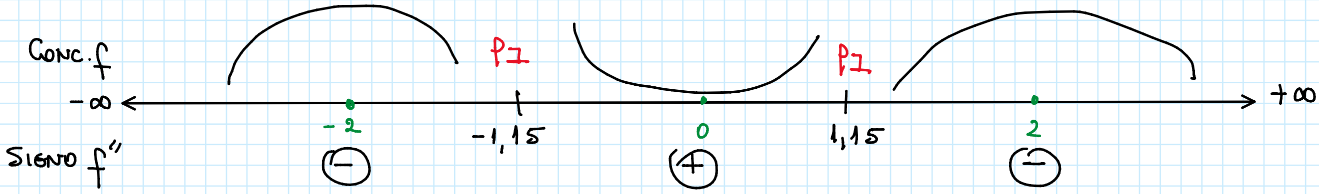
$0 = -3x^2 + 4.$

$$\frac{-4}{-3} = x^2$$

$$\sqrt{\frac{4}{3}} = |x|$$

$$\sqrt{\frac{4}{3}} = x_1 \qquad -\sqrt{\frac{4}{3}} = x_2.$$

$$\underbrace{1{,}15 \cong x_1 \qquad -1{,}15 = x_2.}_{\text{Cond. a PI.}}$$



$$\text{Signo } f''(-2) = -3(-2)^2 + 4 = -$$

$$\text{Signo } f''(0) = 4 = +$$

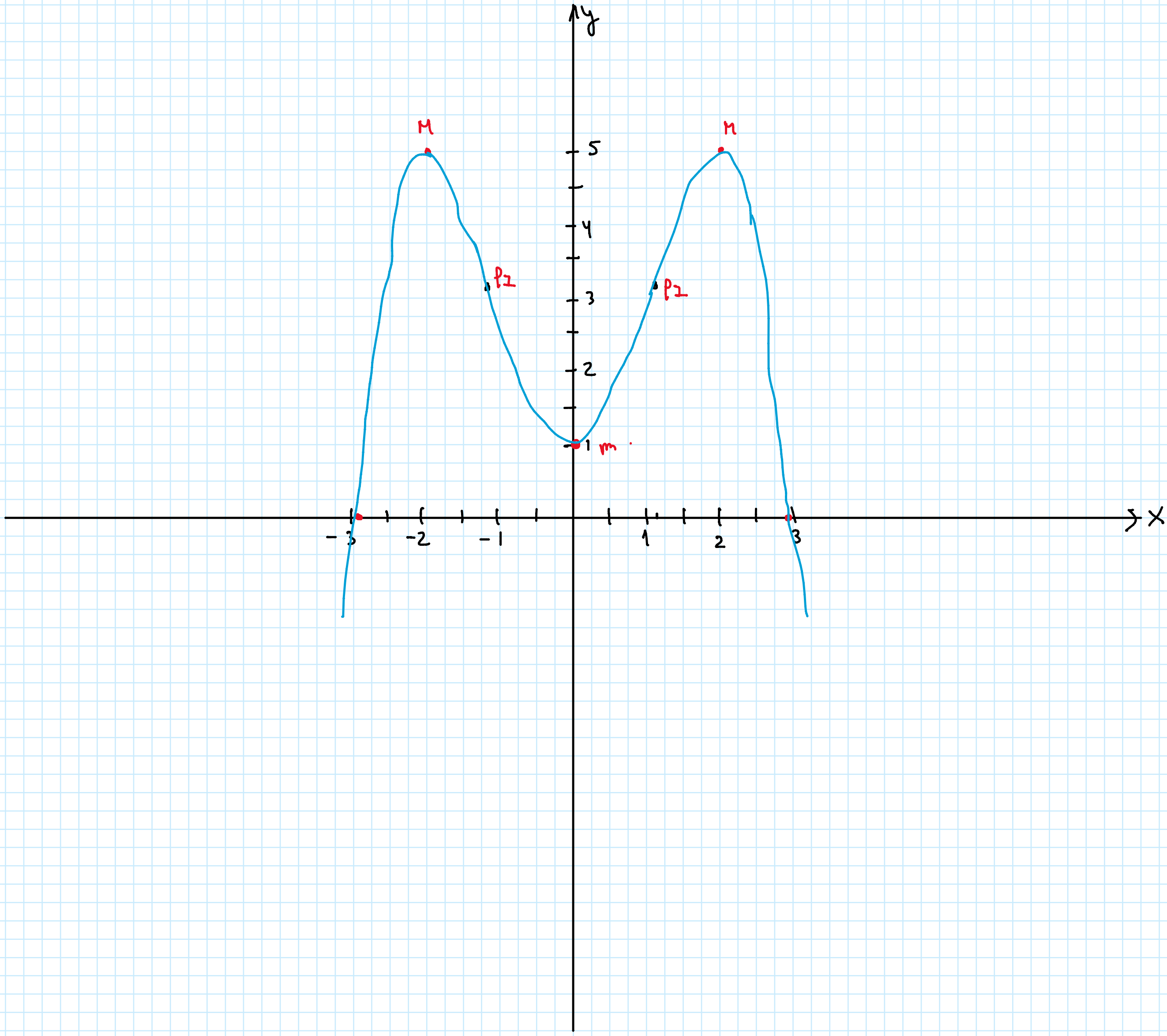
$$\text{Signo } f''(2) = -3 \cdot 2^2 + 4 = -$$

$$\boxed{\begin{array}{l} \text{Conc. Positiva} = (-1{,}15;\ 1{,}15) \\ \text{Conc. Negativa} = (-\infty; -1{,}15) \cup (1{,}15; +\infty) \end{array}}$$

$$\boxed{\begin{array}{l} PI = (-1{,}15 ; f(-1{,}15)) = (-1{,}15 ; 3{,}2) \\ \quad (1{,}15 ; f(1{,}15)) = (1{,}15 ; 3{,}2) \end{array}}$$

$$f\left(-\sqrt{\frac{4}{3}}\right) = -\frac{1}{4}\left(-\sqrt{\frac{4}{3}}\right)^4 + 2\left(-\sqrt{\frac{4}{3}}\right)^2 + 1 = 29/9 = 3{,}\overline{2}$$

$$f\left(\sqrt{\frac{4}{3}}\right) = -\frac{1}{4}\left(\sqrt{\frac{4}{3}}\right)^4 + 2\left(\sqrt{\frac{4}{3}}\right)^2 + 1 = 3{,}\overline{2}$$



y
x
M
M
P1
P1
m
5
4
3
2
1
-3
-2
-1
1
2
3