Limites-Asíntotas-Continuidad

sábado, 6 de abril de 2024

$\log_{10}$ $\quad \ln_e \quad \ln$

$f: \ln(x)$ — argumento $\qquad$ Dom $(0;+\infty)$

$f: \ln(x-1)$ $\qquad x-1>0$

$x>1 \qquad Dom=(1;+\infty)$

Analizo el Dom de $\ln\left(\frac{1}{x}\right)$

$$\overset{+}{\frac{1}{x}}>0$$

$x>0$

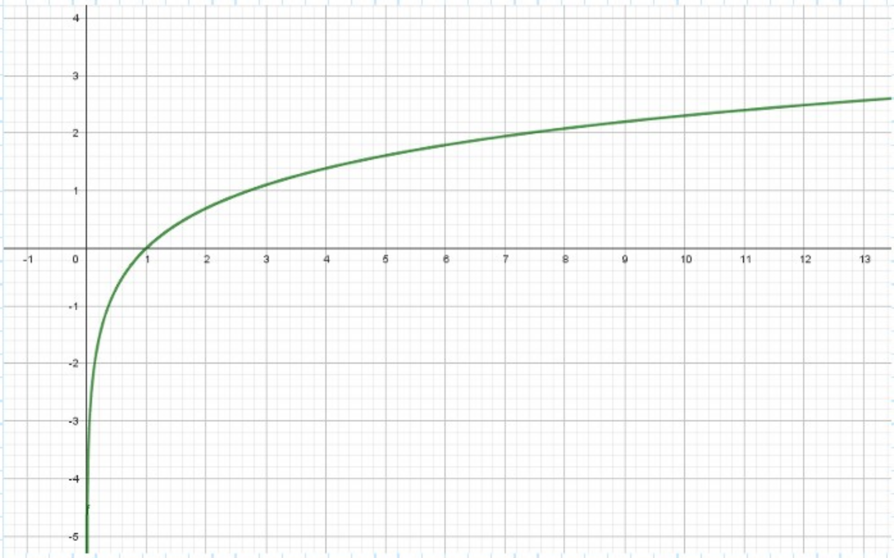
$Dom=(0;+\infty)$

1) $\lim\limits_{x\to+\infty}\left[\ln\left(\frac{1}{x}\right)\right]=$

$$\ln \lim_{x\to+\infty} \underbrace{\frac{1}{\underset{\to+\infty}{x}}}_{\to 0^+} = \ln 0^+ = -\infty$$

2) $\lim\limits_{x\to 0^+}\left[\ln\left(\frac{1}{x}\right)\right]=$

$$\ln \lim_{x\to 0^+} \overbrace{\frac{1}{\underset{\to 0^+}{x}}}^{\to+\infty} = +\infty$$



4) $\lim\limits_{x\to-1^+}\left[\ln\left(\frac{1}{x+1}\right)\right]=$

$$\ln \lim_{x\to-1^+} \overbrace{\frac{1}{\underset{0^+}{(x+1)}}}^{\to+\infty} = +\infty$$

5) $\lim\limits_{x\to+\infty}\left[\ln\left(\frac{3x-4}{3x+1}\right)\right]=$

$$\ln \lim_{x\to+\infty}\left(\frac{\overset{\infty}{3x-4}}{\underset{\infty}{3x+1}}\right) \text{ -INDET.} = \ln \lim_{x\to+\infty} \frac{\overset{3}{\frac{3\cancel{x}}{\cancel{x}}}-\overset{\to 0}{\frac{4}{x}}}{\underset{3}{\frac{3\cancel{x}}{\cancel{x}}}+\underset{\to 0}{\frac{1}{x}}} = \ln 1 = 0$$

10) $\lim\limits_{x\to+\infty}\left[\frac{\ln(x)}{\ln\sqrt{x}}\right]=$

$\sqrt[2]{x^1}=x^{1/2}$

$\sqrt[3]{x^2}=x^{2/3}$

OP. 1

$$\frac{\lim\limits_{x\to+\infty}\ln x}{\lim\limits_{x\to+\infty}\ln\sqrt{x}} = \frac{\ln \overbrace{\lim\limits_{x\to+\infty} x}^{\to+\infty}}{\ln \lim \sqrt{x}} = \frac{+\infty}{+\infty} \text{ ind.}$$

$$\frac{\lim_{x\to+\infty}}{\lim_{x\to+\infty} \ln\sqrt{x}} = \frac{\lim_{x\to+\infty}}{\ln\left(\lim_{x\to+\infty}\sqrt{x}\right)} = \frac{+\infty}{+\infty}$$

OP 2 — PROP. LOGARITMO

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

11) $\lim_{x\to+\infty}\left[x\cdot\ln\left(1+\frac{1}{x}\right)\right] = \infty \cdot 0$ ind.

PROP. LOGARITMO.

$$\ln(a)^b = b\cdot\ln(a) \qquad \begin{cases}\ln_e e = 1\\ \log_{10} 10 = 1\end{cases}$$

$$\lim_{x\to+\infty} \ln\left(1+\frac{1}{x}\right)^x = \ln \underbrace{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x}_{e} = \ln e = 1$$

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5) Siendo $f(x)=\begin{cases} e^{x-1} & si\ x<1 \\ \ln x & si\ x>1\end{cases}$ hallar $\lim_{x\to1} f(x)$

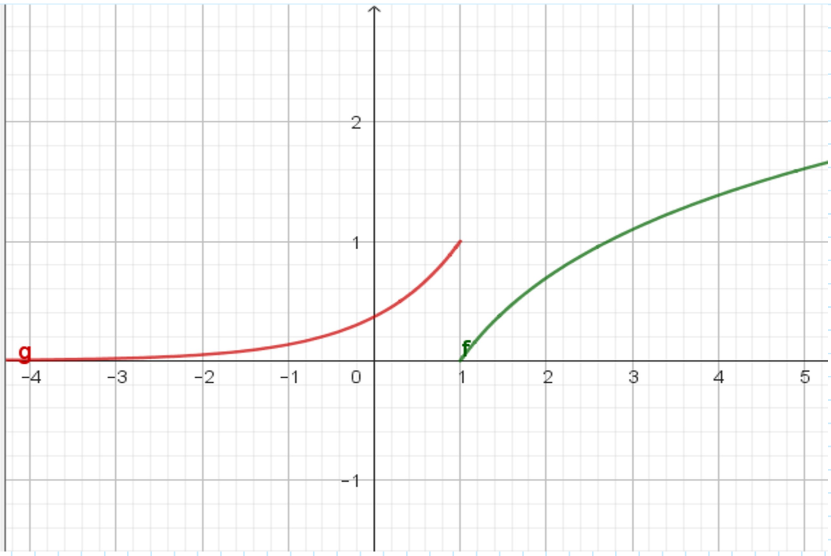
e^{x-1} $\quad 1^- \quad 1 \quad 1^+ \quad$ $\ln x$

$$L_1 = \lim_{x\to1^-} e^{x-1} = 1 \qquad (x-1 \to 0^-)$$

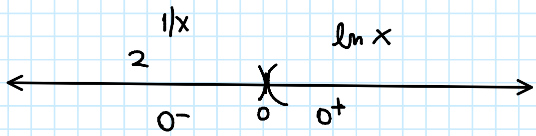
$$L_2 = \lim_{x\to1^+} \ln x = \ln\left(\lim_{x\to1^+} x\right) = 0$$

$$L_1 \neq L_2 \Rightarrow \nexists \lim_{x\to1} f(x)$$

- $f(x) = \ln(x), \quad (x > 1)$
- $g(x) = e^{x-1}, \quad (x < 1)$



6) Siendo $f(x) = \begin{cases} 2^{\frac{1}{x}} & si\ x < 0 \\ \ln x & si\ x > 0 \end{cases}$ hallar $\lim\limits_{x \to 0} f(x)$

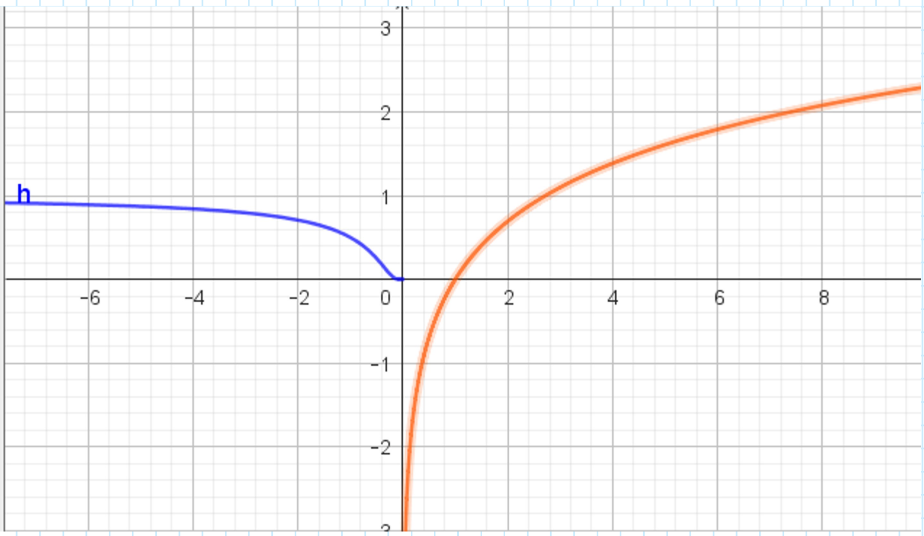


$$L_1 = \lim_{x \to 0^+} \ln x = \ln \lim_{x \to 0^+} x = -\infty$$

$$L_2 = \lim_{x \to 0^-} 2^{1/x} = 0$$

$$L_1 \neq L_2 \Rightarrow \nexists \lim_{x \to 0} f(x)$$

- $f(x) = \ln(x), \quad (x > 1)$
- $g(x) = e^{x-1}, \quad (x < 1)$
- $h(x) = 2^{\frac{1}{x}}, \quad (x < 0)$
- $p(x) = \ln(x)$



"Indeterminación 1^∞"

Límites Básicos

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e \qquad \lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$

$$14)\ \lim_{x\to3}\left(\frac{x^2-x-2}{x+1}\right)^{\frac{1}{x-3}} = 1^{\infty} \text{ ind.}$$

$$= \lim_{x\to3}\left(1+\frac{x^2-x-2}{x+1}-1\right)^{\frac{1}{x-3}}$$

$$= \lim_{x\to3}\left(1+\frac{x^2-x-2-1(x+1)}{x+1}\right)^{\frac{1}{x-3}}$$

$$= \lim_{x\to3}\left(1+\frac{x^2-x-2-x-1}{x+1}\right)^{\frac{1}{x-3}}$$

$$= \lim_{x\to3}\left(1+\frac{x^2-2x-3}{x+1}\right)^{\frac{1}{x-3}}$$

$$\lim_{T\to0}\left(1+T\right)^{\frac{1}{T}} = e$$

Cambio de variable.

$$T=\frac{x^2-2x-3}{x+1} \quad \frac{0}{4}$$

Pienso que 3 la RAÍZ.

$x\to3 \quad T\to0$

$$T=\frac{(x-3)\cancel{(x+1)}}{\cancel{x+1}}$$

	1	-2	-3
3	↓	3	3
	1	1	0

Otro modo.

Comienzo con la división

$$\lim_{x\to3}(x-2)^{\frac{1}{x-3}} =$$

$$\lim_{x\to3}(1+x-2-1)^{\frac{1}{x-3}}$$

$$\lim_{x\to3}(1+x-3)^{\frac{1}{x-3}}$$

$$\begin{array}{rl} x^2-x-2 & \underline{|\ x+1} \\ -\ x^2+x & x-2 \\ \hline -2x-2 & \\ -\ -2x-2 & \\ \hline 0 & \end{array}$$

$$\frac{x^2-x-2}{x+1} = x-2+\cancel{\frac{0}{x+1}}\ 0$$

$$\begin{array}{rl} 9 & \underline{|\ 2} \\ 1 & 4 \end{array}$$

$$\frac{9}{2}=\frac{4.2+1}{2}$$

$$\frac{9}{2}=\frac{4.\cancel{2}}{\cancel{2}}+\frac{1}{2}$$

$$\frac{9}{2}=4+\frac{1}{2}$$

$$9)\ \lim_{x\to+\infty}\left(\frac{x+1}{x}\right)^{2x} = \text{“}1^{\infty}\text{”}$$

$$\lim_{x\to+\infty}\left(\frac{\cancel{x}+\cancel{x}+1-\cancel{x}}{x}\right)^{2x} = \lim_{x\to+\infty}\left(\frac{\cancel{x}}{\cancel{x}}+\frac{1}{x}\right)^{2x}$$

$$\lim_{x\to+\infty}\left(\frac{\cancel{x}}{\cancel{x}}+\frac{1}{x}\right)^{\frac{2}{x}}$$

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{2x} = \lim_{x\to+\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{2}$$

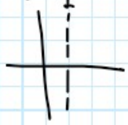
$$= \Bigg[\underbrace{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x}}_{e}\Bigg]^{2} = e^{2}$$

ASÍNTOTAS

AV: $\lim_{x\to a} f(x)=\infty$ $\quad [x=a$ es A.V$]$

a → candidato a A.V Análisis del Dom.

X=1

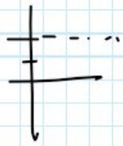


$x = N^{o}$

$x=1$

AH: $\lim_{x\to\infty} f(x)=b$ $\quad [\,y=b$ es AH$]$

y=2



$y=N^{o}$

$y=2$

Si posee AH no posee A.O.

Si no posee AH "puede tener AO" o no.

AO $\quad y = mx + b$

$$m = \lim_{x\to\infty}\left[\frac{f(x)}{x}\right] \quad \text{ó} \quad \lim_{x\to\infty}\left[f(x)\cdot\frac{1}{x}\right]$$

$$b = \lim_{x\to\infty}\left[f(x) - m\cdot x\right]$$

EJERCICIO 2 $\quad f(x)=\dfrac{3x^2}{x+1}$

$\text{Dom} = \mathbb{R} - \{-1\}$ ← candidato a A.V

$x+1 \neq 0 \qquad\qquad x+1=0$

$\qquad\qquad\qquad\qquad x=-1$

AV $\quad \lim_{x\to -1}\dfrac{\overbrace{3x^2}^{\to 3}}{\underbrace{x+1}_{\to 0}} = \infty \qquad x=-1$ es A.V

AH $\quad \lim \dfrac{\overbrace{3x^2}^{\infty}}{\ldots}$ ind. $= \lim \dfrac{3\cancel{x^2}^{\uparrow 3}}{\cancel{x^2}} = \infty \quad$ No posee A.H

$\underline{AH}$ $\lim_{x\to\infty} \frac{3x^2}{x+1}$ ind. $= \lim_{x\to\infty} \frac{\frac{3\cancel{x^2}}{\cancel{x^2}}}{\frac{\cancel{x}}{x^2}+\frac{1}{x^2}} = \infty$ no posee A.H

$\underline{A.O}$

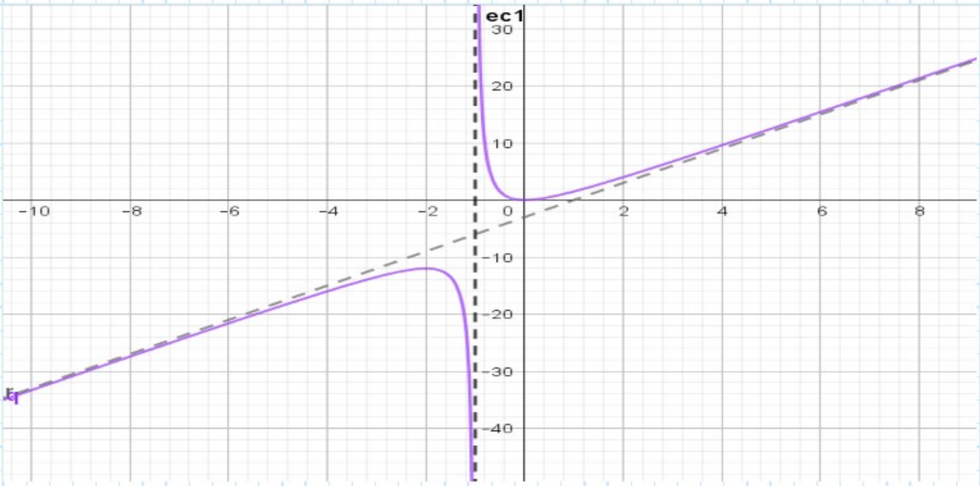
$$m = \lim_{x\to\infty}\left(\frac{3x^2}{x+1}\cdot\frac{1}{x}\right) = \lim_{x\to\infty}\left(\frac{3x^2}{x^2+x}\right) = \lim_{x\to\infty}\left(\frac{\frac{3\cancel{x^2}}{\cancel{x^2}}}{\frac{\cancel{x^2}}{\cancel{x^2}}+\frac{x}{x^2}}\right) =$$

$$= \lim_{x\to\infty}\left(\frac{3}{1+\frac{1}{x}}\right) = 3 \; m$$

$$b = \lim_{x\to\infty}\left(\frac{3x^2}{x+1} - 3x\right) = \lim_{x\to\infty}\left(\frac{3x^2-3x(x+1)}{x+1}\right)$$

$$= \lim_{x\to\infty}\left(\frac{\cancel{3x^2}-\cancel{3x^2}-3x}{x+1}\right) = \lim_{x\to\infty}\frac{-3x}{x+1} = \lim_{x\to\infty}\frac{-3x/x}{x/x+1/x} = -3 \; b$$

Posee A.O $y = 3x-3$.



EJERCICIO 6 $f(x) = \frac{3x^3+9x^2-12}{2x^2+2x-4}$

$Dom = \mathbb{R} - \{-2;1\}$

Factorizado $\frac{3(x+2)}{2}$

$\frac{3}{2}(x+2)$

$\frac{3}{2}x+3$

$2x^2+2x-4=0$

$x_1 = 1$ $x_2 = -2$

Cand. a AV

AV

Analizo en $x=-2$

$$\lim_{x\to -2} \frac{3x^3+9x^2-12}{2x^2+2x-4} \overset{\text{ind.}}{=} \lim_{x\to -2} \frac{\cancel{(x-1)}\cancel{(x+2)}\cdot 3(x+2)}{2\cancel{(x+2)}\cancel{(x-1)}} = 0$$

No posee AV en $x=-2$ (AGUJERITO $(-2;0)$)

C. AUX.

	3	9	0	-12
-2	↓	-6	-6	12
	3	3	-6	0
1	↓	3	6	
	3	6	0	

	2	2	-4
-2	↓	-4	4
	2	-2	0

$(x+2)(2x-2)$

$(x+2)^2\cdot(x-1)$

$2\cdot(x+2)(x-1)$

Analizo en $x=1$

$$\lim_{x\to 1} \frac{3x^3+9x^2-12}{2x^2+2x-4} \cdot \lim_{x\to 1} \frac{\cancel{(x-1)}\cancel{(x+2)}\cdot 3(x+2)}{2\cancel{(x+2)}\cancel{(x-1)}} = \frac{9}{2}$$

Agus. $(1;\ 9/2)$

No posee AV.

AH

$$\lim_{x\to\infty} \frac{3x^3+9x^2-12}{2x^2+2x-4} \overset{\text{ind.}}{=} \lim_{x\to\infty} \frac{\frac{3x^3}{x^3}+\frac{9x^2}{x^3}-\frac{12}{x^3}}{\frac{2x^2}{x^3}+\frac{2x}{x^3}-\frac{4}{x^3}}$$

$$= \lim_{x\to\infty} \frac{3+\frac{9}{x}-\frac{12}{x^3}}{\frac{2}{x}+\frac{2}{x^2}-\frac{4}{x^3}} = \infty$$

No posee A.H.

A.O

$$m = \lim_{x\to\infty}\left(\frac{3x^3+9x^2-12}{2x^2+2x-4}\cdot\frac{1}{x}\right)$$

$$m = \lim_{x\to\infty}\left(\frac{3x^3+9x^2-12}{2x^3+2x^2-4x}\right) \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty}\left(\frac{\frac{3x^3}{x^3}+\frac{9x^2}{x^3}-\frac{12}{x^3}}{\frac{2x^3}{x^3}+\frac{2x^2}{x^3}-\frac{4x}{x^3}}\right)$$

$$= \lim_{x\to\infty}\left(\frac{3+\frac{9}{x}-\frac{12}{x^3}}{2+\frac{2}{x}-\frac{4}{x^2}}\right) = \frac{3}{2}.$$

$$b = \lim_{x\to\infty}\left(\underbrace{\frac{3x^3+9x^2-12}{2x^2+2x-4}}_{\infty} - \underbrace{\frac{3x}{2}}_{\infty}\right)$$

$2x^2+2x-4 = 2(x^2+x-2)$

$$x\to\infty \quad \left(\frac{\dots}{2x^2+2x-4}\right) \quad \frac{\infty}{\infty}$$

$$b = \lim_{x\to\infty}\left(\frac{3x^3+9x^2-12-3x(x^2+x-2)}{2x^2+2x-4}\right)$$

$$b = \lim_{x\to\infty}\left(\frac{\cancel{3x^3}+9x^2-12-\cancel{3x^3}-3x^2+6x}{2x^2+2x-4}\right)$$

$$b = \lim_{x\to\infty}\left(\frac{6x^2+6x-12}{2x^2+2x-4}\right) \quad \text{"}\frac{\infty}{\infty}\text{"}$$

$$b = \lim_{x\to\infty}\left(\frac{\frac{6x^2}{x^2}+\frac{6x}{x^2}-\frac{12}{x^2}}{\frac{2x^2}{x^2}+\frac{2x}{x^2}-\frac{4}{x^2}}\right)$$

$$b = \lim_{x\to\infty}\left(\frac{6+\underbrace{\frac{6}{x}}_{\to 0}-\underbrace{\frac{12}{x^2}}_{\to 0}}{2+\underbrace{\frac{2}{x}}_{\to 0}-\underbrace{\frac{4}{x^2}}_{\to 0}}\right)$$

$$b = 3 \qquad\qquad y = \frac{3}{2}x+3$$

CONTINUIDAD

f es CONTINUA en $x=a$.

(1) $\exists f(a)$

(2) $\exists \lim_{x\to a} f(x)$ (FINITO)

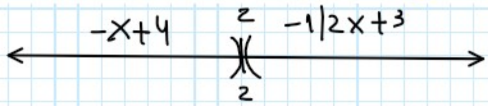
(3) $f(a) = \lim_{x\to a} f(x)$

DISCONTINUA
- ESENCIAL no se cumple COND (2)
- EVITABLE se cumple COND. (2)

EJERCICIO 5:

Estudiar la continuidad de y graficar En $x=2$

$$f(x)=\begin{cases}-x+4 & x<2\\ 2 & x=2\\ -\frac{1}{2}x+3 & x>2\end{cases}$$



Analizo en $x=2$

(1) ¿$\exists f(2)$? $\qquad f(2)=2$ ✓

(2) $L_1 = \lim_{x\to 2^+} -\frac{1}{2}x+3 = 2$

(2) $L_1 = \lim\limits_{x \to 2^+} -\frac{1}{2}x + 3 = 2$

$L_2 = \lim\limits_{x \to 2^-} -x + 4 = 2$

$L_1 = L_2$

$\exists \lim\limits_{x \to 2} f(x) = 2$

(3) $\underset{2}{f(2)} = \underset{2}{\lim\limits_{x \to 2} f(x)}$

f es CONTINUA en $x = 2$

- **$f(x) = -x + 4, \quad (x < 2)$**
- A = (2, 2)
- **$g(x) = \frac{-1}{2} x + 3, \quad (x > 2)$**