# Derivadas

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$f(x) = (2x-3)^2$ FUNCIÓN DERIVADA. $\boxed{f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{[2(x+\Delta x)-3]^2 - (2x-3)^2}{\Delta x}$$

C. AUX. $f(x+\Delta x) = [2(x+\Delta x)-3]^2$

"$\frac{0}{0}$" ind.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(2x+2\Delta x-3)^2 - (2x-3)^2}{\Delta x}$$

$(2x)^2 = 2^2 . x^2 = 4x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(2x+2\Delta x-3)(2x+2\Delta x-3) - (4x^2+2.2x.(-3)+(-3)^2)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{4x^2} + 4x\Delta x \cancel{-6x} + 4x\Delta x + 4\Delta x^2 - 6\Delta x \cancel{-6x} - 6\Delta x \cancel{+9} \cancel{-4x^2} \cancel{+12x} \cancel{-9}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{8x\Delta x - 12\Delta x + 4\Delta x^2}{\Delta x}$$ "$\frac{0}{0}$"

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(8x-12+4\Delta x)}{\cancel{\Delta x}}$$ (Factor común $\Delta x$)

$$f'(x) = \lim_{\Delta x \to 0} 8x - 12 + 4\Delta x$$

$$\boxed{f'(x) = 8x - 12}$$

Reglas de derivación.

$f(x)$ y $g(x)$ 2 funciones.

$[f(x) \pm g(x)]' = f(x)' \pm g'(x)$

$[k f(x)]' = k . f(x)'$

$[f(x) . g(x)]' = f'(x) . g(x) + f(x) . g'(x)$

$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) . g(x) - f(x) . g'(x)}{[g(x)]^2}$

En TABLA
$n \in \mathbb{N}$
$k \in \mathbb{R}$

Ojo No es lo mismo.
$7^x \neq x^7$

9-1 8

$[\;\;]$ $[g(x)]$ $m \in \mathbb{N}$ $k \in \mathbb{R}$ $7^x \neq x^7$

1) $f(x) = x^9 \rightarrow f'(x) = 9 \cdot x^{9-1} \rightarrow f'(x) = 9 \cdot x^8$

2) $f(x) = 7^x + 5 \rightarrow f'(x) = (7^x)' + (5)' \rightarrow f'(x) = 7^x \cdot \ln 7 + 0 \rightarrow f'(x) = 7^x \ln 7.$

3) $f(x) = x^{17} - 39 \rightarrow f'(x) = (x^{17})' - (39)' \rightarrow f'(x) = 17 \cdot x^{16}$

4) $f(x) = x^3 + 3^x \rightarrow f'(x) = (x^3)' + (3^x)' \rightarrow f'(x) = 3x^2 + 3^x \ln 3$

5) $f(x) = x^{12} - x^2 + x^6$ $\quad f'(x) = 12x^{11} - 2x + 6x^5$

6) $f(x) = \frac{x^{12}}{x^3+1}$

7) $f(x) = \frac{x^3}{x^2-1}$

8) $f(x) = \frac{x^2-4}{x^2+1}$

9) $f(x) = \frac{x^2+2}{x^3-1}$

10) $f(x) = \frac{3x}{\ln(x)}$

11) $f(x) = \frac{2\ln(x)}{x}$

$f(x) = 3x^{11} - 7x^3 + 8$

$f'(x) = (3x^{11})' - (7x^3)' + (8)'$

$f'(x) = 3(x^{11})' - 7(x^3)' + (8)'$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}.$$

6)

$$f(x) = \frac{x^{12}}{x^3+1}$$

$$f'(x) = \frac{12x^{11} \cdot (x^3+1) - x^{12} \cdot 3x^2}{(x^3+1)^2}$$

$$f'(x) = \frac{12x^{14} + 12x^{11} - 3x^{14}}{(x^3+1)^2}.$$

$$\boxed{f'(x) = \frac{9x^{14} + 12x^{11}}{(x^3+1)^2}}$$

7) $f(x) = \frac{x^3}{x^2-1}$

$$f'(x) = \frac{(x^3)'(x^2-1) - x^3 \cdot (x^2-1)'}{(x^2-1)^2}.$$

$$f'(x) = \frac{3x^2 \cdot (x^2-1) - x^3 \cdot (2x)}{(x^2-1)^2}$$

$$f'(x) = \frac{3x^4 - 3x^2 - 2x^4}{(x^2-1)^2}$$

$$\boxed{f'(x) = \frac{x^4 - 3x^2}{(x^2-1)^2}.}$$

$$f'(x) = \frac{x^4 - 3x^2}{(x^2-1)^2}$$

$$f'(x) = \frac{x^2 \cdot (x^2-3)}{(x^2-1)^2}$$

8) $f(x) = \frac{x^2-4}{x^2+1}$

$$f'(x) = \frac{(x^2-4)'(x^2+1) - (x^2-4)\cdot(x^2+1)'}{(x^2+1)^2}$$

$$f'(x) = \frac{2x(x^2+1) - (x^2-4)\cdot 2x}{(x^2+1)^2}$$

Con distributiva.

$\cancel{2x^3} + 2x - \cancel{2x^3} + 8x$

$$f'(x) = \frac{2x\left[\cancel{x^2}+1-\cancel{x^2}+4\right]}{(x^2+1)^2}$$

$$f'(x) = \frac{2x \cdot 5}{(x^2+1)^2}$$

$$f'(x) = \frac{10x}{(x^2+1)^2}$$

10) $f(x) = \frac{3x}{ln(x)}$

$$f'(x) = \frac{(3x)'\ln(x) - 3x\cdot(\ln(x))'}{(\ln(x))^2}$$

$$f'(x) = \frac{3\cdot 1\cdot \ln(x) - 3\cancel{x}\cdot\frac{1}{\cancel{x}}}{\ln^2(x)}$$

$$f'(x) = \frac{3\ln x - 3}{\ln^2(x)}$$

$$f'(x) = \frac{3(\ln(x)-1)}{\ln^2(x)}$$

$\frac{3\cancel{\ln x}}{\ln^{\cancel{2}} x} - \frac{3}{\ln^2 x}$ SI

$\frac{3\cancel{\ln x} - 3}{\ln^{\cancel{2}} x}$ NO

$\ln(x)$

22) $f(x) = \frac{x^2+5x+6}{x+3} + (x^2+4)\cdot(x^2-4)$

Operación principal $+$

$$f'(x) = \left(\frac{x^2+5x+6}{x+3}\right)' + \left((x^2+4).(x^2-4)\right)'$$

$$f'(x) = \underbrace{\frac{(x^2+5x+6)'(x+3) - (x^2+5x+6)(x+3)'}{(x+3)^2}}_{\text{COCIENTE}} + \underbrace{(x^2+4)'(x^2-4) + (x^2+4).(x^2-4)'}_{\text{PRODUCTO}}$$

$$f'(x) = \frac{(2x+5)(x+3) - (x^2+5x+6).1}{(x+3)^2} + (2x)(x^2-4) + (x^2+4).2x$$

$$f'(x) = \frac{2x^2+6x+\cancel{5x}+15-x^2-\cancel{5x}-6}{(x+3)^2} + 2x\left[x^2\cancel{-4}+x^2\cancel{+4}\right]$$

$$f'(x) = \frac{x^2+6x+9}{(x+3)^2} + 2x.2x^2$$

$x^2+6x+9 = (x+3)^2$

$$f'(x) = 1 + 4x^3$$

23) $f(x) = \overbrace{\frac{2x}{\ln(x)}}^{1} + \overbrace{2.e^x.sen(30°)}^{2} - \overbrace{\frac{2x\cos(30°)}{\sqrt{3}.\frac{\ln x}{2}}}^{3}$

CONSTANTE

$$\frac{2\cos(30)}{\frac{\sqrt{3}}{2}} \quad \underbrace{\frac{x}{\ln x}}_{\text{cociente}}$$

$$f'(x) = \underbrace{(1)'}_{\text{COCIENTE}} + \underbrace{(2)'}_{k f'} - \underbrace{(3)'}_{k\text{ . COCIENTE}}$$

$2.\frac{x}{\ln x}$

$\cos(30°) = \frac{\sqrt{3}}{2}$

$$f(x) = 2\frac{x}{\ln x} + 2\,sen(30°)\,e^x - \frac{2\cancel{\cos(30°)}^{\sqrt{3}/2}}{\cancel{\frac{\sqrt{3}}{2}}} . \frac{x}{\ln x}$$

$$f(x) = \cancel{2\frac{x}{\ln x}} + 2\underbrace{sen(30°)}_{\frac{1}{2}}\,e^x \cancel{- 2\frac{x}{\ln x}}$$

$$f(x) = \cancel{2}.\frac{1}{\cancel{2}}.e^x$$

$$f(x) = e^x$$

$$f'(x) = e^x$$

$f'(x) = e^x$

24) $f(x) = \frac{x-1}{\sqrt{x}+1} - (\sqrt{x} - 1) - \frac{e^x+1}{\cancel{e^x}} \cdot \frac{1}{\frac{1}{\cancel{e^x}}}$

$f'(x) = \left(\frac{x-1}{\sqrt{x}+1}\right)' - (\sqrt{x} - 1)' - (e^x + 1)'$

$f'(x) = \frac{(x-1)'(\sqrt{x}+1) - (x-1)(\sqrt{x}+1)'}{(\sqrt{x}+1)^2} - \frac{1}{2\sqrt{x}} - e^x$

$f'(x) = \frac{1.(\sqrt{x}+1) - (x-1) \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} - \frac{1}{2\sqrt{x}} - e^x$

$f'(x) = \frac{\sqrt{x}+1 - \frac{x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} - \frac{1}{2\sqrt{x}} - e^x$

$f'(x) = \frac{\sqrt{x}+1 - \frac{x+1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} - \frac{1 \cdot \sqrt{x}}{2\sqrt{x}\sqrt{x}} - e^x$

$f'(x) = \ldots\ldots - \frac{\sqrt{x}}{2x} - e^x$

25) $f(x) = \frac{2^{x+2} . 2^{x-3}}{4^x . 4^{x-1}} \cdot 0{,}5^{-x+3} \cdot (\sqrt{8})^x$

$2^{x+2} = 2^x . 2^2 = 4 . 2^x$

$\sqrt{8} = 8^{1/2}$

$f(x) = \frac{2^x . 2^2 . 2^x . 2^{-3}}{4^x . 4^x . 4^{-1}} \cdot 0{,}5^{-x} . (0{,}5)^3 \cdot 8^{1/2 x}$

(*) $2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$

$f(x) = \frac{4 \cdot \frac{1}{8} . (2^x)^2}{\frac{1}{4} . (4^x)^2} \cdot \left[\left(\frac{1}{2}\right)^{-1}\right]^x . \left(\frac{1}{2}\right)^3 \cdot \left[8^{1/2}\right]^x$

$(4^x)^2 = \left[(2^2)^x\right]^2 = \left[(2^x)^2\right]^2 = (2^x)^4$

(*) $\frac{4^{-1}}{1} = \frac{1}{4}$

$f(x) = 2 \frac{(2^x)^2}{(2^x)^4} \cdot 2^x \cdot \frac{1}{8} \cdot (8^{1/2})^x$

(*) $(0{,}5)^{-x} = \left[(0{,}5)^{-1}\right]^x = \left[\left(\frac{1}{2}\right)^{-1}\right]^x$

$\left[(2^3)^{1/2}\right]^x$

$f(x) = \cancel{2} \cdot \frac{1}{(2^x)^{\cancel{2}}} \cdot \cancel{2^x} \cdot \frac{1}{\cancel{8}_{4}}$

$8 = 2^3$

$2^{3 . \frac{1}{2} . x}$

$f(x) = \frac{1}{(2^x)^1} \cdot \frac{1}{4} \cdot (2^x)^{3/2}$

$$f(x) = \frac{1}{(2^x)'} \cdot \frac{1}{4} \cdot (2^x)^{3/2}$$

$$2^{\cdot\frac{\cdot}{2}}$$

$$f(x) = \frac{1}{4} \cdot (2^x)^{3/2 - 1}$$

$$f(x) = \underbrace{\frac{1}{4}}_{k} \cdot \underbrace{(2^x)^{\frac{1}{2}}}_{f(x)}$$ Regla cadena.

$$(2^x)^{1/2} = \sqrt{2^x}$$

$$f(x) = \sqrt{\phantom{x}}$$

$$g(x) = 2^x$$

$$\left[\ln\left(\mathrm{sen}(e^x)\right)\right]^2$$

$$f(x) = \frac{1}{4} \cdot (2^{1/2})^x$$

$$f(x) = \frac{1}{4} (\sqrt{2})^x$$

$$f'(x) = \frac{1}{4} \cdot (\sqrt{2})^x \cdot \ln\sqrt{2}.$$

Regla de la cadena.

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

Ej: $f(x) = \underbrace{(2x-3)}_{g}^{2\,f}$ Ej. realizado por definición

$$f'(x) = 2(2x-3) \cdot 2$$

$$f'(x) = 4(2x-3)$$

$$f'(x) = 8x - 12 \checkmark$$

28) $f(x) = \frac{x^3}{2} \cdot \sqrt[3]{6x^2 \cdot \sqrt[4]{\frac{32x^2}{243}}}$

Factorizo

$6 = 2.3.$

$32 = 2^5$

$243 = 3^5$

$$\begin{array}{r|l} 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$$

$$f(x) = \frac{x^3}{2} \cdot \sqrt[3]{2.3\,x^2 \cdot \sqrt[4]{\frac{2^5 . x^2}{3^5}}}$$

$$\frac{2^5}{3^5} = \left(\frac{2}{3}\right)^5$$

$$f(x) = \frac{x^3}{2} \cdot \sqrt[3]{2.3.x^2 \cdot \sqrt[4]{\left(\frac{2}{3}\right)^5 . x^2}}$$

$$\ldots \frac{x^3}{\phantom{2}} \cdot \sqrt[3]{\ldots^2 \left[\ldots \ldots^2\right]^{1/4}}$$

$$f(x)=\frac{x^3}{2}\cdot\sqrt[3]{2\cdot3\cdot x^2\cdot\left[\left(\frac{2}{3}\right)^5\cdot x^2\right]^{1/4}}$$

$$f(x)=\frac{x^3}{2}\cdot\sqrt[3]{2\cdot3\cdot x^2\left[\left(\frac{2}{3}\right)^5\right]^{1/4}\cdot\left(x^2\right)^{1/4}}$$

$$\cancel{2}\cdot\frac{1}{\cancel{4}_2}=\frac{1}{2}$$

$$f(x)=\frac{x^3}{2}\cdot\sqrt[3]{2\cdot3\cdot x^2\cdot\left(\frac{2}{3}\right)^{5/4}\cdot x^{1/2}}$$

$$f(x)=\frac{x^3}{2}\cdot\sqrt[3]{2\cdot3\cdot\frac{2^{5/4}}{3^{5/4}}\cdot x^{5/2}}$$

$$f(x)=\frac{x^3}{2}\cdot\sqrt[3]{2^{9/4}\cdot3^{-1/4}\cdot x^{5/2}}$$

$$f(x)=\frac{x^3}{2}\cdot\left(2^{9/4}\cdot3^{-1/4}\cdot x^{5/2}\right)^{1/3}$$

$$f(x)=\frac{x^3}{2}\left[\left(2^{9/4}\right)^{1/3}\cdot\left(3^{-1/4}\right)^{1/3}\cdot\left(x^{5/2}\right)^{1/3}\right]$$

$$f(x)=\frac{x^3}{2}\cdot2^{3/4}\cdot3^{-1/12}\cdot x^{5/6}$$

$$f(x)=x^{3+\frac{5}{6}}\cdot2^{3/4-1}\cdot\left(\frac{1}{3}\right)^{1/12}$$

$$\frac{1^{1/12}}{3^{1/12}}=\frac{\sqrt[12]{1^1}}{\sqrt[12]{3^1}}=\frac{1}{\sqrt[12]{3}}.$$

$$f(x)=x^{23/6}\cdot2^{-1/4}\cdot\frac{1}{\sqrt[12]{3}}$$

$$f(x)=x^{23/6}\cdot\left(\frac{1}{2}\right)^{1/4}\cdot\frac{1}{\sqrt[12]{3}}$$

$$f(x)=x^{23/6}\cdot\sqrt[4]{\frac{1}{2}}\cdot\frac{1}{\sqrt[12]{3}}.$$

K.