SÁBADO 16 DE MARZO

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Análisis Matemático I

Depto. de Ciencias Exactas

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LÍMITES Parte A

Ejercitación

IND. $\frac{\to 0}{\to 0}$ $\frac{1}{0}$ "$\frac{0}{0}$"

A) Hallar los límites de las siguientes funciones: FACTORIZANDO

1) $\lim\limits_{x\to 1}\left(\frac{x^3-1}{x-1}\right) = 3$

2) $\lim\limits_{x\to 2}\left(\frac{x^4-16}{-x+2}\right) = -32.$

3) $\lim\limits_{x\to 0}\left(\frac{3x^2-5x}{2x^2-x}\right) = 5$

4) $\lim\limits_{x\to 0}\left(\frac{3x^3-4x^2+5x^4}{2x^2-7x^3}\right) = -2.$

5) $\lim\limits_{x\to 2}\frac{x^7-128}{x-2} = 448$

6) $\lim\limits_{x\to 3}\left(\frac{x-3}{x^3-27}\right) =$

7) $\lim\limits_{x\to 3}\left(\frac{x^2-6x+9}{-x^2+7x-12}\right) =^* 0$

(*) 8) $\lim\limits_{x\to 2}\left(\frac{x^2-5x+6}{2x^2-6x+4}\right) = -1/2$

9) $\lim\limits_{x\to -3}\left(\frac{x^2+6x+9}{9-x^2}\right) =$

10) $\lim\limits_{x\to 3}\left(\frac{x^2-7x+12}{-x^2+6x-9}\right) = \infty$

11) $\lim\limits_{x\to 2}\left(\frac{2x^2-8x+8}{-x^3+8}\right) =$

12) $\lim\limits_{x\to 1}\left(\frac{x^3+2x^2+2x-5}{-x^3+6x^2-7x+2}\right) =^* 9/2$

13) $\lim\limits_{x\to 1}\left(\frac{2x^3-6x+4}{x^3+x^2-5x+3}\right) =$

14) $\lim\limits_{x\to 3}\left(\frac{x^3-9x^2+27x-27}{-x^3+5x^2-3x-9}\right) =$

15) $\lim\limits_{x\to 2}\left(\frac{x^3-\frac{7}{2}x^2+2x+2}{x^3-\frac{13}{3}x^2+\frac{16}{3}x-\frac{4}{3}}\right) =^* 3/2$

16) $\lim\limits_{x\to 2}\left(\frac{x^3-8x-x^2+12}{3x^3+8x+4-11x^2}\right) =$

17) $\lim\limits_{x\to 3}\left(\frac{9-x^2}{x-3}\right) =$

18) $\lim\limits_{x\to y}\left(\frac{x^3-y^3}{x-y}\right) = 3y^2$

19) $\lim\limits_{x\to y}\left(\frac{x-y}{x^4-y^4}\right) = \frac{1}{4y^3}$

$\frac{\to 0}{\to 0}$ ind.

B) Hallar los límites de las siguientes funciones: RACIONALIZANDO.

1) $\lim\limits_{x\to 2}\frac{\sqrt{x}-\sqrt{2}}{x^3-8} = \frac{\sqrt{2}}{48}$

2) $\lim\limits_{x\to 0}\frac{1-\sqrt{x+1}}{x} =$

3) $\lim\limits_{x\to 9}\frac{81-x^2}{\sqrt{x}-3} =$

4) $\lim\limits_{x\to -3}\frac{-x-3}{\sqrt{x+3}} =$

5) $\lim\limits_{x\to 4}\frac{3-\sqrt{2x+1}}{3-\sqrt{x+5}} =$

6) $\lim\limits_{x\to 1}\frac{2x-\sqrt{x+3}}{2\sqrt{x+8}-6} =$

7) $\lim\limits_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{x-a} =$ con $a > 0$

8) $\lim\limits_{x\to 0}\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} =$ con $a > 0 \ \wedge \ b > 0$

9) $\lim\limits_{x\to 5}\frac{3-\sqrt{x+4}}{x-5} =$

10) $\lim\limits_{x\to 3}\frac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9} =$

11) $\lim\limits_{x\to 5^-}\frac{x^3-125}{\sqrt{5-x}} =$

12) $\lim\limits_{x\to -2}\frac{\sqrt{18+x}+2x}{\sqrt{6+x}+x} =$

1) $$\lim_{x\to1}\left(\frac{x^3-1}{x-1}\right)=$$

$$\lim_{x\to1}\frac{\overset{1}{x^3}-1}{\underset{1}{x}-1}=\frac{\to0}{\to0}\quad \text{ind.}$$

$$\lim_{x\to1}\frac{(\overset{1}{x^2}+\overset{1}{x}+1)\cancel{(x-1)}}{\cancel{x-1}}=3$$

9 | 3
3 | 3
1 |

$9=3^2=3.3$

EXPRESAR COMO PRODUCTO

RESTA.

DIF. DE CUADRADADOS.

$$x^2-1^2=(x-1).(x+1)$$

$$x^2\cancel{+x}\cancel{-x}-1$$

$$x^2-\underset{3^2}{9}=(x+3).(x-3)$$

RUFFINI RAÍZ

Polinomio completo y ordenado.

	x^3	x^2	x	
	1	0	0	-1
1	↓	1	1	1
	1	1	1	0

$$x^3-1=(x^2+x+1)(x-1)$$

$$(x-\text{RAÍZ})(x^2+x.\text{RAIZ}+(\text{RAÍZ})^2)$$

2) $$\lim_{x\to2}\left(\frac{x^4-16}{-x+2}\right)=$$

$$\lim_{x\to2}\frac{\overset{16}{x^4}-16}{\underset{-2}{-x}+2}=\frac{\to0}{\to0}\quad \text{ind.}$$

x^4-16 → DIF DE CUADR. / RUFFINI / FORMULA.

$$(x^2)^2-(4)^2=(x^2-4).(x^2+4)$$

$$x^4-16=(x-2)(x+2)(x^2+4)$$

$$\lim_{x\to2}\frac{(x-2)(x+2)(x^2+4)}{-x+2}$$

FACTOR COMÚN

$$-x+2=-1.(x-2)$$

$$\lim \frac{\cancel{(x-2)}(\overset{4}{x+2})(\overset{4}{x^2}+4)}{\ldots}\quad \overset{4\cdot 8}{}\quad = -32$$

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)(x^2+4)}{-1\cdot\cancel{(x-2)}} = -32$$

FACTOR COMÚN

$-x+2 = -1\cdot(x-2)$

3) $$\lim_{x\to 0}\left(\frac{3x^2-5x}{2x^2-x}\right) =$$

$$\lim_{x\to 0} \frac{3x^2-5x}{2x^2-x} = \frac{\to 0}{\to 0} \text{ ind.}$$

$$\lim_{x\to 0} \frac{\cancel{x}\cdot(3x-5)}{\cancel{x}\cdot(2x-1)} = 5$$

FACTOR COMÚN

$x\cdot(3x-5) = 3x^2-5x$

$x\cdot(2x-1) = 2x^2-x$

CUADRÁTICA.

$$\underbrace{x_1 = 5/3 \qquad x_2 = 0}_{\text{RAÍCES}}$$

(con calculadora o FÓRMULA RESOLVENTE)

$\otimes$ $a\cdot(x-x_1)\cdot(x-x_2)$

FORMA FACTORIZADA.

$ax^2+bx+c = \otimes$

A

$3\cdot(x-5/3)\,(x-0)$

$(3x-5)\cdot x$

No puedo usar RUFFINI

	3	-5	0
0			

4) $$\lim_{x\to 0}\left(\frac{3x^3-4x^2+5x^4}{2x^2-7x^3}\right) = \frac{\to 0}{\to 0} \text{ ind.}$$

$$\lim_{x\to 0} \frac{\cancel{x^2}(3x-4+5x^2)}{\cancel{x^2}(2-7x)} = -2$$

5) $$\lim \frac{x^7-128}{} = \frac{\to 0}{} \text{ ind.}$$

$x^7 - 2^7$

$(x-2)(x^2+2x+2^2)$ MAL !!

5) $\lim_{x\to 2} \frac{x^7-128}{x-2} = \frac{\to 0}{\to 0}$ ind.

$x-2$

$(x-2)(x^2+2x+2^2)$ MAL !!

RUFFINI

	1	0	0	0	0	0	0	−128
2	↓	2	4	8	16	32	64	128
	1	2	4	8	16	32	64	0

$(x-2)\cdot(x^6+2x^5+4x^4+8x^3+16x^2+32x+64)$

$\lim_{x\to 2} \frac{\cancel{(x-2)}\cdot(x^6+2x^5+4x^4+8x^3+16x^2+32x+64)}{\cancel{x-2}} = 448$

64 64 64 64 64 64

6) $\lim_{x\to 3} \left(\frac{x-3}{x^3-27}\right) =$ 1/27

7) $\lim_{x\to 3} \left(\frac{x^2-6x+9}{-x^2+7x-12}\right) =$*

(6) $\lim_{x\to 3} \frac{x-3}{x^3-27}$ $\frac{\to 0}{\to 0}$ ind

3 27

	1	0	0	−27
3	↓	3	9	27
	1	3	9	0

$(x-3)(x^2+3x+9)$

$\lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(x^2+3x+9)} = \frac{1}{27}$

1

9 9

(7) $\lim_{x\to 3} \frac{x^2-6x+9}{-x^2+7x-12} = \frac{\to 0}{\to 0}$ ind.

9 −18 +9

−9 21

TRINOMIO CUADRADO PERFECTO

$x^2 = x$

$9 = 3$

$2 \cdot x \cdot 3 = 6$

BINOMIO AL CUADRADO

$(x-3)^2$

$(x-3)(x-3)$

$x^2 - 3x - 3x + 9$

$x^2 - 2\cdot 3x + 9$.

$\lim_{x\to 3} \frac{(x-3)^{\cancel{2}}}{-1\cdot\cancel{(x-3)}(x-4)} = 0$

0

−1

Con calculadora
(da 1 sola raíz.

1. $(x-3)(x-3)$

$a=1 \quad b=-6 \quad c=9$

$$x_1,x_2=\frac{-(-6)\pm\sqrt{(-6)^2-4.1.9}}{2.1}$$

$$x_1,x_2=\frac{6\pm\sqrt{36-36}}{2}$$

$x_1=\frac{6+0}{2}$ $\qquad$ $x_2=\frac{6-0}{2}$

$x_1=3$ $\qquad$ $x_2=3$.

$$12)\ \lim_{x\to1}\left(\frac{x^3+2x^2+2x-5}{-x^3+6x^2-7x+2}\right)=* \quad \frac{\to0}{\to0} \text{ ind.}$$

	1	2	2	−5
1		1	3	5
	1	3	5	0

$$\lim_{x\to1}\frac{\cancel{(x-1)}.(x^2+3x+5)}{\cancel{(x-1)}(-x^2+5x-2)}=\frac{9}{2}$$

	−1	6	−7	2
1	↓	−1	5	−2
	−1	5	−2	0

$-x^2=-1.x^2$

$$15)\ \lim_{x\to2}\left(\frac{x^3-\frac{7}{2}x^2+2x+2}{x^3-\frac{13}{3}x^2+\frac{16}{3}x-\frac{4}{3}}\right)=* \quad \frac{\to0}{\to0} \text{ ind.}$$

8, −14, 4, 4, 2

8, $-\frac{52}{3}$, $\frac{32}{3}$, $-\frac{4}{3}$

	1	−7/2	2	2
2	↓	2	−3	−2
	1	−3/2	−1	0
2	↓	2	1	
	1	1/2	0	

$$\lim_{x\to2}\frac{\cancel{(x-2)}(x^2-3/2x-1)}{\cancel{(x-2)}(x^2-7/3x+2/3)}=\frac{\to0}{\to0}$$

4, −3

4, −14, −12

$\lim_{x\to 2} \dfrac{\cancel{(x-2)}(x^2-7/3x+2/3)}{} \to 0$

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x+1/2)}{\cancel{(x-2)}(x-1/3)} = \frac{5/2}{5/3} = \frac{3}{2}$$

	1	-13/3	16/3	-4/3
2	↓	2	-14/3	4/3
	1	-7/3	2/3	0
2	↓	2	-2/3	
	1	-1/3	0	

18) $\lim_{x\to y} \left(\dfrac{x^3-y^3}{x-y}\right) = \dfrac{\to 0}{\to 0}$ ind.

$$\lim_{x\to y} \frac{\cancel{(x-y)}(x^2+yx+y^2)}{\cancel{x-y}} = 3y^2$$

	1	0	0	$-y^3$
y	↓	y	y^2	y^3
	1	y	y^2	0

$(x-y)(x^2+yx+y^2)$

19) $\lim_{x\to y} \left(\dfrac{x-y}{x^4-y^4}\right) = \dfrac{\to 0}{\to 0}$ ind.

$x^4-y^4 = (x^2-y^2)(x^2+y^2)$

$x^4-y^4 = (x-y)(x+y)(x^2+y^2)$

$$\lim_{x\to y} \frac{\cancel{x-y}}{\cancel{(x-y)}\underbrace{(x+y)}_{2y}\underbrace{(x^2+y^2)}_{2y^2}} = \frac{1}{4y^3}$$

10) $\lim_{x\to 3} \left(\dfrac{x^2-7x+12}{-x^2+6x-9}\right) = \dfrac{\to 0}{\to 0}$

$x^2-7x+12 = 1(x-4)(x-3)$

$-x^2+6x-9 = -1.(x-3)^2$

$\lim \dfrac{(x-4)\cancel{(x-3)}}{} = \dfrac{-1}{} = \dfrac{1}{} = (\infty) \quad \dfrac{1}{0.1} = 10$

$$\lim_{x\to 3} \frac{\overset{1}{(x-4)}\cancel{(x-3)}}{-1\underbrace{(x-3)^{\cancel{2}}}_{\to 0}} = \frac{-1}{\to -1\cdot 0} = \frac{1}{\to 0} = \infty \qquad \frac{1}{0{,}1} = 10$$

$$\frac{1}{0{,}01} = 100$$

$$\frac{N^o}{\to 0} \qquad \frac{1}{0{,}001} = 1000$$

B) Hallar los límites de las siguientes funciones:

1) $\lim_{x\to 2} \frac{\overset{\sqrt{2}}{\sqrt{x}}-\sqrt{2}}{\underbrace{x^3}_{8}-8} = \frac{\to 0}{\to 0}$ ind.

$\sqrt{x} = x^{1/2}$

$\frac{1}{x} = x^{-1}$

$(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})$

CONJUGADO

$(\sqrt{x}+9).(\sqrt{x}-9)$

	1	0	0	-8
2		2	4	8
	1	2	4	$\boxed{0}$

DIF. de cuadrados

$$\lim_{x\to 2.} \frac{\sqrt{x}-\sqrt{2}}{x^3-8} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$$

$$\lim_{x\to 2} \frac{(\cancel{\sqrt{x}})^{\cancel{2}}-(\cancel{\sqrt{2}})^2}{(x^3-8)\,(\sqrt{x}+\sqrt{2})}$$

$$\lim_{x\to 2} \frac{\overset{0}{x-2}}{\underbrace{(x^3-8)}_{0}.(\sqrt{x}+\sqrt{2})} \quad \frac{\to 0}{\to 0}$$

$$\lim_{x\to 2.} \frac{\overset{1}{\cancel{x-2}}}{\cancel{(x-2)}\underbrace{(x^2+2x+4)}_{12}.\underbrace{(\sqrt{x}+\sqrt{2})}_{2.\sqrt{2}}} = \frac{1}{24\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{24(\cancel{\sqrt{2}})^{\cancel{2}}} = \frac{\sqrt{2}}{48}$$

2) $\lim_{x\to 0} \frac{\overset{1}{1}-\overset{1}{\sqrt{x+1}}}{\underbrace{x}_{0}} = \frac{\to 0}{\to 0}$

DIF. de cuadrados.

¡ojo! cambio signo

$$\lim_{x\to 0} \frac{(1-\sqrt{x+1})}{x} \cdot \frac{1+\sqrt{x+1}}{1+\sqrt{x+1}} = \lim_{x\to 0} \frac{1^2-(\cancel{\sqrt{x+1}})^{\cancel{2}}}{x\,(1+\sqrt{x+1})}$$

$x\to 0 \quad x \qquad 1+\sqrt{x+1} \qquad x\to 0 \quad x\,(1+\sqrt{x+1})$

$$\lim_{x\to 0}\frac{\cancel{1}\overbrace{-x}^{\to 0}\cancel{-1}}{\underbrace{x}_{0}\,(1+\sqrt{x+1})} = \lim_{x\to 0}\frac{\overset{-1}{-\cancel{x}}}{\cancel{x}\,(\underbrace{1}_{1}+\underbrace{\sqrt{x+1}}_{1})} = -\frac{1}{2}$$

3) $\displaystyle\lim_{x\to 9}\frac{81-\overbrace{x^2}^{81}}{\underbrace{\sqrt{x}}_{3}-3} = \frac{\to 0}{\to 0}$ ind.

3) $\displaystyle\lim_{x\to 9}\frac{81-x^2}{\sqrt{x}-3} =$

4) $\displaystyle\lim_{x\to -3}\frac{-x-3}{\sqrt{x+3}} =$

5) $\displaystyle\lim_{x\to 4}\frac{3-\sqrt{2x+1}}{3-\sqrt{x+5}} =$

6) $\displaystyle\lim_{x\to 1}\frac{2x-\sqrt{x+3}}{2\sqrt{x+8}-6} =$

$$\lim_{x\to 9}\underbrace{\frac{81-x^2}{\sqrt{x}-3}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}}_{\text{DIF. de cuadr.}}$$

$$\lim_{x\to 9}\frac{(81-x^2)\cdot(\sqrt{x}+3)}{(\cancel{\sqrt{x}})^{\cancel{2}}-3^2}$$

$$\lim_{x\to 9}\frac{\overbrace{(81-x^2)}^{\to 0}\cdot(\overbrace{\sqrt{x}+3}^{6})}{\underbrace{x}_{9}-9} = \frac{\to 0}{\to 0}$$

$81-x^2 = (9-x)(9+x)$

9^2-x^2

$$\lim_{x\to 9}\frac{(9-x)(9+x)(\sqrt{x}+3)}{(x-9)}$$

$$\lim_{x\to 9}\frac{\overbrace{-1}^{-1}\,\cancel{(-9+x)}\,\overbrace{(9+x)}^{18}\,\overbrace{(\sqrt{x}+3)}^{6}}{\cancel{x-9}} = -108$$

4) $\displaystyle\lim_{x\to -3}\frac{\overbrace{-x}^{-(-3)}-3}{\sqrt{x+3}} = \frac{\to 0}{\to 0}$ ind.

$\sqrt{x+3} \neq \sqrt{x}+\sqrt{3}$

$$\lim_{x\to -3}\frac{-x-3}{\sqrt{x+3}}\cdot\frac{\sqrt{x+3}}{\sqrt{x+3}}$$

$$x \to -3 \quad \sqrt{x+3} \qquad \sqrt{x+3}$$

$$\lim_{x\to -3} \frac{(-x-3)\cdot(\sqrt{x+3})}{(\sqrt{x+3})^2}$$

$$\lim_{x\to -3} \frac{-1\cdot\cancel{(x+3)}(\sqrt{x+3})}{\cancel{x+3}} = 0$$

5) $\lim_{x\to 4} \frac{3-\sqrt{2x+1}}{3-\sqrt{x+5}} = \frac{\to 0}{\to 0}$ ind.

$$\lim_{x\to 4} \frac{3-\sqrt{2x+1}}{3-\sqrt{x+5}} \cdot \frac{3+\sqrt{2x+1}}{3+\sqrt{2x+1}} \cdot \frac{3+\sqrt{x+5}}{3+\sqrt{x+5}}$$

$$\lim_{x\to 4} \frac{\left[3^2-(\sqrt{2x+1})^2\right](3+\sqrt{x+5})}{\left[3^2-(\sqrt{x+5})^2\right]\cdot(3+\sqrt{2x+1})}$$

$$\lim_{x\to 4} \frac{(9-2x-1)(3+\sqrt{x+5})}{(9-x-5)(3+\sqrt{2x+1})}$$

$$\lim_{x\to 4} \frac{(8-2x)(3+\sqrt{x+5})}{(4-x)(3+\sqrt{2x+1})}$$

$$\lim_{x\to 4} \frac{2\cancel{(4-x)}\overbrace{(3+\sqrt{x+5})}^{6}}{\cancel{(4-x)}\underbrace{(3+\sqrt{2x+1})}_{6}} = \frac{12}{6} = 2$$