Sábado 23 de marzo

C) Hallar el valor de los siguientes límites. En los casos corresponda, aplicar Cambio de Variable:

1) $\lim_{x\to 0} \frac{sen\,(4x)}{5x} =$

2) $\lim_{x\to 0} \frac{sen(x+3)}{x+3} =$

3) $\lim_{x\to 1} \frac{3x-3}{sen(x-1)} =$

4) $\lim_{x\to 0} \frac{tg(x)}{x} =$

5) $\lim_{x\to 0} \frac{cos(x)}{x} =$

6) $\lim_{x\to a} \frac{sen(x-a)}{x-a} =$

7) $\lim_{x\to 0} \frac{sen(x)}{tg(x)} =$

8) $\lim_{x\to 0} \frac{sen\left(\frac{x}{2}\right)}{3x} =$

9) $\lim_{x\to 0} \frac{1}{x\cdot cosec(x)} =$

10) $\lim_{x\to 0} \frac{sen(x)}{cosec(x)} =$

$\lim_{x\to 0} \frac{sen(x)}{x} = 1$ $\lim_{x\to 0} \frac{x}{sen\,x} = 1$

1) $\lim_{x\to 0} \frac{sen\,(\overset{0}{4x})}{\underset{\to 0}{5x}} = \frac{\to 0}{\to 0}$ ind.

$\lim_{x\to 0} \frac{1}{5} \cdot \frac{sen(4x)}{x} = \frac{1}{5} \lim_{x\to 0} \frac{sen(4x)}{x} \cdot \frac{4}{4}$

$\frac{1}{5} \cdot 4 \lim_{x\to 0} \frac{sen(4x)}{4x} = \frac{4}{5} \lim_{x\to 0} \frac{sen(4x)}{4x}$

$\frac{4}{5} \underbrace{\lim_{T\to 0} \frac{sen\,T}{T}}_{1} = \frac{4}{5} \cdot 1 = \frac{4}{5}$

Cambio de variable
$T = 4x$
$x \to 0$
$T \to 0$

2) $\lim_{x\to 0} \frac{\overbrace{sen(\overset{3}{x+3})}^{sen(3)}}{\underset{\to 3}{x+3}} = \frac{sen(3)}{3}$ no esta indeterminado.

3) $\lim_{x\to 1} \frac{\overset{\to 0}{3x-3}}{\underbrace{sen(\overset{0}{x-1})}_{\to 0}} = \frac{\to 0}{\to 0}$ ind.

CAMBIO VARIABLE
$T = x-1$
$x \to 1$
$T \to 0$

$\lim_{x\to 1} \frac{3(x-1)}{sen(x-1)} = 3 \lim_{x\to 1} \frac{x-1}{sen(x-1)}$

$= 3 \cdot \underbrace{\lim_{T\to 0} \frac{T}{sen\,T}} = 3 \cdot 1 = 3$

$\underbrace{\lim_{T\to 0} \frac{sen\,T}{T}}_{1}$

4) $\lim_{x\to 0} \frac{\overbrace{tg(\overset{0}{x})}^{\to 0}}{x} = \frac{\to 0}{\to 0}$

$tg\,x = \frac{sen\,x}{cos\,x}$


$\lim_{x\to 0} tg\,x \cdot \frac{1}{x} = \lim_{x\to 0} \frac{sen\,x}{cos\,x} \cdot \frac{1}{x} = \lim_{x\to 0} \frac{sen\,x}{x} \cdot \frac{1}{cos\,x}$

$= \underbrace{\lim_{x\to 0} \frac{sen\,x}{x}}_{1} \cdot \lim_{x\to 0} \frac{1}{\underbrace{cos\,x}_{1}} = 1 \cdot 1 = 1$

$f(x) = \frac{1}{x}$

$\frac{K}{\to 0} = \infty \; (K \neq 0) \quad \frac{1}{\to 0}$

x	$\frac{1}{x}$
0,1	10
0,01	100
0,001	1000
⋮	⋮
	$+\infty$

x	$\frac{1}{x}$
-0,1	-10
-0,01	-100
-0,001	-1000
⋮	⋮
	$-\infty$

$\lim_{x\to 0^+} \frac{1}{x} = +\infty \quad \lim_{x\to 0^-} \frac{1}{x} = -\infty$

5) $\lim_{x\to 0} \frac{\overbrace{cos(x)}^{\to 1}}{\underbrace{x}_{\to 0}} = \infty$

Para que el límite exista los límites laterales deben ser IGUALES

$\lim_{x\to 0} \frac{1}{x} = \infty$

6) $\lim_{x\to a} \frac{sen(\overbrace{x-a}^{0})}{\underbrace{x-a}_{0}} = \frac{\to 0}{\to 0}$

CAMBIO DE VARIABLE.

$T = x - a$

$x \to a$.

$T \to 0$

$\lim_{T\to 0} \frac{sen\,T}{T} = 1$

7) $\lim_{x\to 0} \frac{\overbrace{sen(x)}^{\to 0}}{\underbrace{tg(x)}_{\to 0}} = \frac{\to 0}{\to 0}$

$\lim_{x\to 0} sen(x) \cdot \frac{1}{tg\,x} \cdot \frac{x}{x} = \lim_{x\to 0} \frac{sen(x)}{x} \cdot \frac{x}{tg\,x} =$

$= \underbrace{\lim_{x\to 0} \frac{sen(x)}{x}}_{1} \cdot \lim_{x\to 0} \frac{x}{tg\,x} = 1 \cdot \lim_{x\to 0} \frac{x}{\frac{sen\,x}{cos\,x}} =$

$x : \frac{sen\,x}{cos\,x}$

$1 \cdot \lim\; x \cdot cos\,x \;\; = 1 \cdot \lim \frac{x}{\;} \cdot cos\,x$

$$1\cdot\lim_{x\to0} x\cdot\frac{\cos x}{\operatorname{sen} x} = 1\cdot\underbrace{\lim_{x\to0}\frac{x}{\operatorname{sen} x}}_{1}\cdot\cos x$$

$$x : \frac{\operatorname{sen} x}{\cos x}$$

$$x\cdot\frac{\cos x}{\operatorname{sen} x}$$

$$1\cdot\underbrace{\lim_{x\to0}\frac{x}{\operatorname{sen} x}}_{1}\cdot\underbrace{\lim_{x\to0}\cos x}_{1} = 1$$

8) $$\lim_{x\to0}\frac{\overbrace{sen\left(\frac{x}{2}\right)}^{\to0}}{\underbrace{3x}_{\to0}} = \frac{\to0}{\to0} \text{ ind.}$$

$$\frac{1}{3}\lim_{x\to0}\frac{\operatorname{sen}\left(\frac{x}{2}\right)}{x\cdot\frac{2}{2}} = \frac{1}{3}\cdot\lim_{x\to0}\frac{\operatorname{sen}\left(\frac{x}{2}\right)}{\frac{x}{2}\cdot 2}$$

$$\frac{1}{3}\cdot\frac{1}{2}\lim_{x\to0}\frac{\operatorname{sen}\left(\frac{x}{2}\right)}{\frac{x}{2}} = \frac{1}{6}\lim_{x\to0}\frac{\operatorname{sen}\left(\frac{x}{2}\right)}{\frac{x}{2}}$$

CAMBIO DE VARIABLE

$T = x/2$

$x\to0$

$T\to0$

$$= \frac{1}{6}\underbrace{\lim_{T\to0}\frac{\operatorname{sen} T}{T}}_{1} = \frac{1}{6}$$

$$\operatorname{cosec} x = \frac{1}{\operatorname{sen} x}$$

9) $$\lim_{x\to0}\frac{1}{\underbrace{x}_{0}\cdot\underbrace{cosec(x)}_{\infty}} =$$

$$\lim_{x\to0}\frac{1}{x\cdot\frac{1}{\operatorname{sen} x}} = \lim_{x\to0}\frac{1}{\frac{x}{\operatorname{sen} x}} = \lim_{x\to0}\frac{\operatorname{sen} x}{x} = 1$$

10) $$\lim_{x\to0}\frac{\overbrace{sen(x)}^{0}}{\underbrace{cosec(x)}_{\infty}} = 0$$

$$\operatorname{sen}^2 x = (\operatorname{sen} x)^2 \neq \operatorname{sen} x^2$$

$$\lim_{x\to0}\frac{\operatorname{sen} x}{\frac{1}{\operatorname{sen} x}} = \lim_{x\to0}\operatorname{sen}^2 x = 0$$

$$\operatorname{sen} x : \frac{1}{\operatorname{sen} x} = \operatorname{sen}^2 x$$

$$x^2 \neq 2^x$$

FUNCIÓN EXPONENCIAL

EXPONENTE

FUNCIÓN EXPONENCIAL

$$\boxed{f:\mathbb{R}\rightarrow\mathbb{R} \;/\; f(x) = \underbrace{a}_{\text{BASE}}^{\overset{\text{EXPONENTE}}{x}} \qquad a>0 \;\wedge\; a\neq 1}$$

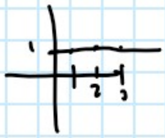
DEFINICIÓN

$$\left(\frac{-1}{2}\right)^{-1} = (-2)^{1} = -2$$

$$\left(\frac{-1}{2}\right)^{-1/2} = (-2)^{1/2} = \sqrt[2]{-2} \quad \nexists$$

$$f(x) = 1^{x}$$

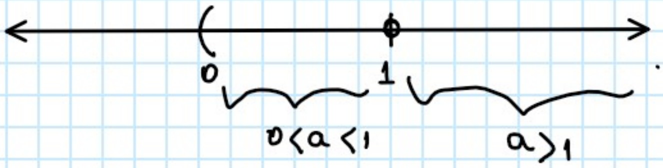
x	f(x)
1	1
2	1
3	1



$$\left(\frac{-1}{2}\right) = a$$

$$f(x) = \left(\frac{-1}{2}\right)^{x}$$

x	f(x)
-1	-2
-1/2	
0	
1	



$$f(x) = 2^{x}$$

x	f(x)
-2	1/4
-1	1/2
0	1
1	2
2	4
⋮	⋮

$a>1$

CRECIENTE

$(0;1)$

$$\lim_{x\to+\infty} 2^{x} = +\infty$$

$$\lim_{x\to-\infty} 2^{x} = 0$$

$$f(x) = \left(\frac{1}{2}\right)^{x}$$

x	f(x)
-2	4
-1	2
0	1
1	1/2
2	1/4

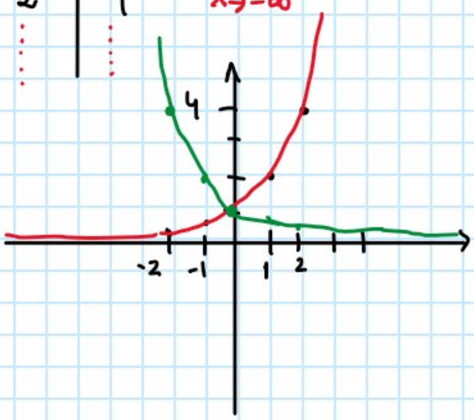
$0<a<1$

DECRECIENTE

$(0;1)$

$$\lim_{x\to+\infty} \left(\frac{1}{2}\right)^{x} = 0$$

$$\lim_{x\to-\infty} \left(\frac{1}{2}\right)^{x} = +\infty$$



A) Hallar los límites de las siguientes funciones:

$e = 2{,}718281\ldots > 1$

A) Hallar los límites de las siguientes funciones: $e = 2{,}718281\ldots\ldots > 1$

1) $\lim\limits_{x \to +\infty} e^{x} =$

2) $\lim\limits_{x \to -\infty} e^{x} =$

3) $\lim\limits_{x \to +\infty} \left(\frac{1}{2}\right)^{x} =$

4) $\lim\limits_{x \to -\infty} \left(\frac{1}{2}\right)^{x} =$

5) $\lim\limits_{x \to +\infty} 2^{-x} =$

6) $\lim\limits_{x \to -\infty} 2^{-x} =$

7) $\lim\limits_{x \to +\infty} e^{x^2} =$

8) $\lim\limits_{x \to -\infty} e^{x^2} =$

9) $\lim\limits_{x \to +\infty} e^{-x^2} =$

10) $\lim\limits_{x \to -\infty} e^{-x^2} =$

11) $\lim\limits_{x \to -\infty} e^{-x} =$

12) $\lim\limits_{x \to +\infty} e^{-x} =$

13) $\lim\limits_{x \to +\infty} e^{x^3} =$

14) $\lim\limits_{x \to -\infty} e^{x^3} =$

$f(x) = e^{x}$ f. exponencial. Base $e > 1$ CREC.

1) $\lim\limits_{x \to +\infty} e^{x} = +\infty$

2) $\lim\limits_{x \to -\infty} e^{x} = 0$

Base $= 1/2$ $\quad 0 < 1/2 < 1$ F. decreciente

3) $\lim\limits_{x \to +\infty} \left(\frac{1}{2}\right)^{x} = 0$

4) $\lim\limits_{x \to -\infty} \left(\frac{1}{2}\right)^{x} = +\infty$

$$2^{-x} = 2^{-1.x} = \left(2^{-1}\right)^{x} = \left(\frac{1}{2}\right)^{x}$$

5) $\lim\limits_{x \to +\infty} 2^{-x} = \lim\limits_{x \to +\infty} \left(\frac{1}{2}\right)^{x} = 0$

6) $\lim\limits_{x \to -\infty} 2^{-x} = \lim\limits_{x \to -\infty} \left(\frac{1}{2}\right)^{x} = +\infty$

7) $\lim\limits_{x \to +\infty} e^{x^2} = +\infty$

x	
10	$e^{(10)^2} =$
100	$e^{(100)^2} =$
1000	$e^{(1000)^2} =$
$\vdots$	$\vdots$
$+\infty$	$+\infty$

8) $\lim\limits_{x \to -\infty} e^{x^2} = +\infty$

x	
-10	$e^{(-10)^2}$
-100	$e^{(-100)^2}$
-100	$\vdots$
$\vdots$	$\vdots$
$-\infty$	$+\infty$

B) Resolver los siguientes límites:

1) $\lim_{x\to 0} 4^{\frac{1}{x}} =$ 2) $\lim_{x\to 0^+} 3^{-\frac{2}{x}} =$ 3) $\lim_{x\to 0^-} e^{-\frac{3}{x}} =$ 4) $\lim_{x\to 2^+} \left(\frac{1}{3}\right)^{\frac{3}{x-2}} =$

(1) $\lim_{x\to 0} 4^{\frac{1}{x}} =$

$L1 = \lim_{x\to 0^+} 4^{\frac{1}{x}\ +\infty} = +\infty$

$L2 = \lim_{x\to 0^-} 4^{\frac{1}{x}\ -\infty} = 0$

$\nexists \lim_{x\to 0} 4^{1/x}$

$L1 \neq L2$

Recordar LÍMITES LATERALES

$\rightarrow 0 \leftarrow$

0,1	$4^{\left(\frac{1}{0,1}\right)} = 1.048.576$
0,01	
0,001	
0^+	$+\infty$

$-0,1 \quad 9,537 \cdot 10^{-7}$

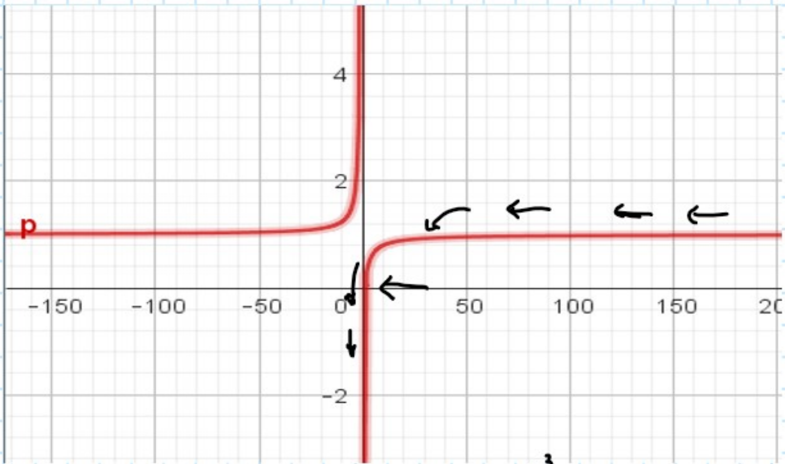
2) $\lim_{x\to 0^+} 3^{-\frac{2}{x}} = 0$ ($-\infty$; 0^+)

$\frac{K}{\to 0} = \infty$

3^{-10} $\quad 3^{\frac{-2}{0,1}}$

3^{-100} $\quad 3^{\frac{-2}{0,01}}$

3^{-1000}



3) $\lim_{x\to 0^-} e^{-\frac{3}{x}} = +\infty$ ($+\infty$)

$e^{\frac{-3}{-0,1}} = 1,06 \cdot 10^{13}$

$e^{\frac{-3}{-0,01}} =$: grande.

4) $\lim_{x\to 2^+} \left(\frac{1}{3}\right)^{\frac{3}{x-2}} = 0$ ($+\infty$; 0^+)

$\left(\frac{1}{3}\right)^{\frac{3}{2,1-2}} = 0,0\ldots$

$\left(\frac{1}{3}\right)^{\frac{3}{2,01-2}} =$

C) Hallar los límites de las siguientes funciones

1) $\lim_{x\to\infty} \frac{3x^2-4x^4}{-x^4-6x^2-x} = 4$

2) $\lim_{x\to\infty} \frac{-x^4+3x^5}{-x^7-6x^6-x^5} = 0$

3) $\lim_{x\to\infty} \frac{x+2x^5}{x^4-6x^2-x} = \infty$

4) $\lim_{x\to+\infty} \frac{x-2}{x^3+2x-5} =$

5) $\lim_{x\to 2^+} \left(\frac{x^2+1}{x-2} - \frac{x^3+x-2}{x^2-2x}\right) =$

6) $\lim_{x\to 1} \left(\frac{3}{x^3-1} - \frac{1}{x-1}\right) =$

7) $\lim_{x\to 3^-} \left(\frac{x^2}{x^2-4} - \frac{2x^2}{x^2-x-6}\right) =$

8) $\lim_{x\to 1} \left(\frac{3}{x-1} - \frac{2}{x^2-1}\right) =$

9) $\lim_{x\to-\infty} \frac{3x-7x^5}{5x^2+2x} =$

10) $\lim_{x\to+\infty} \frac{-3x^4}{3x-6x^2+2} =$

$-4.\infty^4$

1) $\lim_{x\to\infty} \frac{3x^2-4x^4}{-x^4-6x^2-x} = \frac{-\infty}{-\infty} = \text{"}\frac{\infty}{\infty}\text{"}$ ind.

$\frac{x^2}{x^4} = x^{2-4} = x^{-2} = \frac{1}{x^2}$

$\frac{K}{\to\infty} = 0$

$\frac{K}{\to 0} = \infty$

$$\lim_{x\to\infty} \frac{\frac{3x^2}{x^4} - \frac{4x^4}{x^4}}{-\frac{x^4}{x^4} - \frac{6x^2}{x^4} - \frac{x}{x^4}} = \lim_{x\to\infty} \frac{\underset{\to 0}{\frac{3}{x^2}} - \underset{\to -4}{4}}{\underset{-1}{-1} - \underset{\to 0}{\frac{6}{x^2}} - \underset{\to 0}{\frac{1}{x^3}}} = 4$$

2) $\lim_{x\to\infty} \frac{-x^4+3x^5}{-x^7-6x^6-x^5} = \frac{\infty}{\infty}$ ind.

$$\lim_{x\to\infty} \frac{-\frac{x^4}{x^7} + \frac{3x^5}{x^7}}{-\frac{x^7}{x^7} - \frac{6x^6}{x^7} - \frac{x^5}{x^7}} = \lim_{x\to\infty} \frac{\underset{\to 0}{-\frac{1}{x^3}} + \underset{\to 0}{\frac{3}{x^2}}}{-1 - \underset{\to 0}{\frac{6}{x}} - \underset{\to 0}{\frac{1}{x^2}}} = \frac{0}{-1} = 0$$

3) $\lim_{x\to\infty} \frac{x+2x^5}{x^4-6x^2-x} = \frac{\infty}{\infty}$ ind.

$$\lim_{x\to\infty} \frac{\frac{x}{x^5} + \frac{2x^5}{x^5}}{\frac{x^4}{x^5} - \frac{6x^2}{x^5} - \frac{x}{x^5}} = \lim_{x\to\infty} \frac{\underset{\to 0}{\frac{1}{x^4}} + 2}{\underset{\to 0}{\frac{1}{x}} - \underset{\to 0}{\frac{6}{x^3}} - \underset{\to 0}{\frac{1}{x^4}}} = \frac{2}{\to 0} = \infty$$