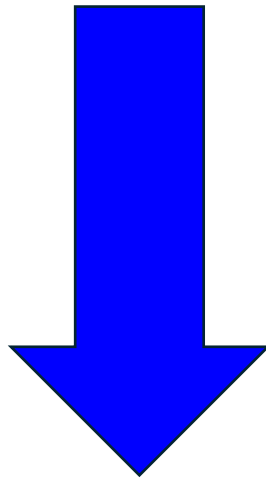


Derivar la siguiente función y llevar el resultado a su mínima expresión

$$f(x) = \frac{e^{\sin x} - e^{-\sin x}}{e^{\sin x} + e^{-\sin x}}$$

Resolución y video



video

$$f(x) = \frac{e^{\sin x} - e^{-\sin x}}{e^{\sin x} + e^{-\sin x}}$$

$$f'(x) = \frac{(e^{\sin x} - e^{-\sin x})' \cdot (e^{\sin x} + e^{-\sin x}) - (e^{\sin x} - e^{-\sin x}) \cdot (e^{\sin x} + e^{-\sin x})'}{(e^{\sin x} + e^{-\sin x})^2} =$$

$$(e^{\sin x})' = e^{\sin x} \cdot \cos x$$

$$(e^{-\sin x})' = e^{-\sin x} \cdot (-\cos x) = -e^{-\sin x} \cdot \cos x$$

$$= \frac{(e^{\sin x} \cdot \cos x + e^{-\sin x} \cdot \cos x) \cdot (e^{\sin x} + e^{-\sin x}) - (e^{\sin x} - e^{-\sin x}) \cdot (e^{\sin x} \cos x - e^{-\sin x} \cdot \cos x)}{(e^{\sin x} + e^{-\sin x})^2} =$$

$$= \frac{\cos x \cdot (e^{\sin x} + e^{-\sin x}) \cdot (e^{\sin x} + e^{-\sin x}) - \cos x \cdot (e^{\sin x} - e^{-\sin x}) \cdot (e^{\sin x} - e^{-\sin x})}{(e^{\sin x} + e^{-\sin x})^2} =$$

$$= \frac{\cos x \cdot [(e^{\sin x} + e^{-\sin x}) \cdot (e^{\sin x} + e^{-\sin x}) - (e^{\sin x} - e^{-\sin x}) \cdot (e^{\sin x} - e^{-\sin x})]}{(e^{\sin x} + e^{-\sin x})^2} =$$

$$(e^{\sin x} + e^{-\sin x}) \cdot (e^{\sin x} + e^{-\sin x}) = (e^{\sin x})^2 + 1 + 1 + (e^{-\sin x})^2 = e^{2 \sin x} + 2 + e^{-2 \sin x}$$

$$e^{\sin x} \cdot e^{-\sin x} = e^{\sin x + (-\sin x)} = e^0 = 1$$

$$\begin{aligned} (e^{\sin x} - e^{-\sin x}) \cdot (e^{\sin x} - e^{-\sin x}) &= (e^{\sin x})^2 - e^0 - e^0 + (e^{-\sin x})^2 = \\ &= e^{2 \sin x} - 2 + e^{-2 \sin x} \end{aligned}$$

$$= \frac{\cos x \cdot [e^{2 \sin x} + 2 + e^{-2 \sin x} - (e^{2 \sin x} - 2 + e^{-2 \sin x})]}{(e^{\sin x} + e^{-\sin x})^2} =$$

$$= \frac{\cos x \cdot [e^{2 \sin x} + 2 + e^{-2 \sin x} - e^{2 \sin x} + 2 - e^{-2 \sin x}]}{(e^{\sin x} + e^{-\sin x})^2} =$$

$$= \frac{\cos x \cdot [2 + 2]}{(e^{\sin x} + e^{-\sin x})^2} = \frac{\cos x \cdot [4]}{(e^{\sin x} + e^{-\sin x})^2} = \boxed{\frac{4 \cos x}{(e^{\sin x} + e^{-\sin x})^2}}$$