INTEGRALES INDEFINIDAS

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POR PARTES
$$\begin{cases} \text{I. L. P. E. T (elijo U)} \\ \int u \cdot dv = u \cdot v - \int v \cdot du \end{cases}$$

59) $\int \underbrace{e^x}_{u} \cdot \underbrace{sen(x)\,dx}_{dv} =$

derivo
$u = e^x$
$1 \cdot du = e^x \cdot dx$

$dv = sen(x)\,dx$
$v = -\cos(x)$
integro

$$\int e^x \cdot sen(x) \cdot dx = e^x \cdot -\cos(x) - \int -\cos(x) \cdot e^x \cdot dx$$

$$\int e^x \cdot sen(x) \cdot dx = -e^x \cdot \cos(x) + \int \underbrace{\cos(x)}_{dv} \cdot \underbrace{e^x}_{u} \cdot \underbrace{dx}_{dv}$$

$u = e^x$ $\quad$ $dv = \cos x\,dx$
$du = e^x\,dx$ $\quad$ $v = sen(x)$

$$\int e^x \cdot sen(x) \cdot dx = -e^x \cdot \cos(x) + \left[e^x \cdot sen(x) - \int sen(x) \cdot e^x\,dx \right]$$

$$\int e^x \cdot sen(x) \cdot dx = -e^x \cdot \cos(x) + e^x \cdot sen(x) - \int sen(x) \cdot e^x\,dx$$

$$\int e^x \cdot sen(x) \cdot dx + \int sen(x) \cdot e^x \cdot dx = -e^x \cdot \cos(x) + e^x \cdot sen(x)$$

$$2 \cdot \int e^x \cdot sen(x)\,dx = -e^x \cdot \cos(x) + e^x \cdot sen(x)$$

$$\boxed{\int e^x \cdot sen(x)\,dx = \frac{-e^x \cdot \cos(x) + e^x \cdot sen(x)}{2} + c}$$

POR SUSTITUCIÓN

42) $\int [x^2 + 1] \cdot (x-1)^{\frac{2}{3}}\,dx =$

$$\int \left[x^2 \cdot (x-1)^{2/3} + (x-1)^{2/3} \right] dx$$

$$\int x^2 \cdot (x-1)^{2/3}\,dx + \int (x-1)^{2/3}\,dx$$

$h = x-1 \rightsquigarrow h+1 = x$
$dh = dx$

$$\int x^2 \cdot h^{2/3} \cdot dh \quad + \int h^{2/3}\, dx$$

$$\int (h+1)^2 \cdot h^{2/3} \cdot dh \quad + \frac{3}{5} h^{5/3} \quad + c$$

$$\int (h^2+2h+1)\ h^{2/3} \cdot dh \quad + \frac{3}{5} (x-1)^{5/3} \quad + c$$

$$\int (h^{8/3} + 2h^{5/3} + h^{2/3})\, dh$$

$$\int h^{8/3}\, dh + 2\int h^{5/3}\, dh + \int h^{2/3}\, dh$$

$$\frac{3}{11} \cdot h^{11/3} + \cancel{2} \cdot \frac{3}{\cancel{8}_4} h^{8/3} + \frac{3}{5} h^{5/3}$$

$$\frac{3}{11}(x-1)^{11/3} + \frac{3}{4}(x-1)^{8/3} + \underbrace{\frac{3}{5}(x-1)^{5/3} + \frac{3}{5}(x-1)^{5/3}} + c$$

$$\frac{3}{11}(x-1)^{11/3} + \frac{3}{4}(x-1)^{8/3} + \frac{6}{5}(x-1)^{5/3} + c.$$

42) $\int [x^2+1] \cdot (x-1)^{\frac{2}{3}}\, dx =$

$h = x-1 \rightarrow h+1 = x$

$dh = dx$

$$\int [(h+1)^2+1] \cdot h^{2/3} \cdot dh.$$

$$\int ((h+1)^2 \cdot h^{2/3} + h^{2/3})\, dh$$

$$\int ((h^2+2h+1) \cdot h^{2/3} + h^{2/3})\, dh.$$

$$\int (h^{8/3} + 2h^{5/3} + h^{2/3} + h^{2/3})\, dh.$$

$$\int \left(h^{8/3} + 2h^{5/3} + 2h^{2/3}\right) dh$$

$$\int h^{8/3}\, dh + 2\int h^{5/3}\, dh + 2\int h^{2/3}\, dh.$$

$$\frac{3}{11} h^{11/3} + \cancel{2}\cdot\frac{3}{\cancel{8}\,4} h^{8/3} + 2\cdot\frac{3}{5}\cdot h^{5/3} + c.$$

$$\boxed{\frac{3}{11}(x-1)^{11/3} + \frac{3}{4}(x-1)^{8/3} + \frac{6}{5}(x-1)^{5/3} + c.}$$

63) $\int \frac{x+2}{x^2-2x+1} dx =$

$$\underbrace{x^2}_{(x)^2} - \underbrace{2x}_{2.x.1} + \underbrace{1}_{(1)^2} = (x-1)^2 \quad \text{RAÍZ DOBLE. o múltiple}$$

$$\int \frac{x+2}{x^2-2x+1} = \int \frac{A}{(x-1)^2} + \frac{B}{x-1} = \frac{A+B(x-1)}{(x-1)^2}.$$

$$x+2 = A+B(x-1)$$

• Si $x=1$

$$1+2 = A + \cancel{B(1-1)}$$

$$\boxed{3 = A}$$

• Si $x=0$

$$0+2 = 3 + B(0-1)$$

$$2 = 3 - B$$

$$B = 3-2$$

$$\boxed{B=1}$$

$$\int \frac{x+2}{x^2-2x+1} dx = \int \left[\frac{3}{(x-1)^2} + \frac{1}{x-1}\right] dx$$

$$= 3\int \frac{1}{(x-1)^2} dx + \int \frac{1}{x-1} dx$$

$x-1 = T$

$dx = dT$

$$= 3\int \frac{1}{T^2} \cdot dT + \int \frac{1}{T} \cdot dT$$

$$= 3\int T^{-2}\,dT + \ln|T| + c$$

$$= 3 \cdot \frac{T^{-1}}{-1} + \ln|T| + c$$

$$= \boxed{\frac{-3}{x-1} + \ln|x-1| + c}$$

65) $\int \frac{x^3+4x^2-4}{x^2+3x-4}\,dx =$

$$\begin{array}{rrrr|l} x^3 & +4x^2 & +0x & -4 & x^2+3x-4 \ \text{D.} \\ \ominus\ x^3 & +3x^2 & -4x & \downarrow & x+1 \ \text{C} \\ \hline - & x^2 & +4x & -4 & \\ & \ominus\ x^2 & +3x & -4 & \\ \hline & - & x & & \text{R.} \end{array}$$

$$\begin{array}{r|l} 9 & 2 \ \text{D} \\ \text{R}\ 1 & 4 \ \text{C} \end{array}$$

$$\frac{9}{2} = \frac{2.4+1}{2}$$

$$\frac{9}{2} = \frac{\cancel{2}.4}{\cancel{2}} + \frac{1}{2}$$

$$\frac{9}{2} = \underset{\text{C}}{4} + \frac{1\ \text{R}}{2\ \text{D.}}$$

$$\frac{x^3+4x^2-4}{x^2+3x-4} = \overset{\text{C}}{x+1} + \frac{\overset{\text{R.}}{x}}{x^2+3x-4\ \text{D.}}$$

$$\int \frac{x^3+4x^2-4}{x^2+3x-4}\,dx = \int\left(x+1 + \frac{x}{x^2+3x-4}\right)dx$$

$$= \overbrace{\int x.dx + \int 1.dx}^{\text{inmediatas}} + \int \frac{x}{x^2+3x-4}\,dx \ (*)$$

→ ahora el polinomio del numerador tiene menos grado que el del denominador.

D.F.S calculo las raíces para saber en que caso estoy.

$$\underbrace{x_1 = 1 \qquad x_2 = -4}_{\text{Raíces reales SIMPLES}}$$

$$= \frac{1}{2}x^2 + x + (*)$$

$$(*)\ \frac{x}{x^2+3x-4} = \frac{A}{x-1} + \frac{B}{x+4} = \frac{A(x+4)+B(x-1)}{(x-1)(x+4)}$$

$$x = A(x+4) + B(x-1)$$

. Si $x = 1$

$$1 = A(1+4) + \cancel{B(1-1)}$$

$$1 = A \cdot 5$$

$$\boxed{\frac{1}{5} = A}$$

. Si $x = -4$.

$$-4 = \cancel{A(-4+4)} + B(-4-1)$$

$$-4 = B \cdot (-5)$$

$$\frac{-4}{-5} = B$$

$$\boxed{B = 4/5}$$

$$\int \frac{x}{x^2+3x-4}\,dx = \int \left(\frac{1/5}{x-1} + \frac{4/5}{x+4}\right) dx$$

$$= \frac{1}{5}\int \frac{1}{x-1}\,dx + \frac{4}{5}\int \frac{1}{x+4}\,dx$$

$$= \boxed{\frac{1}{5}\ln|x-1| + \frac{4}{5}\ln|x+4| + c}. \quad (*)$$

$$\int \frac{x^3+4x^2-4}{x^2+3x-4}\,dx = \frac{1}{2}x^2 + x + \frac{1}{5}\ln|x-1| + \frac{4}{5}\ln|x+4| + c$$ Rta

SUSTITUCIÓN

21) $\int x \cdot [x^2+1]^{-1}\,dx =$ $\int \frac{x \quad dx}{x^2+1}$

$h = x^2 + 1$

$dh = 2\,x\,dx$

$\frac{dh}{2} = x \cdot dx$

$$\int \frac{1}{h} \cdot \frac{dh}{2} = \frac{1}{2}\int \frac{1}{h}\,dh$$

$$= \frac{1}{2}\ln|h| + c$$

$$= \frac{1}{2}\ln|x^2+1| + c.$$

44) $\int [x^2-3] \cdot (x+2)^6 dx =$

$h = x + 2$ $\xrightarrow{\text{despejo } x}$ $h - 2 = x$

$dh = dx$

$$= \int (x^2-3)(h)^6 \cdot dh.$$

$$= \int \left[(h-2)^2 - 3 \right] . h^6 \, dh$$

$$= \int \left[(h-2)^2 . h^6 - 3h^6 \right] dh$$

$$= \int \left[(h^2 - 4h + 4) . h^6 - 3h^6 \right] . dh.$$

$$= \int (h^8 - 4h^7 + 4h^6 - 3h^6) \, dh.$$

$$= \int (h^8 - 4h^7 + h^6) \, dh$$

$$= \frac{1}{9} h^9 - \frac{\cancel{4}}{\cancel{8}_2} h^8 + \frac{1}{7} h^7 + c$$

$$= \boxed{\frac{1}{9} (x+2)^9 - \frac{1}{2} (x+2)^8 + \frac{1}{7} (x+2)^7 + c}$$

74) $\int \frac{x^2}{x^2+2x+1} \cdot dx =$

$$\begin{array}{rrrr|l} - & x^2 & 0x & 0 & x^2+2x+1 \\ & x^2 & +2x & +1 & 1 \\ \hline & & -2x & -1 & \end{array}$$

$$\int \frac{x^2}{x^2+2x+1} dx = \int \left(1 + \frac{-2x-1}{x^2+2x+1} \right) dx$$

$$= \int 1 . dx + \int \frac{-2x-1}{x^2+2x+1} dx \quad (*)$$

$(x^2+2x+1) = \underbrace{(x+1)^2}_{\text{RAÍCES MÚLTIPLES}}$

$$\boxed{\int \frac{x^2}{x^2+2x+1} dx = x + \frac{-1}{x+1} - 2 \ln |x+1| + c.}$$ Rta

$(*)$ $\frac{-2x-1}{x^2+2x+1} = \frac{A}{(x+1)^2} + \frac{B}{x+1} = \frac{A + B(x+1)}{(x+1)^2}$

$-2x-1 = A + B(x+1)$

• Si $x=-1$

$-2(-1)-1 = A + \cancel{B(-1+1)}$

$2-1 = A$

$\boxed{1 = A}$

• Si $x=0$

$-2\cdot 0-1 = 1 + B(0+1)$

$-1 = 1 + B$

$\boxed{-2 = B}$

$$\int\left(\frac{1}{(x+1)^2} + \frac{-2}{x+1}\right)dx = \int\frac{1}{(x+1)^2}\,dx - 2\int\frac{1}{x+1}\,dx$$

$B = x+1$

$dB = dx$

$$\int\frac{1}{B^2}\cdot dB \quad - 2\ln|x+1| \qquad B^{-2}$$

$$\frac{B^{-1}}{-1} - 2\ln|x+1| + c$$

$$-\frac{1}{B} - 2\ln|x+1| + c$$

$$\boxed{\frac{-1}{x+1} - 2\ln|x+1| + c}$$ (*)

70) $\displaystyle\int\frac{x^2}{\underbrace{(x+1)^2}_{\text{DOBLE}}\underbrace{(x+2)}_{\text{SIMPLE}}}dx =$

$$\frac{x^2}{(x+1)^2(x+2)} = \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{c}{x+2} = \frac{A(x+2)+B(x+1)(x+2)+c(x+1)^2}{(x+1)^2(x+2)}$$

$x^2 = A(x+2) + B(x+1)(x+2) + c(x+1)^2$

• Si $x=-1$

$(-1)^2 = A(-1+2) + \cancel{B(-1+1)(-1+2)} + \cancel{c(-1+1)^2}$

$\boxed{1 = A}$

• Si $x=-2$.

$$(-2)^2 = \cancel{A(-2+2)} + \cancel{B(-2+1)(-2+2)} + C(-2+1)^2$$

$$\boxed{4 = C}$$

• Si $x=0$

$$0^2 = 1(0+2) + B(0+1)(0+2) + 4(0+1)^2$$

$$0 = 2 + 2B + 4$$

$$\frac{-6}{2} = B$$

$$\boxed{-3 = B}$$

$$\int \frac{x^2}{(x+1)^2(x+2)}\,dx = \int\left(\frac{1}{(x+1)^2} + \frac{-3}{x+1} + \frac{4}{x+2}\right)dx$$

$$= \int \frac{1}{(x+1)^2}\,dx - 3\int \frac{1}{x+1}\,dx + 4\int \frac{1}{x+2}\,dx$$

$T = x+1$

$dT = dx$

$$= \int \frac{1}{T^2}\cdot dT - 3\ln|x+1| + 4\ln|x+2| + c.$$

$$= \int T^{-2}\,dT - 3\ln|x+1| + 4\ln|x+2| + c.$$

$$= \frac{T^{-1}}{-1} - 3\ln|x+1| + 4\ln|x+2| + c.$$

$$= \boxed{\frac{-1}{x+1} - 3\ln|x+1| + 4\ln|x+2| + c.}$$

72) $\int \frac{x^3+\frac{9}{2}x^2-\frac{29}{3}x-1}{2x-3}\,dx =$

$$\begin{array}{l|l} x^3 + \frac{9}{2}x^2 - \frac{29}{3}x - 1 & 2x-3 \\ \hline \ominus\; x^3 - \frac{3}{2}x^2 & \frac{1}{2}x^2 + 3x - \frac{1}{3} \\ \hline \quad 6x^2 - \frac{29}{3}x & \\ \ominus\; 6x^2 - 9x & \\ \hline \quad -\frac{2}{3}x - 1 & \end{array}$$

$$-\frac{2}{3}x \quad -1$$

$$-\quad -\frac{2}{3}x \quad +1$$

$$-2$$

$$\frac{x^3+\frac{9}{2}x^2-\frac{29}{3}x-1}{2x-3} = \frac{1}{2}x^2+3x-\frac{1}{3} \qquad -\frac{2}{2x-3}$$

$$\int\left(\frac{1}{2}x^2+3x-\frac{1}{3} \quad -\frac{2}{2x-3}\right)dx = \frac{1}{2}\int x^2\,dx + 3\int x\,dx - \frac{1}{3}\int dx - \int\frac{2}{2x-3}\,dx$$

$$h = 2x-3$$

$$dh = 2\,dx$$

$$\int \frac{1}{h}\cdot dh$$

$$\ln|h|$$

$$\ln|2x-3|$$

$$= \frac{1}{2}\cdot\frac{1}{3}x^3 + 3\cdot\frac{1}{2}x^2 - \frac{1}{3}x - \ln|2x-3| + c$$

$$= \boxed{\frac{1}{6}x^3 + \frac{3}{2}x^2 - \frac{1}{3}x - \ln|2x-3| + c}$$