Ejercicio 66

$$\int \frac{-x^2+3}{x^4-6x^3+9x^2}\,dx$$

Factorizamos el denominador

$$x^4-6x^3+9x^2 = x^2.(x^2-6x+9)$$

$$x^2-6x+9=0$$

$$x_1=3 \quad x_2=3$$

$$x^2-6x+9 = 1(x-3)(x-3) = (x-3)^2$$

$$\frac{-x^2+3}{x^2(x-3)^2} = \frac{A}{x^2}+\frac{B}{x}+\frac{C}{(x-3)^2}+\frac{D}{x-3}$$

$$\frac{-x^2+3}{x^2(x-3)^2} = \frac{A(x-3)^2+Bx(x-3)^2+Cx^2+Dx^2(x-3)}{x^2(x-3)^2}$$

$$\boxed{-x^2+3 = A(x-3)^2+Bx(x-3)^2+Cx^2+Dx^2(x-3)}$$

- $si\ x=0 \quad 3=A.9 \quad A=\frac{1}{3}$
- $si\ x=3 \quad -6=C.9 \quad C=-\frac{2}{3}$
- $si\ x=2 \quad -1=A+B.2+C.4+D.4(-1) \quad \frac{4}{3}=2B-4D$
- $si\ x=1 \quad 2=\frac{1}{3}.4+B.4-\frac{2}{3}+D(-2) \quad 4B-2D=\frac{4}{3}$

$$\begin{cases} 2B-4D=\frac{4}{3} \\ 4B-2D=\frac{4}{3} \end{cases}$$

Hay que resolver el sistema de ecuaciones para hallar los valores:

$$D=-\frac{2}{9} \qquad B=\frac{2}{9}$$

$$\frac{-x^2+3}{x^2(x-3)^2} = \frac{\frac{1}{3}}{x^2}+\frac{\frac{2}{9}}{x}+\frac{-\frac{2}{3}}{(x-3)^2}+\frac{-\frac{2}{9}}{x-3}$$

$$\int \frac{-x^2+3}{x^4-6x^3+9x^2}\,dx = \int \left[\frac{\frac{1}{3}}{x^2}+\frac{\frac{2}{9}}{x}+\frac{-\frac{2}{3}}{(x-3)^2}+\frac{-\frac{2}{9}}{x-3}\right]dx =$$

$$= \frac{1}{3}\int x^{-2}dx+\frac{2}{9}\int \frac{1}{x}dx-\frac{2}{3}\int (x-3)^{-2}dx-\frac{2}{9}\int \frac{1}{x-3}dx =$$

$$= \frac{1}{3} \cdot \frac{x^{-1}}{-1} + \frac{2}{9}\ln|x| - \frac{2}{3} \cdot \frac{(x-3)^{-1}}{-1} - \frac{2}{9}\ln|x-3| + C =$$

$$= \boxed{-\frac{1}{3} \cdot x^{-1} + \frac{2}{9}\ln|x| + \frac{2}{3} \cdot (x-3)^{-1} - \frac{2}{9}\ln|x-3| + C}$$

Ejercicio 68

$$\int \frac{x^2+2}{(x+1)^3(x-2)}\,dx$$

$$\frac{x^2+2}{(x+1)^3(x-2)} = \frac{A}{(x+1)^3} + \frac{B}{(x+1)^2} + \frac{C}{x+1} + \frac{D}{x-2}$$

$$\frac{x^2+2}{(x+1)^3(x-2)} = \frac{A(x-2) + B(x+1)(x-2) + C(x+1)^2(x-2) + D(x+1)^3}{(x+1)^3(x-2)}$$

$$\boxed{x^2+2 = A(x-2) + B(x+1)(x-2) + C(x+1)^2(x-2) + D(x+1)^3}$$

- $si\ x = -1 \quad 3 = A(-3) \quad A = -1$
- $si\ x = 2 \quad 6 = D.27 \quad D = \frac{2}{9}$
- $si\ x = 1 \quad 3 = -1(-1) - 2B - 4C + \frac{16}{9} \qquad \frac{2}{9} = -2B - 4C$
- $si\ x = 0 \quad 2 = -1(-2) + B(-2) + C(-2) + \frac{2}{9} \qquad -\frac{2}{9} = -2B - 2C$

$$\begin{cases} -2B - 4C = \frac{2}{9} \\ -2B - 2C = -\frac{2}{9} \end{cases}$$

$$C = -\frac{2}{9} \qquad B = \frac{1}{3}$$

$$\frac{x^2+2}{(x+1)^3(x-2)} = \frac{-1}{(x+1)^3} + \frac{\frac{1}{3}}{(x+1)^2} + \frac{-\frac{2}{9}}{x+1} + \frac{\frac{2}{9}}{x-2}$$

$$\int \frac{x^2+2}{(x+1)^3(x-2)}dx = \int \left(\frac{-1}{(x+1)^3} + \frac{\frac{1}{3}}{(x+1)^2} + \frac{-\frac{2}{9}}{x+1} + \frac{\frac{2}{9}}{x-2} \right) dx =$$

$$= -\int (x+1)^{-3}dx + \frac{1}{3}\int (x+1)^{-2}dx - \frac{2}{9}\ln|x+1| + \frac{2}{9}\ln|x-2| + C =$$

$$= -\frac{(x+1)^{-2}}{-2} + \frac{1}{3}\cdot\frac{(x+1)^{-1}}{-1} - \frac{2}{9}\ln|x+1| + \frac{2}{9}\ln|x-2| + C =$$

$$\boxed{= \frac{1}{2}\cdot(x+1)^{-2} - \frac{1}{3}\cdot(x+1)^{-1} - \frac{2}{9}\ln|x+1| + \frac{2}{9}\ln|x-2| + C}$$