Derivadas

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C) Derivar las siguientes funciones:

1) $f(x) = x^{sen\,x}$

2) $f(x) = x^{x+1}$

3) $f(x) = [sen\,x]^{\ln x}$

4) $f(x) = [\ln x]^{sen\,x}$

5) $f(x) = x^{e^x}$

6) $f(x) = [ln(x)]^{x^3}$

7) $f(x) = [\cos(x)]^{\sqrt{x}}$

$$\underbrace{f(x)^{g(x)}} \neq x^n \neq a^x$$

PROCEDIMIENTO. PARA DERIVAR UNA FUNCIÓN ELEVADA A OTRA FUNCIÓN.

(1) $f(x) = x^{sen\,x}$

PROP LOG $\lg a^b = b.\log a.$ $\ln a^b = b.\ln a.$

$$\ln f(x) = \ln x^{sen\,x}$$

$$\ln f(x) = sen(x) \cdot \ln(x)$$

$$\frac{1}{f(x)} \cdot f'(x) = [sen(x)]' \cdot \ln(x) + sen(x) \cdot (\ln(x))'$$

$$\frac{f'(x)}{f(x)} = \cos(x) \cdot \ln(x) + sen(x) \cdot \frac{1}{x}$$

$$f'(x) = \left(\cos(x) \cdot \ln(x) + sen(x) \cdot \frac{1}{x}\right) \cdot f(x)$$

$$\boxed{f'(x) = \left(\cos(x) \cdot \ln(x) + sen(x) \cdot \frac{1}{x}\right) \cdot x^{sen\,x}}$$

7) $f(x) = [\cos(x)]^{\sqrt{x}}$

$$\ln f(x) = \ln [\cos(x)]^{\sqrt{x}}$$

$$\ln f(x) = \sqrt{x} \cdot \ln(\cos(x))$$

$$\frac{1}{f(x)} \cdot f'(x) = (\sqrt{x})' \cdot \ln(\cos(x)) + \sqrt{x} \cdot (\ln(\cos(x)))'$$

$$\frac{f'(x)}{f(x)} = \frac{1}{2\sqrt{x}} \cdot \ln(\cos(x)) + \sqrt{x} \cdot \frac{1}{\cos(x)} \cdot (-sen(x))$$

$$f'(x) = \left[\frac{\ln(\cos(x))}{2\sqrt{x}} - \sqrt{x} \cdot tg(x)\right] \cdot f(x)$$

$$\boxed{f'(x) = \left[\frac{\ln(\cos(x))}{2\sqrt{x}} - \sqrt{x} \cdot tg(x)\right] \cdot [\cos(x)]^{\sqrt{x}}}$$

3) $f(x) = [sen\,x]^{\ln x}$

$$\ln f(x) = \ln (sen\,x)^{\ln x}$$

$$\ln f(x) = \ln x \cdot \ln(sen\,x)$$

$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{x} \cdot \ln(sen\,x) + \ln x \cdot \frac{1}{sen\,x} \cdot \cos(x)$$

$$\frac{f'(x)}{f(x)} = \frac{\ln(\operatorname{sen} x)}{x} + \ln x \cdot \operatorname{cotg} x$$

$$f'(x) = \left(\frac{\ln(\operatorname{sen} x)}{x} + \ln x \cdot \operatorname{cotg} x\right) \cdot f(x)$$

$$\boxed{f'(x) = \left(\frac{\ln(\operatorname{sen} x)}{x} + \ln x \cdot \operatorname{cotg} x\right) \cdot (\operatorname{sen} x)^{\ln x}}$$

5) $f(x) = x^{e^x}$

$$\ln f(x) = \ln x^{e^x}$$

$$\ln f(x) = e^x \cdot \ln x$$

$$\frac{1}{f(x)} \cdot f'(x) = e^x \cdot \ln x + e^x \cdot \frac{1}{x}$$

$$f'(x) = \left(e^x \cdot \ln x + \frac{e^x}{x}\right) \cdot f(x)$$

$$\boxed{f'(x) = \left(e^x \cdot \ln x + \frac{e^x}{x}\right) x^{e^x}}$$

REGLA DE L'HOPITAL

a

Si $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$, en donde f y g son derivables en un entorno de a y existe $\lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$ entonces este límite coincide con $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$

$$\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}}$$

Para aplicar la regla de L'H hay que un límite de la forma. $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$ donde "a" puede ser un número o infinito y de esa forma aparecen las indeterminaciones "$\frac{0}{0}$" ó "$\frac{\infty}{\infty}$"

EJEMPLOS

$$\lim_{x\to 1} \frac{\overbrace{\ln(\underbrace{2x^2-1}_{\to 1})}^{\to 0}}{\underbrace{\operatorname{tg}(\underbrace{x-1}_{0})}_{\to 0}} = \frac{\to 0}{\to 0} \text{ ind.}$$

$$\lim_{x\to 1} \frac{\frac{1}{2x^2-1} \cdot 4x}{\frac{1}{\ldots}} = \lim_{x\to 1} \left[\frac{\frac{\overset{4}{4x}}{2x^2-1}\,\big|_1}{1}\right]^{4} = 4$$

$$\lim_{x\to 1} \frac{2x^2-1}{\frac{1}{\cos^2(x-1)}} = \lim_{x\to 1} \frac{2x^2-1}{\frac{1}{\cos^2(x-1)}} = 4 \checkmark$$

(b) $$\lim_{x\to 0} \frac{x - \operatorname{sen} x}{x^3} \quad \text{“}\frac{0}{0}\text{” ind.}$$

$$\lim_{x\to 0} \frac{1-\cos x}{3x^2} = \text{“}\frac{0}{0}\text{” ind. Vuelvo a aplicar L'H.}$$

$$\lim_{x\to 0} \frac{-(-\operatorname{sen} x)}{6x} = \text{“}\frac{0}{0}\text{” ind.}$$

$$\lim_{x\to 0} \frac{\cos x}{6} = \frac{1}{6}$$

$$\frac{1}{x} = x^{-1}$$

(c) $$\lim_{x\to 1} \frac{1-\cos(x-1)}{(\ln x)^2} \quad \text{“}\frac{0}{0}\text{” ind.}$$

$$\lim_{x\to 1} \frac{\operatorname{sen}(x-1)}{2\cdot \ln x \cdot \frac{1}{x}} = \text{“}\frac{0}{0}\text{” ind.} \qquad \lim_{x\to 1} \frac{\operatorname{sen}(x-1)}{\underbrace{2}_{k}\cdot \underbrace{\frac{\ln x}{x}}_{f}}$$

$$\lim_{x\to 1} \frac{\cos(x-1)}{2\cdot\left[\frac{\frac{1}{\cancel{x}}\cdot \cancel{x} - \ln x}{x^2}\right]} = \lim_{x\to 1} \frac{\cos(x-1)}{2\left[\frac{1-\ln x}{x^2}\right]} = \frac{1}{2}$$

(d) $$\lim_{x\to 0} \frac{\cos^2 x - 1}{x^2} \quad \text{“}\frac{0}{0}\text{” ind.}$$

$$\lim_{x\to 0} \frac{\cancel{2}\cdot\cos x\cdot(-\operatorname{sen} x)}{\cancel{2}\cdot x} \quad \frac{\to 0}{\to 0} \text{ ind. “L'H} \qquad \lim_{x\to 0} \frac{-\cos x \cdot \operatorname{sen} x}{x} \qquad -(\cos x\cdot \operatorname{sen} x)$$

$$\lim_{x\to 0} \frac{-(-\operatorname{sen} x\cdot \operatorname{sen} x + \cos x \cdot \cos x)}{1} = \lim_{x\to 0} \operatorname{sen}^2 x - \cos^2 x = -1 \checkmark$$

(2) Indeterminación infinito − infinito

$$\lim_{x\to 0}\left(\operatorname{cotg} x - \frac{1}{x}\right) = \text{“}\infty - \infty\text{”}$$

$$\operatorname{cotg} x = \frac{\cos x}{\operatorname{sen} x}$$

$$\lim_{x\to 0}\left(\frac{\cos x}{\operatorname{sen} x} - \frac{1}{x}\right) \quad \text{Denominador común}$$

$$\lim_{x\to 0}\left(\frac{\cos x}{\operatorname{sen} x}-\frac{1}{x}\right) \qquad \text{Denominador común}$$

$$\lim_{x\to 0}\frac{x\cdot\cos x-\operatorname{sen} x}{x\cdot\operatorname{sen} x} \qquad \text{"}\frac{0}{0}\text{" aplico L'H}$$

$$\lim_{x\to 0}\frac{\cancel{\cos x}+x\cdot(-\operatorname{sen} x)\cancel{-\cos x}}{\operatorname{sen} x+x\cdot\cos x} \qquad \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x\to 0}\frac{-\operatorname{sen} x+x(-\cos x)}{\cos x+\cos x+x\cdot(-\operatorname{sen} x)}=\lim_{x\to 0}\frac{-\operatorname{sen} x-x\cos x}{2\cos x-x\operatorname{sen} x}=\frac{0}{2}=0\ \checkmark$$

(3) "Indeterminación $0\cdot\infty$"

Estas indeterminaciones se transforman de la siguiente manera.

$$\lim_{x\to a} A\cdot B=\lim_{x\to a}\frac{A}{\frac{1}{B}}$$

Ejemplo

$$\lim_{x\to 0^+} x\cdot\ln x=0^+\cdot(-\infty)$$

$$\lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} \qquad \text{ind "}\frac{\infty}{\infty}\text{"}$$

$$\lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{-1}{x^2}}=\lim_{x\to 0^+}\frac{1}{\cancel{x}}\cdot(-x^{\cancel{2}})=\lim_{x\to 0^+}-x=0$$

$$\frac{1}{x}=x^{-1}$$

$$(x^{-1})'=-1\cdot x^{-2}=\frac{-1}{x^2}$$

¿Qué pasa si digo que $B=\ln x$?

$$\lim_{x\to 0^+}\frac{x}{\frac{1}{\ln x}} \qquad \text{"}\frac{0}{0}\text{" ind.}$$

$$\lim_{x\to 0^+}\frac{1}{\frac{-\frac{1}{x}}{\ln^2 x}}=\lim_{x\to 0^+}\frac{1}{\frac{-1}{x\ln^2 x}}=\lim_{x\to 0^+}-x\,\ln^2 x$$

(4) Indeterminaciones 0^0 ∞^0 1^∞

Realizamos las siguientes operaciones.

$$\lim_{x\to a} u^v$$

$$A=u^v$$

$$\ln A=\ln u^v$$

$$A=e^{\ln u^v}$$

$\log_{10}$

$\ln_e x=10$

$e^{10}=x$

$$\lim e^{\ln u^v}$$

$x \to 0$

$$\lim_{x\to 0} e^{\lim_{x\to 0} \ln u^v}$$

$$e^{\lim_{x\to 0} \ln u^v}$$

EJEMPLO

$$\lim_{x\to 0^+} (2x)^{\frac{1}{x}} = 0^{\infty} \text{ ind.}$$

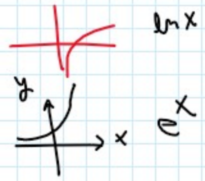
$$e^{\lim_{x\to 0^+} \frac{1}{x} \cdot \ln(2x)} = e^{-\infty} = 0$$

$A = (2x)^{\frac{1}{x}}$

$\ln A = \ln (2x)^{\frac{1}{x}}$

$\ln A = \frac{1}{x} \cdot \ln(2x)$

$A = e^{\frac{1}{x} \cdot \ln(2x)}$



(b) $$\lim_{x\to 0} (\cos 2x)^{3/x^2} = 1^{\infty} \text{ ind.}$$

$A = (\cos 2x)^{3/x^2}$

$\ln A = \frac{3}{x^2} \ln(\cos 2x)$

$A = e^{\frac{3}{x^2} \ln(\cos 2x)}$

$$e^{\lim_{x\to 0} \frac{3}{x^2} \cdot \ln((\cos 2x))} \quad \text{ind. } \infty \cdot 0$$

$$e^{\lim_{x\to 0} \frac{3 \ln(\cos 2x)}{x^2}}$$

$$e^{\lim_{x\to 0} \frac{3 \cdot \frac{1}{\cos 2x} \cdot (-\sin 2x) \cdot 2}{2x}}$$

$$e^{\lim_{x\to 0} \frac{-6 \operatorname{tg}(2x)}{2x}}$$

$$e^{\lim_{x\to 0} \frac{-6 \frac{1}{\cos^2(2x)} \cdot \cancel{2}}{\cancel{2}}} = e^{-6}$$

C. AUXILIAR

$$\lim_{x\to 0} \frac{\ln(\cos 2x)}{\frac{x^2}{3}} = \frac{3 \cdot \ln(\cos 2x)}{x^2} \quad \text{“}\frac{0}{0}\text{”}$$

$\frac{1}{\frac{3}{x^2}} = \frac{x^2}{3}$

$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$

$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$

GUÍA

26) $$\lim_{x\to\infty} \left(1 - \frac{3}{x}\right)^{2x} = 1^{\infty} \text{ ind.}$$

$A = \left(1 - \frac{3}{x}\right)^{2x}$

$\ln A = \ln\left(1 - \frac{3}{x}\right)^{2x}$

$\ln A = 2x \cdot \ln\left(1 - \frac{3}{x}\right)$

$A = e^{2x \cdot \ln\left(1 - \frac{3}{x}\right)}$

$$\lim_{x\to\infty} e^{2x \cdot \ln\left(1-\frac{3}{x}\right)} = e^{\lim_{x\to\infty} 2x \cdot \ln\left(1-\frac{3}{x}\right)}$$

PROP. LÍMITE

$$e^{\boxed{6}}$$

$$\lim_{x\to\infty} 2x \ln\left(1 - \frac{3}{x}\right) = 2 \cdot \lim_{x\to\infty} x \cdot \ln\left(1 - \frac{3}{x}\right) \quad \text{ind. “}\infty \cdot 0\text{”}$$

1 -2

$$\lim_{x\to\infty} 2x \cdot \ln\Big(\underbrace{1-\frac{3}{x}}_{0}\Big) = 2\cdot\lim_{x\to\infty} \underbrace{x}_{\infty} \cdot \ln\underbrace{\Big(1-\frac{3}{x}\Big)}_{0} \quad \text{ind. } \infty\cdot 0$$

$$2\cdot\lim_{x\to\infty} \frac{\overbrace{\ln\left(1-\frac{3}{x}\right)}}{\underbrace{\frac{1}{x}}_{0}} \quad \text{"}\frac{0}{0}\text{"}$$

$$\frac{3}{x} = 3\cdot\frac{1}{x} = 3\cdot x^{-1} = 3\cdot x^{-2} = \frac{3}{x^2}$$

$$\frac{0\cdot x - 3\cdot 1}{x^2} = \frac{-3}{x^2}$$

$$2\cdot\lim_{x\to\infty} \frac{\frac{1}{1-\frac{3}{x}}\cdot\frac{-3}{x^2}}{\frac{-1}{x^2}} = 2\cdot\lim_{x\to\infty} \frac{\frac{-3}{\left(1-\frac{3}{x}\right)x^2}}{\frac{-1}{x^2}} = 2\lim_{x\to\infty} \frac{3}{\left(1-\frac{3}{x}\right)\cancel{x^2}}\cdot\cancel{x^2}$$

$$2\lim_{x\to\infty} \frac{3}{1-\underbrace{\frac{3}{x}}_{0}} = \boxed{6}$$

19) $\lim\limits_{x\to 1^+} \underbrace{(x^2-1)}_{0}\cdot \underbrace{ln(\underbrace{x-1}_{0^+})}_{-\infty} =$ "$0\cdot\infty$" ind.

$$\lim_{x\to 1^+} \frac{\overbrace{\ln(x-1)}^{\infty}}{\underbrace{\frac{1}{\underbrace{x^2-1}_{0}}}_{\infty}} = \text{"}\frac{\infty}{\infty}\text{" ind.}$$

$$\lim_{x\to 1^+} \frac{\frac{1}{x-1}}{\frac{\cancel{0\,(x^2-1)} - 1\cdot 2x}{(x^2-1)^2}} = \lim_{x\to 1^+} \frac{(x^2-1)^2}{(x-1)\cdot(-2x)} \quad \text{"}\frac{0}{0}\text{" L'H} =$$

$$= \lim_{x\to 1^+} \frac{2(x^2-1)\cdot 2x}{-2x + (x-1)\cdot(-2)} = \lim_{x\to 1^+} \frac{\cancel{4}^{\,2}x(x^2-1)}{-\cancel{2}(\underbrace{x+x}_{2x}-1)} = \lim_{x\to 1^+} \frac{2\overbrace{(x^3-x)}^{0}}{\underbrace{1-2x}_{1}} = 0$$