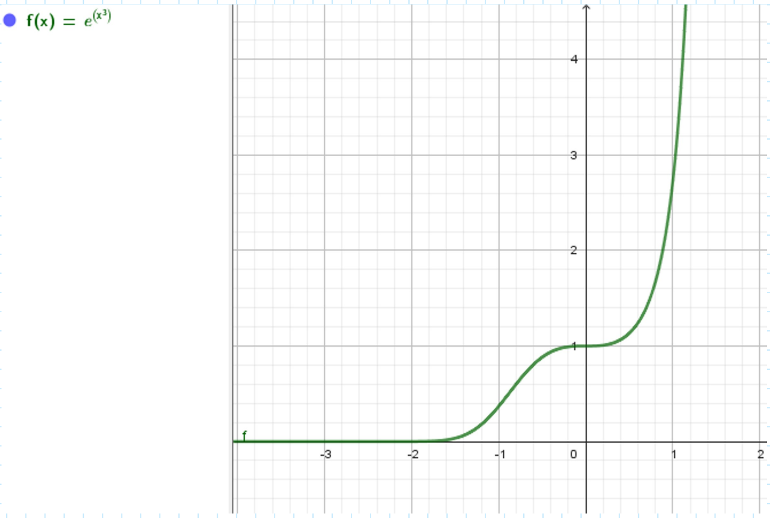
Estudio de funciones

sábado, 18 de mayo de 2024

$32)\ f(x) = e^{x^3}$



$28)\ f(x) = 2x \cdot e^{-x}$

- Dom = $\mathbb{R}$
- ASINTOTAS

AV no posee (no tengo valores excluidos del Dom, NO TENGO CANDIDATO)

AH

$$\lim_{x \to +\infty} \underbrace{2x}_{+\infty} \cdot \overbrace{e^{-x}}^{0} = \text{ind.} \qquad \lim_{x \to -\infty} \underbrace{2x}_{-\infty} \cdot \overbrace{e^{-x}}^{+\infty} = \infty$$

$$\lim_{x \to +\infty} \frac{2x}{e^x} = \frac{\infty}{\infty} \ \text{ind.}$$

$$L'H \quad \lim_{x \to +\infty} \frac{2 \to 2}{\underbrace{e^x}_{\to \infty}} = 0$$

Posee AH en $y=0$ cuando $x \to +\infty$

AO: Probamos para $x \to -\infty$

$$m = \lim_{x \to -\infty} \frac{2\cancel{x} \cdot e^{-x}}{\cancel{x}}$$

$$m = \lim_{x \to -\infty} \frac{2 \to}{\underbrace{e^x}_{0}} = \infty \quad \text{No posee A.O}$$

- CORTE CON LOS EJES.

$\cap x \qquad y=0 \qquad f(x)=0$

$$2 \cdot x \cdot e^{-x} = 0$$

$$\frac{2x}{e^x} = 0$$

NUNCA es CERO

$$2x = 0$$

$$x = 0$$

$\cap y \qquad x=0 \qquad f(0)$

$$f(0) = 2 \cdot 0 \cdot e^{-0}$$

$$f(0) = 0$$

$$\cap y = (0;0)$$

• CORTE CON LOS EJES:

$\cap x$ $\quad y=0 \quad f(x)=0$

$$2 \cdot x \cdot e^{-x} = 0$$

$$\frac{2x}{e^x} = 0$$

NUNCA es CERO

$$2x = 0$$

$$x = 0$$

$$\cap x = (0;0)$$

$\cap y$ $\quad x=0 \quad f(0)$

$$f(0) = 2 \cdot 0 \cdot e^{-0}$$

$$f(0) = 0$$

$$\cap y = (0;0)$$

• CRECIMIENTO / EXTREMOS

$$f(x) = 2x \cdot e^{-x} \quad \text{ó} \quad f(x) = \frac{2x}{e^x}$$

$$f'(x) = \frac{2 \cdot e^x - 2x \cdot e^x}{(e^x)^2}$$

$$f'(x) = \frac{2 \cdot \cancel{e^x}(1-x)}{(e^x)^{\cancel{2}}}$$

$$f'(x) = \frac{2(1-x)}{e^x}$$

$$f'(x) = 0$$

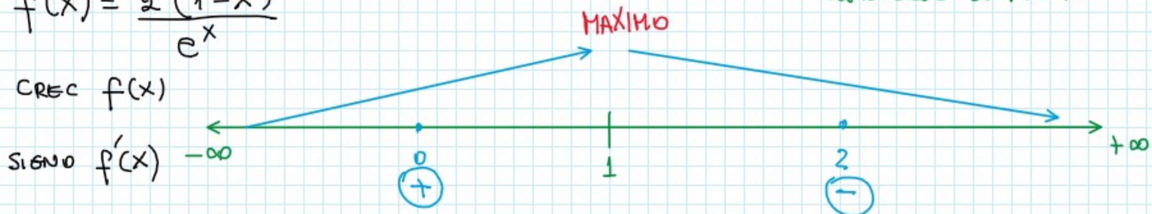
$$\frac{2(1-x)}{e^x} = 0$$

NO SON CERO NUNCA.

$$2 \cdot (1-x) = 0$$

$$1 - x = 0$$

$$\underbrace{1 = x}_{\text{Candidato a Máx/m.}}$$



$$f'(0) = \frac{\overset{+}{2}\,\overset{+}{(1-0)}}{e^0} = +$$

$$f'(2) = \frac{\overset{+}{2}\,\overset{-}{(1-2)}}{\underset{+}{e^2}}$$

$$f(1) = 2 \cdot 1 \cdot e^{-1} = \frac{2}{e} \quad (0{,}73)$$

MÁXIMO $= \left(1; \frac{2}{e}\right)$ Para graficar $(1; 0{,}7)$
CRECE $= (-\infty; 1)$
DECRECE $= (1; +\infty)$

CONCAVIDAD / PUNTO DE INFLEXIÓN

$$f'(x) = \frac{2 \cdot (1-x)}{e^x} = \frac{2 - 2x}{e^x}$$

$$f''(x) = \frac{-2e^x - (2-2x) \cdot e^x}{(e^x)^2} = -2e^x - (2 \cdot (1-x)) \cdot e^x$$

$$f'' = \ldots \cancel{x}\,(1 + 1 - x)$$

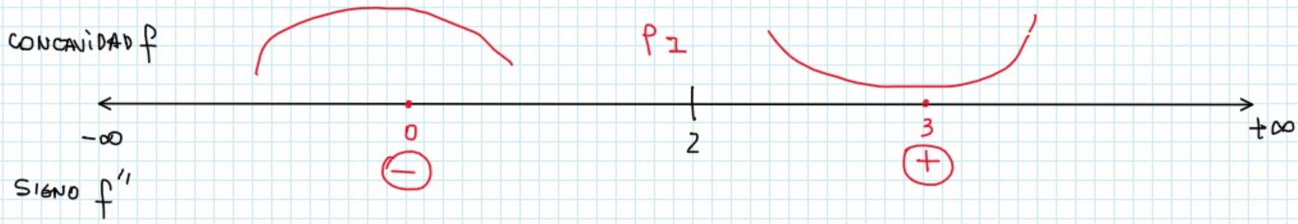
$$(e^x)^2$$

$$f''(x) = \frac{-2\cancel{e^x}(1+1-x)}{(e^x)^{\cancel{2}}}$$

$$f''(x) = \frac{\overset{\neq 0}{-2}\cdot(2-x)}{\underbrace{e^x}_{\text{NO ES CERO}}}$$

$0 = 2-x$

$x = 2$ cand. a PI



$$f''(0) = \frac{\overset{-}{-2}\cdot\overset{+}{(2-0)}}{\underset{+}{e^0}} = \ominus \qquad f''(3) = \frac{\overset{-}{-2}\cdot\overset{-}{(2-3)}}{\underset{+}{e^3}} = \oplus$$

$$f(2) = 2\cdot 2\cdot e^{-2} = \frac{4}{e^2} \qquad 0{,}5$$

$PI = (2\,;0{,}5)$

CONCAVIDAD HACIA ARRIBA: $(2\,;+\infty)$

ABAJO: $(-\infty\,;2)$

RESUMEN

Posee AH en $y=0$ cuando $x \to +\infty$

$\cap x = (0;0)$ $\quad \cap y = (0;0)$

MÁXIMO $= \left(1\,;\frac{2}{e}\right)$ $\quad$ Para graficar $(1\,;0{,}7)$

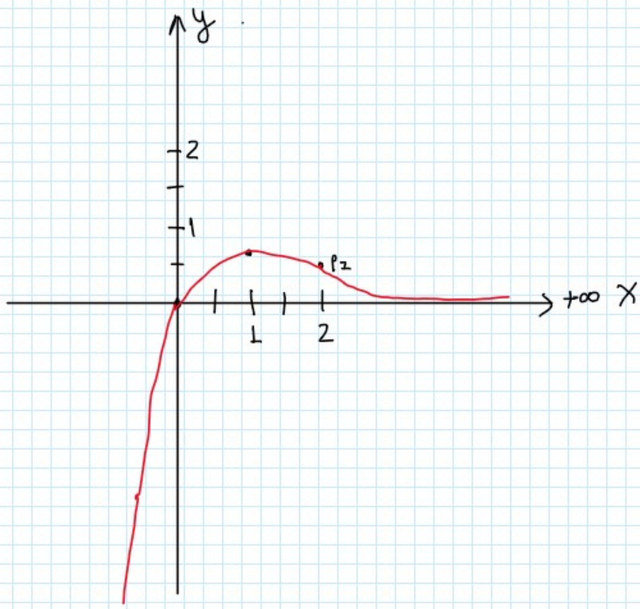
CRECE $= (-\infty\,;1)$

DECRECE $= (1\,;+\infty)$

$PI = (2\,;0{,}5)$

CONCAVIDAD HACIA ARRIBA: $(2\,;+\infty)$

CONCAVIDAD HACIA ABAJO: $(-\infty\,;2)$



34) $f(x) = \frac{x}{\ldots}$

$\boxed{\cdot\ \text{Dom} = (0;1)\cup(1;+\infty)}$ $\qquad \mathbb{R}^+ - \{1\}$

$$34)\ f(x) = \frac{x}{\ln x}$$

- $Dom = (0;1) \cup (1;+\infty)$ $\quad \mathbb{R}^{+} - \{1\}$

$\ln x \neq 0$

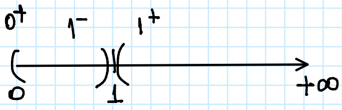
$\ln x = 0$

$e^{0} = x$

$1 = x$

$x \neq 1$

$x > 0$



cond. a AV

$\log_a b = c \Rightarrow a^{c} = b$

$e^{x} = 3$

$\ln e^{x} = \ln 3$

$x \ln e = \ln 3$

$x = \frac{\ln 3}{\underbrace{\ln e}_{1}}$

$x = \ln 3$

- ASÍNTOTAS

AV $\quad \lim\limits_{x\to 1^{+}} \frac{\overset{1}{x}}{\underbrace{\ln x}_{0^{+}}} = +\infty \qquad \lim\limits_{x\to 1^{-}} \frac{\overset{1}{x}}{\underbrace{\ln x}_{0^{-}}} = -\infty$

$\boxed{\text{Posee AV en } x = 1}$

AH $\quad \lim\limits_{x\to +\infty} \frac{\overset{\infty}{x}}{\underbrace{\ln x}_{\infty}} \underset{L'H}{\overset{ind\ \frac{\infty}{\infty}}{=}} \lim\limits_{x\to +\infty} \frac{1}{\frac{1}{x}} = \lim\limits_{x\to +\infty} x = +\infty$

No posee A.H

A.O $\quad m = \lim\limits_{x\to +\infty} \frac{\cancel{x}}{\ln x \cdot \cancel{x}}$

$m = \lim\limits_{x\to +\infty} \frac{1}{\underbrace{\ln x}_{\infty}}$

$m = 0$ $\quad$ No posee A.O

- CORTE CON LOS EJES

$\cap x \quad f(x) = 0$

$\frac{x}{\ln x} = 0$

$x = 0 \notin Dom.$

$\cap y \quad f(0)$

$f(0) = \frac{0}{\ln 0}$

El 0 $\notin$ Dom.

- CRECIMIENTO / EXTREMOS.

$f(x) = \frac{x}{\ln x}$ $\qquad f'(x) = \frac{\ln x - \cancel{x} \cdot 1/\cancel{x}}{(\ln x)^2}$

Ojo!

$$f(x) = \frac{x}{\ln x}$$

$$f'(x) = \frac{\ln x - \cancel{x} \cdot 1/\cancel{x}}{(\ln x)^2}$$

$$f'(x) = \frac{\ln(x) - 1}{\ln^2(x)}$$

¡ojo!

$$\frac{\cancel{\ln x}}{\ln^{\cancel{2}}(x)} - \frac{1}{\ln^2(x)}$$

$$f'(x) = 0$$

$$\frac{\ln(x) - 1}{\ln^2(x)} = 0$$

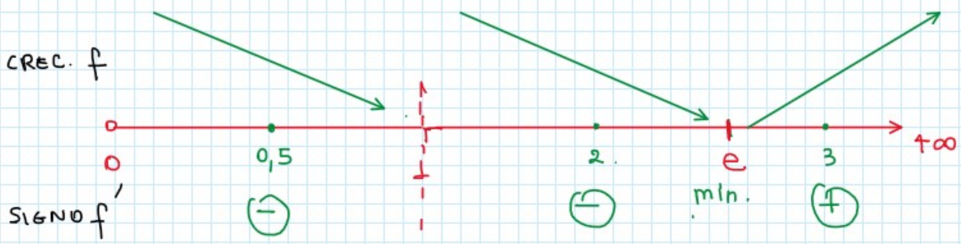
$$\ln(x) - 1 = 0$$

$$\ln(x) = 1$$

$$e^1 = x$$

$$[e = x]$$

Cand. a Máx. / mín.



min = (e ; e)

DECRECE = (0 ; 1) ∪ (1 ; e)

CRECE = (e ; +∞)

$$f'(0{,}5) = \frac{\ln(0{,}5) - 1}{\ln^2(0{,}5)} \quad \frac{-}{+}$$

$$f'(2) = \frac{\ln(2) - 1}{\ln^2(x)} \quad \frac{-}{+}$$

$$f'(\;) = \frac{\ln(3) - 1}{\ln^2(x)} \quad \frac{+}{+}$$

$$f(e) = \frac{e}{\ln e} = e$$

CONCAVIDAD / PI

$$f'(x) = \frac{\ln(x) - 1}{\ln^2(x)}$$

$$f''(x) = \frac{\frac{1}{x} \cdot \ln^2(x) - (\ln(x) - 1) \cdot 2 \ln(x) \cdot \frac{1}{x}}{\ln^4(x)}$$

$$f''(x) = \frac{\frac{1}{x} \cdot \cancel{\ln x} \left[\ln(x) - 2(\ln(x) - 1)\right]}{\ln^{\cancel{4}}(x)}$$

$$f''(x) = \frac{\frac{1}{x} \left[\ln(x) - 2\ln(x) + 2\right]}{\ln^3(x)}$$

$$f''(x) = \frac{\frac{1}{x} \left[2 - \ln(x)\right]}{\ln^3(x)}$$

$\ln_e$ → BASE $\qquad \log_{10}$ → BASE

C

$$\frac{\;}{\ln^3(x)}$$

$$0 = \underbrace{\frac{1}{x}}_{\text{No va a ser cero.}} \cdot \left[2 - \ln x\right]$$

$$2 - \ln x = 0$$

$$2 = \ln x$$

$$\underbrace{e^2 = x}_{\text{cand a PI}}$$

$\ln \to e$ ↓ BASE $\log \to 10$ ↓ BASE

$$\log_a b = c \Rightarrow a^c = b$$

Ejemplo

$$\underbrace{\log_3 9 = 2}_{} \Rightarrow 3^2 = 9.$$

$$\log_{\square} \square \quad \frac{\log 9}{\log 3} = 2$$

CONCAVIDAD f

SIGNO f''

0	0,5	1	2	e^2	8	$+\infty$
	$-$		$+$	PI	$-$	

$$f''(0{,}5) = \frac{\overset{+}{\frac{1}{0{,}5}} \left[\overset{+}{2 - \ln(0{,}5)}\right]}{\ln^3(0{,}5)\,\ominus} = \ominus$$

$$f''(2) = \frac{\overset{+}{\frac{1}{2}} \left[\overset{+}{2 - \ln(2)}\right]}{\ln^3(2)\,\oplus} = \oplus$$

$$f''(8) = \frac{\overset{+}{\frac{1}{8}} \left[\overset{-}{2 - \ln(8)}\right]}{\ln^3(8)\,\oplus} = \ominus$$

$$f(e^2) = \frac{x}{\ln x}$$

$$f(e^2) = \frac{e^2}{2}$$

$$PI = \left(e^2 ; \frac{e^2}{2}\right)$$

CONCAVIDAD POSITIVA: $(1 ; e^2)$

NEGATIVA: $(0 ; 1) \cup (e^2 ; +\infty)$

$$\boxed{\bullet \text{ Dom} = (0;1) \cup (1;+\infty)}$$

$$\mathbb{R}^+ - \{1\}$$

Posee AV en $x = 1$

AV

$$\lim_{x \to 1^+} \frac{x}{\underbrace{\ln x}_{0^+}} = +\infty$$

$$\lim_{x \to 1^-} \frac{x}{\underbrace{\ln x}_{0^-}} = -\infty$$

min = $(e ; e)$ DECRECE = $(0;1) \cup (1;e)$

CRECE = $(e ; +\infty)$

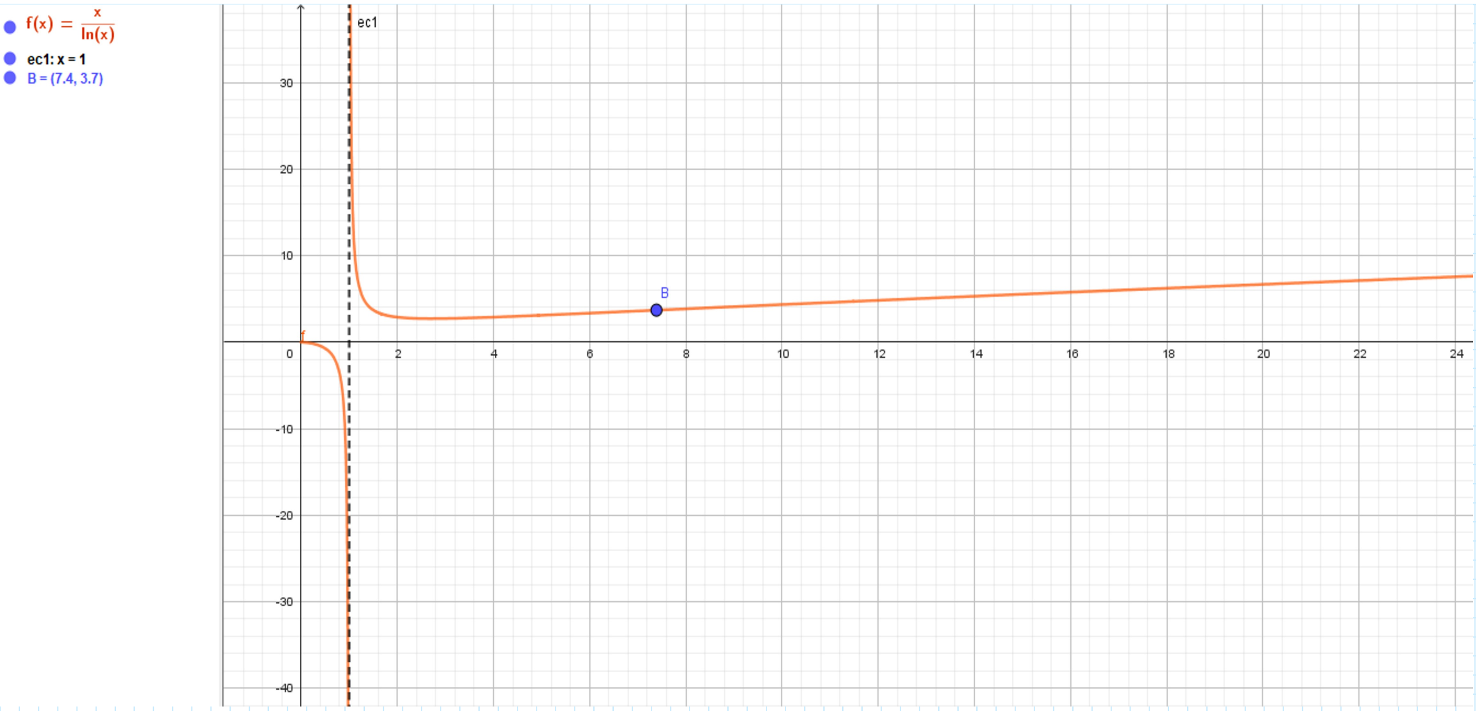
¿Qué pasa cuando $x \to 0^+$?

$$\lim_{x \to 0^+} \frac{\overset{0^+}{x}}{\underbrace{\ln x}_{-\infty}} = 0$$

$$PI = \left(e^2 ; \frac{e^2}{2}\right)$$

CONCAVIDAD POSITIVA: $(1 ; e^2)$

NEGATIVA: $(0 ; 1) \cup (e^2 ; +\infty)$



14) $f(x) = \frac{1}{5}x^5 + x^4 + x^3 - 2x^2 - 4x$

Dom = ℝ

no posee ASINTOTAS por ser f. polinomica.

CORTE CON LOS EJES

$f(0) = 0$

$\cap y = (0;0)$

$f(x) = 0$

$\frac{1}{5}x^5 + x^4 + x^3 - 2x^2 - 4x = 0$

$x\left(\frac{1}{5}x^4 + x^3 + x^2 - 2x - 4\right) = 0$

$\downarrow$

$x = 0$

(Búsqueda BINARIA o DICOTÓMICA)