<u>CÁLCULO DE ÁREAS</u>

Grafique, sombree y calcule el área de las regiones encerradas por las siguientes fórmulas:

<u>EJERCICIO 1</u> $y = x^2 \quad y = 0 \quad x = 3$

<u>EJERCICIO 2</u> $y = x^3 \quad y = 0 \quad -1 \leq x \leq 2$

<u>EJERCICIO 3</u> $f(x) = x^2 \quad g(x) = x$

<u>EJERCICIO 4</u> $y = 4x - 5 \quad y = 0 \quad -3 \leq x \leq -2$

<u>EJERCICIO 5</u> $f(x) = x^3 \quad x = 0 \quad g(x) = 8$

<u>EJERCICIO 6</u> $y_1 = 2^x \quad y_2 = 2^{-x} \quad y_3 = 4$

<u>EJERCICIO 7</u> $y_1 = 2^x \quad y_2 = 2^{-x} \quad y_3 = 0 \quad x = -2 \quad x = 2$

<u>EJERCICIO 8</u> $f(x) = \frac{2-x}{x} \quad g(x) = 0 \quad x = \frac{1}{2} \quad x = 4$

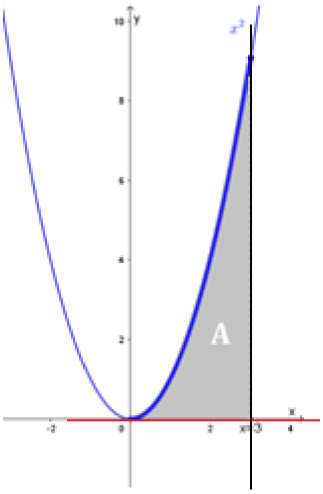
<u>EJERCICIO 9</u> $f(x) = 2x^2 + 1 \quad g(x) = 4 - x \quad h(x) = 0 \quad x = 0$

<u>EJERCICIO 10</u> $f(x) = x^2 - 2x$ y $g(x) = 6x - x^2$

<u>EJERCICIO 11</u> $f(x) = x^3 + 2 \quad g(x) = 14 - 2x \quad h(x) = 1$

EJERCICIO 1

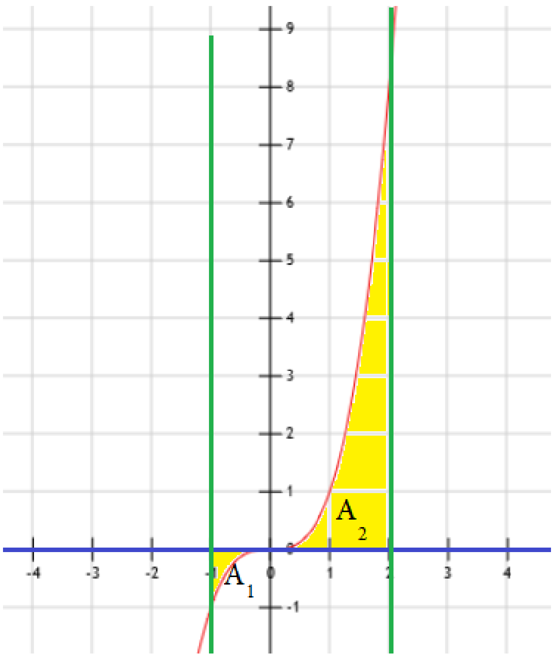
$$y = x^2 \qquad y = 0 \qquad x = 3$$



$$A = \int_0^3 (x^2 - 0)dx = \left.\frac{x^3}{3}\right|_0^3 = \left(\frac{3^3}{3}\right) - \left(\frac{0^3}{3}\right) = 9 - 0 = 9$$

EJERCICIO 2

$$y = x^3 \qquad y = 0 \qquad -1 \leq x \leq 2$$



$$A = A_1 + A_2$$

$$A_1 = \int_{-1}^{0} [0 - x^3]dx = -\int_{-1}^{0} x^3 dx = -\left.\frac{x^4}{4}\right|_{-1}^{0} = \left(-\frac{0^4}{4}\right) - \left(-\frac{(-1)^4}{4}\right) = 0 - \left(-\frac{1}{4}\right) = \frac{1}{4}$$

$$A_2 = \int_0^2 [x^3 - 0]dx = +\frac{x^4}{4}\Big|_0^2 = \left[\left(\frac{2^4}{4}\right) - \left(\frac{0^4}{4}\right)\right] = \left[\left(\frac{16}{4}\right) - (0)\right] = 4$$

$$A = \frac{1}{4} + 4 = \frac{17}{4}$$

EJERCICIO 3

$$\boldsymbol{f(x) = x^2 \quad g(x) = x}$$

Intersecciones

$$f(x) = g(x)$$

$$x^2 = x$$

$$x^2 - x = 0$$

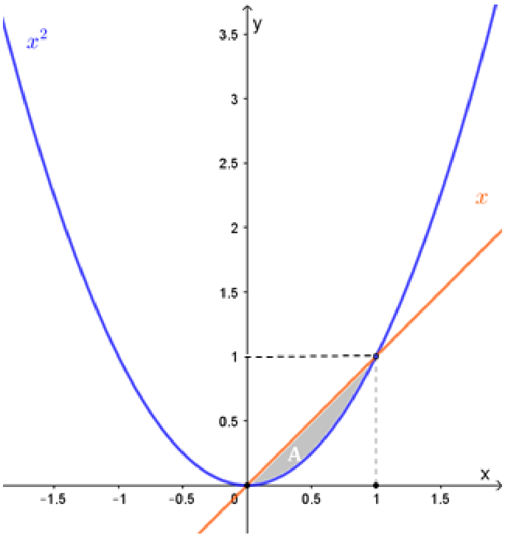
$$x.(x - 1) = 0$$

$$x_1 = 0 \qquad x_2 = 1$$

$$y_1 = f(0) = g(0) = 0$$

$$y_2 = f(1) = g(1) = 1$$

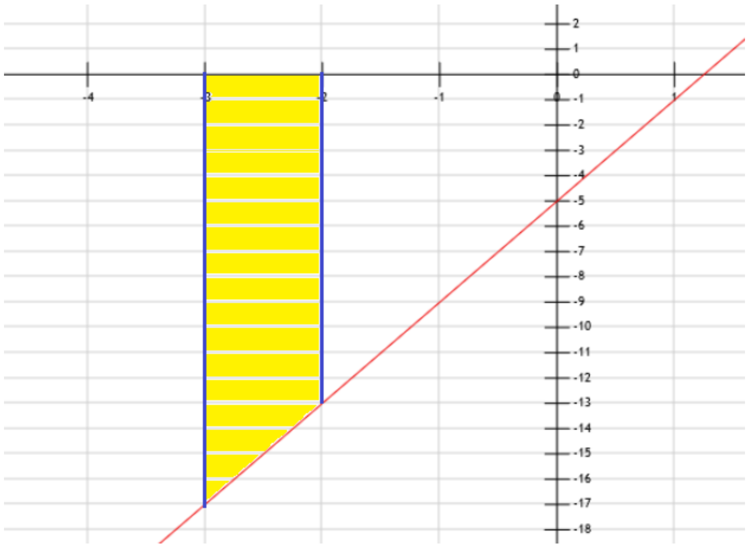
Es decir que las dos funciones tienen que pasar por los puntos $(0;0)$ y $(1;1)$



$$A = \int_0^1 [x - x^2]dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = \left(\frac{1^2}{2} - \frac{1^3}{3}\right) - \left(\frac{0^2}{2} - \frac{0^3}{3}\right) = \left(\frac{1}{2} - \frac{1}{3}\right) - (0) = \frac{1}{6} \approx 0{,}1667$$

EJERCICIO 4

$$\boldsymbol{y = 4x - 5 \qquad y = 0 \qquad -3 \le x \le -2}$$



$$A = \int_{-3}^{-2} [0-(4x-5)]dx = \int_{-3}^{-2} (-4x+5)dx = -2x^2+5x\Big|_{-3}^{-2} =$$

$$= \left(-2(-2)^2+5(-2)\right)-\left(-2(-3)^2+5(-3)\right) =$$

$$= (-8-10)-(-18-15) = -18-(-33) = \boxed{15}$$

<u>EJERCICIO 5</u>

$$\boldsymbol{f(x) = x^3 \quad x = 0 \quad g(x) = 8}$$

Hay que encontrar el área encerrada por

- $f(x) = x^3$
- $g(x) = 8$
- $x = 0$

Intersecciones

$\boxed{f(x) = x^3}$ con $\boxed{g(x) = 8}$

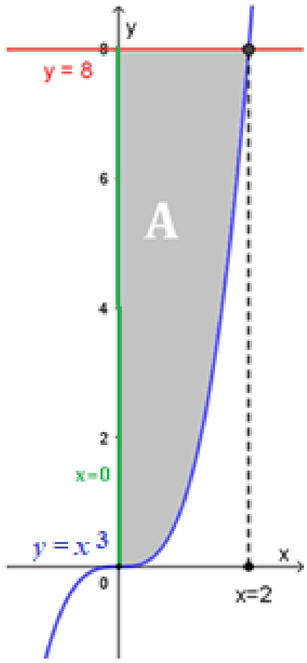
$$f(x) = g(x)$$

$$x^3 = 8$$

$$x = \sqrt[3]{8}$$

$$x = 2$$

Punto de intersección $(2; 8)$



$$A = \int_0^2 [8 - x^3]\,dx = \int_0^2 [8 - x^3]\,dx = \left(8x - \frac{x^4}{4}\right)\Bigg|_0^2 = \left(8 \cdot 2 - \frac{2^4}{4}\right) - \left(8 \cdot 0 - \frac{0^4}{4}\right) =$$

$$= \left(16 - \frac{16}{4}\right) - (0) = 16 - 4 = 12$$

EJERCICIO 6

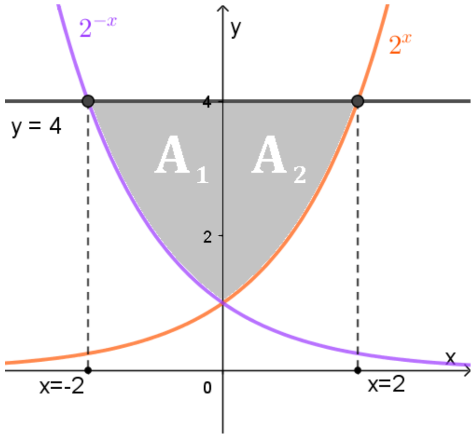
$$\boldsymbol{y_1 = 2^x \quad y_2 = 2^{-x} \quad y_3 = 4}$$

Para graficar realizamos tablas de valores

x	$y_1 = 2^x$
-2	1/2
-1	1/4
0	1
1	2
2	4

x	$y_2 = 2^{-x}$
-2	4
-1	2
0	1
1	1/2
2	1/4

Hay que encontrar el área formada por las funciones $y_1 = 2^x$ $y_2 = 2^{-x}$ y $y_3 = 4$ si graficamos las funciones nos queda:



Intersecciones

- $y_1 = 2^x$ con $y_2 = 2^{-x}$

$$y_1 = y_2$$
$$2^x = 2^{-x}$$
$$x = -x$$
$$2x = 0$$
$$x = 0$$
$$y_1 = 2^0 \qquad y_2 = 2^{-0}$$
$$y_1 = 1 \qquad y_2 = 1$$

Punto de intersección $(\mathbf{0}; \mathbf{1})$

- $y_1 = 2^x$ con $y_3 = 4$

$$y_1 = y_3$$
$$2^x = 4$$
$$\ln(2^x) = \ln(4)$$
$$x.\ln(2) = \ln(4)$$
$$x = \frac{\ln(4)}{\ln(2)}$$
$$x = 2$$
$$y_1 = 2^2 \qquad y_3 = 4$$

Punto de intersección $(\mathbf{2}; \mathbf{4})$

- $y_2 = 2^{-x}$ con $y_3 = 4$

$$y_2 = y_3$$
$$2^{-x} = 4$$
$$\ln(2^{-x}) = \ln(4)$$
$$-x.\ln(2) = \ln(4)$$

$$-x = \frac{\ln(4)}{\ln(2)}$$

$$x = -2$$

$$y_1 = 2^{-(-2)} \qquad y_3 = 4$$

Punto de intersección $(-2; 4)$

$$A_1 = \int_{-2}^{0} [4 - 2^{-x}]\, dx = \cdots$$

$$\boxed{\begin{array}{l} \displaystyle\int 2^{-x} dx = \qquad sustitución \qquad \begin{array}{r} \boldsymbol{z = -x} \\ \boldsymbol{dz = -1.dx} \\ \boldsymbol{-dz = dx} \end{array} \\ \displaystyle\int 2^z.(-dz) = -\int 2^z.dz = -\frac{2^z}{\ln 2} + C = -\frac{2^{-x}}{\ln 2} + C \end{array}}$$

$$A_1 = \int_{-2}^{0} [4 - (2^{-x})] = \left(4x - \left(-\frac{2^{-x}}{\ln 2}\right)\right)\Bigg|_{-2}^{0} = \left(4x + \frac{2^{-x}}{\ln 2}\right)\Bigg|_{-2}^{0} =$$

$$= \left(4.0 + \frac{2^{-0}}{\ln 2}\right) - \left(4(-2) + \frac{2^{-(-2)}}{\ln 2}\right) = \frac{1}{\ln 2} - \left(-8 + \frac{4}{\ln 2}\right) = \frac{1}{\ln 2} + 8 - \frac{4}{\ln 2} = \boxed{8 - \frac{3}{\ln 2}}$$

$$A_2 = \int_{0}^{2} [4 - 2^x]\, dx = \left(4x - \frac{2^x}{ln2}\right)\Bigg|_{0}^{2}$$

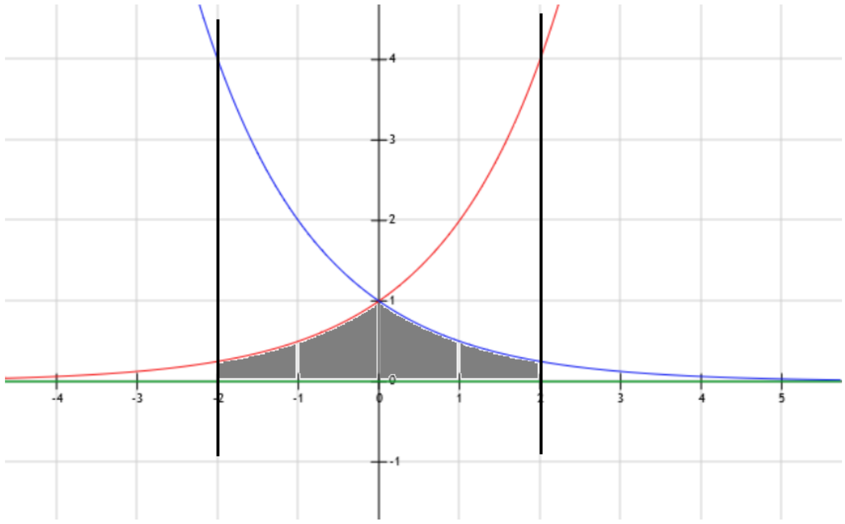
$$= \left(4 \cdot 2 - \frac{2^2}{ln2}\right) - \left(4 \cdot 0 - \frac{2^0}{ln2}\right) = \left(8 - \frac{4}{ln2}\right) - \left(0 - \frac{1}{ln2}\right) =$$

$$= 8 - \frac{4}{ln2} + \frac{1}{ln2} = \boxed{8 - \frac{3}{ln2}}$$

Entonces el área total es: $A = A_1 + A_2 = 2 \cdot \left(8 - \frac{3}{ln2}\right) = 16 - \frac{6}{\ln 2} \approx 2 \cdot 3{,}67 \approx 7{,}34$

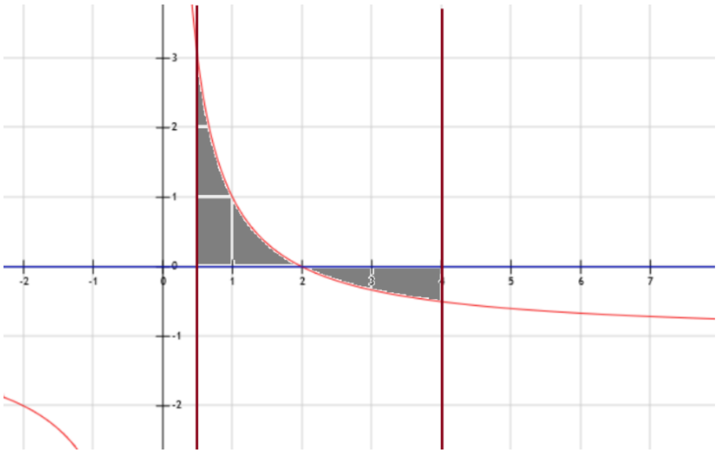
EJERCICIO 7

$$\boldsymbol{y_1 = 2^x} \qquad \boldsymbol{y_2 = 2^{-x}} \qquad \boldsymbol{y_3 = 0} \qquad \boldsymbol{x = -2} \qquad \boldsymbol{x = 2}$$



EJERCICIO 8

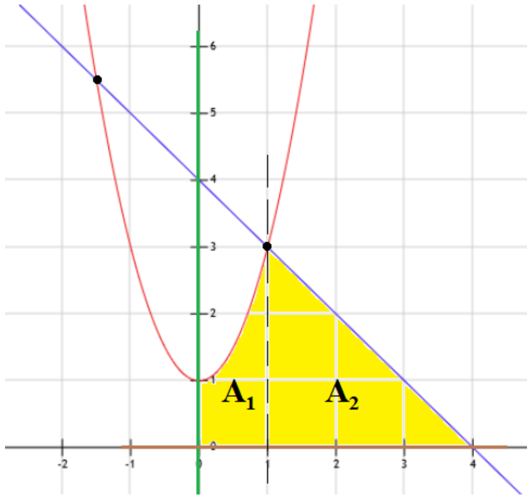
$$f(x) = \frac{2-x}{x} \qquad g(x) = 0 \qquad x = \frac{1}{2} \qquad x = 4$$



EJERCICIO 9

Encontrar el área limitada por:

- $f(x) = 2x^2 + 1$
- $g(x) = 4 - x$
- $h(x) = 0$
- $x = 0$

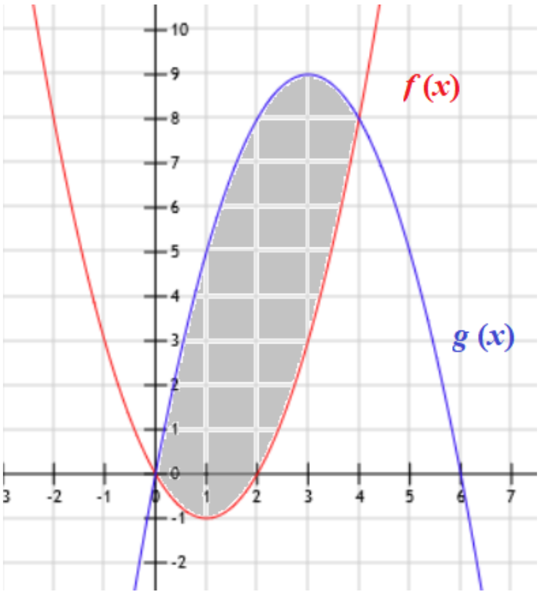


$$A_1 = \frac{5}{3} \qquad A_2 = \frac{9}{2} \qquad A_T = A_1 + A_2 = \frac{37}{6}$$

EJERCICIO 10

Encontrar el área limitada por:

- $f(x) = x^2 - 2x$
- $g(x) = 6x - x^2$

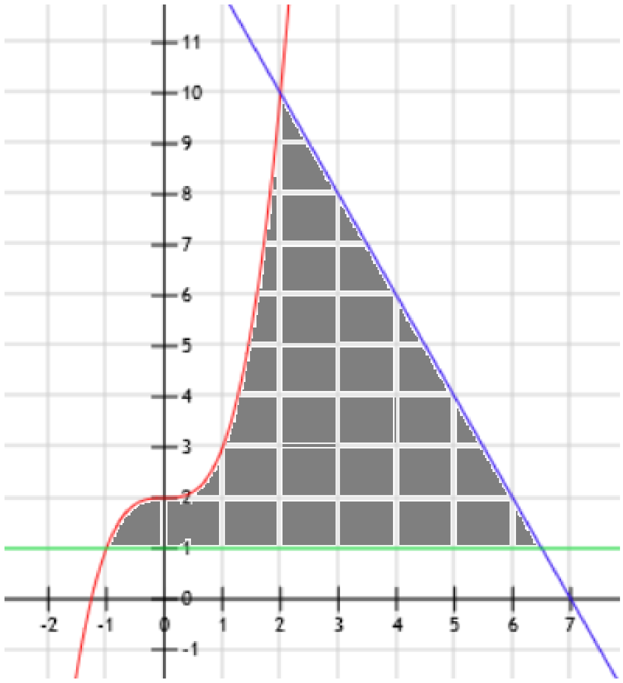


$$A = \int_0^4 [6x - x^2 - (x^2 - 2x)]dx = \int_0^4 [8x - 2x^2]dx = \left(4x^2 - \frac{2}{3}x^3\right)\Big|_0^4 =$$

$$= \left(4.4^2 - \frac{2}{3}4^3\right) - 0 = \frac{64}{3} \approx 21{,}33$$

EJERCICIO 11

- $f(x) = x^3 + 2$
- $g(x) = 14 - 2x$
- $h(x) = 1$



$$g(x) = h(x)$$

$$14 - 2x = 1$$

$$x = -13 : (-2)$$

$$x = 6{,}5$$

$$A_1 = \int_{-1}^{2} [(x^3 + 2) - (1)]dx = \int_{-1}^{2} [x^3 + 1]dx = \left(\frac{x^4}{4} + x\right)\Bigg|_{-1}^{2} =$$

$$= \left(\frac{2^4}{4} + 2\right) - \left(\frac{(-1)^4}{4} + (-1)\right) = \frac{16}{4} + 2 - \frac{1}{4} + 1 = \frac{27}{4}$$

$$A_2 = \int_{2}^{6{,}5} [(14 - 2x) - (1)]dx = \int_{2}^{6{,}5} [13 - 2x]dx = (13x - x^2)\Bigg|_{2}^{6{,}5} =$$

$$= (13.6{,}5 - 6{,}5^2) - (13.2 - 2^2) = 20{,}25$$

Rta.: $A = A_1 + A_2 = \frac{27}{4} + 20{,}25 = 27$