Derivar la siguiente función y llevar el resultado a su mínima expresión

$$f(x) = \frac{e^{\sin x} - e^{-\sin x}}{e^{\sin x} + e^{-\sin x}}$$

Resolución y video

video

$$f(x)=\frac{e^{\sin x}-e^{-\sin x}}{e^{\sin x}+e^{-\sin x}}$$

$$f'(x)=\frac{\left(e^{\sin x}-e^{-\sin x}\right)'\cdot\left(e^{\sin x}+e^{-\sin x}\right)\;-\;\left(e^{\sin x}-e^{-\sin x}\right)\cdot\left(e^{\sin x}+e^{-\sin x}\right)'}{(e^{\sin x}+e^{-\sin x})^2}=$$

$$(e^{\sin x})'=e^{\sin x}.\cos x$$

$$(e^{-\sin x})'=e^{-\sin x}.(-\cos x)=-e^{-\sin x}.\cos x$$

$$=\frac{\left(e^{\sin x}.\cos x+e^{-\sin x}.\cos x\right)\cdot\left(e^{\sin x}+e^{-\sin x}\right)\;-\;\left(e^{\sin x}-e^{-\sin x}\right)\cdot\left(e^{\sin x}\cos x-e^{-\sin x}.\cos x\right)}{(e^{\sin x}+e^{-\sin x})^2}=$$

$$=\frac{\cos x\cdot\left(e^{\sin x}+e^{-\sin x}\right)\cdot\left(e^{\sin x}+e^{-\sin x}\right)\;-\cos x\cdot\;\left(e^{\sin x}-e^{-\sin x}\right)\cdot\left(e^{\sin x}-e^{-\sin x}\right)}{(e^{\sin x}+e^{-\sin x})^2}=$$

$$=\frac{\cos x\cdot\left[\left(e^{\sin x}+e^{-\sin x}\right)\cdot\left(e^{\sin x}+e^{-\sin x}\right)\;-\;\left(e^{\sin x}-e^{-\sin x}\right)\cdot\left(e^{\sin x}-e^{-\sin x}\right)\right]}{(e^{\sin x}+e^{-\sin x})^2}=$$

$$\left(e^{\sin x}+e^{-\sin x}\right)\cdot\left(e^{\sin x}+e^{-\sin x}\right)=\left(e^{\sin x}\right)^2+1+1+\left(e^{-\sin x}\right)^2=e^{2\sin x}+2+e^{-2\sin x}$$

$$e^{\sin x}.e^{-\sin x}=e^{\sin x+(-\sin x)}=e^0=1$$

$$\left(e^{\sin x}-e^{-\sin x}\right)\cdot\left(e^{\sin x}-e^{-\sin x}\right)=\left(e^{\sin x}\right)^2-e^0-e^0+\left(e^{-\sin x}\right)^2=$$

$$=e^{2\sin x}-2+e^{-2\sin x}$$

$$=\frac{\cos x\cdot\left[e^{2\sin x}+2+e^{-2\sin x}\quad-\quad\left(e^{2\sin x}-2+e^{-2\sin x}\right)\right]}{(e^{\sin x}+e^{-\sin x})^2}=$$

$$=\frac{\cos x\cdot\left[e^{2\sin x}+2+e^{-2\sin x}\quad-e^{2\sin x}+2\;-e^{-2\sin x}\right]}{(e^{\sin x}+e^{-\sin x})^2}=$$

$$=\frac{\cos x\cdot[+2+2]}{(e^{\sin x}+e^{-\sin x})^2}=\frac{\cos x\cdot[4]}{(e^{\sin x}+e^{-\sin x})^2}=\boxed{\frac{4\cos x}{(e^{\sin x}+e^{-\sin x})^2}}$$