DERIVADAS

sábado, 20 de abril de 2024

7) Hallar la ecuación de las rectas tangente y normal a la función

$f(x) = -\frac{1}{2}x^2 - 2x + 6$ en el punto $x = -4$. Grafique la función y las rectas

$y = mx + b$

$y_T = f'(x_0) \cdot x + b$

$y = m(x - x_0) + y_0$

$\frac{y - y_0}{x - x_0} = m$

$f'(x) = \frac{-1}{\cancel{2}} \cdot \cancel{2} \cdot x - 2$ · Derivo $f'(x)$

$f'(x) = -x - 2$ · Evalua la derivada en $x = x_0$

$f'(-4) = -(-4) - 2$ · Obtengo $f'(x_0)$ pend. de la recta tg.

$f'(-4) = 2$ pend. de la recta tg a f en $x = -4$

Si $x = -4$

$f(-4) = \frac{-1}{2}(-4)^2 - 2(-4) + 6$

$f(-4) = -8 + 8 + 6$

$f(-4) = 6$

$(-4, 6)$ pto de Tangencia.

Evalue a $f(x_0)$

Obtengo el pto de tangencia.

CAMINO 1

$y = m x + b$

($y = 6$, $m = 2$, $x = -4$)

$6 = 2 \cdot (-4) + b$

$6 = -8 + b$

$6 + 8 = b$

$14 = b$

Armo la ec. de la recta tg a f en $x = -4$

$\boxed{y = 2x + 14}$

CAMINO 2

$y = 2(x - (-4)) + 6$

$y = 2(x + 4) + 6$

$y = 2x + 8 + 6$

$\boxed{y = 2x + 14}$

· Calculo la recta Normal.

$m_T = 2$ $\quad m_N = -\frac{1}{2}$

Además tengo el pto de tg $(-4; 6)$

$y = \frac{-1}{2}(x - (-4)) + 6$

$y = -\frac{1}{2}(x + 4) + 6$

$y = \frac{-1}{2}x - 2 + 6$

$\boxed{y_N = \frac{-1}{2}x + 4}$

$f(x) = -\frac{1}{2}x^2 - 2x + 6$ $\quad a = \frac{-1}{2}$ $\quad b = -2$ $\quad c = 6$

OR-OR = $(0;6)$

$a<0$

RAICES $\quad x_1, x_2 = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot \left(-\frac{1}{2}\right) \cdot 6}}{2 \cdot \left(-\frac{1}{2}\right)}$ $\quad x_1 = -6 \quad (-6;0)$ $\quad x_2 = 2. \quad (2;0)$

Vértice $\quad X_V = \dfrac{x_1 + x_2}{2} \qquad X_V = \dfrac{-b}{2 \cdot a}.$

$X_V = \dfrac{-6+2}{2} = -2 \qquad X_V = \dfrac{-(-2)}{2 \cdot \left(-\frac{1}{2}\right)} = \dfrac{2}{-1} = -2.$

$y_V = f(x_V)$

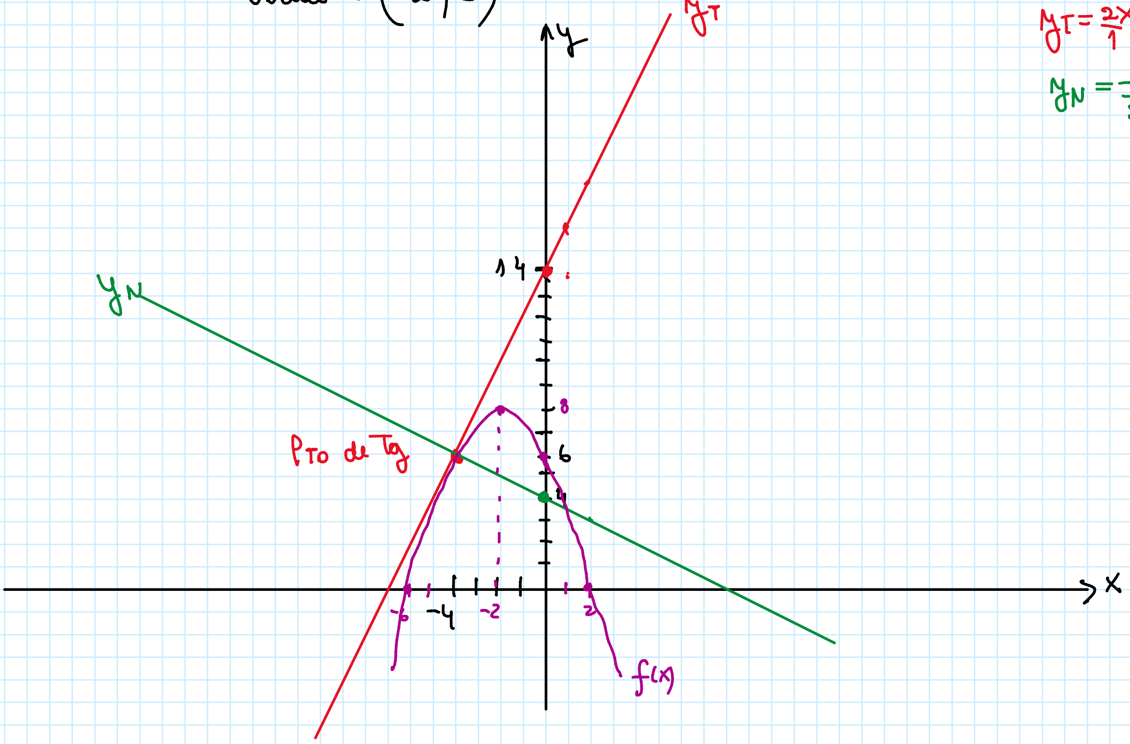
$f(-2) = -\frac{1}{2}(-2)^2 - 2(-2) + 6$

$f(-2) = -\frac{1}{2} \cdot 4 + 4 + 6$

$f(-2) = -2 + 4 + 6$

$f(-2) = 8$

Vértice $= (-2;8)$



$y_T = \frac{2}{1}x + 14$

$y_N = -\frac{1}{2}x + 4$

9) Hallar la ecuación de las rectas tangente y normal a la función $f(x) = \frac{3x-6}{x+1}$ en el punto $x = 2$. Grafique la función y las rectas

$f'(x) = \dfrac{(3x-6)'(x+1) - (3x-6)(x+1)'}{(x+1)^2}.$

$f'(x) = \dfrac{3(x+1) - (3x-6) \cdot 1}{(x+1)^2}$

$f'(2) = \dfrac{3(2+1) - (3 \cdot 2 - 6) \cdot 1}{(2+1)^2}$

$$f'(2) = \frac{3(2+1) - (3.2-6).1}{(2+1)^2}$$

$$f'(2) = \frac{9}{9} = 1 \qquad \boxed{m_T = 1}$$

$$f(2) = \frac{3.2-6}{2+1} = 0 \qquad \text{Pto de tg } (2;0)$$

Armo R_T $\quad y = 1(x-2)+0$

$$\boxed{y_T = x-2}$$

Armo R_N $\quad y = -1(x-2)+0$

$$\boxed{y = -x+2}$$

Analizamos $f(x)$ $\qquad Dom = \mathbb{R} - \{-1\}$ $\quad$ cand. A.V

$$f(x) = \frac{3x-6}{x+1}$$

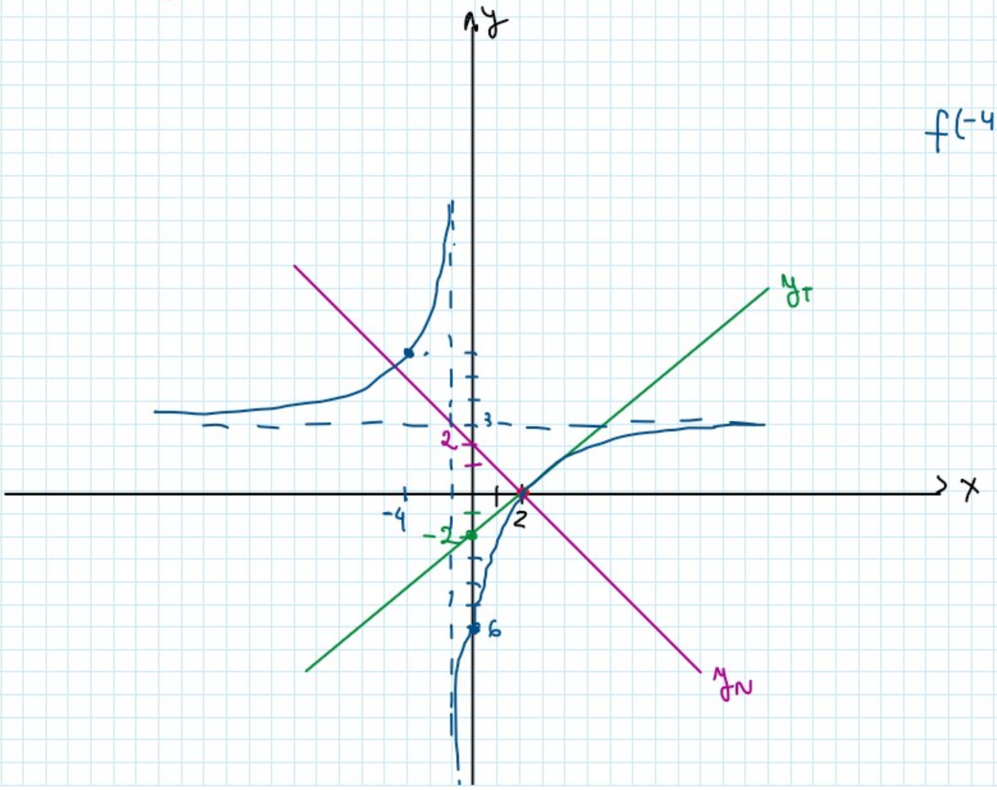
OR-OR $\quad$ x=0 siempre. $\qquad f(0) = \frac{3.0-6}{0+1} = -6 \qquad (0;-6)$

RAIZ $y=0$ $\qquad 0 = \frac{3x-6}{x+1} \qquad 0 = 3x-6 \qquad \frac{6}{3} = x \qquad 2 = x \qquad (2;0)$

ASINTOTAS

A.V $\quad \lim_{x\to -1^+} \frac{\overbrace{3x-6}^{-9}}{\underbrace{x+1}_{\to 0^+}} = -\infty \qquad \lim_{x\to -1^-} \frac{\overbrace{3x-6}^{-9}}{\underbrace{x+1}_{0^-}} = +\infty \qquad$ AV en $x=-1$

AH $\quad \lim_{x\to\infty} \frac{3x-6}{1x+1} = 3 \qquad$ AH $y=3$



$$f(-4) = \frac{3(-4)-6}{-4+1} = \frac{-18}{-3} = +6$$

A) Derivar usando la tabla de derivadas y las reglas de derivación

$sen^2(x) = (sen(x))^2 \neq sen\, x^2$

3) $f(x) = cos^6[sen^3(5x)^4]$

$f(x) = \left\{ cos\left[sen^3(5x)^4 \right] \right\}^6$

$f(x) = \{ cos [sen (5x)^4]^3 \}^6$

$f'(x) = 6 \{ cos [sen (5x)^4]^3 \}^5 \cdot -sen [sen (5x)^4]^3 \cdot 3 [sen (5x)^4]^2 \cdot cos(5x)^4 \cdot 4(5x)^3 \cdot 5$

$f'(x) = -360 \ldots\ldots$

$\frac{x}{2} = \frac{1}{2}x \qquad -x = -1 \cdot x$

6) $f(x) = e^{3x} \cdot sen^2\left(\frac{x}{2}\right) + \frac{2^{3x}}{cos^2(1-x)}$

$$f'(x) = (e^{3x})' \cdot sen^2\left(\frac{x}{2}\right) + e^{3x} \cdot \left(sen^2\left(\frac{x}{2}\right)\right)' + \frac{(2^{3x})' \cdot (cos^2(1-x)) - (2^{3x}) \cdot (cos^2(1-x))'}{(cos^2(1-x))^2}$$

$$\boxed{f(x) = e^{3x} \cdot 3 \cdot sen^2\left(\frac{x}{2}\right) + e^{3x} \cdot 2 \cdot sen\left(\frac{x}{2}\right) \cdot cos\left(\frac{x}{2}\right) \cdot \frac{1}{2} + \frac{2^{3x} \cdot \ln 2 \cdot 3\,(cos^2\, 1-x) - (2^{3x}) \cdot 2\,(cos(1-x)) \cdot (-sen(1-x)) \cdot (-1)}{(cos^2(1-x))^2}}$$

5) $f(x) = ln\left(\frac{\sqrt{x^2+a^2}-x}{\sqrt{x^2+a^2}+x}\right)$

$a = cte$

$$f'(x) = \frac{1}{\frac{\sqrt{x^2+a^2}-x}{\sqrt{x^2+a^2}+x}} \cdot \frac{(\sqrt{x^2+a^2}-x)' \cdot (\sqrt{x^2+a^2}+x) - (\sqrt{x^2+a^2}-x) \cdot (\sqrt{x^2+a^2}+x)'}{(\sqrt{x^2+a^2}+x)^2}$$

$$f'(x) = \frac{1}{\frac{\sqrt{x^2+a^2}-x}{\cancel{\sqrt{x^2+a^2}+x}}} \cdot \frac{\left(\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}} - 1\right) \cdot (\sqrt{x^2+a^2}+x) - (\sqrt{x^2+a^2}-x) \cdot \left(\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}} + 1\right)}{(\sqrt{x^2+a^2}+x)^{\cancel{2}}}$$

cambio el orden   cambio el orden.

$$f'(x) = \frac{\left(\frac{x-\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}\right) \cdot \underbrace{(\sqrt{x^2+a^2}+x)}_{1} - \underbrace{(\sqrt{x^2+a^2}-x)}_{1} \cdot \left(\frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}\right)}{(\sqrt{x^2+a^2}-x) \cdot (\sqrt{x^2+a^2}+x)}$$

$$f'(x) = \frac{\ldots}{\underbrace{\left(\sqrt{x^2+a^2}-x\right)\cdot\left(\sqrt{x^2+a^2}+x\right)}_{\text{dif de cuadrados.}}}$$

$$f'(x) = \frac{\frac{x^2-\left(\sqrt{x^2+a^2}\right)^2}{\sqrt{x^2+a^2}} - \frac{\left(\sqrt{x^2+a^2}\right)^2 - x^2}{\sqrt{x^2+a^2}}}{\left(\sqrt{x^2+a^2}\right)^2 - x^2}$$

$$f'(x) = \frac{\frac{\cancel{x^2}-\cancel{x^2}-a^2}{\sqrt{x^2+a^2}} - \frac{\cancel{x^2}+a^2-\cancel{x^2}}{\sqrt{x^2+a^2}}}{\cancel{x^2}+a^2-\cancel{x^2}}$$

$$f'(x) = \frac{\frac{-a^2-a^2}{\sqrt{x^2+a^2}}}{a^2}$$

$$f'(x) = \frac{-2\cancel{a^2}}{\sqrt{x^2+a^2}} \cdot \frac{1}{\cancel{a^2}} \cdot \frac{\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$$

$$f'(x) = \frac{-2\left(\sqrt{x^2+a^2}\right)}{\left(\cancel{\sqrt{x^2+a^2}}\right)^{\cancel{2}}} = \boxed{\frac{-2\left(\sqrt{x^2+a^2}\right)}{x^2+a^2}}$$

6) $f(x) = \dfrac{2^{-x}.\ x^2}{e^{-x}.\ \ln 2}$

$(k.f)' = k.f'$

$$f'(x) = \frac{\left(2^{-x}.x^2\right)'\left(e^{-x}.\ln 2\right) - \left(2^{-x}.x^2\right).\left(e^{-x}.\overset{\text{cte}}{\ln 2}\right)'}{\left(e^{-x}.\ln 2\right)^2}$$ Planteo

$$f'(x) = \frac{\left(2^{-x}.\ln 2\,(-1).\,x^2 + 2^{-x}.\,2x\right)\left(e^{-x}.\ln 2\right) - \left(2^{-x}.x^2\right).\left(-\ln 2.\,e^{-x}\right)}{\left(e^{-x}.\ln 2\right)^2}$$

$2^{-x}.\ln 2\,(-1).\,x.\,x$

$2^{-x}.\,2x$

$$f'(x) = \frac{\cancel{\left(e^{-x}\ln 2\right)}\left[x.2^{-x}\left(-x\ln 2+2\right)\right) + \left(2^{-x}.\,x^2\right)\right]}{\left(e^{-x}.\ln 2\right)^{\cancel{2}}}$$

$$\boxed{f'(x) = \frac{x.2^{-x}\left(-x\ln 2+2+x\right)}{e^{-x}.\ln 2}}$$

9) $f(x) = \ln\left(\dfrac{\sqrt{\cos^2 x+1}-1}{\sqrt{\cos^2 x+1}+1}\right)$

$$f'(x) = \frac{1}{\frac{\sqrt{\cos^2 x+1}-1}{\sqrt{\cos^2 x+1}+1}} \cdot \frac{\left(\sqrt{\cos^2 x+1}-1\right)'\left(\sqrt{\cos^2 x+1}+1\right) - \left(\sqrt{\cos^2 x+1}-1\right)\left(\sqrt{\cos^2 x+1}+1\right)'}{\left(\sqrt{\cos^2 x+1}+1\right)^2}$$

$$f'(x) = \frac{\frac{-2\cos x\,\mathrm{sen}\,x}{2\sqrt{\cos^2x+1}}\cdot\left(\sqrt{\cos^2x+1}+1\right) - \left(\sqrt{\cos^2x+1}-1\right)\cdot\left(\frac{-2\cos x\,\mathrm{sen}\,x}{2\sqrt{\cos^2x+1}}\right)}{\frac{\sqrt{\cos^2x+1}-1}{\cancel{\sqrt{\cos^2x+1}+1}} \qquad \left(\sqrt{\cos^2x+1}+1\right)^{\cancel{2}}}$$

$$f'(x) = \frac{\frac{-2\cos x\,\mathrm{sen}\,x}{2\sqrt{\cos^2x+1}}\left[\left(\sqrt{\cos^2x+1}+1\right)-\left(\sqrt{\cos^2x+1}-1\right)\right]}{\left(\sqrt{\cos^2x+1}\right)^2-1}$$

$$f'(x) = \frac{\frac{-\cancel{2}\cos x\,\mathrm{sen}\,x}{\cancel{2}\sqrt{\cos^2x+1}}\left[\cancel{\sqrt{\cos^2x+1}}+1 \cancel{-} \cancel{\sqrt{\cos^2x+1}}+1\right]}{\left(\cancel{\sqrt{\ }}\cos^2x+1\right)^{\cancel{2}}-1}$$

$$f'(x) = \frac{-2\cdot\cancel{\cos x}\cdot\mathrm{sen}\,x}{\sqrt{\cos^2x+1}}\cdot\frac{1}{\cos^{\cancel{2}}x\cancel{+1}\cancel{-1}}$$

$$f'(x) = \frac{-2\,\mathrm{sen}\,x}{\sqrt{\cos^2x+1}\cdot\cos x}\cdot\frac{\sqrt{\cos^2x+1}}{\sqrt{\cos^2x+1}}$$

$$f'(x) = \frac{-2\,\mathrm{sen}\,x\cdot\sqrt{\cos^2x+1}}{\left(\cancel{\sqrt{\ }}\cos^2x+1\right)^{\cancel{2}}\cdot\cos x}$$

$$f'(x) = \frac{-2\,\mathrm{sen}\,x\cdot\sqrt{\cos^2x+1}\,.}{\left(\cos^2x+1\right)\cdot\cos x}$$

$$f'(x) = \frac{-2\,\mathrm{tg}\,x\sqrt{\cos^2x+1}}{\cos^2x+1}$$

$$f'(x) = \frac{-2\,\mathrm{tg}\,x\cdot\sqrt{\cos^2x+1}}{1-\mathrm{sen}^2x+1}$$

$$f'(x) = \frac{-2\,\mathrm{tg}\,x\cdot\sqrt{\cos^2x+1}}{2-\mathrm{sen}^2x}$$

$\mathrm{sen}^2x+\cos^2x=1$

$\cos^2x = 1-\mathrm{sen}^2x$

$\mathrm{sen}^2x = 1-\cos^2x$

$\cos x = \sqrt{1-\mathrm{sen}^2x}$

$\mathrm{sen}\,x = \sqrt{1-\cos^2x}$