Exam, TFY4235 Computational physics

Number

Introduction

SIR, and the more advanced SEIIaR, are mathematical models that aim to capture how pandemics spread throughout a simulation. This paper documents the implementation and results of the simulation of these models in Python, as described in [1].

Implementation

All the different models used in this text follow the same basic form. The goal is to find $x(t)$, given initial conditions $x(t_0)$, and a equation of the form

$$f(x(t); \text{args}) = \frac{\mathrm{d}x(t)}{\mathrm{d}t}.$$

In the first part, $x = (S, I, R)$, while later $x = (S_{ij}, E_{ij}, I_{ij}, Ia_{ij}, R_{ij})$ where (ij) are different population groups. This is accomplished using function `integrate` in `utilities.py`. It takes as arguments the initial conditions `x0`, the functions `f` and `step`, the list `args` as well as the time step `dt` and total time `T` to simulate. It then creates a discrete approximation of $x(t)$ by taking time steps given by the function `step`. `step` is the particular *scheme* used, for example Runge-Kutta (4,5), while `f` defines the system.

The equations that give the asymptotic behavior are both of the type $x = f(x)$, and can thus be approximated by recursion, given that they converge. For $\mathcal{R}_0$ close to one, they converge increasingly slowly, and the program may reach maximum recursion depth. For the parameters in this exercise, however, this was not a problem

Results

Deterministic SIR model

The first model is the deterministic SIR model, given by a set of coupled ODEs [1]. In this text, the Runge-Kutta (4, 5) scheme was used, as it is both a simple yet precise scheme. Figure 1 demonstrates that S and R approaches the expected asymptotes, and that I grows exponentially in the beginning.

References

[1] NTNU, Institutt for Fysikk. *Exam, TFY4235 Computational Physics.* 2021.

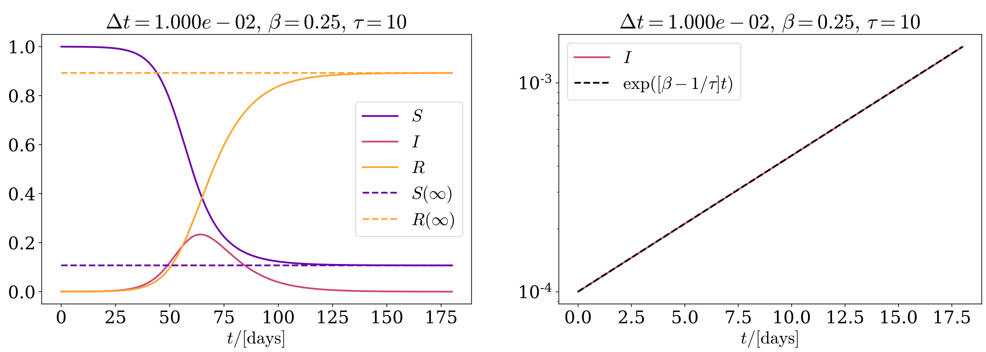

Figure 1: On the left, the fraction of the population that is in each group, over time. The plot on the right shows how the infection spreads exponentially in the begining