Introduction

This text explores several different methods for solving differential equations to model a pendulum. The methods are anlyzed for different time steps, to see the different expences of running the methods.

Theory

Using the small angle approximation for a pendulum, we get the differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\theta(t) = -\frac{g}{l}\theta(t)$$

This can be rewritten in the form

$$\dot{y} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\theta \\ \dot{\theta}\end{pmatrix} = f\begin{pmatrix}\theta \\ \dot{\theta}\end{pmatrix} = \begin{pmatrix}\dot{\theta} \\ -\frac{g}{l}\theta,\end{pmatrix}$$

making explicit differential equation solvers straight forward. Eulers method is

$$y_{n+1} = f(y_n)\Delta t,$$

and Runge Kutta 4 is.

$$\begin{aligned}
k_1 &= f(y_n), \\
k_2 &= f(y_n + k_1/2)\Delta t, \\
k_3 &= f(y_n + k_2/2)\Delta t, \\
k_4 &= f(y_n + k_3)\Delta t, \\
y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).
\end{aligned}$$